Initial implementation of an assessment model for the South African chokka squid (*Loligo reynaudii*) resource

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Abstract

The simulation model is adjusted further to provide an assessment model of chokka squid, taking into account mainly some of the data recommendations made by the 2024 Panel. Initial model fits hit a problem, so that the results are restricted to data from 2009 to 2013 only at this stage.

Keywords: assessment model, pack category data, abundance indices, squid

Introduction

The 2024 International Workshop Panel recommended that stock assessments for squid be based on the monthly age-structured model used in a simulation exercise to replicate historical industry binned catch-at-length data, and suggested further adaptations to be made to the model. The first attempt at adapting the simulation model into an assessment model presented at the 2024 IWS was restricted to a very small dataset (Brandão and Butterworth (2024)). This paper presents a follow up on adapting the simulation model to an assessment model, taking into account some of the recommendations by the Panel (excerpts of these are given in Brandão and Butterworth (2025a)). The main aim of this paper has been to concentrate on fitting the model to the available data suggested by the 2024 Panel, with most of the methodological recommendations left for future work.

Data

The data currently available to be used in the assessment model fitting process are given in Brandão and Butterworth (2025b). Further data are available but were not extracted in time for this round, but will be incorporated in future model fits. The sources of data taken into consideration are the monthly catches for the jig and for the trawl fisheries, nominal jig CPUE indices disaggregated by month and pack size category, and summer and autumn survey abundance indices and pack size data for the jig fishery. Observed proportions of males in each pack category collected from pilot studies in 2024 are used as a check that the predicted proportions are in the range of the observed values. Data for the fishing year period of 2009 to 2024 is considered, where a fishing year y is defined to be from May of year y to April of year $y + 1$.

For the moment, the fact that the survey indices have not been calibrated for vessel and gear has been ignored.

ASSESSMENT METHODOLOGY

Appendix A provides the methodology which adjusts the simulation model to an assessment model. Figure 1 gives a schematic representation of the population dynamics of squid, which is further explained in the Appendix.

Previously, the assumption was made that the parameter values obtained in the simulation exercise to replicate the average pack catch-at-length data would be fixed and used in the assessment model. However, these model parameter values were those that attained a good fit to the observed pack data proportions, averaged over all years, irrespective of whether they were biologically meaningful/appropriate. Those parameter values that were deemed inappropriate (mainly the natural mortality and the variation in length-at-age) have consequently been changed to more appropriate values after discussion with local scientists. These values may need further changes. The estimable parameters in the model are the total recruitment to the population each year (R_y^T) and the proportion of the exploitable resource harvested by the industry each year (F_y^f).

The assumptions made for the assessment model are given below.

a) The life cycle of squid is assumed to be 18 months. For simplicity, every month is taken to comprise 30 days.

b) The fishing year for commercial catches is taken to be from May to April, with closed seasons applying in later years. Catches are taken mid-month.

c) A knife edge selectivity at the value $L_0 = 12.8$ cm ML is assumed for both the jig and the trawl fisheries. A uniform selectivity is assumed for surveys.

d) Continuous recruitment into the population is assumed. The functional form for recruitment over the course of the season is given by fitting a cyclic spline to recruitment values assumed to occur at five levels in a ratio of 1.9:1.0:3.5:0.8:2.2:0.0:1.7:1.4:0.6:0.8:1.4: 1.4 from October to the following September (Figure 2).

e) The standard deviation of length-at-age *in days* σ_g is modelled to be constant with age with 5.688 cm ML for males and 2.205 cm ML for females. However, for young squid (up to the age of 175 days), the standard deviation of length-at-age is modelled to be proportional to the expected length-at-age *days* (Equation A.6) for a CV of 0.21 for males and 0.12 for females.

f) Age-specific natural mortality (M_a) is assumed, with a juvenile mortality of 0.25 month^{-1} up to age 175 days and an adult mortality of 0.1 month^{-1}.

g) Deterministic growth curves and weight-at-length relationships are assumed with parameter values given in Table 1. The data to estimate the growth curve parameters, given in Table1, covered only the middle to the late part of the range of the life cycle of squid. Therefore, for juveniles the growth curve is assumed to be linear from the origin to 175 days (corresponding to ML of 8.67 cm for males and 6.81 cm for females at that time). The use of alternative growth curves and of platoons still need to be investigated.

RESULTS AND DISCUSSION

In attempting to fit the assessment model to the available data, a problem was experienced in that the program failed to update the parameter values after the first iteration. A solution has not been found to correct this, but it was found that fitting to a few years of data only worked. The problem might be some memory capacity issue, and the program needs to be written more efficiently to avoid this problem. Initial attempts at this have not fixed the problem and with the constraints of time to produce some results, results here are restricted to data for five years from 2009 to 2013. Results for other five-year periods will be presented in working papers unless a solution has been found in the meantime.

Results are shown for a weighting ($w_F = 0.002$) applied to the penalty function related to the fishing proportions to match the catches taken (Equation A.31). Table 2 shows the total negative log-likelihood values obtained and the contribution to the negative log-likelihood from each data type.

The annual estimated total recruitment numbers into the population and the recruitment proportions for each cohort are shown in Figure 2. The estimated proportion of the exploitable resource harvested by the jig and by the trawl fisheries are given in Figure 3, while Figure 4 shows monthly total biomass.

The fits to the nominal jig CPUE are given in Figure 5, with the difference between observed and predicted values shown as bubble plots. The blue bubbles represent positive differences, while white bubbles represent negative differences. Fits to the summer and to the autumn survey abundance indices are shown in Figure 6.

Figures 7 to 11 compare the observed and the sex-disaggregated predicted monthly proportions by the weight of each pack category for 2009 to 2013 respectively. The predicted proportions for the medium pack category are still fairly constant over months. Figure 12 shows the estimated monthly catches per pack category for 2009 and the estimated proportions. This shows that the constant pattern for the medium pack is evident only in terms of proportions.

ACKNOWLEDGEMENTS

Jean Githaiga-Mwicigi, Granville Louw, Tracey Fairweather and Deon Durholtz of DFFE are thanked for providing the data.

REFERENCE

Brandão, A. and Butterworth, D.S. 2024. Adapting the simulation model to an assessment model for the chokka squid resource of South Africa. IWS document: MARAM/IWS/2024/SQUID/P6.

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Lorenzen, K. 1996. The relationship between body weight and natural mortality in juvenile and adult fish: a comparison of natural ecosystems and aquaculture. *Journal of Fish biology*, 49(4): 627-642.Lorenzen, K. 2000. Allometry of natural mortality as a basis for assessing optimal release size in fishstocking programmes. *Canadian Journal of Fisheries and Aquatic Sciences*, 57: 2374–2381.

Table 1. Biological parameter values for chokka squid. Growth parameters for males and females are given for the Francis parameterisation of the von Bertalanffy growth model (Mwanangombe *et al*. 2021), as well as in terms of the von Bertalanffy growth model formulation. The parameter values for the weight-length relationship for males and females were obtained from Lipiński *et al.* (2021), Supplementary Table S4.

Francis growth model	**Males**	**Females**
L1 (mean length (cm) at age 200 days)	12.220	9.942
L2 (mean length (cm) at age 300 days)	23.685	17.328
L3(mean length (cm) at age 400 days)	31.748	20.345
von Bertalanffy growth model		
L_∞ (cm)	50.858	22.428
κ (days^{-1})	0.00352	0.008953
t_0 (days)	121.93	134.58
Weight (in gm) – length (in cm) relationship $\left(\boldsymbol{W = cL^d}\right)$		
c	0.217	0.158
d	2.19	2.37

Table 2. Contributions to the negative log-likelihood when the model is fitted to the monthly pack size-disaggregated nominal CPUE, survey indices and pack data (Brandão and Butterworth, 2025b).

	Contribution to -ln *L*
-ln *L*: pack	-461.66
-ln *L*: CPUE	106.68
-ln *L*: survey SC autumn)	-1.138
-ln *L*: survey (WC summer)	12.69
-ln *L*: F_y related penalty	0.0075
-ln *L*: Total	-343.42

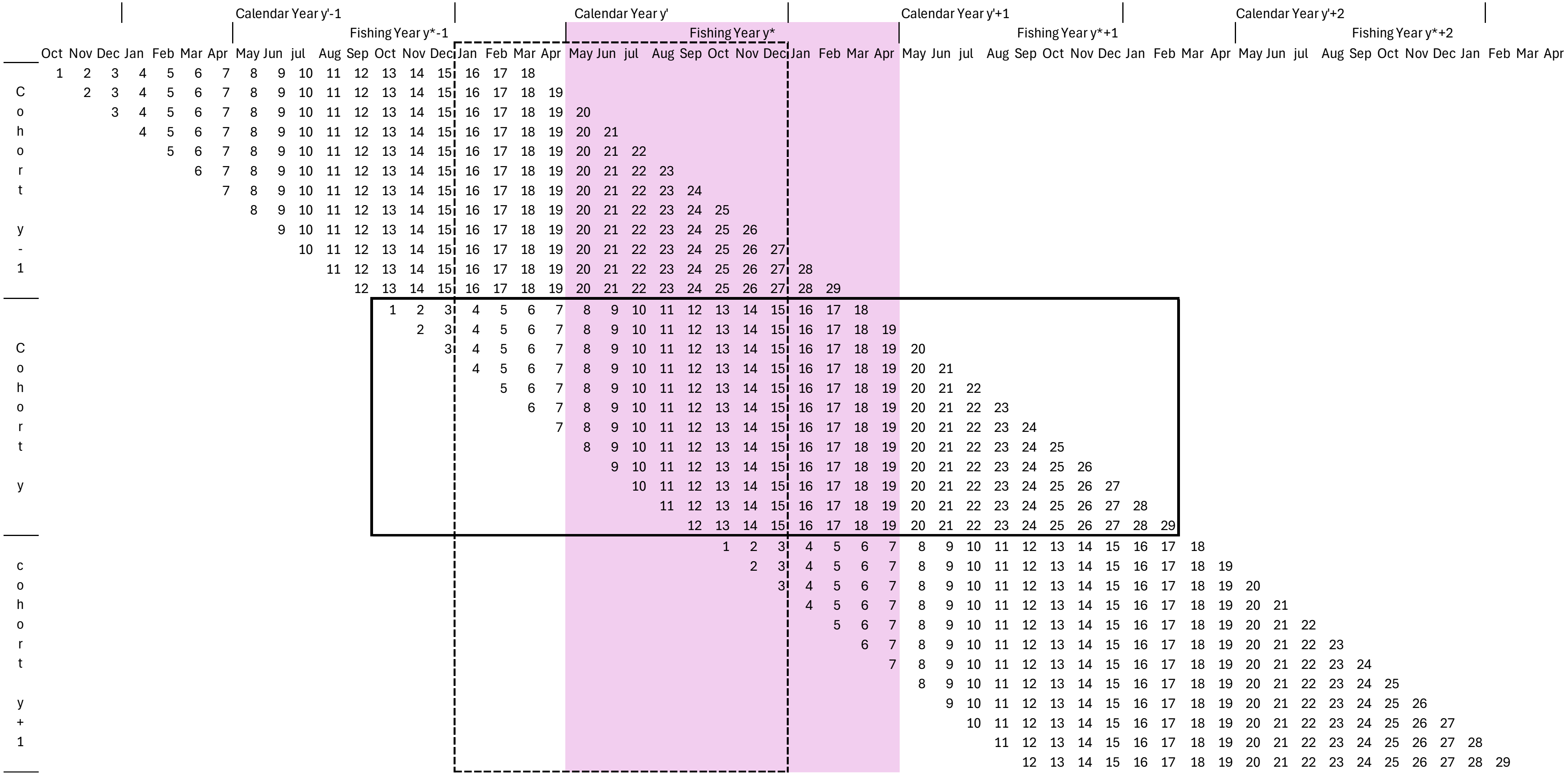
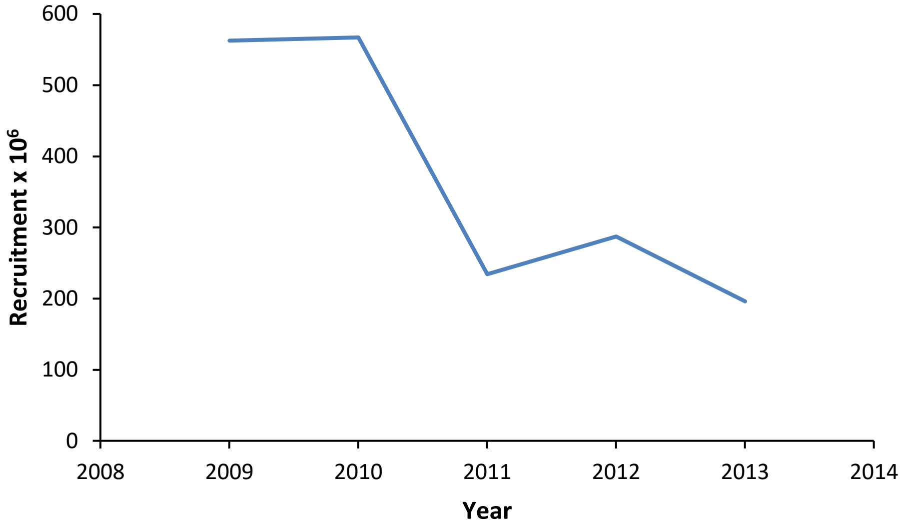


Figure 1. A schematic representation of the population dynamics of squid. The block with thick borders represents a cohort year y which represents cohorts born in the period from October of fishing year $y^* - 1$ to September of fishing year y^* for their life cycle, where a fishing year y^* is defined as the period from May of year y' to April of year $y' + 1$. The block with dashed borders represents a calendar year y' and the coloured block represents a fishing year y^*. The squid that are in the population in fishing year y^* consists of mostly squid born in cohort-year y plus some of the older squid born in cohort year $y - 1$ and very young squid born in cohort year $y + 1$.



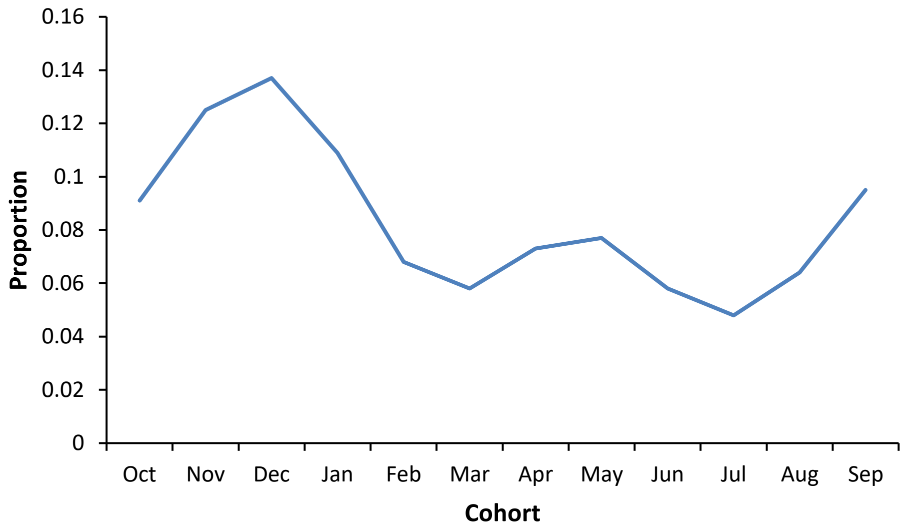


Figure 2. Estimated total recruitment (R_y^T) numbers added to the population (top) and recruitment proportions into the population (R_{bm}^B) (bottom).

Figure 3. Estimated proportion of the exploitable resource harvested by the jig and the trawl fisheries ($F^f_{y^*}$).

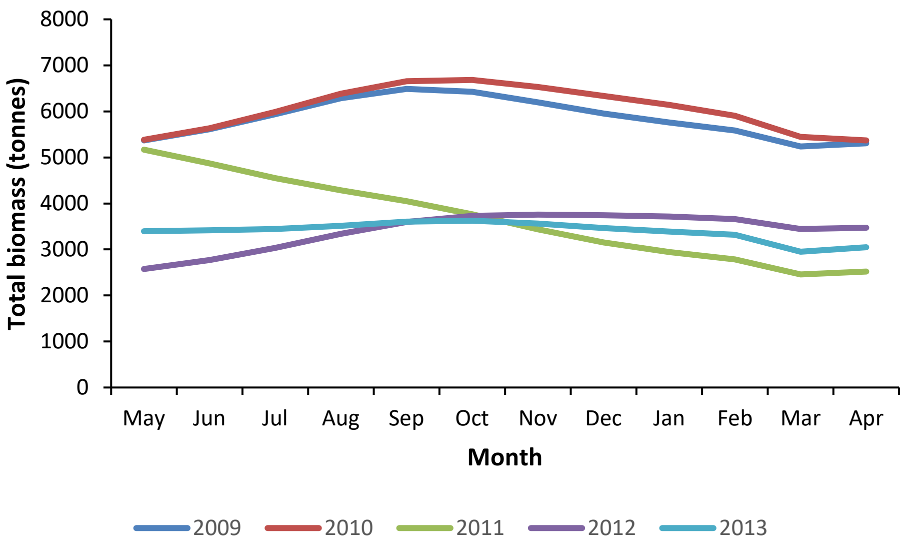


Figure 4. Total monthly biomass estimates for chokka squid for fishing years 2009 to 2013.

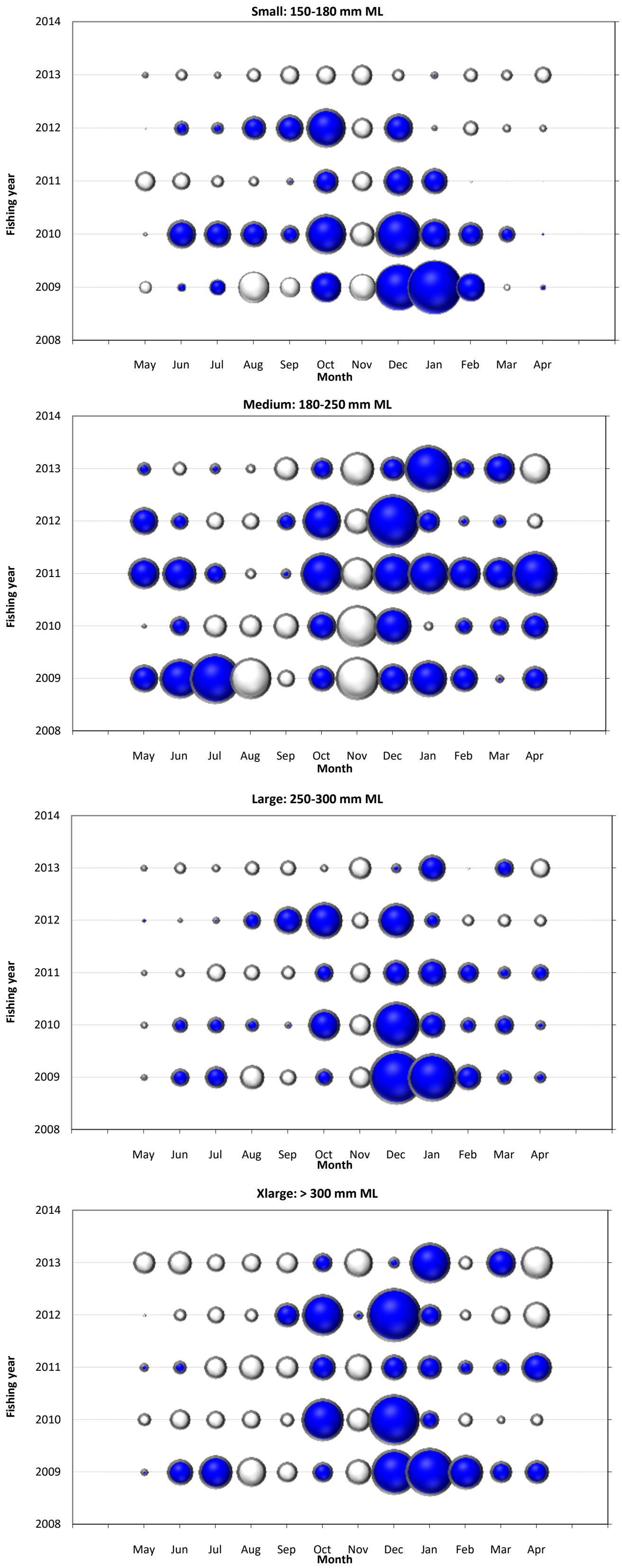


Figure 5. Bubble plot of the difference between the observed nominal jig CPUE indices by month and by pack size category to which the model is fit, and the predicted values (the predicted values are exploitable biomass multiplied by the estimated catchability q^{Jig} to express them in CPUE units). Blue bubbles represent positive differences, while white bubbles represent a negative one.

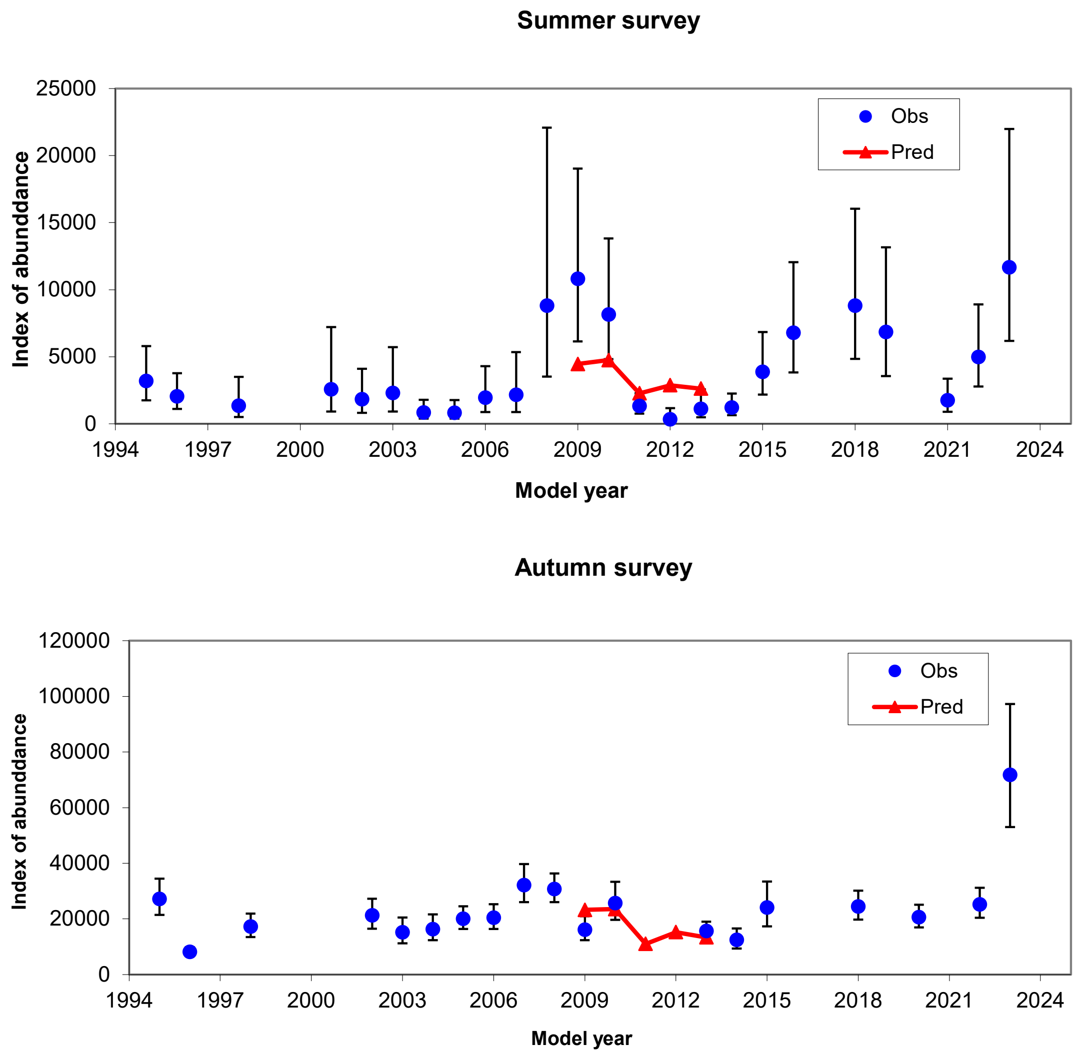


Figure 6. Comparison of observed and model-predicted trends for the survey abundance indices of the West coast summer surveys and the South coast autumn surveys. Note that the 95% confidence intervals shown for observed values have been computed as: estimate*exp(±1.96*CV).

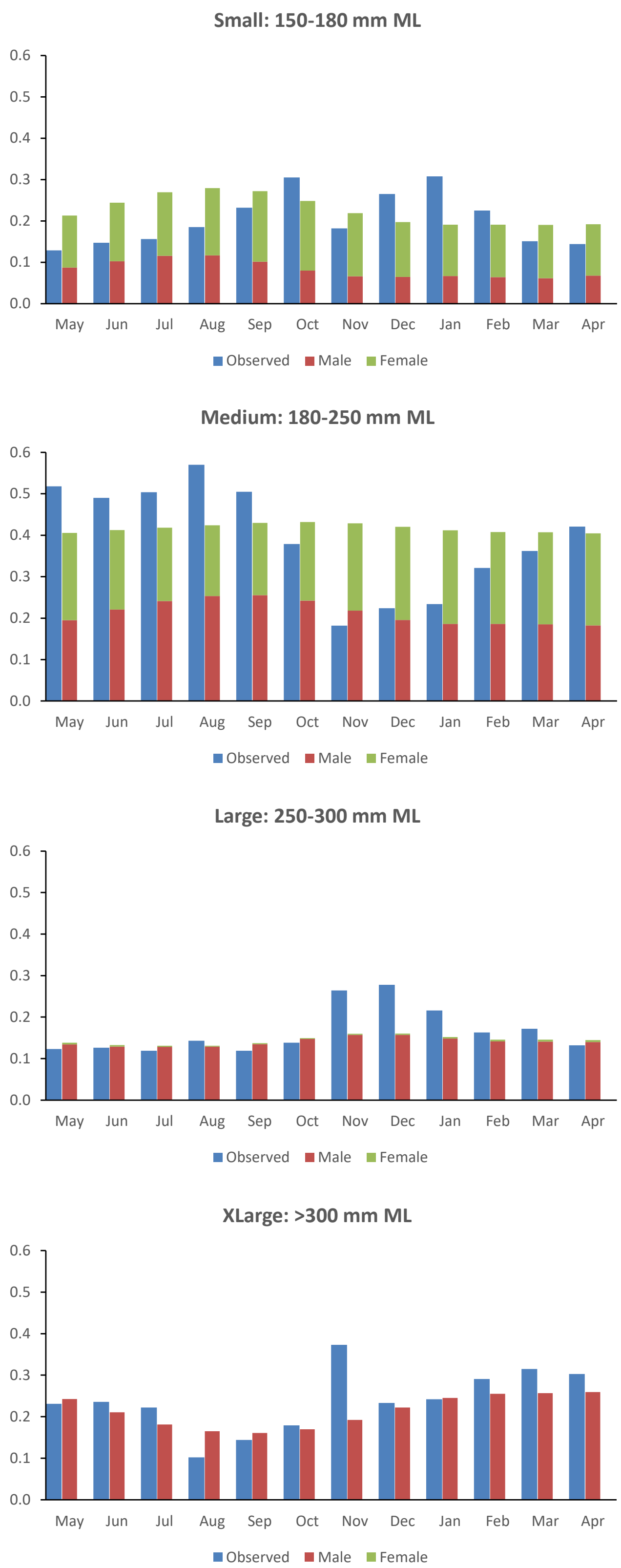


Figure 7. Comparison of the 2009 observed proportions by weight of each pack category in each month and the sex-disaggregated model estimated proportions.

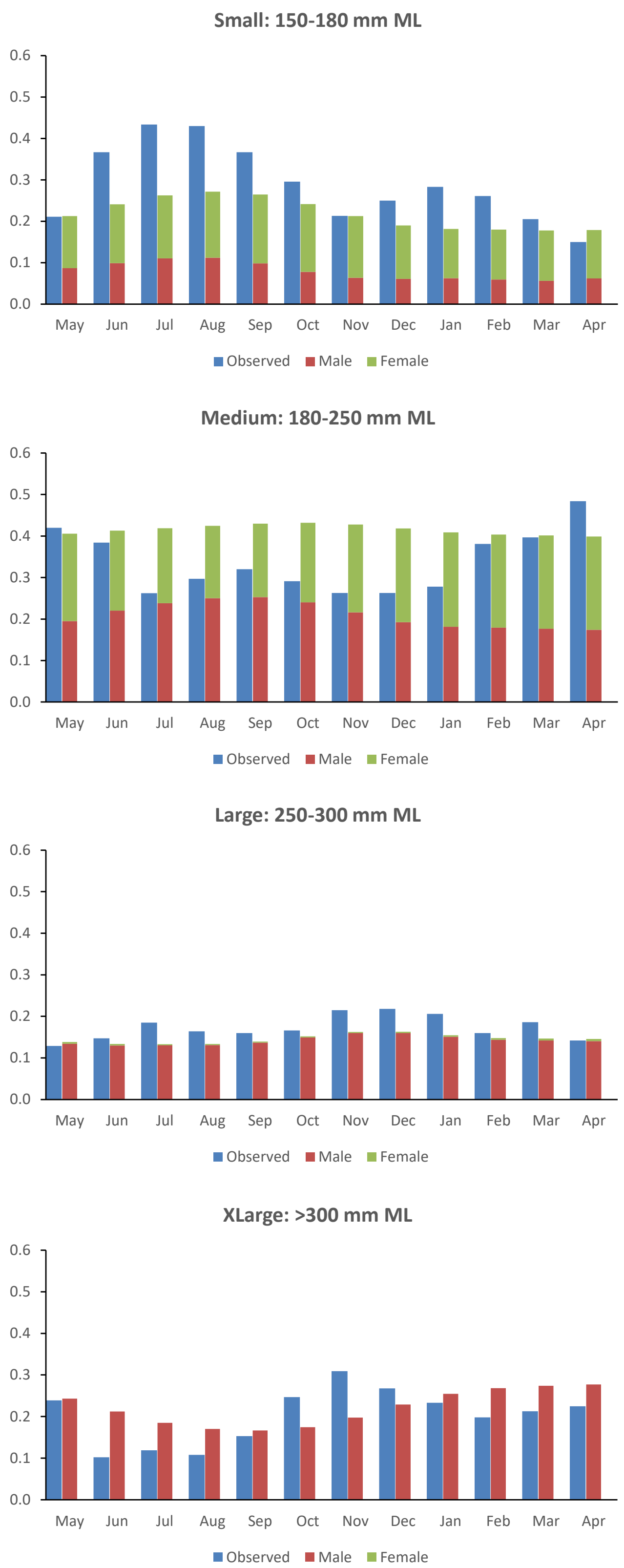


Figure 8. Comparison of the 2010 observed proportions by weight of each pack category in each month and the sex-disaggregated model estimated proportions.

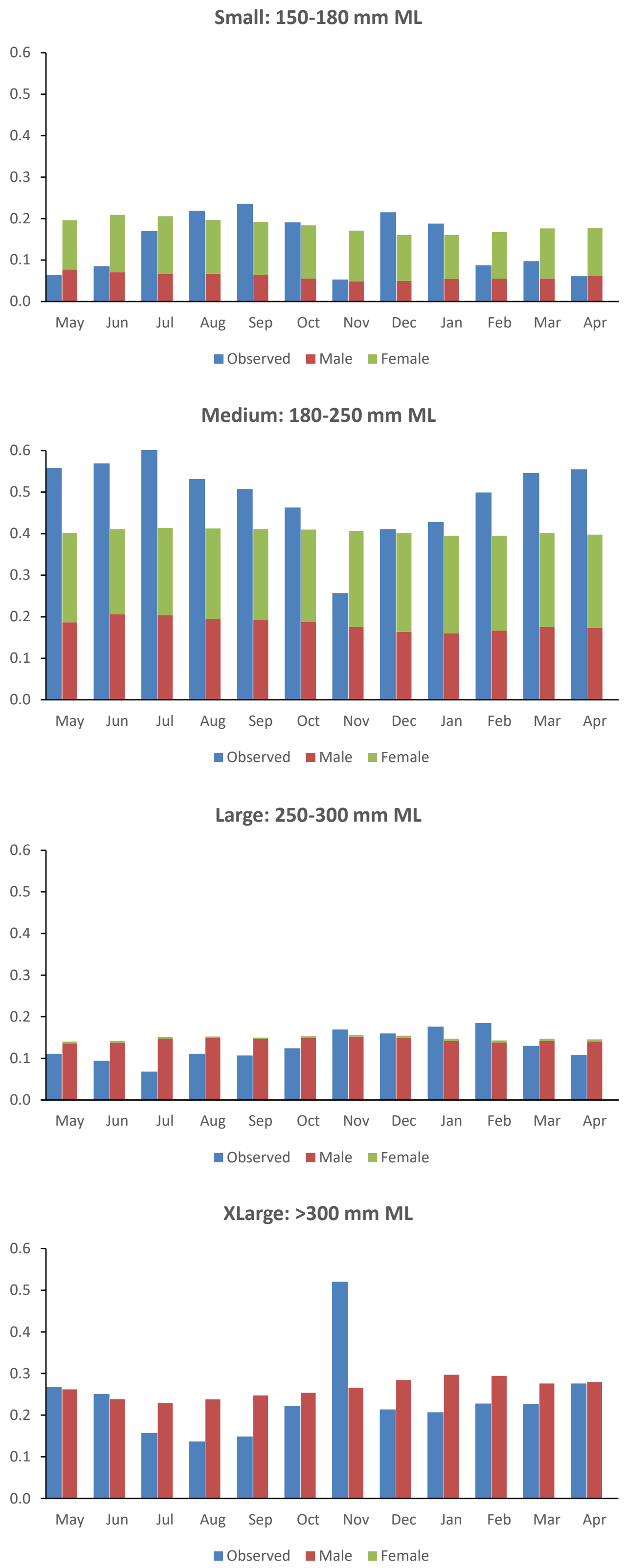


Figure 9. Comparison of the 2011 observed proportions by weight of each pack category in each month and the sex-disaggregated model estimated proportions.

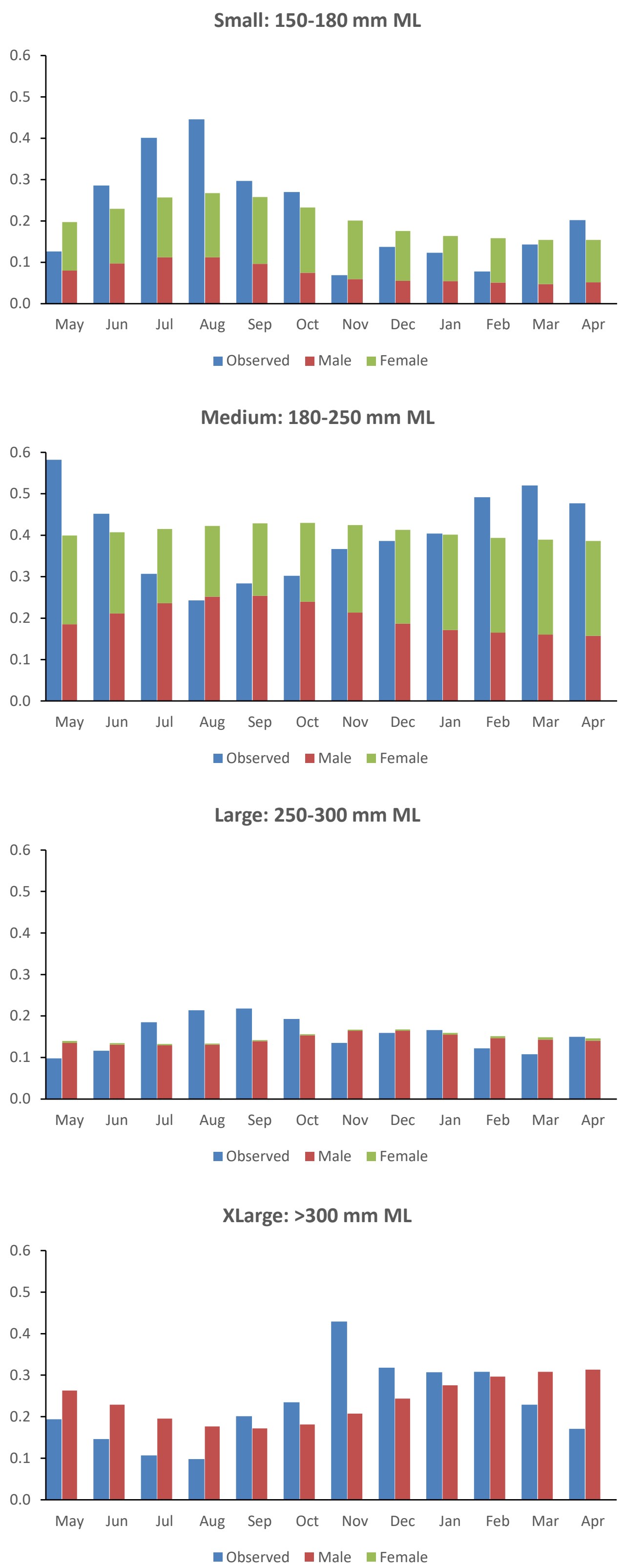


Figure 10. Comparison of the 2012 observed proportions by weight of each pack category in each month and the sex-disaggregated model estimated proportions.

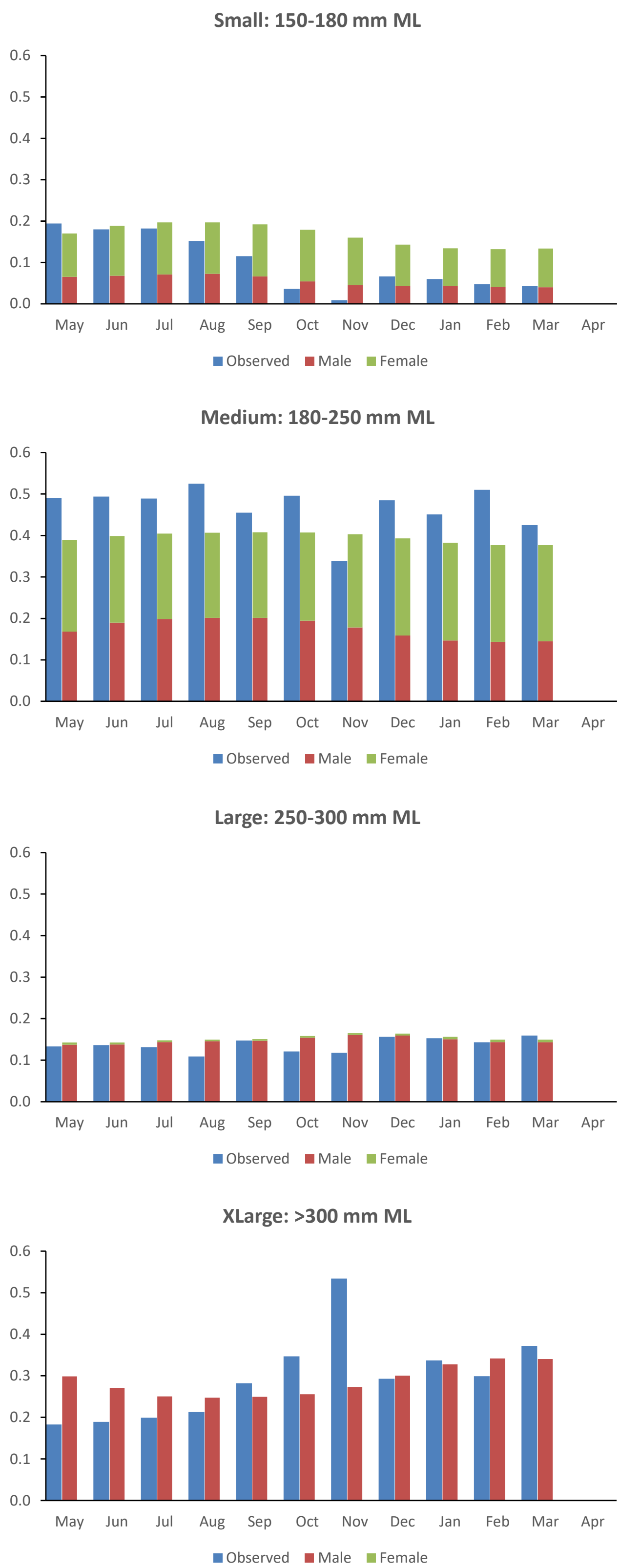


Figure 11. Comparison of the 2013 proportions by weight of each pack category in each month and the sex-disaggregated model estimated proportions.

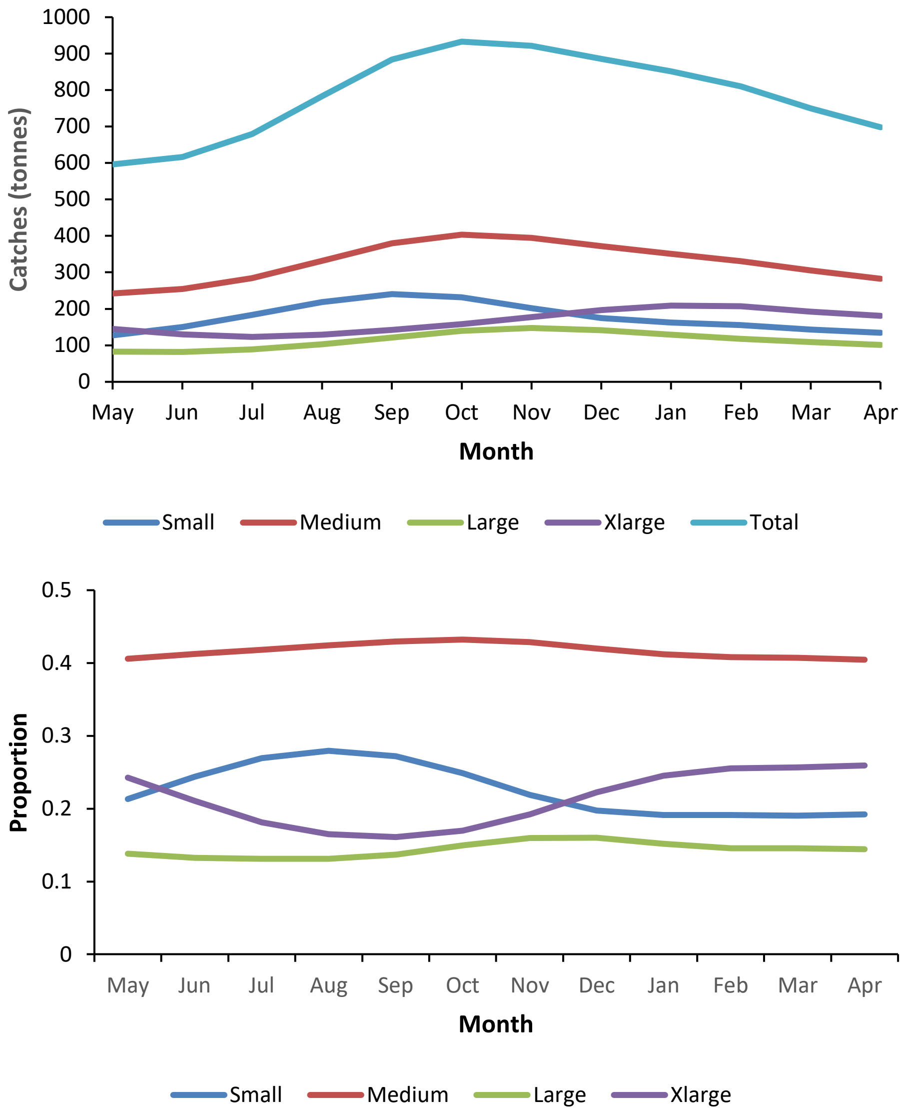


Figure 12. Estimated monthly catches per pack size category (and the total) for 2009 (top). The monthly proportions per pack size category are shown on the bottom plot.

APPENDIX A

Assessment model for chokka squid

THE BASIC DYNAMICS

The following assumptions are made with regards to the squid population

- The life cycle of squid is assumed to be 18 months.
- For simplicity, every month is taken to comprise 30 days.
- Continuous recruitment into the population is assumed.
- Catches are taken mid-month.

In what follows below, three "years" are referred to. A calendar year (referenced as y'), a cohort year (referenced as y) represents cohorts born in the period from October of fishing year y^*-1 to September of fishing year y^*, and a fishing year (referenced by y^*) is defined as the period from May of year y' to April of year $y'+1$. Figure 1 gives a schematic representation of the population dynamics of squid. The block with thick borders represents a cohort year y. The block with dashed borders represents a calendar year y' and the coloured block represents a fishing year y^*. The squid that are in the population in fishing year y^* consists of mostly squid born in cohort year y, plus some of the older squid born in cohort year $y-1$ and very young squid born in cohort year $y+1$. The months that relate to the life cycle of a cohort (i.e. 18 months) are referenced as mth, those that relate to the months of a fishing year (i.e. 12 months) are referenced as fm.

The chokka squid cohort population dynamics are given by the equations

$$N^g_{y,bm,mth+1} = \begin{cases} R^g_{y,bm}e^{-M_a/2} & if\ mth = bm \\ \left(N^g_{y,bm,mth}e^{-M_a/2} - C^g_{y,bm,mth}\right) e^{-M_a/2} & if\ mth \neq bm \end{cases} \qquad (A.1)$$

where

$N^g_{y,bm,mth}$ is the number of chokka squid of gender g in the cohort born in mid-month bm of age a in days at the start of month mth in cohort year y,

$C^g_{y,bm,mth}$ is the number of chokka squid of gender g in the cohort born in month bm that are taken by the fishery in mid-month mth in cohort year y,

$R^g_{y,bm}$ is the number of recruits of chokka squid of gender g in the cohort born in mid-month bm in cohort year y, and

M_a is the natural mortality rate for chokka squid of age a days.

The number of male/female recruits in the cohort born in mid-month bm in cohort year y, $R^g_{y,bm}$, is given by

$$R^g_{y,bm} = R^T_y R^B_{bm} \rho^g \qquad (A.2)$$

where

R_y^T is the total number of recruits into the population in cohort year y,

R_{bm}^B is the recruitment proportion in the cohort born in mid-month bm, and

ρ^g is the proportion of males/females chokka squid born in the population.

Recruitment into the population is assumed to be continuous, with the total number of recruits in each cohort year R_y^T being an estimable parameter.

The number of males/females chokka squid in the cohort *bm* caught in mid-month mth in in cohort year y is given by

$$C_{y,bm,mth}^{g} = \sum_{f}\sum_{\ell} F_y^f S_\ell^f N_{y,bm,mth}^{g} e^{-M_a/2} p_{bm,mth,\ell}^{g} = \sum_{f}\sum_{\ell} C_{y,bm,mth,\ell}^{g,f} \qquad (A.3)$$

where

$C_{y,bm,mth,\ell}^{,f}$ is the number of chokka squid of gender g born in month bm that are caught by fleet f in mid-month mth that fall in length bin ℓ in cohort year *y*, which is given by

$$C_{y,bm,mth,\ell}^{,f} = \begin{cases} 0 & \text{if } mth \text{ is a closed season month or if } mth = bm \\ F_y^f S_\ell^f N_{y,bm,mth}^{g} e^{-M_a/2} p_{bm,mth,\ell}^{g} & \text{otherwise} \end{cases}, (A.4)$$

F_y^f is the proportion of the exploitable (i.e. selected) resource harvested by industry in cohort year y. This relates to the estimable proportion of the exploitable resource in fishing year y^* (F_y^{*f}) by

$$F_y^f = \begin{cases} F_{y-1}^{*f} & \text{if } mth \text{ in cohort year } y \text{ falls in fishing year } y^* - 1 \\ F_y^{*f} & \text{if } mth \text{ in cohort year } y \text{ falls in fishing year } y^* \\ F_{y+1}^{*f} & \text{if } mth \text{ in cohort year } y \text{ falls in fishing year } y^* + 1 \end{cases},$$

S_ℓ^f is the commercial selectivity at length ℓ, and

$p_{bm,mth,\ell}^{g}$ is the proportion of a cohort born in month bm which is to be found in length bin ℓ in mid-month mth. The $p_{bm,mth,\ell}^{g}$ are calculated under the assumption that length-at-age (in days) is normally distributed about a mean given by the von Bertalanffy equation, i.e.

$$\ell(days) \sim \boldsymbol{N}^*\left[\ell_\infty\left\{1 - e^{-\kappa(days - t_0)}\right\};\ \sigma_g^2\right] \qquad (A.5)$$

where

$\boldsymbol{N}^*$ is a normal distribution truncated at ± 3 standard deviations (to avoid negative values), and

σ_g is the standard deviation of length-at-age *in days,* or this can be modelled to be proportional to the expected length-at-age *days* for a specified CV, i.e.

$$\sigma_g = \mathrm{CV}^{m/f}\ell_\infty\left\{1 - e^{-\kappa(days - t_0)}\right\}. \qquad (A.6)$$

In the case where the variation in length with age is assumed to be constant, the variation for the initial ages up to some specified age is assumed to follow that given by Equation A.5.

Note that $\sum_\ell p^g_{bm,mth,\ell} = 1$ when summed over all length bins ℓ.

The total **number** of chokka squid caught by fleet f in mid-month mth that are to be found in length bin ℓ in cohort year y is given by

$$C^f_{y,mth,\ell} = \sum_{bm}\sum_{g} F^f_y S^f_\ell N^g_{y,bm,mth} e^{-M_a/2} p^g_{bm,mth,\ell} = \sum_{bm}\sum_{g} C^{g,f}_{y,bm,mth,\ell}. \qquad (A.7)$$

The total **mass** of chokka squid caught by fleet f in mid-month mth that are found in length bin ℓ in cohort year y is given by

$$CM^f_{y,mth,\ell} = \sum_{bm}\sum_{g} w^g_a F^f_y S^f_\ell N^g_{y,bm,mth} e^{-M_a/2} p^g_{bm,mth,\ell} = \sum_{bm}\sum_{g} w^g_a C^{g,f}_{y,bm,mth,\ell}. \qquad (A.8)$$

where

w^g_a is the mass of chokka squid of gender g at age a days for the cohort born in month bm at mid-month mth, where

$$w^g_a = c^g[\ell(days)]^{d^g}. \qquad (A.9)$$

From $CM_{y,mth,\ell}$, the total mass of squid caught by fleet f in month fm (taken to occur in the middle of the month) in fishing year y^* that fall in pack category p ($CM_{y^*,fm,p}$) is given by

$$CM^f_{y^*,fm,p} = \sum_{\substack{\ell\in p \\ fm \text{ in } y-1 \in y^*}} CM^f_{y-1,fm,\ell} + \sum_{\substack{\ell\in p \\ fm \text{ in } y \in y^*}} CM^f_{y,fm,\ell} + \sum_{\substack{\ell\in p \\ fm \text{ in } y+1 \in y^*}} CM^f_{y+1,fm,\ell} \qquad (A.10)$$

where for a particular pack category p, the summation is over those lengths that fall within that pack category. The proportion of the catch by mass by fleet f in fishing year y^* and month fm in each pack category p ($p^{pred,f}_{y^*,fm,p}$) is calculated by normalising $CM^f_{y^*,fm,p}$ such that

$$\sum_p p^{pred,f}_{y^*,fm,p} = 1. \qquad (A.11)$$

FISHING SELECTIVITY

Two alternatives are considered for commercial fishing selectivity (the second is a limiting case of the first). At the moment the same fishing selectivity is assumed for both fleets in the fishery.

Linear

The commercial fishing selectivity, S_ℓ^f, is assumed to be linear between lengths L_0 and L_1, such that

$$S_\ell^f = \begin{cases} 0 & \ell \le L_0 \\ x & L_0 < \ell < L_1 \\ 1 & \ell \ge L_1 \end{cases} \qquad (A.12)$$

where with x reflects the linear increase from 0 at $\ell = L_0$ to 1 at $\ell = L_1$.

Knife-edge

The commercial fishing selectivity, S_ℓ^f, is assumed to be knife-edge at length L_0, such that

$$S_\ell^f = \begin{cases} 0 & \ell < L_0 \\ 1 & \ell \ge L_0 \end{cases}. \qquad (A.13)$$

The survey-specific selectivity for survey s ($S_\ell^{surv,s}$) is assumed to be uniform for both.

NATURAL MORTALITY

Two options were considered for age-specific natural mortality.

Lorenzen's model

Lorenzen (1996) modelled natural mortality rate (M) using the proportional relationship between M and a power function of body weight (w) such that

$$M(w) = M_u w^{-b_1} \qquad (A.14)$$

where M_u is the natural mortality at unit weight and an estimate for the exponent parameter b_1 of 0.305 was obtained for ocean systems (Lorenzen, 1996 and 2000).

Applying the length-weight relationship $\left(w = cL^d\right)$, M can be modified to vary with length

$$M(L) = M_u\left(c^g L^{d^g}\right)^{-b_1}. \qquad (A.15)$$

In the application here, given a von Bertalanffy growth curve, the natural mortality at age a days (M_a^g) is given by

$$M_a^g = M_\infty^g\left(1 - e^{-\kappa(days - t_0)}\right)^{-b_1 d^g}, \qquad (A.16)$$

where M_∞^g is the natural mortality rate at the asymptotic length L_∞.

Natural mortality ogive

A four-parameter logistic curve is assumed to describe the natural mortality-at-age, which is given by

$$M_a = M_j + \frac{M_{max} - M_j}{\left(1 + e^{-(a - a_{mid})/\omega}\right)} \qquad (A.17)$$

where

M_j is the asymptotic value of natural mortality for juvenile squid

M_{max} is the asymptotic value of natural mortality for older squid

a_{mid} is the midpoint age of the ogive, and

ω is the steepness of the ogive curve.

BIOMASS

The biomass of male/female chokka squid for a cohort born in mid-month bm at the start of month mth in cohort year y is given by

$$B^g_{y,bm,mth} = w^g_a N^g_{y,bm,mth} \tag{A.18}$$

where

w^g_a is the mass of male/female chokka squid at age a days for the cohort born in month bm at the start of month mth.

The total biomass of male/female chokka squid in fishing year y^* is given by

$$B^g_{y^*} = \sum_{\substack{bm,fm \\ fm \text{ in } y-1 \in y^*}} B^g_{y-1,bm,fm} + \sum_{\substack{bm,fm \\ fm \text{ in } y \in y^*}} B^g_{y,bm,fm} + \sum_{\substack{bm,fm \\ fm \text{ in } y+1 \in y^*}} B^g_{y+1,bm,fm}\,. \tag{A.19}$$

The model estimate of the exploitable biomass of chokka squid at the start of month fm in fishing year y^* for each fleet is then given by

$$B^{exp,f}_{y^*,fm} = \sum_{\substack{g,bm,\ell \\ fm \text{ in } y-1 \in y^*}} S^f_\ell w^g_a N^g_{y-1,bm,fm} p^g_{bm,fm,\ell} + \sum_{\substack{g,bm,\ell \\ fm \text{ in } y \in y^*}} S^f_\ell w^g_a N^g_{y,bm,fm} p^g_{bm,fm,\ell} + \sum_{\substack{g,bm,\ell \\ fm \text{ in } y+1 \in y^*}} S^f_\ell w^g_a N^g_{y+1,bm,fm} p^g_{bm,fm,\ell}\,. \tag{A.20}$$

The model estimate of the exploitable biomass of chokka squid at the start of month fm in fishing year y^* for each fleet disaggregated by pack category size is given by

$$B^{exp,f}_{y^*,fm,p} = \sum_{\substack{\ell \in p,g,bm \\ fm \text{ in } y-1 \in y^*}} S^f_\ell w^g_a N^g_{y-1,bm,fm} p^g_{bm,fm,\ell} + \sum_{\substack{\ell \in p,g,bm \\ fm \text{ in } y \in y^*}} S^f_\ell w^g_a N^g_{y,bm,fm} p^g_{bm,fm,\ell} + \sum_{\substack{\ell \in p,g,bm \\ fm \text{ in } y+1 \in y^*}} S^f_\ell w^g_a N^g_{y+1,bm,fm} p^g_{bm,fm,\ell}\,. \tag{A.21}$$

The model estimate of the survey biomass of chokka squid at the start of month fm in fishing year y^* for each fleet is given by

$$B_{y^*,fm}^{surv,s} = \sum_{\substack{g,bm,\ell \\ fm \text{ in } y-1\in y^*}} S_\ell^{surv,s} w_a^g N_{y-1,bm,fm}^g p_{bm,fm,\ell}^g + \sum_{\substack{g,bm,\ell \\ fm \text{ in } y\in y^*}} S_\ell^{surv,s} w_a^g N_{y,bm,fm}^g p_{bm,fm,\ell}^g + \sum_{\substack{g,bm,\ell \\ fm \text{ in } y+1\in y^*}} S_\ell^{surv,s} w_a^g N_{y+1,bm,fm}^g p_{bm,fm,\ell}^g \qquad (A.22)$$

where the month fm is taken to be January for the summer survey, and April for the autumn survey.

EXTENSION TO AN ASSESSMENT MODEL

The total **mass** of chokka squid caught in fishing year y^* is given by

$$CM_{y^*}^f = F_{y^*}^{*f}\left[\begin{array}{l} \sum_{bm}\sum_{g}\sum_{fm \text{ in } y-1\in y^*}\sum_{\ell} w_a^g S_\ell^f N_{y-1,bm,mth}^g e^{-M_a/2} p_{bm,mth,\ell}^g + \\ \sum_{bm}\sum_{g}\sum_{fm \text{ in } y\in y^*}\sum_{\ell} w_a^g S_\ell^f N_{y,bm,mth}^g e^{-M_a/2} p_{bm,mth,\ell}^g + \\ \sum_{bm}\sum_{g}\sum_{fm \text{ in } y+1\in y^*}\sum_{\ell} w_a^g S_\ell^f N_{y+1,bm,mth}^g e^{-M_a/2} p_{bm,mth,\ell}^g \end{array}\right]. \qquad (A.23)$$

THE LIKELIHOOD FUNCTION

The model takes account of all available CPUE and survey abundance indices and commercial pack size data in the fitting process to estimate model parameters.

CPUE relative abundance data

The likelihood is calculated assuming that the observed abundance indices are lognormally distributed about their expected value. For monthly, pack size disaggregated indices this is given as

$$I_{y^*,fm,p}^f = \hat{I}_{y^*,fm,p}^f e^{\varepsilon_{y*,fm,p}^f}$$

or

$$\varepsilon_{y^*,fm,p}^f = ln\left(I_{y^*,fm,p}^f\right) - ln\left(\hat{I}_{y^*,fm,p}^f\right) \qquad (A.24)$$

where

$I_{y^*,fm,p}^f$ is the abundance index for fleet f in month fm and pack size p for fishing year y^*,

$\hat{I}_{y^*,fm,p}^f$ $= \hat{q}^f \hat{B}_{y^*,fm,p}^{exp,f}$ is the corresponding model estimate, where

$\hat{B}_{y^*,fm,p}^{exp,f}$ is the model estimate of exploitable biomass of the resource for fishing year y^*, and

$\hat{q}^f$ is the catchability coefficient for the commercial CPUE abundance indices for fleet f, whose maximum likelihood estimate is given by

$$ln\hat{q}^f = \frac{1}{n^f}\sum_{y^*}\sum_{fm}\sum_{p}\left(ln\, I^f_{y^*,fm,p} - ln\hat{B}^{exp,f}_{y^*,fm,p}\right) \qquad (A.25)$$

where

n^f is the number of data points in the abundance index for fleet f, and

$\varepsilon^f_{y^*,fm,p}$ is normally distributed with mean zero and standard deviation σ^f (assuming homoscedasticity of residuals, so that $\hat{\sigma}^f_{y^*} = \hat{\sigma}^f$), whose maximum likelihood estimate is given by

$$\hat{\sigma}^f = \sqrt{\frac{1}{n^f}\sum_{y^*}\sum_{fm}\sum_{p}\left(ln\, I^f_{y^*,fm,p} - ln\hat{q}^f\hat{B}^{exp,f}_{y^*,fm,p}\right)^2}. \qquad (A.26)$$

For the case when CPUE indices that are not disaggregated by pack size (i.e. the trawl fishery), the equations above will not have the p subscript and there will be no summation over p. The contribution by the CPUE data to the negative log-likelihood function (ignoring constants) which is minimised in the fitting procedure is thus

$$-\,ln\, L_{CPUE} = \sum_{f}\sum_{y^*}\sum_{fm}\sum_{p}\left\{\frac{1}{2(\sigma^f)^2}\;\left(ln\, I^f_{y^*,fm,p} - lnq^f\hat{B}^{exp,f}_{y^*,fm,p}\right)^2 + n^f\left(ln\sigma^f\right)\right\}. \qquad (A.27)$$

Survey abundance data

Survey abundance indices are treated as relative indices in a similar way to the CPUE indices, with the biomass available to the survey ($B^{surv,s}_{y^*,fm}$) replacing the commercial exploitable biomass ($B^{exp,f}_{y^*,fm,p}$) and the p subscript and summation falling away, the superscript s denoting survey s. The month subscript fm is taken to be January for the summer surveys and April for the autumn surveys. The associated $\hat{\sigma}^s_{y^*}$ for survey indices are input and are given by

$$\left(\hat{\sigma}^s_{y^*}\right)^2 = ln\left(1 + \left(CV^s_{y^*}\right)^2\right) \qquad (A.26)$$

where the $CV^s_{y^*}$ are the coefficients of variation of the survey abundance estimates for survey s in fishing year y^*.

The contribution of the survey abundance data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\,ln\, L_{surv} = \sum_{s}\sum_{y^*}\left[ln\,\sigma^s_{y^*} + \frac{\left(\varepsilon^s_{y^*,fm}\right)^{2^2}}{2\left(\sigma^s_{y^*}\right)}\right]. \qquad (A.27)$$

The catchability coefficient q^s for the survey abundance index s is estimated by its maximum likelihood value and is given by

$$ln\hat{q}^s = \frac{\sum_{y^*} \frac{1}{\left(\hat{\sigma}_y^s\right)^2} \left(ln\, I_{y^*,fm}^s - ln\hat{B}_{y^*,fm}^{surv,s}\right)}{\sum_{y^*} \frac{1}{\left(\hat{\sigma}_{y^*}^s\right)^2}}. \qquad (A.28)$$

FITTING TO PACK CATCH-AT-LENGTH DATA

The model above provides predicted proportions of the catch by mass in fishing year y^* for pack category p and month fm for fleet f $\left(p_{y^*,fm,p}^{pred,f}\right)$. Pack size data is only available for the jig fleet and therefore $f\ =$ jig in this instance. The contribution of the pack catch-at-length to the negative log-likelihood function is taken to be given by[1]

$$-\,ln\, L_{pack} = \sum_{y^*} \sum_{fm} \sum_{p} \left\{ ln\left(\sigma_{pack}\right) + \left(\frac{1}{2\sigma_{pack}^2}\right) \left(\sqrt{p_{y^*,fm,p}^{obs,jig}} - \sqrt{p_{y^*,fm,p}^{pred,jig}}\right)^2 \right\} \qquad (A.29)$$

where σ_{pack} has a closed form maximum likelihood estimate given by

$$\hat{\sigma}_{pack}^2 = \frac{\sum_{y^*} \sum_{fm} \sum_{p} \left(\sqrt{p_{y^*,fm,p}^{obs,jig}} - \sqrt{p_{y^*,fm,p}^{pred,jig}}\right)^2}{\sum_{y^*} \sum_{fm} \sum_{p} 1}. \qquad (A.30)$$

PENALTY FUNCTION RELATED TO F_y

The fishing proportions $F_{y^*}^f$ are considered estimable parameters. The relationship between $F_{y^*}^f$ and the total catches given by Equation (A.23) is used to provide a penalty in the negative log-likelihood function, in which the sum of squares of the difference between, $CM_{y^*}^f$, and the far right-hand side of Equation (A.23) (the predicted catches) are minimised. The contribution from this penalty function that is added to the negative log-likelihood ($-lnL$) is given by

$$SSQ = w_F \sum_{f} \sum_{y^*} \left(CM_{y^*}^{obs,f} - CM_{y^*}^{pred,f}\right)^2 \qquad (A.31)$$

where

$CM_{y^*}^{pred,f}$ are the predicted catches which are given by the far right-hand side of Equation (A.23),

$CM_{y^*}^{obs,f}$ are the observed catches for fleet f in fishing year y^*, and

w_F is a weight applied to the sum of squares.

The total negative log-likelihood function that is minimised is then given by

$$-\ln L = -\,ln\, L_{CPUE} - ln\, L_{surv} - ln\, L_{pack} + SSQ. \qquad (A.32)$$

[1] This rests on the assumption that the square roots of proportions from a Poisson-like distribution have an approximately Normal distribution.

ESTIMABLE PARAMETERS

The parameters estimated by the model are the total number of recruits into the population in cohort year y (R_y^T) and the proportion of the exploitable resource in fishing year y^* (F_y^{*f}).