# Correcting the recruitment process error variance for bias due to recruitment estimation error.

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## 1 Summary

The recruitment process error variance ${\sigma_R}^2$ can be calculated from recruitment estimates produced by stock assessments such as age structured production models and general integrated stock assessments. These recruitment estimates are subject to estimation error and this has implications for the reliability of the estimate of ${\sigma_R}^2$. Equations presented here show that recruitment estimation error most likely causes positive bias in estimates of ${\sigma_R}^2$. Two possible bias correction procedures are proposed, based on the estimation error variance derived from either the Hessian matrix or Markov chains from Bayesian analyses. Application to South African sardine shows that the scale of the effect can be very substantial. For CTS the bias corrected estimate of ${\sigma_R}^2$ is about half of the uncorrected estimate (referred to as the 'putative estimates'). This has implications for the risk measure (the simulated probability of resource biomass falling below a low level), a very important performance statistic for Management Strategy Evaluations (MSEs) currently being carried out for the development of a management procedure for South African sardine. It is proposed that an appropriate bias correction such as is proposed here is applied to the 'putative' recruitment process error variance estimate before using these in MSEs.

## 2 Method

Sardine recruitment $R_y$ is assumed to be driven by a component $\widetilde{R_y}$ that is a function $Q$ of spawning biomass $SSB_y$, say $\widetilde{R_y} = Q(SSB_y)$ and another component that is year specific and causes recruitment to deviate from $\widetilde{R_y}$, $R_y = \widetilde{R_y}e^{\xi_y}$. $\xi_y$ is typically assumed to be a normally distributed random variate with mean 0 and variance ${\sigma_R}^2$ (${\sigma_R}^2 = VAR(\xi_y)$). Here ${\sigma_R}^2$ is referred to as the recruitment process error variance. In stock assessment analyses an estimate of $R_y$, say $\widehat{R_y}$, is used to obtain an estimate of $\xi_y$, $\widehat{\xi_y}$, from the equation;

$$\widehat{\xi_y} = ln(\widehat{R_y}/\widetilde{R_y}). \quad (1)$$

The 'putative' estimator

$$\widehat{\sigma_R}^2 = Var(\widehat{\xi_y}) = \sum\nolimits_{\forall y} \widehat{\xi_y}^2 \Big/ (n-1) \quad (2)$$

is then used, for n= the number of years for which there are recruitment estimates. But, whereas $R_y = \widetilde{R_y}e^{\xi_y}$,

$$\widehat{R_y} = \widetilde{R_y}e^{\xi_y + \eta_y}, \quad (3)$$

where $\eta_y$ is an estimation error term, reasonably assumed to be normally distributed, which has a mean of 0 and a variance of ${\sigma_\eta}^2$. Therefore

$$\widehat{\xi_y} = \xi_y + \eta_y \quad (4)$$

and

$$Var(\widehat{\xi_y}) = Var(\xi_y) + Var(\eta_y) + 2Cov(\xi_y, \eta_y) \quad (5)$$

For the plausible situation in which process and estimation errors are independent, the 'putative' estimate of the recruitment process error variance is

$$\widehat{\sigma_R}^2 = \sigma_R{}^2 + \sigma_\eta{}^2. \quad (6)$$

This suggests that a more reliable estimate of $\sigma_R{}^2$ (than the 'putative' estimate) is the bias corrected quantity $\sigma_R{}^2 = \widehat{\sigma_R}^2 - \sigma_\eta{}^2$. Two possible sources of information about $\sigma_\eta{}^2$, the estimation error variance, are (i) the standard errors of estimation of $\widehat{R_y}$ provided by the inverse of the Hessian matrix, and (ii) the variance in $\widehat{R_y}$ derived from converged Markov chains from Bayesian analyses. The standard errors based on method (i) are given in Table A1 of FISHERIES/2025/OCT/SWG-PEL/53, or can be obtained for method (ii) from Figures 2a to 5b of FISHERIES/2025/OCT/SWG-PEL/56 which provide 60 elements of a converged Markov chain, sufficient to provide variance estimates. Here Method (i) is implemented. The values given in Table A1 of FISHERIES/2025/OCT/SWG-PEL/53 are the standard errors of estimation of $\widehat{R_y}$, effectively the year specific estimates of the estimation standard errors $\sigma_{\widehat{R_y}}$ or $\sqrt{\widehat{Var(\widehat{R_y})}}$ in notation suitable for calculations that follow. To allow calculations to proceed further it is assumed that $\widehat{R_y} = E(\widehat{R_y})$, and $\hat{\mu} = ln\widehat{R_y}$. Since possible realizations of recruitment in year y due to estimation error are assumed to be lognormally distributed, the following equations follow from standard theory about the lognormal distribution;

$$E(\widehat{R_y}) = e^{\hat{\mu}+\sigma_\eta{}^2/2} \quad (7)$$

$$\widehat{Var(\widehat{R_y})} = (e^{\sigma_\eta{}^2} - 1)e^{2\hat{\mu}+\sigma_\eta{}^2} \quad (8)$$

Solving for $\sigma_{\eta,y}{}^2$;

$$\sigma_{\eta,y}{}^2 = ln\left( \widehat{Var(\widehat{R_y})} \Big/ E(\widehat{R_y})^2 + 1 \right). \quad (9)$$

# 3 Results

Using the WTS values from Table A1 of FISHERIES/2025/OCT/SWG-PEL/53 (see Table 1 here for WTS) produces the estimates of $\sigma_{\eta,y}{}^2$ plotted in Figure 1[1]. These show considerable variability over time, with an average of 0.571. From the data in Table 1, the putative estimate of $\sigma_R{}^2$, $\widehat{\sigma_R}^2$=2.84 and $\widehat{\sigma_R} = 1.68$ for WTS. The bias correction implied by application of Method (i), using the average over all years of $\sigma_{\eta,y}{}^2$ of 0.571 reduces the estimate of $\sigma_R{}^2$ from 2.84 to 2.27 and $\sigma_R$ from 1.68 to 1.51.

For CTS, application of Method (i) based on the data from Table A1 of FISHERIES/2025/OCT/SWG-PEL/53 (see the data presented in Table 2 reduces the estimate of $\sigma_R$ from 1.05 to 0.50.

*The bias correction implied by application of Method (i), and the data from Table 2*

Table 2 (provided by Table A1 of FISHERIES/2025/OCT/SWG-PEL/53), and using the average over all years of $\sigma_{\eta,y}{}^2$ of 0.966, reduces the estimate of $\sigma_R{}^2$ from 1.02 to 0.25 and $\sigma_R$ from 1.05 to 0.50.

[1] It is evident from Figure 1 that the $\sigma_\eta{}^2$ values are year specific, hence the introduction of the y subscript in $\sigma_{\eta,y}{}^2$.

Table 3 presents the size of the correction across a wider range of values of $\widehat{\sigma_R}$ and $\overline{\sigma_\eta}$.

# 4 Discussion and Conclusion

Estimates of ${\sigma_R}^2$ based on estimates of recruitment using a 'putative' estimator will be positively biassed when the process error and estimation error are independent, as seems plausible. Two different sources of estimation error information are highlighted, viz. (i) from standard errors derived from the inverse of the Hessian matrix, and (ii) from Markov chains. Both of these are available for South African sardine. A method based on the Hessian based standard errors shows the size of the bias in the recruitment process error variance ${\sigma_R}^2$ given by the 'putative' estimate, $\widehat{{\sigma_R}^2}$. This bias may be relatively small, as is the case for WTS using available data, or can be very substantial, as is the case for CTS. For CTS the bias correction reduces the recruitment process error variance estimates by half. This has implications for risk calculations in MSE's for a revised sardine OMP. It is proposed that a bias correction procedure such as the one described here be applied when these MSEs are carried out.

# 5 References

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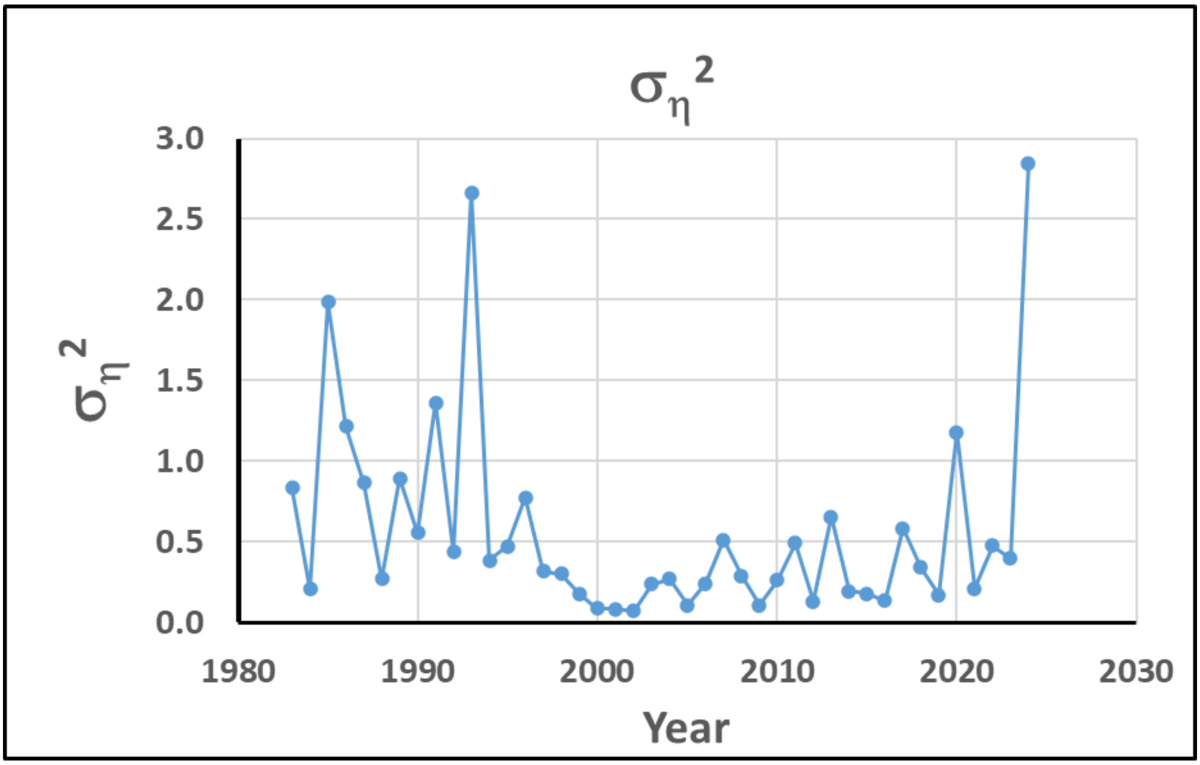


Figure 1. Error variance estimates ${\sigma_\eta}^2$ calculated using Method (i) described here and information provided in Table A1 of FISHERIES/2025/OCT/SWG-PEL/53.

*Table 1. Table of estimates of WTS recruitment and its standard error as reported in FISHERIES/2025/OCT/SWG-PEL/56. The estimate itself is taking here to be the expectation of recruitment for the purpose of calculations reported here.*

| Year | $E(\widehat{R_y})$ | $\sqrt{\widehat{Var(\widehat{R_y})}}$ | Year | $E(\widehat{R_y})$ | $\sqrt{\widehat{Var(\widehat{R_y})}}$ |
|---|---|---|---|---|---|
| 1983 | 0.312 | 0.356 | 2004 | 2.098 | 1.169 |
| 1984 | 1.346 | 0.644 | 2005 | 5.989 | 2.026 |
| 1985 | 0.055 | 0.138 | 2006 | 2.007 | 1.049 |
| 1986 | 0.026 | 0.040 | 2007 | 3.051 | 2.490 |
| 1987 | 0.075 | 0.088 | 2008 | 3.867 | 2.247 |
| 1988 | 0.677 | 0.378 | 2009 | 10.194 | 3.423 |
| 1989 | 0.462 | 0.554 | 2010 | 2.726 | 1.489 |
| 1990 | 1.177 | 1.015 | 2011 | 3.206 | 2.556 |
| 1991 | 1.555 | 2.649 | 2012 | 5.965 | 2.219 |
| 1992 | 5.720 | 4.226 | 2013 | 0.584 | 0.561 |
| 1993 | 0.026 | 0.095 | 2014 | 1.861 | 0.861 |
| 1994 | 6.693 | 4.588 | 2015 | 1.449 | 0.640 |
| 1995 | 3.338 | 2.593 | 2016 | 3.175 | 1.211 |
| 1996 | 6.292 | 6.811 | 2017 | 0.995 | 0.884 |
| 1997 | 7.271 | 4.442 | 2018 | 2.409 | 1.540 |
| 1998 | 6.570 | 3.905 | 2019 | 7.446 | 3.195 |
| 1999 | 14.673 | 6.449 | 2020 | 1.127 | 1.690 |
| 2000 | 36.235 | 10.881 | 2021 | 2.494 | 1.202 |
| 2001 | 36.965 | 10.893 | 2022 | 3.014 | 2.355 |
| 2002 | 32.898 | 8.864 | 2023 | 4.400 | 3.065 |
| 2003 | 3.205 | 1.662 | 2024 | 1.124 | 4.520 |

*Table 2. Table of estimates of CTS recruitment and its standard error as reported in FISHERIES/2025/OCT/SWG-PEL/56. The estimate itself is taking here to be the expectation of recruitment for the purpose of calculations reported here. Recruitment values of zero were excluded from the calculations.*

| Year | $E(\widehat{R_y})$ | $\sqrt{\widehat{Var(\widehat{R_y})}}$ | Year | $E(\widehat{R_y})$ | $\sqrt{\widehat{Var(\widehat{R_y})}}$ |
|---|---|---|---|---|---|
| 1983 | | | 2004 | 0.000 | 0.002 |
| 1984 | 0.200 | 0.725 | 2005 | 0.000 | 0.001 |
| 1985 | 1.714 | 0.698 | 2006 | 0.126 | 0.736 |
| 1986 | 3.160 | 1.270 | 2007 | 1.348 | 1.961 |
| 1987 | 0.400 | 0.264 | 2008 | 0.000 | 0.007 |
| 1988 | 0.401 | 0.441 | 2009 | 2.414 | 0.936 |
| 1989 | 2.098 | 0.976 | 2010 | 0.639 | 0.545 |
| 1990 | 0.278 | 1.198 | 2011 | 1.530 | 1.633 |
| 1991 | 2.395 | 1.981 | 2012 | 0.000 | 0.000 |
| 1992 | 3.320 | 2.459 | 2013 | 1.336 | 0.453 |
| 1993 | 0.903 | 0.361 | 2014 | 0.183 | 0.776 |
| 1994 | 4.228 | 1.728 | 2015 | 0.000 | 0.000 |
| 1995 | 0.860 | 1.221 | 2016 | 0.000 | 0.000 |
| 1996 | 9.099 | 4.579 | 2017 | 0.000 | 0.000 |
| 1997 | 1.771 | 2.730 | 2018 | 0.000 | 0.000 |
| 1998 | 2.215 | 2.159 | 2019 | 0.000 | 0.000 |
| 1999 | 0.000 | 0.002 | 2020 | 0.755 | 0.758 |
| 2000 | 0.000 | 0.001 | 2021 | 2.714 | 1.147 |
| 2001 | 0.000 | 0.000 | 2022 | 1.331 | 1.675 |
| 2002 | 0.000 | 0.000 | 2023 | 0.000 | 0.015 |
| 2003 | 0.408 | 0.650 | 2024 | 1.673 | 4.387 |

*Table 3. A table of adjusted estimates of $\sigma_R$ for given values of the putative estimates $\widehat{\sigma_R}$ and the estimation standard deviation in 'log-space', $\sigma_\eta$.*

| | | $\sigma_\eta$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.55** | **0.60** | **0.65** | **0.70** | **0.75** | **0.80** | **0.85** | **0.90** | **0.95** | **1.00** |
| $\widehat{\sigma_R}$ | **0.70** | 0.43 | 0.36 | 0.26 | 0.00 | | | | | | |
| | **0.80** | 0.58 | 0.53 | 0.47 | 0.39 | 0.28 | 0.00 | | | | |
| | **0.90** | 0.71 | 0.67 | 0.62 | 0.57 | 0.50 | 0.41 | 0.30 | 0.00 | | |
| | **1.00** | 0.84 | 0.80 | 0.76 | 0.71 | 0.66 | 0.60 | 0.53 | 0.44 | 0.31 | 0.00 |
| | **1.10** | 0.95 | 0.92 | 0.89 | 0.85 | 0.80 | 0.75 | 0.70 | 0.63 | 0.55 | 0.46 |
| | **1.20** | 1.07 | 1.04 | 1.01 | 0.97 | 0.94 | 0.89 | 0.85 | 0.79 | 0.73 | 0.66 |
| | **1.30** | 1.18 | 1.15 | 1.13 | 1.10 | 1.06 | 1.02 | 0.98 | 0.94 | 0.89 | 0.83 |
| | **1.40** | 1.29 | 1.26 | 1.24 | 1.21 | 1.18 | 1.15 | 1.11 | 1.07 | 1.03 | 0.98 |
| | **1.50** | 1.40 | 1.37 | 1.35 | 1.33 | 1.30 | 1.27 | 1.24 | 1.20 | 1.16 | 1.12 |
| | **1.60** | 1.50 | 1.48 | 1.46 | 1.44 | 1.41 | 1.39 | 1.36 | 1.32 | 1.29 | 1.25 |
| | **1.70** | 1.61 | 1.59 | 1.57 | 1.55 | 1.53 | 1.50 | 1.47 | 1.44 | 1.41 | 1.37 |
| | **1.80** | 1.71 | 1.70 | 1.68 | 1.66 | 1.64 | 1.61 | 1.59 | 1.56 | 1.53 | 1.50 |
| | **1.90** | 1.82 | 1.80 | 1.79 | 1.77 | 1.75 | 1.72 | 1.70 | 1.67 | 1.65 | 1.62 |
| | **2.00** | 1.92 | 1.91 | 1.89 | 1.87 | 1.85 | 1.83 | 1.81 | 1.79 | 1.76 | 1.73 |