

Assessment of the South African sardine resource using data from 1984-2010: results at the posterior mode for a single stock hypothesis

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Abstract

The operating model (OM) for the South African sardine resource has been updated from that used to develop OMP-08 given four more years of data and a revised time series of commercial catches. A Hockey Stick stock recruitment relationship, and the same median juvenile and adult natural mortality rates as in previous assessments are used. When considering the Beverton Holt, Ricker and Hockey stick stock recruitment relationships, AIC model selection criterion slightly favours the Hockey stick relationship over the others. Two base case hypotheses are chosen: one estimates random effects about juvenile and adult natural mortality over time while the other assumes time-invariant annual natural mortality. The recruitment residual standard deviation and autocorrelation for this updated OM are similar to those used in the previous OM. The resource abundance is below the historic average, with a model-estimated 1+ biomass of 1.0 million tons in November 2010, following six years of below average recruitment in the past seven years.

Introduction

The operating model for the South African sardine resource has been updated from the last assessment (Cunningham and Butterworth 2007) to take account of new data collected between 2007 and 2010. A number of key changes to the model and data used have been made.

- In 2007 no ageing data were used in the assessment (Anon. 2007, Cunningham and Butterworth 2007). The November survey ageing data have now been updated and are available for further years and some ageing of commercial catches has taken place (see de Moor *et al.* 2011). The model is fit to commercial and survey proportion-at-length data for quarters / years for which ageing information is not available.
- Commercial catches from the primarily juvenile anchovy bycatch fishery have been considered separately to those from the primarily adult directed and redeye bycatch fisheries. Juvenile catches have been calculated using cut-off lengths which vary by year and month (de Moor *et al.* 2011), so that selectivity need not be estimated for age 0. Commercial selectivities for ages 2 to 5+ are estimated within the model.
- The manner in which bias on the November 1+ biomass and May recruit surveys is modelled has been updated. Previously a single parameter was estimated for each survey and an informative prior distribution was given for both parameters. Bias is now estimated separately first for the hydroacoustic survey, using the same prior as had formally been developed for the November survey. A second bias parameter is estimated for the proportion of the stock abundance covered during the recruit survey relative to the November survey. Finally, for the two-stock hypothesis, the proportion of the recruit abundance covered from the “east” stock in comparison to the “west” stock is also estimated. The assumption is made that full coverage of the sardine abundance is obtained during the November survey.

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- The model has been extended to include random effects, which are autocorrelated, for juvenile and adult natural mortality.

Over the past three years a hypothesis of two sardine stocks has been explored. Although in initial testing a two discrete stock hypothesis was considered to be compatible with the data (de Moor and Butterworth 2009), de Moor and Butterworth (2011c) have shown more recently that such an hypothesis is implausible on the grounds that it would imply only about one fifth of the recruits of the “east” stock is surveyed during the annual May hydroacoustic survey. A two-mixing-stocks hypothesis with movement from the “west” stock to the “east” stock is also to be used as an operating model in simulation testing OMP-13, as discussed in a separate document to this workshop.

Initial results for the updated operating model (assessment) of the South African sardine resource were presented by de Moor and Butterworth (2011a,b) for a single stock and de Moor and Butterworth (2011c) for a two stock hypothesis. This document presents the updated base case operating models for a single sardine stock hypothesis, assuming a Hockey Stick stock recruitment relationship. One base case model estimates random effects about juvenile and adult natural mortality over time while the other assumes constant (time-invariant) adult natural mortality. A few robustness tests are also developed. Results are given at the posterior mode only. A separate document will show the full posterior distributions. These operating models are to be used in developing and simulation testing OMP-13.

Population Dynamics Model

The generalised operating model for the South African sardine resource, applying to both the single and two stock hypotheses, is detailed in Appendix A. The data used in this assessment are listed in de Moor *et al.* (2011). A glossary of terms used in this model is provided in Appendix C.

Informative prior distributions were constructed for the multiplicative bias in the hydroacoustic survey as well as the additional variance associated with these surveys (see Appendix B). The prior distributions for the growth curve parameters were informed by the von Bertalanffy growth curve estimated by available ageing data (Durholtz and Mtengwane pers. commn). The priors for the remaining estimable parameters were chosen to be relatively uninformative (see Appendices A and B for details), although the bounds on the survey selectivities and upper bound on commercial selectivities were chosen to constrain some parameter estimates (see below).

Initial results estimated similar autocorrelation coefficients for annual residuals of adult and juvenile natural mortality. As a result a single autocorrelation coefficient is now estimated for both adult and juvenile natural mortality.

Multiplicative bias associated with the November survey is taken to be that associated with the hydroacoustic survey. The assumption is made that full coverage of the sardine abundance is obtained during the November survey. Multiplicative bias associated with the May recruit survey is taken to be that associated with the hydroacoustic survey multiplied by that associated with the proportion of the recruit abundance covered by the recruit survey in comparison to the November survey. Given that not all of the recruitment is assumed to be available to the survey by mid-May, this latter ratio is constrained by a maximum of 1. In the two-stock hypothesis, the multiplicative bias associated with the May recruit survey of the “east” stock is taken to be that associated with the May recruit survey of the “west” stock (as described in the preceding sentences) multiplied by the bias associated with the proportion of the “east” stock recruit abundance covered by the recruit survey compared to proportion of the “west” stock recruit abundance covered in the same survey. Further details are provided in Appendices A and B.

Initial results showed that the likelihood profile for the standard deviation in the annual residuals about adult natural mortality, σ_{ad} , is monotonic with a better (smaller) objective function value obtained for smaller σ_{ad} values. This was primarily a result of the large contribution to the likelihood from the log-prior on the residuals about adult natural mortality. In contrast, the fit to the hydroacoustic survey estimates of November 1+ biomass improve for higher σ_{ad} values. Due to the need to fit the latter time series well, the range for σ_{ad} has been constrained to [0.2, 0.5] as for anchovy. The same prior distribution was chosen to apply to juvenile natural mortality as well.

Stock recruitment relationship

The following alternative stock recruitment relationships have been considered (Table 1):

- S_{HS} – hockey stick stock-recruitment curve, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity
- S_{BH} – Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity
- S_R – Ricker stock-recruitment curve, with uniform priors on steepness and carrying capacity

In all three of the alternatives above the standard deviation about the curve is estimated assuming a difference between peak (2000-2004) and non-peak years.

Natural mortality

A number of combinations of juvenile and median adult natural mortality values are tested, covering the range 0.4 to 1.2 year⁻¹, for the case where a Hockey Stick stock recruitment relationship is assumed. For realism, only combinations with $\bar{M}_j^S \geq \bar{M}_{ad}^S$ are tested.

Constant adult natural mortality

As projecting forward and simulation testing a new OMP using an operating model including random effects about juvenile and adult natural mortality is novel, the previous approach of a constant natural mortality with time is also tested:

S_{cstM} – constant annual adult natural mortality, i.e. no random effects model

ResultsNatural mortality

Table 2 lists the various contributions to the objective function at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. Given the choice of prior distributions the ratio k_r^S / k_N^S is by definition less than 1. Combinations of natural mortality which result in $k_r^S / k_N^S < 0.5$ are considered less plausible.

There is little change in the posterior distribution as \bar{M}_j^S is changed for a given \bar{M}_{ad}^S (<2 likelihood points, improving as \bar{M}_j^S increases). Given \bar{M}_j^S , the posterior distribution indicated an improved fit to the data for increasing \bar{M}_{ad}^S , with a slight deviation from this ‘rule’ for $\bar{M}_j^S = 2.1$. The lowest log-joint posterior mode was obtained for $\bar{M}_{ad}^S = 1.0$, with $\bar{M}_j^S = 1.0$ or $\bar{M}_j^S = 1.2$, while $\bar{M}_{ad}^S = 0.8$ with $\bar{M}_j^S = 1.0$ or $\bar{M}_j^S = 1.2$ gave a similar result. For consistency between assessments, the base case hypothesis assumes $\bar{M}_j^S = 1.0$ and $\bar{M}_{ad}^S = 0.8$.

Stock recruitment relationship

Table 3 lists the various contributions to the objective function at the posterior mode for the alternative stock-recruitment relationships considered. From a frequentist viewpoint, this is strictly a random effects model as regards the annual variations in adult natural mortality and recruitment. However, the REML process to get unbiased estimates of the variances for these two effects has not been implemented as the key operating model(s) for use in developing OMP-13 will be Bayesian. Thus the use of AIC to compare between alternative stock-recruitment relationships is approximate. AIC suggests that the preferred stock-recruitment relationship is the Hockey stick. The alternative stock recruitment relationships are plotted in Figures 1 and 2. A much higher standard deviation about the curve is estimated for “peak” (2000-2004) years in comparison to non-peak years (Table 4).

Base case (S_{HS}) results at posterior mode

The estimated parameter values and other key outputs are listed in Table 4 together with the individual contributions to the objective function at the posterior mode.

The population model fits to the time series of abundance estimates of November 1+ biomass and May recruitment are shown in Figures 3 and 4, respectively. In both cases the fits to the survey data are reasonably good. The model does not predict as high a peak in 1+ biomass as is shown by the point estimates from the survey results, though the predicted 1+ biomass is well within the 95% CI for the biomass estimated by the survey. The model under-predicts recruitment in May 2010 as it is unable to reconcile the conflicting data of an above average recruitment estimate in May 2010, with almost no increase in the November 1+ biomass estimate from 2009 to 2010.

Figures 5 and 6 show the fits to the time series of survey and commercial proportion-at-age data, respectively. The model estimated survey and commercial selectivities at age are plotted in Figures 7 and 8, respectively. It is clear that the bounds of the uniform prior distribution are constraining the survey selectivities at ages 1, 2, 3 and 5. However, given the survey design, survey selectivity should not differ by age, and thus these bounds have been retained. In addition the commercial selectivity at age 5+ is constrained by a maximum of 2 prior to 2007. The model under-predicts proportions-at-age 5. However, allowing yet higher selectivities at age 5+ seems unrealistic. A negative trend in the residuals for age 2 of the survey data is also evident. The below-par fit to these data is considered acceptable in the light of the relatively low confidence placed in the ageing data.

Figure 9 shows the residuals from the fit to the survey proportion-at-length data while Figure 10 shows the residuals from the fit to the quarterly commercial proportion-at-length data. Trends in these residuals are evident, but again because of the relatively low confidence in the ageing data, attempts to introduce further features into the model to reduce these patterns was considered unjustified.

The model estimated annual juvenile and adult natural mortality is plotted in Figure 11 together with the estimated residuals. Some autocorrelation between these residuals is estimated by the model ($\rho = 0.54$), with the standard deviation in these residuals on the lower bound of the uniform prior distribution (Table 4). The historic annual harvest rates are plotted in Figure 12.

One new aspect of this operating model, compared to historic models, is that it has incorporated a random effects model for juvenile and adult natural mortality. At the posterior mode of S_{HS} , juvenile natural mortality is estimated to vary between 0.96 and 1.11 and adult natural mortality is estimated to vary between 0.74 and 0.98. Although this variability is not as large as that estimated for anchovy, 7 out of the past 8 years are estimated to have had above average natural mortality. The increase in natural mortality after the peak in abundance observed in the early 2000s implies that loss of sardine to predation exceeded 3 million tons for four consecutive years (Table 5). The autocorrelation in the residuals about natural mortality will affect future projections.

The alternative base case (S_{cstM})

The fit of the model predictions from S_{cstM} to the data are also shown in Figures 3 to 10, with historic annual harvest rates plotted in Figure 12. The overall fit to the data is slightly worse than for S_{HS} (Table 4). The difference between the largest and smallest annual losses to predation is 3.8 million tons compared to 4.3 million tons under S_{HS} (Table 5).

Convergence

In many of the results presented in this document, a positive definite Hessian could not be obtained with ADMB, indicating convergence to the posterior mode has not yet been confirmed. For some cases, a positive definite Hessian could be obtained for a slightly inferior overall fit. Sometimes a positive definite Hessian could be obtained if some parameters were fixed at their estimated value (e.g., $(\lambda_{j,N/r}^s)^2$, $\tilde{S}_{j,a}$ and N_{1983}). However, this “solution” to the problem does not generalise readily as fixing these parameters does not guarantee a positive definite Hessian will be obtained for all alternatives.

Summary

This document has detailed the updated operating model for the South African sardine resource. Two base case hypotheses have been chosen, one assuming a random effects model for juvenile and adult natural mortality, S_{HS} , and one assuming constant natural mortality, S_{cstM} . A Hockey stick stock recruitment relationship is assumed for these base case hypotheses. Under these base case hypotheses at the joint posterior mode, the resource abundance is 0.952 to 1 million tons in November 2010. This is below the long-term average of 1.2 – 1.3 million tons, but above the 1991-1994 average of 0.58 - 0.61 million tons. Six out of the last seven years have resulted in below average recruitment. A wide range of robustness tests will need to be considered as well as retrospective runs for the base case hypotheses.

References

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Table 1. The alternative stock-recruitment relationships considered. The parameter h_j^S denotes the “steepness” of the stock-recruitment relationship of stock j , which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass K_j^S (shown in units of thousands of tons). For the hockey stick model, $X_j = \sum_{a=1}^4 \bar{w}_{j,a}^S e^{-M_j^S - (a-1)\bar{M}_{ad}^S} + \bar{w}_{j,5+} e^{-M_j^S - 3\bar{M}_{ad}^S} \frac{1}{1 - e^{-\bar{M}_{ad}^S}}$, where $\bar{w}_{j,a}^S$ is the average of $w_{j,y,a}^S$ as defined in Appendix A. For the hockey stick model, a_j^S denotes the maximum recruitment (in billions) and b_j^S denotes the spawner biomass below which the expectation for recruitment is reduced below the maximum.

Test	Stock recruitment relationship	$f(SSB_{y,N}^S) =$	Parameters
S _{BH}	Beverton Holt	$\frac{\alpha_j^S SSB_{j,y}^S}{\beta_j^S + SSB_{j,y}^S}$	$h_j^S \sim U(0.2, 1.5)$ $K_j^S \sim U(0, 10)$ $\alpha^S = \frac{4h^S}{5h^S - 1} \frac{K^S}{X}$ $\beta^S = \frac{K^S(1 - h^S)}{5h^S - 1}$
S _R	Ricker	$\alpha_j^S SSB_{j,y}^S e^{-\beta_j^S SSB_{j,y}^S}$	$h_j^S \sim U(0.2, 1.5)$ $K_j^S \sim U(0, 10)$ $\alpha^S = \frac{1}{X} \left(\frac{h^S}{0.2} \right)^{1/0.8}$ $\beta^S = \frac{\ln(h^S / 0.2)}{0.8K^S}$
S _{HS}	Hockey stick	$\begin{cases} a_j^S & , \text{if } SSB_{j,y}^S \geq b_j^S \\ \frac{a_j^S}{b_j^S} SSB_{j,y}^S & , \text{if } SSB_{j,y}^S < b_j^S \end{cases}$	$\ln(a^S) \sim U(0, 4.4)$ ¹ $\frac{b^S}{K^S} \sim U(0, 1)$ $K^S = a^S X^2$

¹ Given the lack of *a priori* information on the scale of a^S , a log-scale was used, with a maximum corresponding to about 10 million tons.

² For consistency, K relates throughout to corresponding MLEs. These will be less than the corresponding average pre-exploitation levels because of the lognormal distributions assumed for recruitment.

Table 2. The contributions to the objective function at the posterior mode for a range of combinations of juvenile, \bar{M}_j^S , and adult, \bar{M}_{ad}^S , natural mortality for models assuming the Hockey Stick stock recruitment relationship. The ratio of the multiplicative bias in the recruit survey to that in the November survey, k_r^S/k_N^S , is given for diagnostic purposes. Shaded cells represent unrealistic choices for this ratio.

\bar{M}_j^S	\bar{M}_{ad}^S	Posterior	-ln(Likelihood)						-ln(Prior)				k_N^S	k_r^S	k_r^S/k_N^S
			Nov	Rec	Survey Prop-at-age	Com Prop-at-age	Com Prop-at-length	Survey Prop-at-length	k_{ac}^S	ϵ_y^S	$L_{j,\infty}$	$\epsilon_y^{j/ad}$			
0.6	0.4	87.94	18.63	23.94	1.41	1.29	60.72	2.39	-1.61	11.30	1.62	-31.75	0.74	0.74	1.00
0.6	0.6	84.32	17.71	22.22	1.59	1.61	60.78	2.32	-1.64	11.28	1.62	-33.16	0.72	0.72	1.00
0.8	0.4	87.22	18.55	23.74	1.39	1.24	60.56	2.45	-1.61	11.24	1.61	-31.95	0.73	0.73	1.00
0.8	0.6	84.16	17.73	22.14	1.58	1.58	60.69	2.34	-1.65	11.30	1.62	-33.17	0.72	0.72	1.00
0.8	0.8	83.50	17.28	21.81	1.96	1.99	60.71	2.17	-1.65	10.97	1.66	-33.40	0.72	0.56	0.78
1	0.4	87.08	18.71	23.83	1.37	1.22	60.54	2.49	-1.63	11.54	1.61	-32.61	0.73	0.73	1.00
1	0.6	84.09	17.71	22.22	1.58	1.57	60.63	2.34	-1.65	11.19	1.62	-33.13	0.72	0.67	0.94
1	0.8	83.48	17.26	21.89	1.96	1.97	60.70	2.18	-1.65	10.86	1.67	-33.35	0.72	0.52	0.73
1	1	83.35	17.19	21.67	2.26	2.33	60.73	2.07	-1.65	10.40	1.72	-33.38	0.71	0.43	0.60
1.2	0.4	86.12	18.44	23.39	1.38	1.16	60.40	2.52	-1.62	11.06	1.61	-32.23	0.73	0.73	1.00
1.2	0.6	84.01	17.68	22.30	1.58	1.56	60.60	2.34	-1.65	11.06	1.62	-33.08	0.72	0.63	0.88
1.2	0.8	83.45	17.23	21.95	1.95	1.97	60.69	2.18	-1.65	10.74	1.67	-33.28	0.72	0.49	0.68
1.2	1	83.34	17.18	21.73	2.26	2.33	60.72	2.07	-1.65	10.30	1.72	-33.31	0.71	0.40	0.56
1.2	1.2	83.64	17.22	21.62	2.51	2.67	60.88	2.02	-1.65	9.83	1.77	-33.22	0.71	0.33	0.47

Table 3. The contributions to the objective function at the posterior mode for alternative stock recruitment relationships.

	S_{HS}	S_{BH}	S_R
Objective function	83.48	84.49	83.99
$-\ln(L_{Nov})$	17.26	17.07	16.89
$-\ln(L_{Rec})$	21.89	22.55	22.50
$-\ln(L_{SurPropA})$	1.96	1.93	1.94
$-\ln(L_{ComPropA})$	1.97	1.97	1.96
$-\ln(L_{ComPropLmin})$	7.16	7.15	7.15
$-\ln(L_{ComPropL})$	53.54	53.60	53.66
$-\ln(L_{SurPropLmin})$	0.25	0.25	0.25
$-\ln(L_{SurPropL})$	1.93	1.89	1.89
$-\ln(\text{Prior } k_{\text{acoustic}})$	-1.65	-1.65	-1.65
$-\ln(\text{Prior rec residuals})$	10.86	11.47	11.10
$-\ln(\text{Prior Linf})$	1.67	1.67	1.67
$-\ln(\text{Prior Mad residuals})$	-33.35	-33.41	-33.37
# parameters	102	102	102
AIC	537.90	541.97	539.95
h^A	0.63	0.49	0.59
K^A	1904	3590	2062
a^A	41.4		
b^A	608		

Table 4. Key model parameter values and model outputs estimated at the joint posterior mode for SHS and ScstM. Values fixed on input are given in **bold**. Numbers are reported in billions and biomass in thousands of tonnes.

Parameter	S_{HS}	S_{cstM}	Parameter	S_{HS}	S_{cstM}
Objective function	83.48	119.91	$S_{q,1}$,	0.05 from '84-06, then 0.15 from '07-10	0.05 from '84-06, then 0.14 from '07-10
$-\ln L^{Nov}$	17.26	18.41	$S_{q,2}$,	0.52	0.53
$-\ln L^{rec}$	21.89	22.26	S_3	0.86	0.82
$-\ln L^{sur propa}$	1.96	1.92	S_4	1.00	1.00
$-\ln L^{com propa}$	1.97	2.01	S_{5+}	2.00 from '84-06, then 1.38 from '07-10	2.00 from '84-06, then 1.02 from '07-10
$-\ln L^{coml proph\min}$	7.16	7.16	N_{1983}^S	3.79	3.80
$-\ln L^{coml propl}$	53.54	53.58	\bar{B}_{Nov}^S ³	613.5	575.8
$-\ln L^{sur proph\min}$	0.25	0.25	K_{normal}^S	2072	1904
$-\ln L^{surl propl}$	1.93	1.95	a^S	45.0	41.4
$-\ln(priors)$	-22.47	12.39	b^S	660	608
\bar{M}_j^S	1.0	1.0	σ_r^S	0.40	0.40
\bar{M}_{ad}^S	0.8	0.8	$\sigma_{r,peak}^S$	1.06	1.13
$k_{j,N}^S = k_{ac}^S$	0.72	0.73	η_{2009}^S	0.75	0.63
k_{cov}^S	0.73	0.76	s_{cor}^S	0.41	0.46
$k_{j,r}^S$	0.52	0.56	L_∞	20.4	20.4
$k_{j,r}^S / k_{j,N}^S$	0.73	0.76	κ	0.56	0.56
$S_{j,1}^{survey}$	1.1	1.1	t_0	-1.7	-1.7
$S_{j,2}^{survey}$	0.9	0.9	\mathcal{G}_1	0.25	0.25
$S_{j,3}^{survey}$	0.9	0.9	\mathcal{G}_2	0.10	0.10
$S_{j,4}^{survey}$	1.0	1.0	\mathcal{G}_3	0.06	0.06
$S_{j,5+}^{survey}$	1.1	1.1	\mathcal{G}_4	0.05	0.05
$(\lambda_N^S)^2$	0.0	0.0	\mathcal{G}_{5+}	0.25	0.26
$(\lambda_r^S)^2$	0.0	0.0			
σ_j	0.2	N/A			
σ_{ad}	0.2	N/A			
p	0.54	N/A			

³ OMP-04 and OMP-08 were developed using Risk defined as “the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years”.

Table 5. The annual estimated sardine loss to predation (in '000t), $P_{j,y}^S$ in Appendix D, compared to the annual sardine catch (in '000t).

Year	Catch	S_{HS}		S_{cstM}	
		Loss to M	Catch: Loss to M	Loss to M	Catch: Loss to M
1984	27.2	161.5	0.17	162.0	0.17
1985	30.7	190.8	0.16	191.3	0.16
1986	30.6	211.7	0.14	211.8	0.14
1987	33.5	271.6	0.12	272.5	0.12
1988	36.3	251.6	0.14	260.0	0.14
1989	34.7	302.7	0.11	319.8	0.11
1990	57.4	354.5	0.16	376.5	0.15
1991	53.0	409.7	0.13	426.9	0.12
1992	55.1	492.6	0.11	485.3	0.11
1993	51.1	678.0	0.08	652.5	0.08
1994	94.9	758.6	0.13	717.7	0.13
1995	121.2	1175.9	0.10	1128.0	0.11
1996	107.9	1032.2	0.10	988.7	0.11
1997	119.4	1583.0	0.08	1559.2	0.08
1998	133.3	1703.6	0.08	1640.0	0.08
1999	131.9	1802.5	0.07	1714.8	0.08
2000	135.2	1996.3	0.07	1883.5	0.07
2001	191.5	3096.7	0.06	2969.8	0.06
2002	260.9	4332.0	0.06	3969.1	0.07
2003	290.0	4426.8	0.07	3714.5	0.08
2004	373.8	3070.0	0.12	2429.0	0.15
2005	246.7	1601.0	0.15	1262.4	0.20
2006	217.3	1290.9	0.17	1043.4	0.21
2007	139.5	861.8	0.16	798.7	0.17
2008	90.9	656.9	0.14	680.7	0.13
2009	94.3	732.8	0.13	793.6	0.12
2010	112.4	935.2	0.12	986.0	0.11

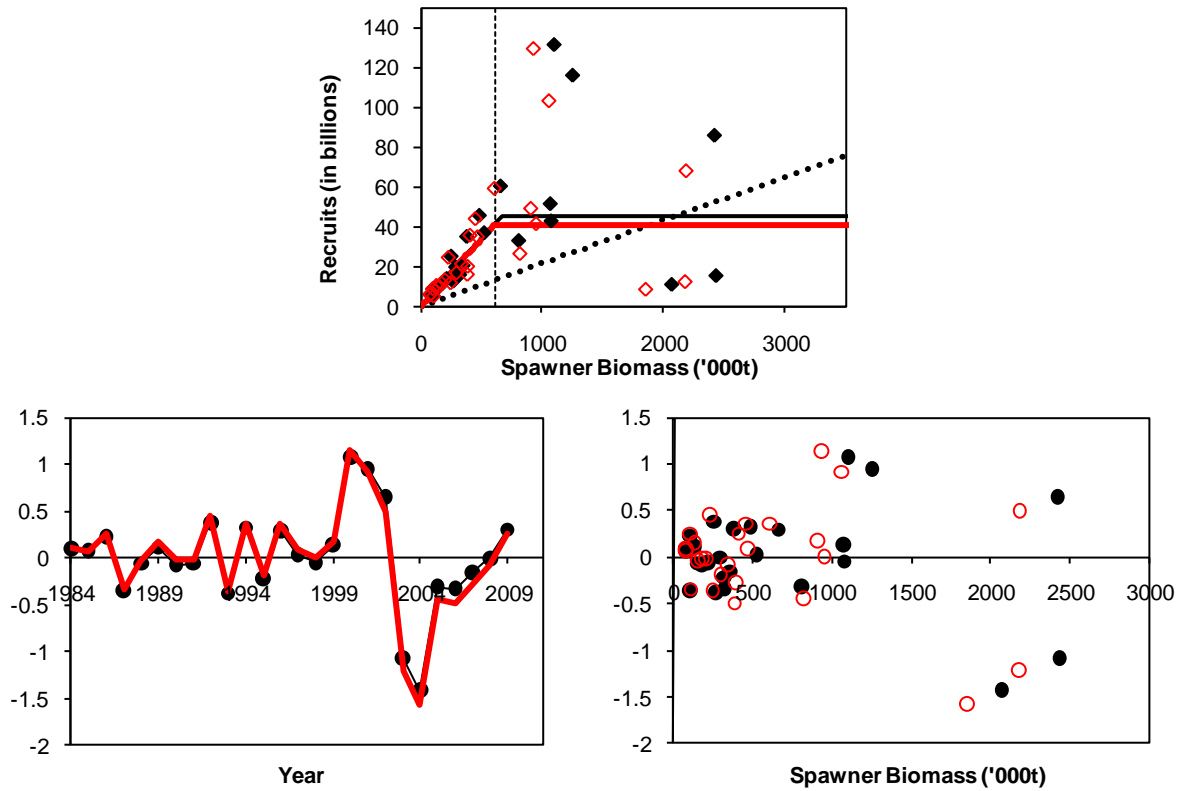


Figure 1. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2009 for S_{HS} (black, filled symbols) and S_{cstM} (red, open symbols) with the Hockey stick stock recruitment relationship. The vertical thin dashed line indicates the average 1991 to 1994 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, against year and against spawner biomass.

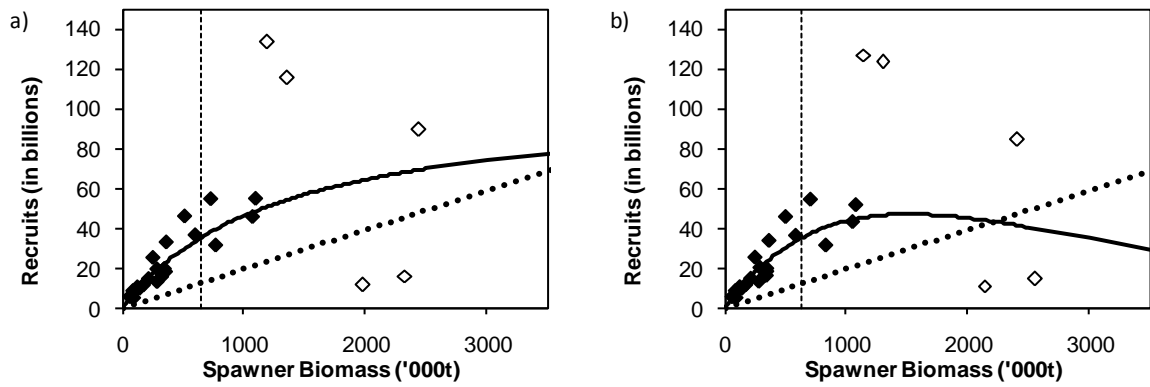


Figure 2. Stock-recruit relationships for a) S_{BH} and b) S_R .

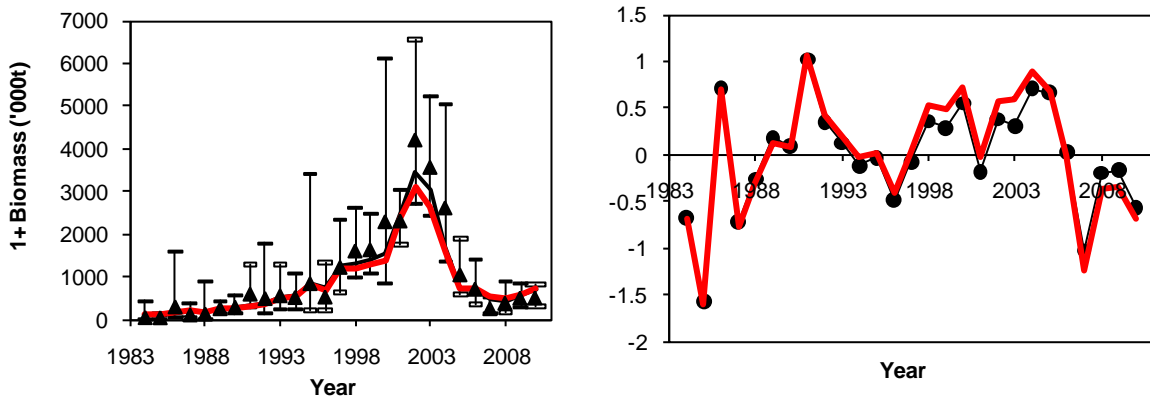


Figure 3. Acoustic survey estimated and model predicted November sardine 1+ biomass from 1984 to 2010 for S_{HS} (black, connecting filled circles in the right hand plot) and S_{cstM} (red). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fits are given in the right hand plot.

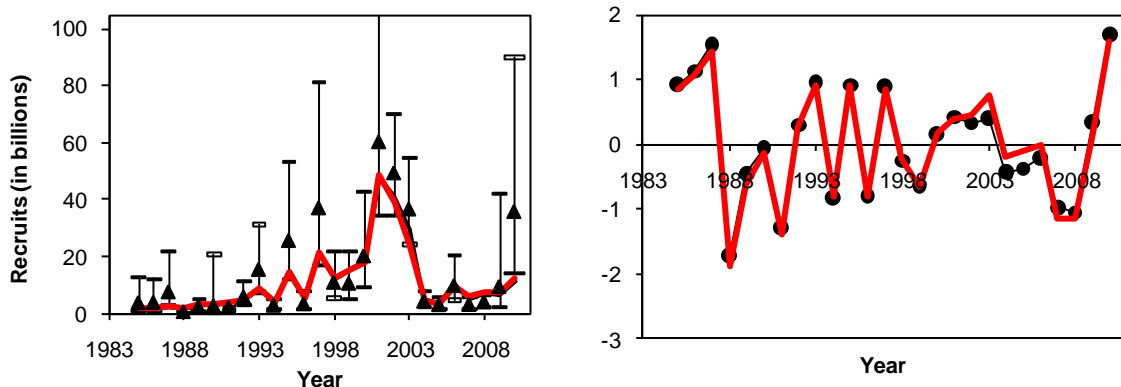


Figure 4. Acoustic survey estimated and model predicted sardine recruitment numbers from May 1985 to May 2010 for S_{HS} (black, connecting filled circles in the right hand plot) and S_{cstM} (red). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

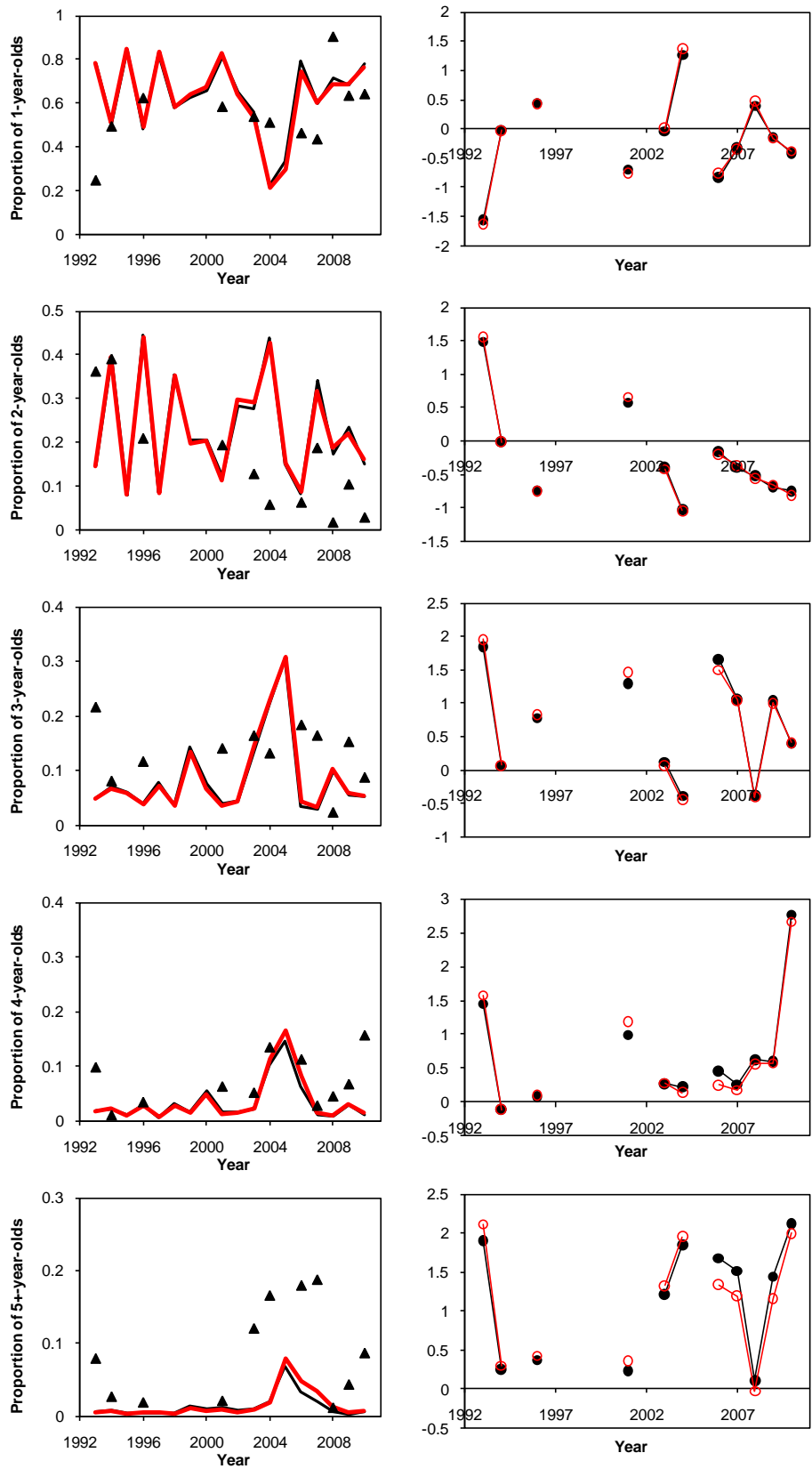


Figure 5. Acoustic survey estimated and model predicted sardine proportion-at-ages 1 (at the top) to 5+ (lowest plot) associated with the November surveys from 1993 to 2010 for S_{HS} (black, connecting filled circles in the right hand plot) and S_{cstM} (red). The residuals from the fits are given in the right hand plots.

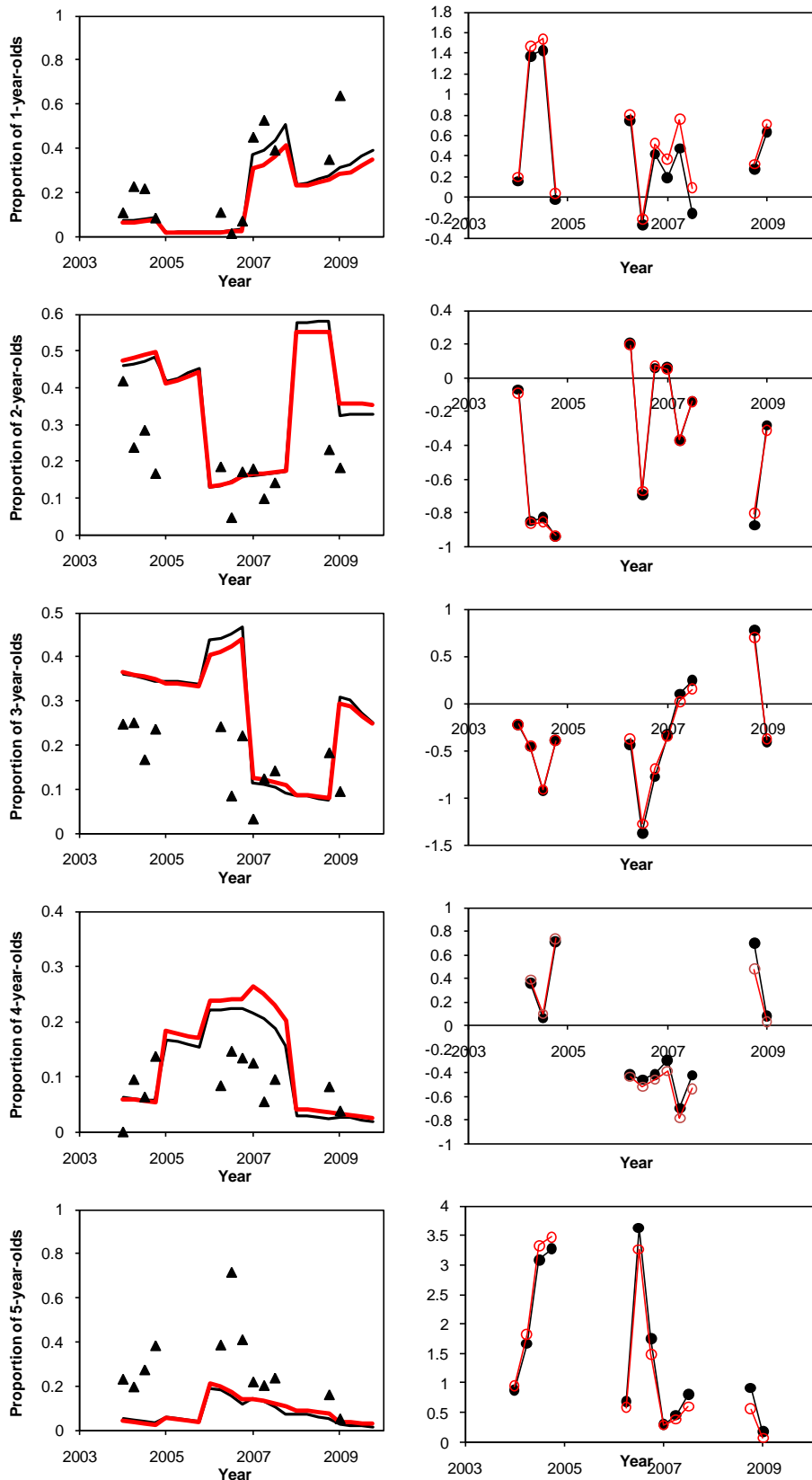


Figure 6. Acoustic survey estimated and model predicted sardine proportion-at-ages 1 (at the top) to 5+ (lowest plot) associated with the quarterly commercial catch from 2004 to 2009 for S_{HS} (black, connecting filled circles in the right hand plot) and S_{cstM} (red). The residuals from the fits are given in the right hand plots.

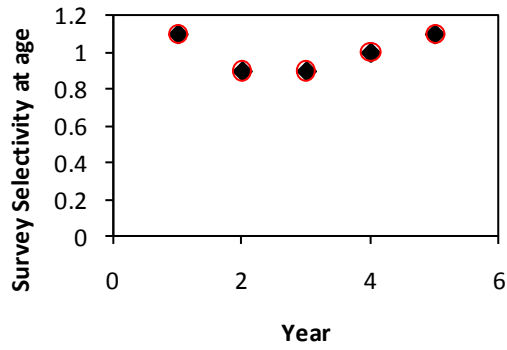


Figure 7. The model estimated November survey selectivity at age for S_{HS} (black filled diamonds) and S_{cstM} (red open circles).

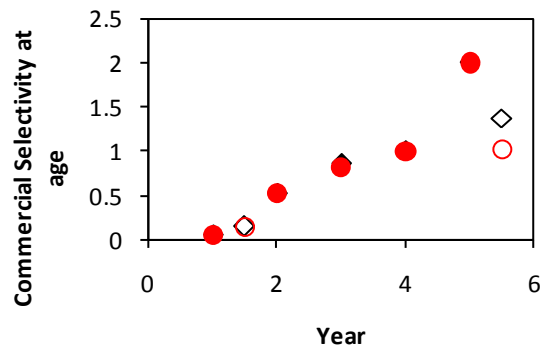


Figure 8. The model estimated commercial selectivity at age for S_{HS} (black diamonds) and S_{cstM} (red circles). The open indices represent the selectivity at ages 1 and 5+ estimated from 2007 to 2010, while the solid indices for these ages represent the selectivity from 1984 to 2006.

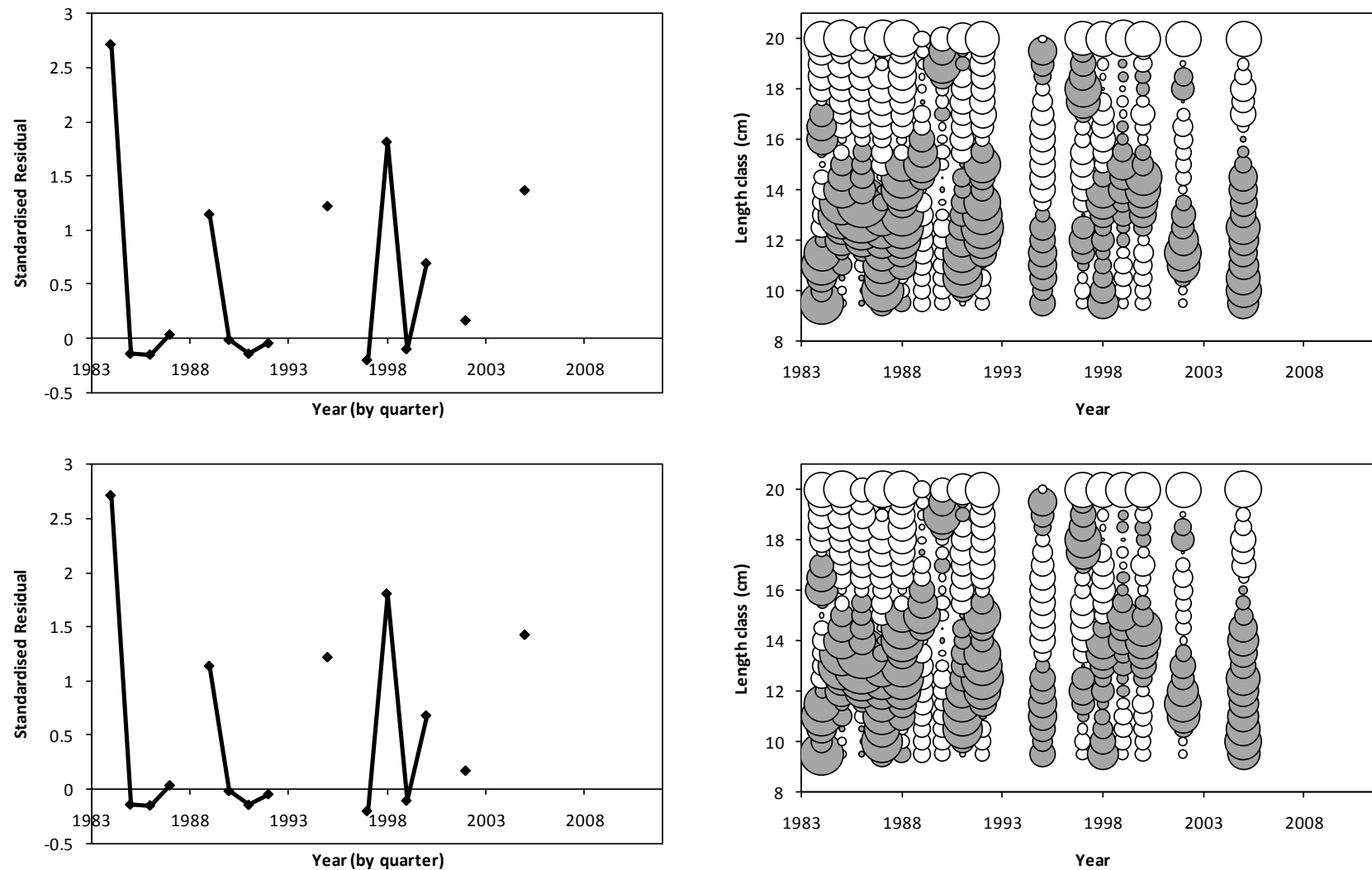


Figure 9. Residuals from the fit of the model predicted proportion-at-length in the November survey to the hydroacoustic survey estimated proportions for S_{HS} (top panels) and S_{CSTM} (lower panels). The left panels show the residuals for the minus length class (9cm) and the right panels show the residuals for the remaining length classes.

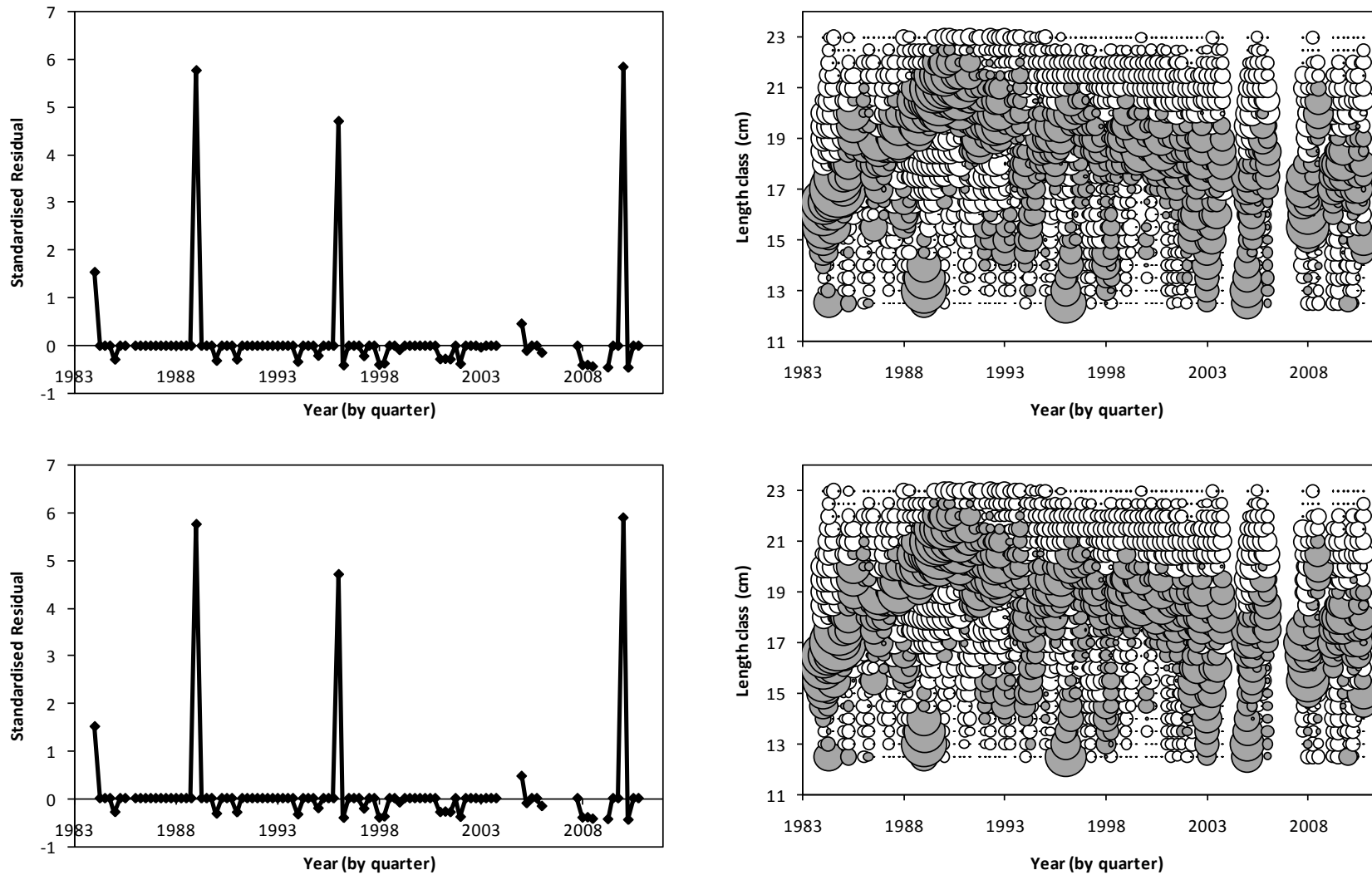


Figure 10. Residuals from the fit of the model predicted proportion-at-length in the commercial catch to the observed proportions for S_{HS} (top panels) and S_{estM} (lower panels). The left panels show the residuals for the minus length class (12cm) and the right panels show the residuals for the remaining length classes.

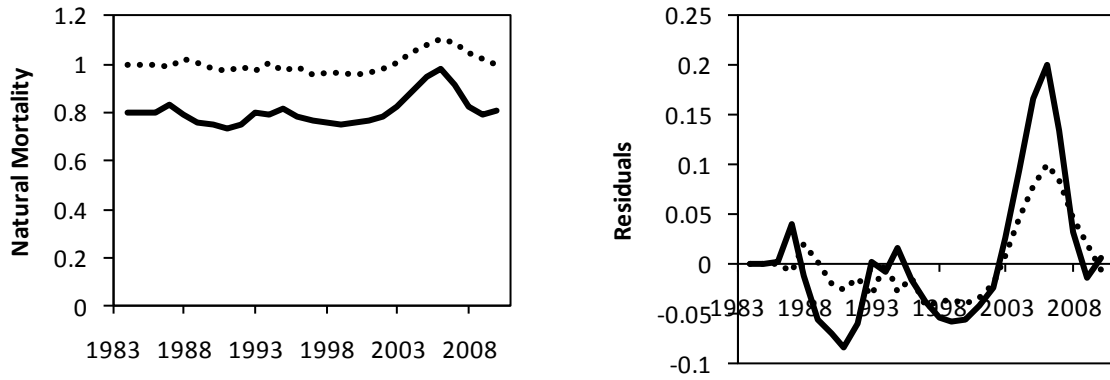


Figure 11. Model estimated annual juvenile (dotted) and adult (solid) natural mortality for S_{HS} . The random effects are plotted in the right hand panel.

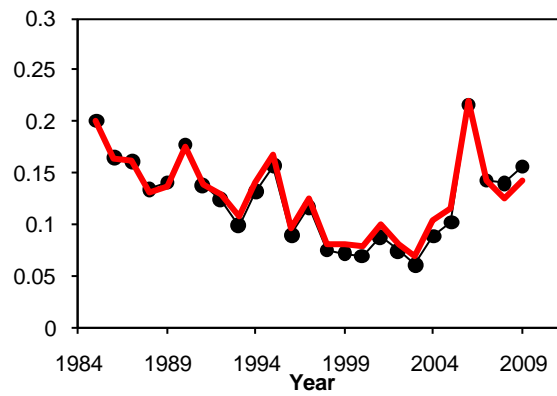


Figure 12. The historic harvest proportion (catch by mass to 1+ biomass) for sardine for S_{HS} (black, connecting filled circles) and S_{cstM} (red).

Appendix A: Bayesian age-structured operating model for the South African sardine resource

Base Case Model Assumptions

- 1) All fish have a theoretical birthdate of 1 November.
- 2) Sardine spawn for the first time when they turn two years old.
- 3) A plus group of age five is assumed.
- 4) Two surveys are held each year: the first takes place in November (known as the November survey) and surveys the adult (1+) stock; the second is in May/June (known as the recruit survey) and surveys juvenile (0-year-old) sardine (also called recruits).
- 5) The November survey provides a relative index of abundance of unknown bias.
- 6) The recruit survey provides a relative index of abundance of unknown bias.
- 7) The survey strategy is such that it results in surveys of invariant bias over time.
- 8) Pulse fishing occurs four times a year, in the middle of each quarter after the birthdate.
- 9) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

Population Dynamics

The basic dynamic equations for sardine, based on Pope's approximation (Pope, 1984), are as follows, where $y_1 = 1984$ and $y_n = 2010$. The numbers-at-age are modelled at 1 November each year.

Catch is taken at four intervals during the year where $q = 1$ is from November $y - 1$ to January y , $q = 2$ from February to April y , $q = 3$ from May to July y and $q = 4$ from August to October y :

Numbers-at-age at 1 November

$$N_{j,y,a}^S = \left(\left(\left(\left(N_{j,y-1,a-1}^S e^{-M_{a-1,y}^S/8} - C_{j,y,1,a-1}^S \right) e^{-M_{a-1,y}^S/4} \right) - C_{j,y,2,a-1}^S \right) e^{-M_{a-1,y}^S/4} - C_{j,y,3,a-1}^S \right) e^{-M_{a-1,y}^S/4} - C_{j,y,4,a-1}^S \right) e^{-M_{a-1,y}^S/8}$$

$$y = y_1, \dots, y_n, \quad a = 1, \dots, 4$$

$$N_{j,y,5+}^S = \left(\left(\left(\left(N_{j,y-1,4}^S e^{-M_{4,y}^S/8} - C_{j,y,1,4}^S \right) e^{-M_{4,y}^S/4} \right) - C_{j,y,2,4}^S \right) e^{-M_{4,y}^S/4} - C_{j,y,3,4}^S \right) e^{-M_{4,y}^S/4} - C_{j,y,4,4}^S \right) e^{-M_{4,y}^S/8}$$

$$+ \left(\left(\left(\left(N_{j,y-1,5+}^S e^{-M_{5+,y}^S/8} - C_{j,y,1,5+}^S \right) e^{-M_{5+,y}^S/4} \right) - C_{j,y,2,5+}^S \right) e^{-M_{5+,y}^S/4} - C_{j,y,3,5+}^S \right) e^{-M_{5+,y}^S/4} - C_{j,y,4,5+}^S \right) e^{-M_{5+,y}^S/8}$$

$$y = y_1, \dots, y_n \quad (\text{A.1})$$

where

$N_{j,y,a}^S$ is the model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j ;

$C_{j,y,a,q}^S$ is the model predicted number (in billions) of sardine of age a of stock j caught during quarter q of year y ;

$M_{a,y}^S$ is the rate of natural mortality (in year⁻¹) of sardine of age a in year y .

Movement

In the two stock hypothesis, movement of west stock ($j=1$) recruits to the east stock ($j=2$) at the beginning of November, i.e. when the recruits turn age 1, is modelled as follows:

$$N_{1,y,1}^S = (1 - move_y) N_{1,y,1}^{S*}$$

$$N_{2,y,1}^S = move_y N_{2,y,1}^{S*} \quad y = y_1, \dots, y_n$$

where $N_{j,y,1}^{S*}$ is simply the numbers-at-age 1 given by equation (A.1) prior to movement, and

$move_y$ is the proportion of west stock recruits which migrate to the east stock at the beginning of November of year y .

Biomass associated with the November survey

$$B_{j,y}^S = k_{j,N}^S \sum_{a=1}^{5+} N_{j,y,a}^S w_{j,y,a}^S \quad y = y_1, \dots, y_n \quad (A.2)$$

where

$B_{j,y}^S$ is the model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year y , associated with the November survey;

$k_{j,N}^S$ is the constant of proportionality (multiplicative bias) associated with the November survey of stock j ; and

$w_{j,y,a}^S$ is the mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y (de Moor et al. 2011).

The multiplicative bias in the November survey is assumed to be equal to that resulting from the acoustic survey only; hence it is assumed that the full distribution of sardine is covered by the survey, i.e.

$$k_{j,N}^S = k_{ac}^S$$

where

k_{ac}^S is the multiplicative bias associated with the acoustic survey (see Appendix B).

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB_{j,y}^S = \sum_{a=2}^{5+} N_{j,y,a}^S w_{j,y,a}^S \quad y = y_1, \dots, y_n \quad (A.3)$$

Proportion at age and length associated with the November survey

The model predicted proportion-at-age in the survey is:

$$P_{j,y,a}^S = \frac{S_{j,a}^{survey} N_{j,y,a}^S}{\sum_{a=1}^{5+} S_{j,a}^{survey} N_{j,y,a}^S} \quad y = y_1, \dots, y_n, \quad a = 1, \dots, 5+ \quad (A.4)$$

where

$S_{j,a}^{survey}$ is the survey selectivity at age a in the November survey for stock j .

The model predicted proportion-at-length associated with the November survey is:

$$P_{j,y,l}^S = \sum_{a=1}^{5+} \sum_{l=1}^{l \min} A_{j,a,l}^{sur} P_{j,y,a}^S \quad y = y_1, \dots, y_n \quad (A.5)$$

$$P_{j,y,l}^S = \sum_{a=1}^{5+} A_{j,a,l}^{sur} P_{j,y,a}^S \quad y = y_1, \dots, y_n, \quad l = l \min + 1, \dots, l \max \quad (A.6)$$

where

$A_{j,a,l}^{sur}$ is the proportion of sardine of age a in stock j that fall in the length group l in November.

The matrix $A_{j,a,l}^{sur}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A_{j,a,l}^{com} \sim N\left(L_{j,\infty} \left(1 - e^{-\kappa_j(a-t_{0,j})}\right), \mathcal{G}_{j,a}\right) \quad (A.7)$$

where

$L_{j,\infty}$ denotes the maximum length of sardine of stock j ;

κ_j denotes the annual growth rate of sardine of stock j ;

$t_{0,j}$ denotes the age at which the growth rate is zero of sardine of stock j ; and

$\mathcal{G}_{j,a}$ denotes the standard deviation about the mean length for age a of stock j .

Catch

Sardine are landed by three major fisheries: the sardine-directed fishery ($fleet=1$), the red-eye-directed fishery ($fleet=2$), and the anchovy-directed fishery ($fleet=3$). Landings from the former two fisheries comprise mainly adult sardine while bycatch from the anchovy-directed fishery is primarily juvenile sardine. The assumption is made that all sardine smaller than a pre-determined cut-off length are 0-year-olds:

$$C_{j,y,1,0}^S = \sum_{fleet=1}^3 \sum_{m=1}^1 \sum_{l < cut_{y,m}} C_{j,y,m,l}^{RLF, fleet}$$

$$C_{j,y,2,0}^S = \sum_{fleet=1}^3 \sum_{m=2}^4 \sum_{l < cut_{y,m}} C_{j,y,m,l}^{RLF, fleet}$$

$$C_{j,y,3,0}^S = \sum_{fleet=1}^3 \sum_{m=5}^7 \sum_{l < lcut_{y,m}} C_{j,y,m,l}^{RLF, fleet}$$

$$C_{j,y,4,0}^S = \sum_{fleet=1}^3 \sum_{m=8}^{10} \sum_{l < lcut_{y,m}} C_{j,y,m,l}^{RLF, fleet} \quad y = y_1, \dots, y_n \quad (A.8)$$

where

$C_{j,y,m,l}^{RLF}$ is the number of fish in length class l landed in month m of year y of stock j (the ‘raised length frequency’); and

$lcut_{y,m}$ is the cut off length for recruits in month m of year y (see de Moor et al. (2011) for details).

The remaining sardine bycatch from the anchovy-directed fishery are assumed to be 1-year olds, while the remaining directed sardine and redeye bycatch are split between ages using a model estimated selectivity:

$$C_{j,y,1,1}^{bycatch} = \sum_{m=1}^1 \sum_{l \geq lcut_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}$$

$$C_{j,y,2,1}^{bycatch} = \sum_{m=2}^4 \sum_{l \geq lcut_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}$$

$$C_{j,y,3,1}^{bycatch} = \sum_{m=5}^7 \sum_{l \geq lcut_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}$$

$$C_{j,y,4,1}^{bycatch} = \sum_{m=8}^{10} \sum_{l \geq lcut_{y,m}} C_{j,y,m,l}^{RLF, fleet=3}$$

$$C_{j,y,q,a}^{bycatch} = 0 \quad y = y_1, \dots, y_n, \quad q = 1, \dots, 4, \quad a = 2, \dots, 5 +$$

$$C_{j,y,1,a}^S = \left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} \right) S_{j,y,1,a} F_{j,y,1}$$

$$C_{j,y,2,a}^S = \left(\left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^S \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} \right) S_{j,y,2,a} F_{j,y,2}$$

$$C_{j,y,3,a}^S = \left(\left(\left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^S \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} - C_{j,y,2,a}^S \right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} \right) S_{j,y,3,a} F_{j,y,3}$$

$$C_{j,y,4,a}^S = \left(\left(\left(\left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^S \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} - C_{j,y,2,a}^S \right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} - C_{j,y,3,a}^S \right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch} \right) S_{j,y,4,a} F_{j,y,4}$$

$$y = y_1, \dots, y_n, \quad a = 0, \dots, 5 + \quad (A.9)$$

$S_{j,y,q,a}$ is the commercial selectivity at age a during quarter q of year y of stock j ; and

$F_{j,y,q}$ is the fished proportion in quarter q of year y for a fully selected age class a of stock j , by the directed and redeye bycatch fisheries.

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion from the directed and redeye bycatch fisheries is estimated by:

$$\begin{aligned}
F_{j,y,1} &= \frac{\sum_{fleet=1}^2 \sum_{m=1}^{12} \sum_{l>=lcut_{y,m}} C_{j,y-1,m,l}^{RLF,fleet} \sum_{fleet=1}^2 \sum_{l>=lcut_{y,m}} C_{j,y,1,l}^{RLF,fleet}}{\sum_{a=1}^{5+} \left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} \right) S_{j,y,1,a}} \\
F_{j,y,2} &= \frac{\sum_{fleet=1}^2 \sum_{m=2}^4 \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+} \left(\left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^S \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} \right) S_{j,y,2,a}} \\
F_{j,y,3} &= \frac{\sum_{fleet=1}^2 \sum_{m=5}^7 \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+} \left(\left(\left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^S \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} - C_{j,y,2,a}^S \right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} \right) S_{j,y,3,a}} \\
F_{j,y,4} &= \frac{\sum_{fleet=1}^2 \sum_{m=8}^{10} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+} \left(\left(\left(\left(N_{j,y-1,a}^S e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^S \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} - C_{j,y,2,a}^S \right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} - C_{j,y,3,a}^S \right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch} \right) S_{j,y,4,a}}
\end{aligned} \tag{A.10}$$

A penalty is imposed within the model to ensure that $F_{j,y,q} < 0.95$.

Recruitment

For the base case assessment of a single stock hypothesis, a Hockey Stick stock-recruitment curve is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve:

$$N_{j,y,0}^S = \begin{cases} a_j^S e^{\varepsilon_{j,y}^S} & , \text{if } SSB_{j,y}^S \geq b_j^S \\ \frac{a_j^S}{b_j^S} SSB_{j,y}^S e^{\varepsilon_{j,y}^S} & , \text{if } SSB_{j,y}^S < b_j^S \end{cases} \quad y = y_1, \dots, y_n \tag{A.11}$$

where

a_j^S is the maximum recruitment of stock j (in billions) (i.e. median of the distribution in question);

b_j^S is the spawner biomass above which there should be no recruitment failure risk in the hockey stick model for stock j ;

$\varepsilon_{j,y}^S$ is the annual lognormal deviation of sardine recruitment.

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$N_{j,y,r}^S = k_{j,r}^S \left((N_{j,y-1,0}^S e^{-M_0^S/8} - C_{j,y,1,0}^S) e^{-M_0^S/4} - C_{j,y,2,0}^S \right) e^{-0.5t_y^S \times M_0^S/12} - \tilde{C}_{j,y,0bs}^S e^{-0.5t_y^S \times M_0^S/12}$$

$$y = y_1, \dots, y_n \quad (\text{A.12})$$

where

$N_{j,y,r}^S$ is the model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y ;

$k_{j,r}^S$ is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}_{j,y,0bs}^S$ is the number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey (see de Moor et al. 2011); and

t_y^S is the time lapsed (in months) between 1 May and the start of the recruit survey in year y (see de Moor et al. 2011).

The multiplicative bias in the recruit survey is assumed to be equal to that resulting from the acoustic survey as well as the proportion of the recruit abundance which the survey covers in comparison to the November survey. In addition, for the two stock hypothesis, the proportion of the east stock recruit abundance covered compared to that of the west stock abundance is also required. Thus

$$k_{1,r}^S = k_{cov}^S \times k_{ac}^S$$

and for the two stock hypothesis, $k_{2,r}^S = k_{covE}^S \times k_{cov}^S \times k_{ac}^S$

where

k_{cov}^S is the multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey; and

k_{covE}^S is the multiplicative bias associated with the coverage of the east stock recruits by the recruit survey in comparison to the west stock recruits during the same survey.

Proportion at age and length associated with the commercial catch

The model predicted proportion-at-age in the commercial catch from the directed and redeye bycatch fisheries is:

$$P_{j,y,q,a}^{com,S} = \frac{C_{j,y,q,a}^S}{\sum_{a=1}^{5+} C_{j,y,q,a}^S} \quad y = y_1, \dots, y_n, \quad q = 1, \dots, 4, \quad a = 1, \dots, 5+ \quad (\text{A.13})$$

The model predicted proportion-at-length in the commercial catch from the directed and redeye bycatch fisheries is:

$$P_{j,y,q,l \min}^{coml,S} = \sum_{a=1}^{5+} \sum_{l=1}^{l \min} A_{j,q,a,l}^{com} P_{j,y,q,a}^{com,S} \quad y = y_1, \dots, y_n, \quad q = 1, \dots, 4 \quad (\text{A.14})$$

$$P_{j,y,q,l}^{coml,S} = \sum_{a=1}^{5+} A_{j,q,a,l}^{com} P_{j,y,q,a}^{com,S} \quad y = y_1, \dots, y_n, \quad q = 1, \dots, 4, \quad l = l \min + 1, \dots, l \max \quad (\text{A.15})$$

where

$A_{j,q,a,l}^{com}$ is the proportion of sardine of age a in stock j that fall in the length group l in quarter q .

The matrix $A_{j,q,a,l}^{com}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A_{j,1,a,l}^{com} \sim N\left(L_{j,\infty} \left(1 - e^{-\kappa_j(a+1/8-t_{0,j})}\right), \mathcal{G}_{j,a}\right) \quad (\text{A.16})$$

$$A_{j,2,a,l}^{com} \sim N\left(L_{j,\infty} \left(1 - e^{-\kappa_j(a+3/8-t_{0,j})}\right), \mathcal{G}_{j,a}\right) \quad (\text{A.17})$$

$$A_{j,3,a,l}^{com} \sim N\left(L_{j,\infty} \left(1 - e^{-\kappa_j(a+5/8-t_{0,j})}\right), \mathcal{G}_{j,a}\right) \quad (\text{A.18})$$

$$A_{j,4,a,l}^{com} \sim N\left(L_{j,\infty} \left(1 - e^{-\kappa_j(a+7/8-t_{0,j})}\right), \mathcal{G}_{j,a}\right) \quad (\text{A.19})$$

Fitting the Model to Observed Data (Likelihood)

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions and a further additional variance parameter. The estimated proportions-at-age are also assumed to be lognormally distributed, with variance inversely proportional to the number of samples used to calculate the ALK and the observed proportion, while the variance for the proportions-at-length are inversely proportional to the observed proportion. Thus the negative log-likelihood function is given by:

$$\begin{aligned} -\ln L = & -\ln L^{Nov} - \ln L^{rec} - \ln L^{sur propa} - \ln L^{com propa} \\ & - \ln L^{sur proplmin} - \ln L^{sur propl} - \ln L^{com proplmin} - \ln L^{com propl} \end{aligned} \quad (\text{A.20})$$

where

$$\begin{aligned} -\ln L^{Nov} = & \frac{1}{2} \sum_j \sum_{y=y1}^{ym} \left\{ \frac{\left(\ln(\hat{B}_{j,y}^S) - \ln(B_{j,y}^S)\right)^2}{\left(\sigma_{j,y,Nov}^S\right)^2 + \left(\phi_{ac}^S\right)^2 + \left(\lambda_{j,N}^S\right)^2} + \ln \left[2\pi \left(\left(\sigma_{j,y,Nov}^S\right)^2 + \left(\phi_{ac}^S\right)^2 + \left(\lambda_{j,N}^S\right)^2 \right) \right] \right\} \\ -\ln L^{rec} = & \frac{1}{2} \sum_j \sum_{y=y1+1}^{ym} \left\{ \frac{\left(\ln(\hat{N}_{j,y,r}^S) - \ln(N_{j,y,r}^S)\right)^2}{\left(\sigma_{j,y,rec}^S\right)^2 + \left(\phi_{ac}^S\right)^2 + \left(\lambda_{j,r}^S\right)^2} + \ln \left[2\pi \left(\left(\sigma_{j,y,rec}^S\right)^2 + \left(\phi_{ac}^S\right)^2 + \left(\lambda_{j,r}^S\right)^2 \right) \right] \right\} \end{aligned}$$

$$\begin{aligned}
 -\ln L^{sur propa} &= w_{propa}^{survey} \frac{1}{2} \sum_j \sum_{ys} \sum_{a=1}^5 \left\{ \frac{n_{s,y} \hat{p}_{j,y,a}^S (\ln(\hat{p}_{j,y,a}^S) - \ln(p_{j,y,a}^S))^2}{(\sigma_{j,p}^S)^2} + \ln[2\pi(\sigma_{j,p}^S)^2 / (n_{s,y} \hat{p}_{j,y,a}^S)] \right\} \\
 -\ln L^{com propa} &= w_{propa}^{com} \frac{1}{2} \sum_{yc} \sum_{qc} \sum_{a=1}^5 \left\{ \frac{n_{y,q}^{com} \hat{p}_{1,y,q,a}^{com,S} (\ln(\hat{p}_{1,y,q,a}^{com,S}) - \ln(p_{1,y,q,a}^{com,S}))^2}{(\sigma_{com}^S)^2} + \ln[2\pi(\sigma_{com}^S)^2 / (n_{y,q}^{com} \hat{p}_{1,y,q,a}^{com,S})] \right\} \\
 -\ln L^{sur propmin} &= w_{propmin}^{sur} \frac{1}{2} \sum_j \sum_{y=y1}^{yn} \left\{ \frac{\hat{p}_{j,y,l}^S \min (\ln(\hat{p}_{j,y,l}^S \min) - \ln(p_{j,y,l}^S \min))^2}{2(\sigma_{j,l}^S)^2} + \ln \left(\frac{\sigma_{j,l}^S}{\sqrt{\hat{p}_{j,y,l}^S \min}} \right) \right\}^4 \\
 -\ln L^{sur prop1} &= w_{prop1}^{sur} \frac{1}{2} \sum_j \sum_{y=y1}^{yn} \sum_{l=1}^{lmax} \left\{ \frac{\hat{p}_{j,y,l}^S (\ln(\hat{p}_{j,y,l}^S) - \ln(p_{j,y,l}^S))^2}{2(\sigma_{j,l}^S)^2} + \ln \left(\frac{\sigma_{j,l}^S}{\sqrt{\hat{p}_{j,y,l}^S}} \right) \right\}^5 \\
 -\ln L^{com propmin} &= w_{propmin}^{com} \frac{1}{2} \sum_j \sum_{y=y1}^{yn} \sum_{q=1}^4 \left\{ \frac{\hat{p}_{j,y,q,l}^{coml,S} \min (\ln(\hat{p}_{j,y,q,l}^{coml,S} \min) - \ln(p_{j,y,q,l}^{coml} \min))^2}{2(\sigma_{j,coml}^S)^2} + \ln \left(\frac{\sigma_{j,coml}^S}{\sqrt{\hat{p}_{j,y,q,l}^{coml,S} \min}} \right) \right\}^6 \\
 -\ln L^{com prop1} &= w_{prop1}^{com} \frac{1}{2} \sum_j \sum_{y=y1}^{yn} \sum_{q=1}^4 \sum_{l=1}^{lmax} \left\{ \frac{\hat{p}_{j,y,q,l}^{coml,S} (\ln(\hat{p}_{j,y,q,l}^{coml,S}) - \ln(p_{j,y,q,l}^{coml}))^2}{2(\sigma_{j,coml}^S)^2} + \ln \left(\frac{\sigma_{j,coml}^S}{\sqrt{\hat{p}_{j,y,q,l}^{coml,S}}} \right) \right\}^5 \quad (A.21)
 \end{aligned}$$

Here

$\hat{B}_{j,y}^S$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y , with associated CV $\sigma_{j,y,Nov}^S$;

$\hat{N}_{j,y,r}^S$ is the acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y , with associated CV $\sigma_{j,y,rec}^S$; and

ϕ_{ac}^S is the CV associated with the factors which cause bias in the acoustic survey estimates and which vary inter-annually rather than remain fixed over time;

$(\lambda_{j,N/r}^S)^2$ is the additional variance (over and above the squares of the survey sampling CV $\sigma_{y,Nov/rec}^S$ that reflects survey inter-transsect variance and of the CV ϕ_{ac}^S associated with the annually varying factors causing bias in the acoustic survey estimates) associated with the November/recruit surveys of stock j ;

$\hat{p}_{j,y,a}^S$ is an estimate of the proportion (by number) of sardine of age a in stock j in the November survey of year y ;

⁴ Note that the years over which the sum occurs excludes those for which survey proportion-at-age data are used.

⁵ Although strictly there may be bias in the proportions of length-at-age data, no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

⁶ Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

$n_{s,y}$ is the number of fish from the November survey trawls in year y used to compile the age-length key for calculating $\hat{p}_{j,y,a}^S$;

$(\sigma_{j,p}^S)^2$ is the overall variance-related parameter for the log-transformed survey proportion-at-age

observations for stock j , $\hat{p}_{j,y,a}^S$ [note variance = $(\sigma_{j,p}^S)^2 / (n_y \hat{p}_{j,y,a}^S)$], and is estimated in the fitting procedure by the closed form solution:

$$(\sigma_{j,p}^S)^2 = \frac{\sum_{ys} \sum_{a=1}^5 n_{s,y} \hat{p}_{j,y,a}^S (\ln(\hat{p}_{j,y,a}^S) - \ln(p_{j,y,a}^S))^2}{\sum_{ys} \sum_{a=1}^5 1};$$

ys denotes the years for which ALKs are available to calculate proportion-at-age in the November survey ('93, '94, '96, '01, '03, '04, '06-'10);

w_{propa}^{survey} is the weighting applied to the survey proportion-at-age data;

$\hat{p}_{y,q,a}^{com,S}$ is an estimate of the proportion (by number) of single-stock or “west stock” sardine of age a in the commercial catch of quarter q of year y (calculated using the raised length frequencies of the directed and redeye-bycatch fisheries – see de Moor et al. 2011);

$n_{y,q}^{com}$ is the number of fish from the commercial trawls in quarter q of year y used to compile the age-length key for calculating $\hat{p}_{y,q,a}^{com,S}$;

$(\sigma_{com}^S)^2$ is the overall variance-related parameter for the log-transformed commercial proportion-at-age observations, $\hat{p}_{y,q,a}^{com,S}$ [note variance = $(\sigma_{com}^S)^2 / (n_{y,q}^{com} \hat{p}_{y,q,a}^{com,S})$], and is estimated in the fitting procedure by the closed form solution:

$$(\sigma_{com}^S)^2 = \frac{\sum_{yc} \sum_{qc} \sum_{a=1}^5 n_{y,q}^{com} \hat{p}_{1,y,q,a}^{com,S} (\ln(\hat{p}_{1,y,q,a}^{com,S}) - \ln(p_{1,y,q,a}^{com,S}))^2}{\sum_{yc} \sum_{qc} \sum_{a=1}^5 1};$$

yc / qc denotes the years/quarters for which ALKs are available to calculate quarterly proportions-at-age in the commercial catch ('04 Q1-4, '06 Q2-4, '07 Q1-3, '08 Q4, '09 Q1);

w_{propa}^{com} is the weighting applied to the commercial proportion-at-age data;

$\hat{p}_{j,y,l}^S$ is the observed proportion (by number) of sardine in length group l in the November survey of year y ;

$w_{proplmin}^{sur}$ is the weighting applied to the survey proportion at length data for the minus length class;

w_{propl}^{sur} is the weighting applied to the remaining survey proportion at length data;

⁷ This is not stock-dependent as only ALKs for the “west” coast are available.

$\sigma_{l\min}^S$ is the variance-related parameter for the log-transformed survey proportion-at-length data of the minus length class, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_{j,l\min}^S = \sqrt{\sum_{y=y1}^{ym} \hat{p}_{j,y,l\min}^S (\ln \hat{p}_{j,y,l\min}^S - \ln p_{j,y,l\min}^S)^2} / \sum_{y=y1}^{ym} 1^8; \text{ and}$$

σ_l^S is the variance-related parameter for the log-transformed survey proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_{j,l}^S = \sqrt{\sum_{y=y1}^{ym} \sum_{l=l\min+1}^{l\max} \hat{p}_{j,y,l}^S (\ln \hat{p}_{j,y,l}^S - \ln p_{j,y,l}^S)^2} / \sum_{y=y1}^{ym} \sum_{l=l\min+1}^{l\max} 1^9.$$

$\hat{p}_{j,y,q,l}^{coml,S}$ is the observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q ($q=1$ for Nov-Jan, $q=2$ for Feb-Apr, $q=3$ for May-Jul, $q=4$ for Aug-Oct) of year y ;

$w_{propl\min}^{com}$ is the weighting applied to the commercial proportion at length data for the minus length class;

w_{propl}^{com} is the weighting applied to the remaining commercial proportion at length data;

$\sigma_{coml\min}^S$ is the variance-related parameter for the log-transformed commercial proportion-at-length data of the minus length class, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_{j,y,q,l\min}^S = \sqrt{\sum_{y=y1}^{ym} \sum_{q=1}^4 \hat{p}_{j,y,q,l\min}^{coml,S} (\ln \hat{p}_{j,y,q,l\min}^{coml,S} - \ln p_{j,y,q,l\min}^{coml})^2} / \sum_{y=y1}^{ym} \sum_{q=1}^4 1^{10}; \text{ and}$$

σ_{coml}^S is the variance-related parameter for the log-transformed commercial proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_{j,coml}^S = \sqrt{\sum_{y=y1}^{ym} \sum_{q=1}^4 \sum_{l=l\min+1}^{l\max} \hat{p}_{j,y,q,l}^{coml,S} (\ln \hat{p}_{j,y,q,l}^{coml,S} - \ln p_{j,y,q,l}^{coml})^2} / \sum_{y=y1}^{ym} \sum_{q=1}^4 \sum_{l=l\min+1}^{l\max} 1^{11}.$$

Fixed Parameters

The following parameters were fixed externally in the model:

Adult natural mortality varies around a median of $\bar{M}_{ad}^S = 0.8$ as follows:

$$M_{a,y}^S = \bar{M}_{ad}^S e^{\varepsilon_{ad,y}} \text{ for } a = 1, \dots, 5+ \text{ with } \varepsilon_y^{ad} = p\varepsilon_{y-1}^{ad} + \sqrt{1-p^2}\eta_y^{ad} \quad (\text{A.8})$$

⁸ Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

⁹ Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

¹⁰ Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

¹¹ Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

where $\eta_y^{ad} \sim N(0, \sigma_{ad}^2)$ and

Juvenile natural mortality varies around a median of $\bar{M}_j^S = 1.0$ as follows:

$$M_{j,y}^S = \bar{M}_j^S e^{\varepsilon_{j,y}} \quad \text{with} \quad \varepsilon_y^j = p\varepsilon_{y-1}^j + \sqrt{1-p^2}\eta_y^j \quad (\text{A.8})$$

where $\eta_y^j \sim N(0, \sigma_j^2)$ and

σ_{ad} - is the standard deviation in the annual residuals about adult natural mortality;

σ_j - is the standard deviation in the annual residuals about juvenile natural mortality; and

p - is the annual autocorrelation coefficient.

Initial testing of the model indicated similar autocorrelation coefficients were estimated separately for random effects in adult and juvenile natural mortality. Sardine of age 4 are taken to be fully selected in both the survey and commercial trawls:

$$S_{j,y,q,4} = 1, \text{ for } y = y_1, \dots, y_n, q = 1, \dots, 4 \text{ and } S_{j,4}^{survey} = 1.$$

$\kappa \times L_\infty = 11.4957$ and $t_0 = -1.7087$ obtained from fitting a von Bertalanffy growth curve to the available ageing data (Durholtz and Mtengwane pers. commn.)

The weighting for the commercial proportion-at-age should be about a quarter of that for the survey proportion-at-age as there could be up to four observations per year which will be strongly positively correlated. However, as the survey proportion-at-age data (in which there is relatively low confidence) appear to conflict with $-\ln L^{Nov}$ (the contribution from the time series of November biomass survey estimates in which there is relatively much higher confidence), we have set these weights low at $w_{propa}^{survey} = 0.02$ and $w_{propa}^{com} = 0.02$ in line with this wider perspective.

The weighting on the proportion-at-length data should be about a quarter of that on the proportion-at-age data as there are 23 length classes and 5 ages. However, again due to the relatively low confidence in the accuracy of the ageing data, we have again set $w_{proplmin}^{com} = w_{propl}^{com} = 0.02$.

The CV associated with factors causing bias in the acoustic survey estimated which vary inter-annually is fixed at the CV of the posterior distribution calculated in Figure B.2, i.e. $\phi_{ac}^S = 0.215/0.969 = 0.222$.

As hydroacoustic estimates of recruitment to the east stock are only available from May 1994 onwards, $move_y = 0$, $y = y_1, \dots, 1993$, for the two stock hypothesis only.

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. For the single stock hypothesis, the variance about the stock recruitment curve is assumed to differ between peak and non-peak years, i.e. the prior pdfs for the recruitment residuals are given by:

$$\varepsilon_{j,y}^S \sim N\left(0, (\sigma_r^S)^2\right), \quad y = y_1, \dots, 1999, 2005, \dots, y_{n-1}$$

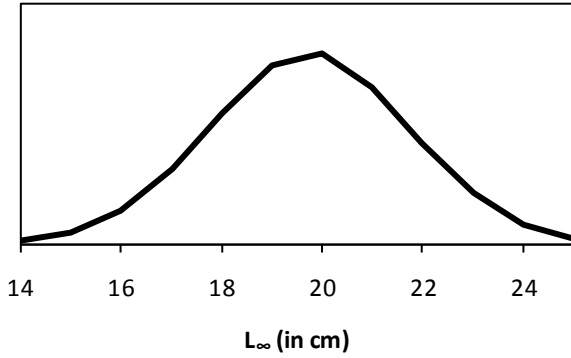
$$\varepsilon_{j,y}^S \sim N\left(0, (\sigma_{r,peak}^S)^2\right), \quad y = 2000, \dots, 2004$$

while for the two stock hypothesis, the variance about the stock recruitment curves is assumed to differ between stocks, but not over years, i.e.

$$\varepsilon_{j,y}^S \sim N\left(0, (\sigma_{j,r}^S)^2\right), \quad y = y_1, \dots, y_{n-1}$$

$$k_{ac}^S \sim N(0.714, 0.077^2), \text{ see Appendix B}$$

$L_{j,\infty} \sim N(19.7416, 2^2)$, where 19.7416 is the value of L_∞ obtained from fitting a von Bertalanffy growth curve to the available ageing data (Durholtz and Mtengwane pers. comm) and a standard deviation of 2 does not allocate too much probability to the 23.5-24 cm length class which is the largest observed historically in the November survey



The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$$move_y \sim U(0,1), \quad y = 1994, \dots, y_n, \text{ for the two stock hypothesis only}$$

$$k_{cov}^S \sim U(0.3,1)$$

$$k_{covE}^S \sim U(0,1)$$

$$(\lambda_{j,N/r}^S)^2 \sim U(0,10)$$

$$S_{j,a}^{survey} \sim U(0.9,1.1)^{12}, \quad a = 1,2,3,5 +$$

$$S_{j,y,q,1} = \tilde{S}_{j,1a}, \quad S_{j,y,q,5+} = \tilde{S}_{j,5+a}, \quad \text{with } \tilde{S}_{j,1+a}, \tilde{S}_{j,1+a} \sim U(0,2) \quad y = y_1, \dots, 2006$$

$$S_{j,y,q,1} = \tilde{S}_{j,1b}, \quad S_{j,y,q,5+} = \tilde{S}_{j,5+b}, \quad \text{with } \tilde{S}_{j,5+a}, \tilde{S}_{j,5+b} \sim U(0,2) \quad y = 2007, \dots, y_n$$

¹² By design, surveys aim to achieve equal selectivity over all ages. Age 1 sardine distributed inshore may be under caught in comparison to other ages. On the other hand older, faster fish may be more able to avoid day-time trawls and thus be under represented in any day-time (about 1/2) trawl samples. It is, however, most likely that selectivity of ages 3 to 5+ is flat (Coetzee pers comm.).

$$S_{j,y,q,a} = \tilde{S}_{j,a}, \text{ with } \tilde{S}_{j,a} \sim U(0,2) \quad y = y_1, \dots, y_n, \quad a = 2,3$$

$\log(a_j^S) \sim U(0,4.4)$ (given the lack of *a priori* information on the magnitude of a^S , a log-scale was used)

$$b_j^S / K_j^S \sim U(0,1)$$

For the single stock hypothesis: $(\sigma_r^S)^2 \sim U(0.16,10)$ and $(\sigma_{r,peak}^S)^2 \sim U(0.16,10)$

While for the two stock hypothesis: $(\sigma_{j,r}^S)^2 \sim U(0.16,10)$

$$N_{1983} \sim U(0,50) \text{ billion}$$

$$N_{j,1983a}^S = N_{1983} e^{-(Finit - \bar{M}_j^S - (a-1)\bar{M}_a^S)}$$

$$N_{j,19835+}^S = N_{1983} \frac{e^{-(Finit - \bar{M}_{ju}^S - 4\bar{M}_{ad}^S)}}{1 - e^{-Finit - \bar{M}_{ad}^S}}, \text{ with } Finit = 0.01$$

$$g_{j,a} \sim U(0.01,2)$$

$$\sigma_{ad} \sim U(0.20,0.5)$$

$$\sigma_j \sim U(0.20,0.5)$$

$$p \sim U(0,1)$$

Further Outputs

Recruitment serial correlation:

$$s_{j,cor}^S = \frac{\sum_{y=y_1}^{y_n-2} \mathcal{E}_{j,y} \mathcal{E}_{j,y+1}}{\sqrt{\left(\sum_{y=y_1}^{y_n-2} \mathcal{E}_{j,y}^2 \right) \left(\sum_{y=y_1}^{y_n-2} \mathcal{E}_{j,y+1}^2 \right)}} \quad (\text{A.22})$$

and the standardised recruitment residual value for 2009:

$$\eta_{j,2009}^S = \frac{\mathcal{E}_{j,2009}^S}{\sigma_{j,r}^S} \quad (\text{A.23})$$

are also required as input into the OM.

Carrying capacity, K_j^S (essentially the $B_{j,N}^S$ value where the replacement line and the stock recruit function intersect) is calculated as follows:

$$K_j^S = a_j^S \left[\sum_{a=1}^4 \bar{w}_{j,a}^S e^{-\sum_{a'=0}^{a-1} M_{a'}^S} + \bar{w}_{j,5+}^S e^{-\sum_{a=0}^4 M_a^S} \frac{1}{1 - e^{-M_{5+}^S}} \right] \quad (\text{A.24})$$

. Here

$\bar{w}_{j,a}^S$ is the mean mass (in grams) of sardine of age a from stock j sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available (de Moor *et al.* 2011).

Note that we work with median rather than mean estimates of K_j^S and thus a bias correction factor for the log-normal distribution above is not used.

APPENDIX B: Calculating the bias in estimates of sardine from the May and November hydro-acoustic surveys

A probability density function (pdf) for the bias in the May and November survey that relate directly to the acoustic survey, rather than, for example the coverage of the stock, k_{ac}^S , was calculated as follows. Ten thousand samples were drawn from the individual pdfs for each source of constant error, together with the median values of the individual pdfs of each source of variable error (see Table B.1, Anon. 2000). Constant error relates to a factor whose value is not known exactly, but whatever it is, it is the same for each year. In contrast variable errors relate to a factor whose true value will change from one year to the next. A second pdf of the factors causing bias in the acoustic survey estimated which vary inter-annually, ϕ_{ac}^S , was then calculated by drawing ten thousand samples from the individual pdfs for each source of variable error. The resultant pdfs on the model predicted biomass (i.e. the inverse of the pdf calculated using the errors provided), together with normal distributions fitted to these pdfs are given in Figures B.1 and B.2. A prior distribution for the multiplicative bias associated with the acoustic survey, k_{ac}^S , is then the normal distribution obtained in Figure B.1, with the mean multiplied by the mean of the normal distribution obtained in Figure B.2, i.e. $k_{ac}^S \sim N(0.969 \times 0.737, 0.077^2)$. The reason to include the 0.969 mean from Figure B.2 here is that the distribution of the annually varying bias factors in combination is not centred on 1; this then takes account of the formulation of equation A.21 treating the impact of these factors as a symmetric variance. There may, however, still be systematic errors relating to the target strength that are unaccounted for in these pdfs. These could be taken into account through sensitivity tests using alternative k_{ac}^S values.

Table B.1. Individual error factors for hydro-acoustic surveys of sardine biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to the model predicted biomass) in this document.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration (On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Variable ¹⁴
(Beam factor)	0.75 ¹³	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Variable ⁸
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

¹³ This was originally reported as 0.8 in Anon 2000, but subsequently corrected (I. Hampton pers. Comm.).

¹⁴ This was recorded in Anon. (2000) as random error denoting that it would be positive or negative rather than purely positive or negative.

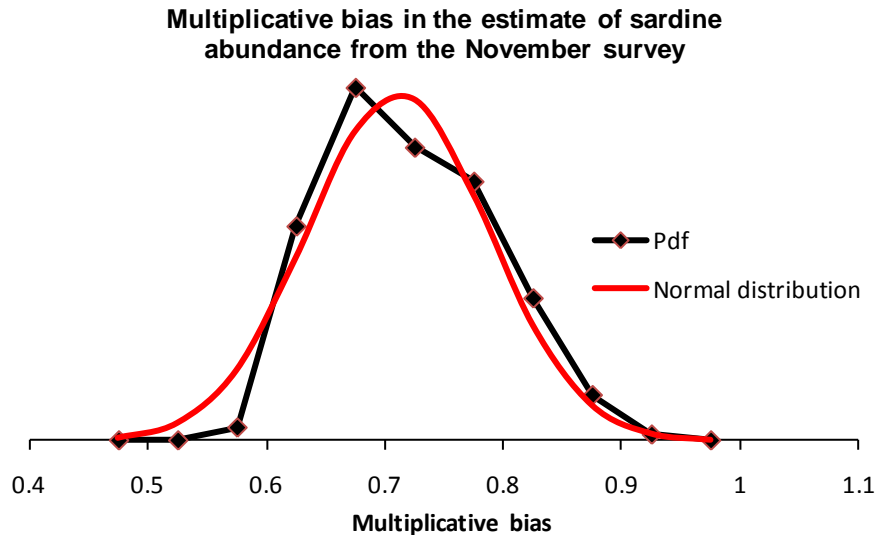


Figure B.1. The probability density function for the overall bias in the estimate of sardine abundance from the November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of constant error, together with the median values of the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $N(0.737, 0.077^2)$.

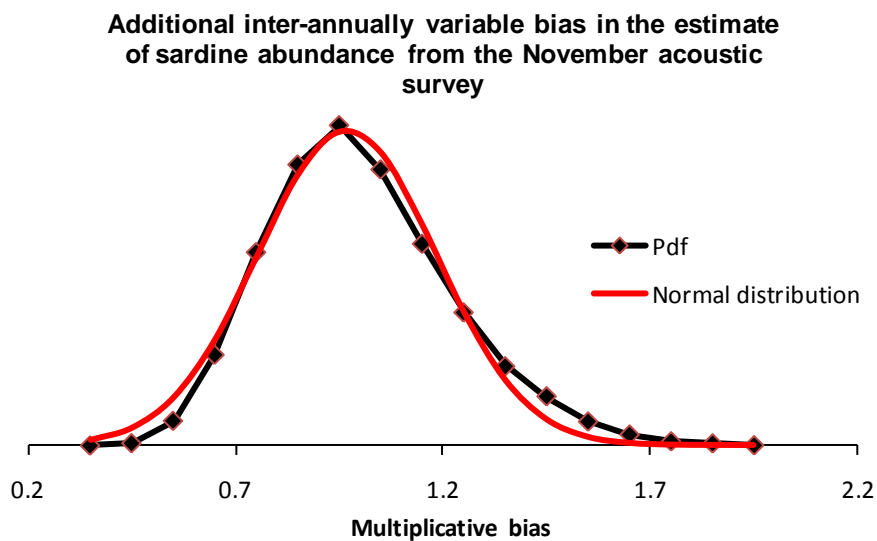


Figure B.2. The probability density function for the factors which cause bias in the sardine acoustic survey estimates and which vary inter-annually, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $N(0.969, 0.215^2)$. The CV of this distribution is thus $\phi_{ac}^S = 0.215/0.969 = 0.222$.

Appendix C: Glossary of parameters used in this document

Annual numbers and biomass:

$N_{j,y,a}^S$ - model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j

$B_{j,y}^S$ - model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year y , associated with the November survey

$SSB_{j,y}^S$ - model predicted spawning stock biomass (in thousand tonnes) of stock j at the beginning of November in year y

$w_{j,y,a}^S$ - mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y

$N_{j,y,r}^S$ - model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y

t_y^S - time lapsed (in months) between 1 May and the start of the recruit survey in year y

$move_y$ - proportion of west stock recruits which migrate to the east stock at the beginning of November of year y

Natural mortality:

$M_{a,y}^S$ - rate of natural mortality (in year⁻¹) of sardine of age a in year y

\bar{M}_{ju}^S - median juvenile rate of natural mortality (in year⁻¹)

\bar{M}_{ad}^S - median adult rate of natural mortality (in year⁻¹)

ε_y^{ad} - annual residuals about adult natural mortality

η_y^{ad} - normally distributed error used in calculating ε_y^{ad}

σ_{ad} - standard deviation in the annual residuals about adult natural mortality

σ_j - standard deviation in the annual residuals about juvenile natural mortality

ρ - annual autocorrelation coefficient in annual residuals about adult natural mortality

Catch:

$C_{j,y,a,q}^S$ - model predicted number (in billions) of sardine of age a of stock j caught during quarter q of year y

$C_{j,y,m,l}^{RLF}$ - number of fish in length class l landed in month m of year y of stock j (the ‘raised length frequency’)

$lcut_{y,m}$ - cut off length for recruits in month m of year y

$C_{j,y,q,a}^{bycatch}$ - the number of fish of age $a \geq 1$ from the anchovy-directed fishery in quarter q of year y

$S_{j,y,q,a}$ - commercial selectivity at age a during quarter q of year y of stock j

$F_{j,y,q}$ - fished proportion in quarter q of year y for a fully selected age class a of stock j , by the directed and redeye bycatch fisheries

$\tilde{C}_{j,y,Obs}^S$ - number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey

Proportions at age:

$p_{j,y,a}^S$ - model predicted proportion-at-age a of stock j in the November survey of year y

$S_{j,a}^{survey}$ - survey selectivity at age a in the November survey for stock j

$p_{j,y,q,a}^{com,S}$ - model predicted proportion-at-age a of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

Recruitment:

a_j^S - maximum recruitment of stock j (in billions)

b_j^S - spawner biomass above which there should be no recruitment failure risk in the hockey stick model for stock j

c^S - constant recruitment (distribution median) during the “peak” years of 2000 to 2004

$\varepsilon_{j,y}^S$ - annual lognormal deviation of sardine recruitment.

$\sigma_{j,r}^S$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve of stock j

$\sigma_{r,peak}^S$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve during peak years in the single stock hypothesis

Proportions at length and growth curve:

$p_{j,y,l}^S$ - model predicted proportion-at-length l of stock j associated with the November survey in year y

$A_{j,a,l}^{sur}$ - proportion of sardine of age a of stock j that fall in the length group l in November

$p_{j,y,q,l}^{coml,S}$ - model predicted proportion-at-length l of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

$A_{j,q,a,l}^{com}$ - proportion of sardine of age a of stock j that fall in the length group l in quarter q

- $L_{j,\infty}$ - maximum length of sardine of stock j
 κ_j - annual growth rate of sardine of stock j
 $t_{0,j}$ - age at which the growth rate is zero of sardine of stock j
 $\vartheta_{j,a}$ - standard deviation about the mean length for age a of sardine of stock j

Likelihoods:

- $\ln L^{Nov}$ - contribution to the negative log likelihood from the model fit to the November 1+ biomass data
- $\ln L^{rec}$ - contribution to the negative log likelihood from the model fit to the May recruit data
- $\ln L^{sur propa}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-age data
- $\ln L^{com propa}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-age data
- $\ln L^{sur propmin}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only
- $\ln L^{sur proppl}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only
- $\ln L^{com propmin}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the minus length class only
- $\ln L^{com proppl}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the remaining length classes
- $\hat{B}_{j,y}^S$ - acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y
- $\sigma_{j,y,Nov}^S$ - survey sampling CV associated with $\hat{B}_{j,y}^S$ that reflects survey inter-transect variance
- $k_{j,N}^S$ - constant of proportionality (multiplicative bias) associated with the November survey of stock j
- k_{ac}^S - multiplicative bias associated with the acoustic survey
- $\hat{N}_{j,y,r}^S$ - acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y
- $\sigma_{j,y,rec}^S$ - survey sampling CV associated with $\hat{N}_{j,y,r}^S$ that reflects survey inter-transect variance
- $k_{j,r}^S$ - constant of proportionality (multiplicative bias) associated with the recruit survey of stock j
- k_{cov}^S - multiplicative bias associated with the coverage of the recruits by the recruit survey in

- comparison to the 1+ biomass by the November survey
- k_{covE}^S - multiplicative bias associated with the coverage of the east stock recruits by the recruit survey in comparison to the west stock recruits during the same survey
- ϕ_{ac}^S - the CV associated with factors which cause bias in the acoustic survey estimates and which vary inter-annually;
- $(\lambda_{j,N/r}^S)^2$ - additional variance (over and above $\sigma_{y,Nov/rec}^S$ and ϕ_{ac}^S) associated with the November/recruit surveys of stock j
- $\hat{p}_{j,y,a}^S$ - estimate of the proportion (by number) of sardine of age a in stock j in the November survey of year y
- $n_{s,y}$ - number of fish from the November survey trawls in year y used to compile the age-length key for calculating $\hat{p}_{j,y,a}^S$
- $(\sigma_{j,p}^S)^2$ - overall variance-related parameter for the log-transformed survey proportion-at-age observations for stock j , $\hat{p}_{j,y,a}^S$ [note variance = $(\sigma_{j,p}^S)^2 / (n_y \hat{p}_{j,y,a}^S)$]
- ys - years for which ALKs are available to calculate proportion-at-age in the November survey ('93, '94, '96, '01, '03, '04, '06-'10);
- w_{propa}^{survey} - weighting applied to the survey proportion-at-age data
- $\hat{p}_{y,q,a}^{com,S}$ - estimate of the proportion (by number) of single-stock or "west stock" sardine of age a in the commercial catch of quarter q of year y
- $n_{y,q}^{com}$ - number of fish from the commercial trawls in quarter q of year y used to compile the age-length key for calculating $\hat{p}_{y,q,a}^{com,S}$
- $(\sigma_{com}^S)^2$ - overall variance-related parameter for the log-transformed commercial proportion-at-age observations, $\hat{p}_{y,q,a}^{com,S}$ [note variance = $(\sigma_{com}^S)^2 / (n_{y,q}^{com} \hat{p}_{y,q,a}^{com,S})$]
- yc/qc - years/quarters for which ALKs are available to calculate quarterly proportions-at-age in the commercial catch ('04 Q1-4, '06 Q2-4, '07 Q1-3, '08 Q4, '09 Q1);
- w_{propa}^{com} - weighting applied to the commercial proportion-at-age data
- $\hat{p}_{j,y,q,l}^{coml,S}$ - observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q of year y ;
- $w_{prophmin}^{com}$ - weighting applied to the commercial proportion at length data for the minus length class
- w_{propl}^{com} - weighting applied to the remaining commercial proportion at length data

$\sigma_{coml\min}^S$ - variance-related parameter for the log-transformed commercial proportion-at-length data of the minus length class

σ_{coml}^S - variance-related parameter for the log-transformed commercial proportion-at-length data

Other:

F_{init} - rate of fishing mortality assumed in the initial year

$s_{j,cor}^S$ - recruitment serial correlation for stock j

$\eta_{j,2009}^S$ - standardised recruitment residual value for 2009 for stock j

K_j^S - carrying capacity for stock j

K_{peak}^S - carrying capacity during peak years (only for single stock hypothesis)

$\bar{w}_{j,a}^S$ - mean mass (in grams) of sardine of age a from stock j sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available

Appendix D: Calculation of Loss to Predation for Sardine

The assessment model assumes catch is taken in four pulses during the year. For simplicity, this catch is totalled and assumed to be taken mid-year when calculating the loss of sardine to predation. The loss in numbers of age a of stock j in year y is calculated by:

$$P_{j,y,a}^S = N_{j,y-1,a-1}^S \left(1 - e^{-M_{a-1,y}^S/2}\right) + \left(N_{j,y-1,a-1}^S e^{-M_{a-1,y}^S/2} - C_{j,y,a-1}^S\right) \left(1 - e^{-M_{a-1,y}^S/2}\right) \quad y = y_1, \dots, y_n$$

$$P_{j,y,5+}^S = N_{j,y-1,4}^S \left(1 - e^{-M_{4,y}^S/2}\right) + \left(N_{j,y-1,4}^S e^{-M_{4,y}^S/2} - C_{j,y,4}^S\right) \left(1 - e^{-M_{4,y}^S/2}\right) \\ + N_{j,y-1,5+}^S \left(1 - e^{-M_{5+,y}^S/2}\right) + \left(N_{j,y-1,5+}^S e^{-M_{5+,y}^S/2} - C_{j,y,5+}^S\right) \left(1 - e^{-M_{5+,y}^S/2}\right) \quad y = y_1, \dots, y_n$$

$$\text{Where } C_{j,y,a}^S = \sum_q C_{j,y,q,a}^S.$$

The loss in biomass of fish of age a of stock j to predation in year y is therefore given by:

$$P_{j,y,a}^S = \left[N_{j,y-1,a-1}^S \left(1 - e^{-M_{a-1,y}^S/2}\right) + \left(N_{j,y-1,a-1}^S e^{-M_{a-1,y}^S/2} - C_{j,y,a-1}^S\right) \left(1 - e^{-M_{a-1,y}^S/2}\right) \right] \frac{1}{2} (w_{j,y-1,a-1} + w_{j,y,a}) \\ y = y_1, \dots, y_n$$

$$P_{j,y,5+}^S = N_{j,y-1,4}^S \left(1 - e^{-M_{4,y}^S/2}\right) + \left(N_{j,y-1,4}^S e^{-M_{4,y}^S/2} - C_{j,y,4}^S\right) \left(1 - e^{-M_{4,y}^S/2}\right) \frac{1}{2} (w_{j,y-1,4} + w_{j,y,5+}) \\ + N_{j,y-1,5+}^S \left(1 - e^{-M_{5+,y}^S/2}\right) + \left(N_{j,y-1,5+}^S e^{-M_{5+,y}^S/2} - C_{j,y,5+}^S\right) \left(1 - e^{-M_{5+,y}^S/2}\right) \frac{1}{2} (w_{j,y-1,5+} + w_{j,y,5+}) \\ y = y_1, \dots, y_n$$

The assumption is made that $w_{j,1983a} = w_{j,1984a}$, $a = 0, \dots, 5+$.

The total loss in sardine biomass of stock j to predation in year y is then given by:

$$P_{j,y}^S = \sum_{a=1}^{5+} P_{j,y,a}^S.$$