

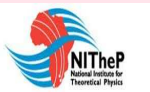
The Status of the Hadron Resonance Gas Model and its Extension in Thermodynamic Quantities

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Mod.Phys.Lett.A26:1197-1209, 2011

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Aim of the study and outline

Aim: To present the influence of the Hagedorn spectrum on the hadronic yields, find the thermodynamic parameters and explain the basic concepts.

Outline

Motivation: The idea of Hagedorn spectrum

Introduction: The spatial evolution of heavy ion collision

The **H**adron **R**esonance **G**as **M**odel (**HRGM**) and its **E**xtension (**EHRGM**)

→ State the expressions for number, energy and entropy density and speed of sound for both models

Show the results of **HRGM** and **EHRGM**

→ Discuss the results for particular thermodynamic quantities

Summary and Conclusion

Motivation: The Hagedorn spectrum

In 1965 Hagedorn¹ postulated that for large masses m the spectrum of hadrons grows exponentially, $\rho_H(m) \sim \exp(m/T_H)$.

The hypothesis was based on the observation increase of energy in collisions no longer raises the temperature of the formed fireball, but results in more and more particles being produced.

There exists uncertainty as to the value of the Hagedorn temperature, T_H which have two origins:

- Sparse information about hadronic resonances above 3 GeV and,
- The analytical form of the Hagedorn spectrum.

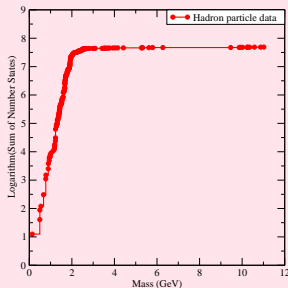
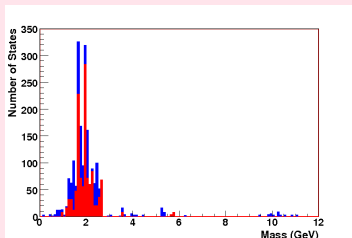
Recently, Hagedorn spectrum is rewritten as

$$\rho_H(m) = \frac{c}{(m^2 + m_0^2)^{5/4}} \exp\left(\frac{m}{T_H}\right). \quad (1)$$

This model uses $m_0 = 0.5$ GeV, it is adopted from *S. Chatterjee et. al. Phys.Rev.C81:044907.*

1) R. Hagedorn, Nuovo Cim. Suppl., 3:147186, 1965.

The number of states for particle data table arranged in terms of their masses with some hadron gas resonances.



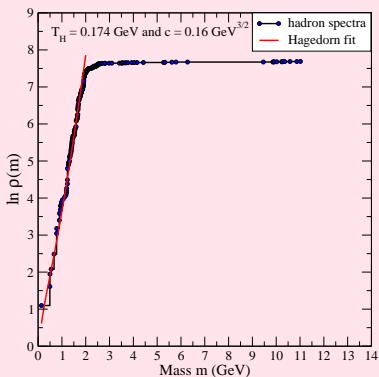
The spectrum of hadrons up to higher masses is given by,

$$\rho(m) = \sum_i g_i \delta(m - m_i), \quad (2)$$

where g_i is the degeneracy factor for hadron state i .

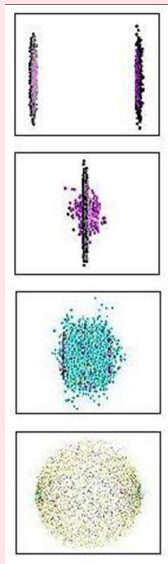
The best fit for c and T_H parameters

The result of the parameters using Eq. 1 for the Hagedorn spectrum state given below:



$T_H = 0.174 \pm 0.011$ GeV and $c = 0.16 \pm 0.02$ GeV^{3/2} (*J. Cleymans and D. Worku Mod.Phys.Lett.A26:11971209, 2011*).

Introduction: The Spatial evolution of a heavy ion collision



- Lorentz-contracted heavy ions approaching ...
 - Relativistic speeds cause the ions to appear disk-like
- Ions interpenetrate, individual particles scatter
- Deconfined quarks and gluons, plasma forms
 - Very short-lived, so not observable
- Formation of hadrons
 - Observable particles, analysis of these reveals information about QGP

The Hadron Resonance Gas Model (**HRGM**)

The thermodynamic properties of the **HRGM** can be determined from the partition function

$$\ln Z(V, T, \mu) = \int dm [\rho_M(m) \ln Z_b(m, V, T, \mu) + \rho_B(m) \ln Z_f(m, V, T, \mu)], \quad (3)$$

where Z_b and Z_f are the partition functions for an ideal gas of bosons and fermions respectively with mass m , $\rho_M(m)$ and $\rho_B(m)$ are the spectral density of mesons and baryons.

The **HRGM** model takes the observed spectrum of hadrons up to some cutoff mass 2 GeV.

Extension of Hadron Resonance Gas Model (**EHRGM**)

In order to explore the stability of predictions from **HRGM**, we develop an **EHRGM** in which:

$$\rho(m) = \sum_i^{m_i \leq 2\text{GeV}} g_i \delta(m - m_i) + \rho_H(m), \quad (4)$$

where ρ_H is the Hagedorn spectrum which is given in Eq. 1.

Expression of thermodynamic quantities in EHRGM

Using HRGM, one can compute the thermodynamic quantities. We consider Boltzmann distribution for our calculation

$$\ln Z = \sum_{i=1}^{m_i \leq 2\text{GeV}} \frac{g_i V T m_i^2}{2\pi^2} K_2 \left(\frac{m_i}{T} \right) \exp \left(\frac{\mu_i}{T} \right), \quad (5)$$

where K_2 is modified Bessel function and $\mu_i = S_i \mu_S + B_i \mu_B + Q_i \mu_Q$.

The particle number density, n using EHRGM is written as

$$n(T, \mu_B, \mu_S, \mu_Q) = \frac{T}{2\pi^2} \sum_{i=1}^{m_i \leq 2\text{GeV}} \exp \left(\frac{\mu_i}{T} \right) \left[g_i m_i^2 K_2 \left(\frac{m_i}{T} \right) + c \int_{m=2\text{GeV}}^{\infty} \frac{m^2}{(m^2 + m_0^2)^{5/4}} \exp \left(\frac{m}{T_H} \right) K_2 \left(\frac{m}{T} \right) dm \right], (6)$$

Very often, it is considered to an isospin symmetric system, where μ_Q is zero.

The energy density

The energy density using EHRGM is given by

$$\varepsilon(T, \mu_B, \mu_S, \mu_Q) = \sum_{i=1}^{m_i \leq 2\text{GeV}} \left[\frac{T^2}{2\pi^2} \exp\left(\frac{\mu_i}{T}\right) \left(g_i m_i^2 \left[3K_2\left(\frac{m_i}{T}\right) + \frac{m_i}{T} K_1\left(\frac{m_i}{T}\right) \right] + A_1 \right) \right], \quad (7)$$

where

$$A_1 = c \int_m^\infty \frac{m^2}{(m^2 + m_0^2)^{5/4}} \exp\left(\frac{m}{T_H}\right) \left[3K_2\left(\frac{m}{T}\right) + \frac{m}{T} K_1\left(\frac{m}{T}\right) \right] dm.$$

The entropy density, s can be computed using the following relation

$$s(T, \mu_B, \mu_S, \mu_Q) = \frac{\varepsilon + p - n_S \mu_S - n_B \mu_B - n_Q \mu_Q}{T}, \quad (8)$$

where p is pressure and n_S , n_B , n_Q are the net number densities for strange, baryonic and electric charge particles respectively

$$n_{S(B)} \equiv \frac{T}{V} \frac{\partial \ln Z}{\partial \mu_{S(B)}}, \quad (9)$$

The speed of sound

In hydrodynamic models, the speed of sound plays an important role in the evolution of a system and is an ingredient in the understanding of the effects of a phase transition.

In our extension we take the L.D. Landau condition, where s/n is fixed in order to define,

$$C_s^2(T, \mu_B, \mu_S) = \left(\frac{\partial P}{\partial \varepsilon} \right)_{s/n}, \quad (10)$$

hence C_s^2 can be rewritten as

$$C_s^2(T, \mu_B, \mu_S) = \frac{\left(\frac{\partial P}{\partial T} \right) + \left(\frac{\partial P}{\partial \mu_S} \right) \left(\frac{d\mu_S}{dT} \right) + \left(\frac{\partial P}{\partial \mu_B} \right) \left(\frac{d\mu_B}{dT} \right)}{\left(\frac{\partial \varepsilon}{\partial T} \right) + \left(\frac{\partial \varepsilon}{\partial \mu_S} \right) \left(\frac{d\mu_S}{dT} \right) + \left(\frac{\partial \varepsilon}{\partial \mu_B} \right) \left(\frac{d\mu_B}{dT} \right)}, \quad (11)$$

The first condition comes from keeping the ratio (s/n) constant.

$$d \left(\frac{s}{n} \right) = 0 \quad \rightarrow \quad nds = sdn. \quad (12)$$

Rearranging the above equation in order to write $\frac{d\mu_B}{dT}$ in terms of $\frac{d\mu_S}{dT}$ one obtains

$$\frac{d\mu_B}{dT} = -\frac{1}{B} \left[A + C \frac{d\mu_S}{dT} \right]. \quad (13)$$

The second condition comes from overall strangeness neutrality, which is

$$n_S = n_{\bar{S}} \quad \rightarrow \quad d(n_S) = d(n_{\bar{S}}) \quad (14)$$

where n_S and $n_{\bar{S}}$ are the strange and antistrange particle densities. The final expression for condition two become

$$\frac{d\mu_B}{dT} = -\frac{1}{E} \left[D + F \frac{d\mu_S}{dT} \right]. \quad (15)$$

Finally, by equating Eq. (13) and Eq. (15) we find

$$\frac{d\mu_S}{dT} = \frac{A * E - B * D}{B * G - C * E} \quad \text{and} \quad \frac{d\mu_B}{dT} = \frac{C * D - A * G}{B * G - C * E}, \quad (16)$$

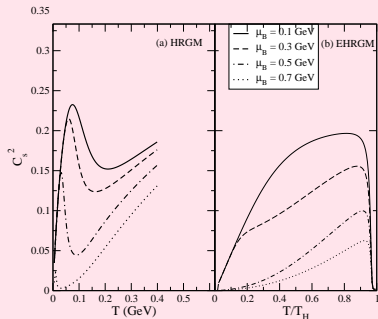
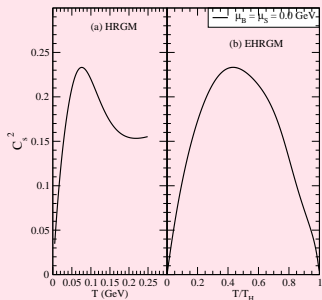
where

$$A = n \frac{\partial s}{\partial T} - s \frac{\partial n}{\partial T}, \quad B = n \frac{\partial s}{\partial \mu_B} - s \frac{\partial n}{\partial \mu_B}, \quad C = n \frac{\partial s}{\partial \mu_S} - s \frac{\partial n}{\partial \mu_S}, \quad D = \frac{\partial L}{\partial T} - \frac{\partial R}{\partial T},$$

$$E = \frac{\partial L}{\partial \mu_B} - \frac{\partial R}{\partial \mu_B}, \quad F = \frac{\partial L}{\partial \mu_S} - \frac{\partial R}{\partial \mu_S}.$$

We define $L = n_S^B + n_S^M$ and $R = n_S^{\bar{B}} + n_S^{\bar{M}}$, which represents the strangeness and antistrangeness density for baryons and mesons.

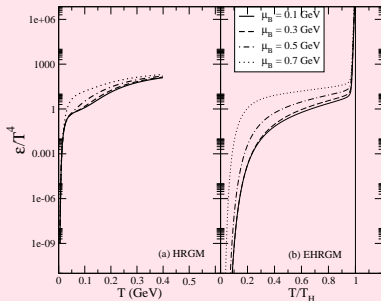
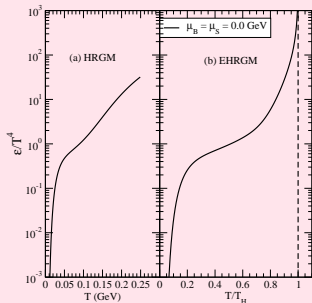
HRGM and EHRGM: The speed of sound versus temperature



→ The value of the squared speed of sound, C_s^2 remains well below the ideal-gas limit for massless particles $C_s^2 = 1/3$.

→ It showed that sharp dip of C_s^2 in the critical region and can be considered as an evidence for the phase transition in the system.

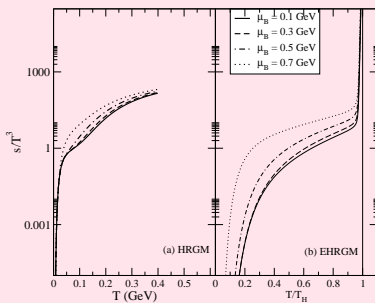
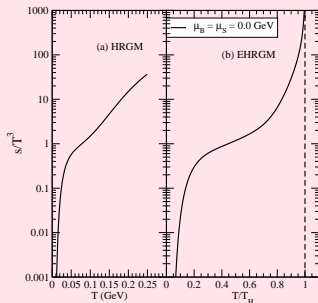
HRGM and EHRGM: Energy density versus temperature



→ The result using **EHRGM** as a function of the temperature show a sudden change and start to increase rapidly at a particular temperature, (i.e. T_H).

→ In a similar way, in *Phys.Rev.C81:044907* and *Eur.Phys.J.C66:207-213* presented with few hadron resonance gases and the shape of graphs shown are similar to our results.

HRGM and EHRGM: The entropy versus temperature



We observe that around $T/T_H \simeq 0.8$, resonances come significantly into play, so that ε and s begin to increase until reach to T_H in these case the resonances provide the dominant part for the those thermodynamic quantities.

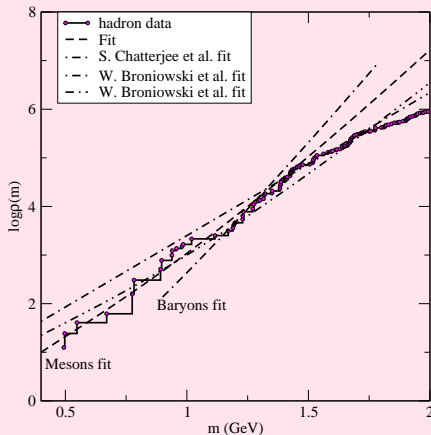
- We presented the **HRGM** to investigate the thermodynamic properties of hadrons, we further extended to the **EHRGM** by involving the Hagedorn spectrum.
- The Hagedorn temperature is determined from the number of hadronic resonances lead to a stable result which is consistent with the critical and the chemical freeze-out temperatures at zero chemical potential.
- We calculated C_s^2 and relevant thermodynamic quantities for a wide range of baryon chemical potentials following the chemical freeze-out curve.

- The **EHRGM** results show unique behavior at the critical point, T_H .
- The thermodynamic quantities obtained using **EHRGM** start rising rapidly at a temperature of about T_H .
- For $T \leq T_H$ hadronic resonances are indeed the most important degrees of freedom in the confined phase.
- The result of C_s^2 can be considered as a sensitive indicator of critical behavior in strongly interacting matter.

Acknowledgements

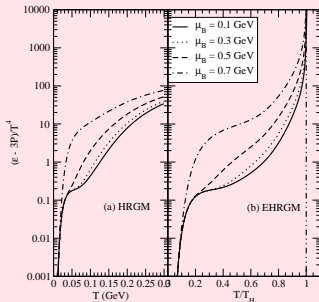
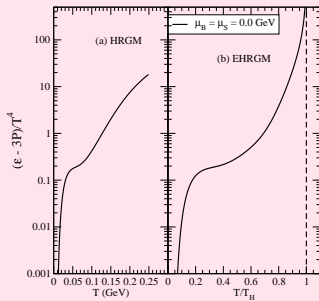
- 1 National Institute for **Theoretical Physics (NiTheP)**
- 2 Centre for **Theoretical and Mathematical Physics (CTMP)**,
Department of Physics, UCT

Backup Slides: The hadron spectrum fits of different authors



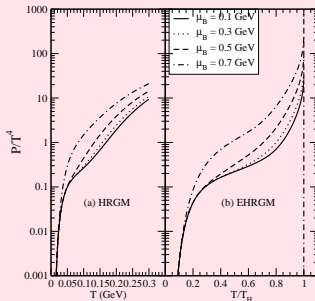
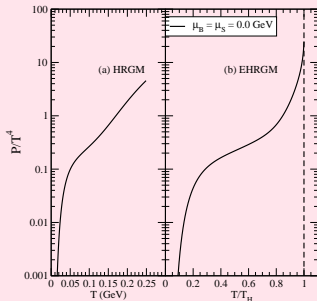
Cumulative number of hadronic resonances as a function of m . The hadron data are included up to 2.0 GeV, including baryons and mesons from the fits of different authors, our fit is also shown here (dashed line).

The interaction measure $\frac{(\varepsilon - 3P)}{T^4}$ at μ



(a) The interaction measure, $(\varepsilon - 3P)/T^4$ in units of T^4 calculated using the **HRGM** as a function of the temperature T at $\mu_B = \mu_S = 0$ GeV. (b) The interaction measure, $(\varepsilon - 3P)/T^4$ in units of T^4 for **EHRGM** as a function of the temperature scaled by the Hagedorn temperature T_H .

HRGM and EHRGM: The pressure versus T at various μ



(a) The pressure, P , in units of T^4 calculated using the **HRGM** as a function of the temperature T at various μ_B . (b) The pressure in units of T^4 for **EHRGM** as a function of the temperature scaled by the Hagedorn temperature T_H .