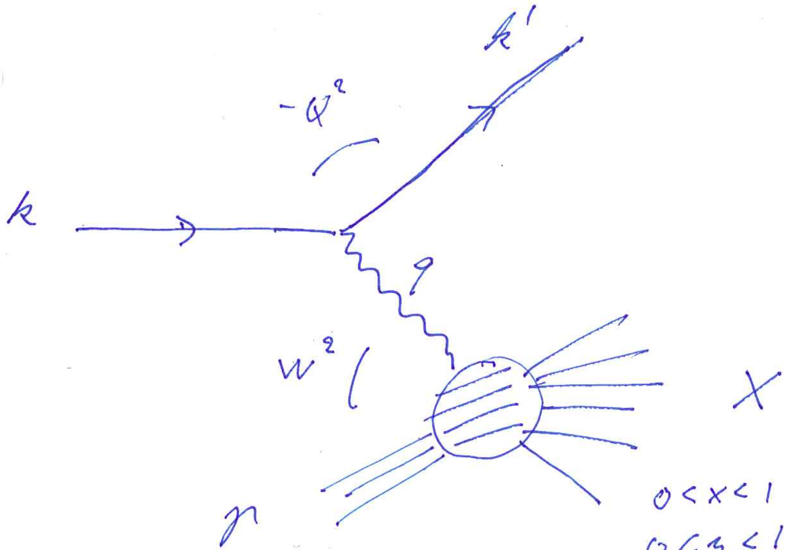
Deep inelastic scattering

Hadron-hadron scattering is difficult. R.P. Feynman: "Colliding hadrons is like scattering Swiss watches to find out how they are built." We would like to study the inner structure of hadrons with a simpler probe → collide leptons on hadronic target. "Deep" = high enough energy / momentum to study inner partonic structure.

First we have to understand kinematics.



$0<x<1$

$0<y<1$

$s = (k+p)^2$

$q = k-k'$, $Q^2 = -q^2$

$W^2 = (P+q)^2$

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2+W^2-m_N^2} = \frac{Q^2}{2m_N\nu}$$

$$y = \frac{P\cdot q}{P\cdot k} = \frac{W^2+Q^2-m_N^2}{s-m_N^2}$$

Some interpretation

W^2: invariant mass of $\gamma^* p$-system. Should think of the whole process as a $\gamma^* p$-collision.

ν = energy loss of electron in target rest frame (energy of γ^*)

x = momentum fraction (of P) of struck parton → more on this later.

y = fraction of electron energy taken by γ^* (in TRF — target rest frame)
↳ determines where k' goes, not very interesting for understanding the hadron.

H.E. limit Q^2 fixed, $W^2 \to \infty$, $x \sim \frac{1}{W^2} \to 0$
"small x physics"

$s > W^2 \Leftrightarrow y<1$, but W^2 is more important for physics of hadron.

The $e\gamma$ vertex is understood from QED and results in the well-known leptonic tensor

$$L_{\mu\nu} = \frac{1}{2}\sum_{r_e r_{e'}}\left[\bar{u}_{r_{e'}}(k')\gamma_\mu u_{r_e}(k)\right]^*\left[\bar{u}_{r_{e'}}(k')\gamma_\nu u_{r_e}(k)\right]$$

$$= 2\left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}\, k\cdot k'\right)$$

The γp part is not known; we will parametrize it by a similar structure

hadronic tensor $W_{\mu\nu}$

The whole cross section (inclusive = summed over all final states X) is

$$d\sigma = \frac{1}{2s}\underbrace{\frac{1}{2}\sum_{r_e r_{e'}}\frac{1}{2}\sum_{s_p s_X}\sum_X \frac{d^3\bar{p}_X}{2E_X(2\pi)^2}(2\pi)^4\delta^4(\Sigma k)|M|^2}\frac{d^3\bar{k}'}{2|\bar{k}'|(2\pi)^3}$$

$$= \frac{e^4}{q^4} L_{\mu\nu}\,(2\pi)W^{\mu\nu}$$

photon propagator $\rightarrow \frac{e^4}{q^4}$

γ^*-~~[illegible]~~ proton (amplitude)^2 summed over X

target rest frame (TRF) $\quad s = 4m_N|\bar{k}|$

$$d\sigma = \frac{\alpha_{em}^2}{2m\,Q^4}\frac{|\bar{k}'|}{|\bar{k}|}L_{\mu\nu}W^{\mu\nu}\,d\Omega_{\bar{k}'}\,d|\bar{k}'|$$

This is often conveniently parametrized in terms of the the virtual photon cross section

$$\sigma^{\gamma^* p} = \frac{e^2}{\underbrace{4m\nu}_{=2\hat{s}}}\,\underline{\varepsilon_\mu^{(\lambda)}(q)\,\varepsilon_\nu^{(\lambda)*}(q)}\,W^{\mu\nu}(2\pi)$$

$\nu = \frac{p\cdot q}{m_N}$ = energy of γ^* in TRF $\quad\rightarrow$ convention! $\quad\approx 2W^2$ @ high W

Flux factor of virtual photon is not well defined quantity. There can be different conventions, since $\sigma^{\gamma^* p}$ is not a measured cross section, but computed from the ep cross section.

Note: this is the total $\gamma^* p$ cross section, not differential. The integral over the phase space of the final state X is included in $W_{\mu\nu}$.

$W_{\mu\nu}$ can depend on 2 Lorentz-invariant kinematical variables; these are normally chosen as $\underline{x, Q^2}$.

The Lorentz index structure follows from the following properties:

- $W_{\mu\nu} = W_{\nu\mu}$
- $q^\mu W_{\mu\nu} = 0$ ($W_{\mu\nu} \sim \langle J_\mu J_\nu \rangle$ and $\partial_\mu J^\mu = 0$) i.e. this follows from the conservation of the electromagnetic current.
- only q^μ, p^μ define $W_{\mu\nu}$

$$\Rightarrow \quad W_{\mu\nu} = 2\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{2}{(p\cdot q)}\left[\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\right] \times F_2(x, Q^2)$$

Interpretation: F_2 : ~~transverse to~~ along direction of p_μ, $\perp$ to q_μ
F_1 : ~~not to. to p_μ~~ the rest, $\perp$ to q_μ

Contracting this with the lepton tensor gives the ep cross section in terms of F_1 and F_2

Projecting on the different polarization projectors of the virtual photon one gets

$$\sigma_L^{\gamma^* p} = \frac{4\pi^2 \alpha_{e.m.}}{Q^2} (F_2 - 2x F_1)$$ ← measuring this at a collider requires doing runs with different beam energies

$$\sigma_T^{\gamma^* p} = \frac{4\pi^2 \alpha_{e.m.}}{Q^2} 2x F_1$$

$$\sigma_{tot}^{\gamma^* p} = \frac{4\pi^2 \alpha_{e.m.}}{Q^2} F_2$$ ← this was measured very well at HERA

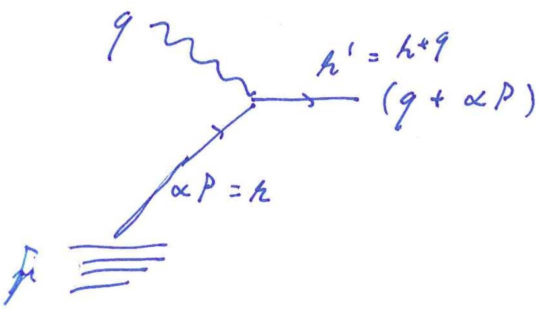
L, T: if $q^\mu = (q^0, \underline{0}, q^z)$, $\varepsilon_L \sim (\varepsilon^0, \underline{0}, \varepsilon^z)$, $\varepsilon_T \sim (0, \underline{\varepsilon}, 0)$

Flux factor: here $4m\nu$. Hand convention: $4m(\nu + \frac{q^2}{2m}) = 4m\nu(1-x)$
Gilman convention $4m\sqrt{\nu^2 + Q^2}$

Partons

IMF (Infinite Momentum Frame)

P^+ large, parton αP^+ → hadron consists of partons that, at high energy, carry a fraction α of the large energy and momentum.



If the ~~quark~~ parton is "quasi-free" it is kicked out of the proton as an on-shell particle

$$0 = (q+\alpha P)^2 = -Q^2 + 2\alpha P\cdot q + \underbrace{\alpha^2 m_N^2}_{\approx 0}$$

$$\alpha = \frac{Q^2}{2P\cdot q} = x_{Bj}$$

So x was defined as a purely kinematical variable. In the parton model x has a physical interpretation as the momentum fraction of the parton.

Assuming that the only partons that the photon couples to are free, massless quarks, one can derive

Callan - Gross (free quarks):

$$F_2 = 2x F_1 \quad \Rightarrow \quad F_L = 0$$

$$\Rightarrow \ \dots \quad W_{\mu\nu} \sim k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k'$$

→ like $L_{\mu\nu}$

I.e. if CG is satisfied, quarks are free, no gluons. Consequently

In QCD: $F_L \sim \alpha_s \, g(x, Q^2)$

I.e. a nonzero F_L arises only from loop/α_s corrections to scattering off free quarks. Conversely: measuring F_L directly measures gluons. Effect of gluons on F_2/F_T is higher order effect.

Dipole picture

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The parton picture is natural in the IMF. In the TRF the physical interpretation of DIS is very different.

$$p^\mu = \overset{0\ \ 12\ \ 3}{(m, \mathbf{0}, 0)} = \overset{+\ \ \ \ -\ \ \ \ \perp}{\left(\tfrac{1}{\sqrt2}m, \tfrac{1}{\sqrt2}m, \mathbf{0}\right)}$$

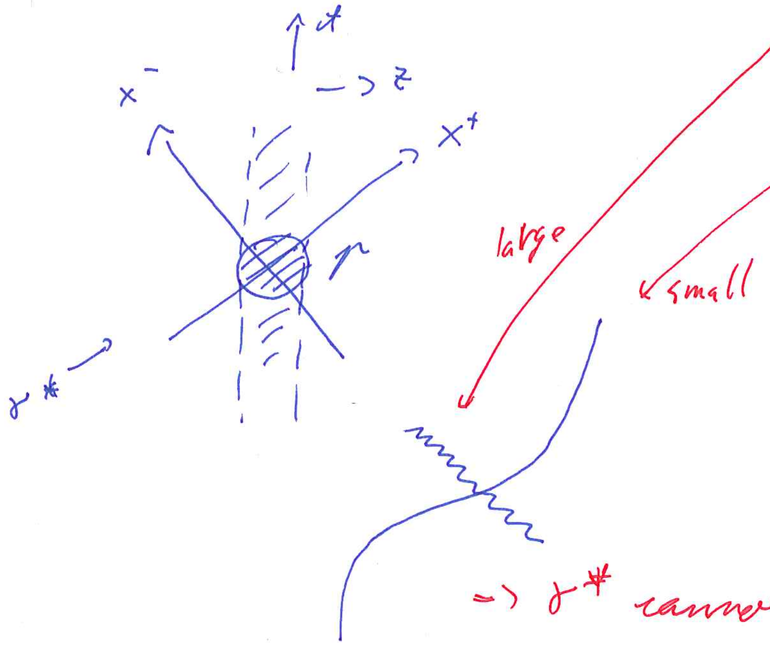
$$q^\mu = (\nu, \mathbf{0}, \sqrt{\nu^2+Q^2}) = \left(q^+, -\frac{Q^2}{2q^+}, \mathbf{0}\right)$$

Note: z-axis is that of γ^*, not of e^-

$$q^+ \approx \sqrt2\,\nu \text{ large}, \quad q^+ = \frac{Q^2}{\sqrt2\, m x}$$

$$q^- = -\frac{mx}{\sqrt2}$$

γ^* wave function: $e^{iq\cdot x} = e^{iq^+x^- + iq^-x^+}$



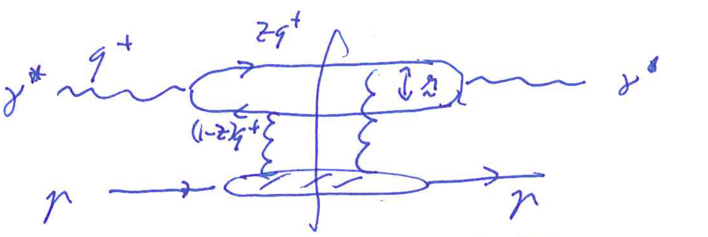
Think Heisenberg, or optics:
A wave like this can "see" degrees of freedom that are
- localized in x^-
- large in x^+

$$\Delta x^+ \sim \frac{1}{q^-} \sim \frac{1}{mx} \gg R_p$$

$\Rightarrow$ γ^* cannot "see" partons localized in p.

The scattering is instantaneous in x^+ compared to natural timescales of γ^*. We should think, instead of partons, of components of the wavefunction of the γ^* that exist before scattering, and then scatter eikonally off the target.

~~At~~ At small x the dominant one is a $q\bar q$ dipole



sea quarks dominate at small x

At leading order: $A \sim \alpha_s\, x g(x, Q^2)$

$$\sigma^{\gamma^* p}_{T,L} = \int d^2\mathbf{r} \int dz \left|\Psi^{T,L}_{\gamma^* q\bar q}(\mathbf{r}, z, Q^2)\right|^2 2\,\mathrm{Im}\,A_{q\bar q p}(\mathbf{r}, x)$$

Can be computed from the diagrams above (tedious). Instead: calculate as component of light cone wavefunction of γ^*.