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Some Numerical Values of the Atomic Scattering Factor

By

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In table I of this note we give the values of a number of atomic scattering factors for X-rays (Atomformfactor f) which we have calculated theoretically. A full account of the method of calculation has been given in a paper in the Philosophical Magazine (1), but, by the courtesy of the Editors of that Journal, we are able to reproduce some of the numbers which we hope may be of some use to those engaged in the analysis of crystal structure.

The f-values given in the table are mostly those of the lighter elements. For the heavier elements, values based on the Thomas-Fermi (2) method of calculating the atomic fields are probably adequate for most purposes[1]). The conception of the atom as an electron gas must, however, be less applicable to the lighter atoms than to the heavier ones, and since we had at our disposal a number of atomic fields for the lighter atoms, calculated by the Hartree method of self-consistent fields (3), it seemed worth while to calculate f-curves from these, and to attempt to devise a method of interpolation which would give f-curves for atoms for which no fields had been calculated. The labour of calculating self-consistent fields for each atom would have been prohibitive. Hartree's method gives to a close approximation the value of $|\psi|^2$, where ψ is the wave function for each closed electron group at any point in the atom. According to the work of Wentzel (4), and of Waller (5), the scattering from the atom can be obtained by treating each element of volume dv as a classically scattering element of charge $|\psi|^2 \cdot dv$. Accordingly, the value of f is given by

$$f = \int_0^\infty 4\pi\, |\psi|^2 \frac{\sin(kr)}{kr} \cdot dr$$

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1) See for example W. L. Bragg and J. West, Z. Krist. **69**, 118. 1928.

where $k = 4\pi(\sin\theta)/\lambda$, θ being half the angle of scattering—or the glancing angle of reflexion, if we consider the X-ray spectra as formed by reflexion from the atomic planes of a crystal—and λ is the wave length of the radiation scattered. Quantitative work on rock salt (6), sylvine (7), and aluminium (8), in which proper corrections for the temperature factor were made, showed that the values of f calculated in this way for Na^+, K^+, Cl^- and Al were in good agreement with the observed values, and this gives some justification for employing the method to calculate other f-curves.

Atomic fields have been calculated by Hartree's method for He, Li, O, F^-, Ne, Na, Na^+, Al^+, Al^{++}, Al^{+++}, Cl^-, K^+, Ca^{++} and Rb^+. Some of the values have been published in the papers already quoted, but for others we are indebted to Prof. Hartree and his fellow workers. On the f-curves for these elements the method of interpolation was based. For a detailed account fo the method, reference must be made to the paper in the Philosophical Magazine, and an outline must suffice here.

Interpolations were made between the values of f for the individual electron groups, and not between those for the atoms as a whole. If f for a single electron of a given group is plotted against $1/(Z - s) \cdot (\sin\theta)/\lambda$ for a series of atoms, Z being the atomic number and s a "screening parameter", it is found possible to make the curves for all the different atoms practically coincident by a proper choice of s. In this way a common f-curve, which is characteristic of the group, is obtained. For a given electron group, the parameter s varies from atom to atom in a manner which is regular enough to allow of the interpolation of s for atoms for which the f-curves are not known. Once such interpolations have been made, the f-curves for the electron group under consideration can be calculated for these atoms, as a function of $(\sin\theta)/\lambda$, from the common f-curve. This process is repeated for each group of electrons in the atom, and so the f-curve for the whole atom can be built up. In the more detailed paper, tables are given by means of which this interpolation can be carried out for a number of electron groups.

A critical discussion of the method indicates that for elements of atomic number less than about 25 the values of f calculated in this way should not differ by more than about 0.1 to 0.2 from those which would have been calculated from the self-consistent field. Of the atoms on which the interpolations are based, only Al^+, Cl^-, K^+, Ca^{++} and Rb^+ contain M-electrons at all, so that data for reliable common curves for these electrons are lacking. Fortunately the contributions of such electrons to f are very small, except for quite small values of $(\sin\theta)/\lambda$, so that the lack of detailed knowledge is of less importance than might be supposed. In table I we have included the values of f for copper, which were calculated by the method of interpolation. In copper the complete M-group is present, so that it is a case very unfavourable to the method. We have also given the values for the same element calculated from an approximation to the self-consistent field given to us by Prof. Hartree, and, for completeness, the values calculated from the Thomas-Fermifield. It will be seen that the general agreement between the three curves is fairly good, although there are considerable divergences. In table I, the values of f for a number of states of ionisation of some of the atoms are given. The values at the smaller angles, where the curves differ, must be regarded as very approximate, since adding another electron to a group causes a general loosening up of that group, which has not as a rule been taken fully into account. The curves must be taken as giving only an indication of the effect of different numbers of outer electrons. Except for small values of $(\sin\theta)/\lambda$, f differs very little for the different states of ionisation. For a discussion of this question, reference may be made to a paper by the writers and R. G. Wood (8), in which the contributions of the different electron groups to f in the case of aluminium are given.

For values of Z greater than 25, the f-curves calculated from the Thomas-Fermifield are probably as accurate as any that can at present be obtained. Some values calculated by this method have already been tabulated by W. L. Bragg and J. West (10) in a paper in this Journal. For the sake of completeness we give in table II a series of values of f, calculated from the Thomas-field for Caesium, for 35 of the more important atoms with Z greater than 22. It will be remembered that the Thomas-f-curve for any atom Z can be obtained from that for a standard atom Z_0, simply by a change of scale both in ordinates and abscissae. To obtain the value of f for $(\sin\theta)/\lambda = x$ for the atom Z we take the value of f from the curve for Z_0 at $(\sin\theta)/\lambda = x \cdot (Z_0/Z)^{1/3}$ and multiply it by Z/Z_0. Thomas worked out the charge distribution for caesium, and the f-curve calculated from this by Bragg and West was taken as the standard. For convenience of reference, the values are reproduced in table III.

It should be noted that the methods of calculating f used in this paper are only valid if the frequency of the scattered radiation is considerably less than the absorption frequencies of any of the electrons of the scattering atom. Now, with the radiations usually employed, this will certainly not be the case for any of the heavier elements included in the table. As the frequency of the radiation approaches and passes the absorption frequency of an electron group, the contribution of that group to f will be greatly modified. The group may cease to contribute, or its contribution may even oppose that from the rest of the atom. The existence of effects of this kind must be kept in mind when using tables of f-values. If molybdenum radiation is employed, for example, they may be expected for scatterers for which Z is greater than about 40.

It is perhaps necessary to emphasise the fact that the f-curves in the tables refer to the free atom, supposed in a state of rest. In a crystal, the packing of the atoms must disturb the outer electrons. This will affect f only for small values of $(\sin\theta)/\lambda$ and is not very important. A much more important matter is the heat motion of the atoms, which will have the effect of reducing f (9), especially for the larger values of $(\sin\theta)/\lambda$. The effect is not a small one. For soft crystals, the amplitudes of vibration may be of the order of 0.2 Å, even at ordinary temperatures, and f for high orders of spectra may be reduced to as little as one half or one third of its value for the atom at rest. This question also is discussed in the more detailed paper, and methods are there suggested for getting some idea of the order of the correction. No simple general method can be given, for heat motions differ not only from crystal to crystal, but also from atom to atom, and from direction to direction in the same crystal. It is very necessary to be alive to the importance of this effect. We have denoted f for the atom at rest by f_0.

Table I.

Atomic Scattering Factors f_0.

$\frac{\sin\Theta}{\lambda}\cdot 10^{-8}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	Remarks
H	1.0	0.81	0.48	0.25	0.13	0.07	0.04	0.03	0.02	0.01	0.00	0.00	*W*
He	2.0	1.88	1.46	1.05	0.75	0.52	0.35	0.24	0.18	0.14	0.11	0.09	*H*
Li^{+}	2.0	1.96	1.8	1.5	1.3	1.0	0.8	0.6	0.5	0.4	0.3	0.3	*H*
Li (neut)	3.0	2.2	1.8	1.5	1.3	1.0	0.8	0.6	0.5	0.4	0.3	0.3	*H*
Be^{++}	2.0	2.0	1.9	1.7	1.6	1.4	1.2	1.0	0.9	0.7	0.6	0.5	*I*
Be (neut)	4.0	2.9	1.9	1.7	1.6	1.4	1.2	1.0	0.9	0.7	0.6	0.5	*I*
B^{+++}	2.0	2.0	1.9	1.8	1.7	1.6	1.4	1.3	1.2	1.0	0.9	0.7	*I*
B (neut)	5.0	3.5	2.4	1.9	1.7	1.5	1.4	1.2	1.2	1.0	0.9	0.7	*I*
C	6.0	4.6	3.0	2.2	1.9	1.7	1.6	1.4	1.3	1.2	1.0	0.9	*I*
N^{+5}	2.0	2.0	2.0	1.9	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.16	*I*
N^{+3}	4.0	3.7	3.0	2.4	2.0	1.8	1.65	1.55	1.5	1.4	1.3	1.15	*I*
N (neut)	7.0	5.8	4.2	3.0	2.3	1.9	1.65	1.55	1.5	1.4	1.3	1.15	*I*

Table I (continuation).

$\frac{\sin \Theta}{\lambda} \cdot 10^{-8}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	Remarks
O (neut)	8.0	7.1	5.3	3.9	2.9	2.2	1.8	1.6	1.5	1.4	1.35	1.25	*H*
O^{-2}	10.0	8.0	5.5	3.8	2.7	2.1	1.8	1.6	1.5	1.4	1.35	1.25	*I* + *H*
F^{-}	10.0	8.7	6.7	4.8	3.5	2.8	2.2	1.9	1.7	1.55	1.5	1.35	*H*
F (neut)	9.0	7.8	6.2	4.45	3.35	2.65	2.15	1.9	1.7	1.6	1.5	1.35	*H*
Ne	10.0	9.3	7.5	5.8	4.4	3.4	2.65	2.2	1.9	1.65	1.55	1.5	*I*
Na^{+}	10.0	9.5	8.2	6.7	5.25	4.05	3.2	2.65	2.25	1.95	1.75	1.6	*H*
Na	11.0	9.65	8.2	6.7	5.25	4.05	3.2	2.65	2.25	1.95	1.75	1.6	*H*
Mg^{++}	10.0	9.75	8.6	7.25	5.05	4.8	3.85	3.15	2.55	2.2	2.0	1.8	*I*
Mg	12.0	10.5	8.6	7.22	5.05	4.8	3.85	3.15	2.55	2.2	2.0	1.8	*I*
Al^{+++}	10.0	9.7	8.9	7.8	6.65	5.5	4.45	3.65	3.1	2.65	2.3	2.0	*H*
Al^{++}	11.0	10.3	9.0	7.75	6.6	5.5	4.5	3.7	3.1	2.65	2.3	2.0	*H*
Al^{+}	12.0	10.9	9.0	7.75	6.6	5.5	4.5	3.7	3.1	2.65	2.3	2.0	*H*
Al	13.0	11.0	8.95	7.75	6.6	5.5	4.5	3.7	3.1	2.65	2.3	2.0	*H* + *I*
Si^{+4}	10.0	9.75	9.15	8.25	7.15	6.05	5.05	4.2	3.4	2.95	2.6	2.3	*H*
Si^{++}	12.0	11.1	9.55	8.2	7.15	6.05	5.05	4.2	3.4	2.95	2.6	2.3	*H* + *I*
Si	14.0	11.35	9.4	8.2	7.15	6.1	5.1	4.2	3.4	2.95	2.6	2.3	*H* + *I*
P^{+5}	10.0	9.8	9.25	8.45	7.5	6.55	5.65	4.8	4.05	3.4	3.0	2.6	*I*
P (neut)	15.0	12.4	10.0	8.45	7.45	6.5	5.65	4.8	4.05	3.4	3.0	2.6	*I*
P^{-3}	18.0	12.7	9.8	8.4	7.45	6.5	5.65	4.85	4.05	3.4	3.0	2.6	*I*
S (neut)	16.0	13.6	10.7	8.95	7.85	6.85	6.0	5.25	4.5	3.9	3.35	2.9	*I*
S^{+6}	10.0	9.85	9.4	8.7	7.85	6.85	6.05	5.25	4.5	3.9	3.35	2.9	*I*
S^{-2}	18.0	14.3	10.7	8.9	7.85	6.85	6.0	5.25	4.5	3.9	3.35	2.9	*I*
Cl	17.0	14.6	11.3	9.25	8.05	7.25	6.5	5.75	5.05	4.4	3.85	3.35	*H* + *I*
Cl^{-}	18.0	15.2	11.5	9.3	8.05	7.25	6.5	5.75	5.05	4.4	3.85	3.35	*H*
A	18.0	15.9	12.6	10.4	8.7	7.8	7.0	6.2	5.4	4.7	4.1	3.6	*I*
K^{+}	18.0	16.5	13.3	10.8	8.85	7.75	7.05	6.44	5.9	5.3	4.8	4.2	*H*
Ca^{++}	18.0	16.8	14.0	11.5	9.3	8.1	7.35	6.7	6.2	5.7	5.1	4.6	*H*
Sc^{+3}	18.0	16.7	14.0	11.4	9.4	8.3	7.6	6.9	6.4	5.8	5.35	4.75	*I*
Ti^{+4}	18.0	17.0	14.4	11.9	9.9	8.5	7.85	7.3	6.7	6.15	5.65	5.05	*I*
Ti^{++}	20.0	18.7	15.5	12.5	10.1	8.5	7.8	7.25	6.7	6.15	5.65	5.05	*I*
Cu^{+}	28.0	27.0	24.0	20.7	17.3	14.0	11.3	9.4	8.0	7.3	7.0	6.7	*I*
Cu^{+}	28.0	26.3	23.0	19.2	15.8	13.0	11.2	9.7	8.4	7.4	6.7	6.5	*H* approx
Cu	29.0	25.8	21.4	17.8	15.2	13.3	11.7	10.2	9.1	8.1	7.3	6.7	*T*
Rb^{+}	36.0	33.6	28.7	24.6	21.4	18.9	16.7	14.6	12.8	11.2	9.9	8.9	*H*
Rb	37.0	33.4	28.2	23.6	20.4	17.9	15.9	14.0	12.4	11.2	10.2	9.3	*T*

I calculated by method of interpolation.

H calculated from Hartree distribution.

T calculated from Thomas model.

W calculated from hydrogen wave-function (ground state).

Table II.

Values of f_0 calculated from the Thomas field.

$\frac{\sin\Theta}{\lambda}\cdot 10^{-8}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
Va	23.0	20.3	16.6	13.5	11.6	10.1	8.7	7.6	6.7	6.0	5.4	4.8
Cr	24.0	21.4	17.4	14.3	12.2	10.6	9.2	8.1	7.2	6.4	5.7	5.2
Mn	25.0	22.1	18.2	14.9	12.8	11.2	9.6	8.5	7.5	6.7	6.0	5.5
Fe	26.0	23.1	19.1	15.6	13.4	11.7	10.2	8.9	7.9	7.1	6.4	5.8
Co	27.0	24.0	19.9	16.3	13.9	12.2	10.7	9.3	8.2	7.4	6.7	6.0
Ni	28.0	24.9	20.7	17.0	14.6	12.8	11,2	9.8	8.7	7.8	7.0	6.4
Cu	29.0	25.8	21.4	17.8	15.2	13.3	11.7	10.2	9.1	8.1	7.3	6.7
Zn	30.0	26.9	22.4	18.6	15.8	14.0	12.2	10.6	9.5	8.5	7.7	7.0
Ga	31.0	27.8	23.3	19.3	16.5	14.5	12.7	11.2	9.9	8.9	8.1	7.3
Ge	32.0	28.7	24.1	20.0	17.1	15.1	13.3	11.7	10.3	9.3	8.4	7.6
As	33.0	29.6	24.9	20.7	17.7	15.6	13.8	12.1	10.7	9.7	8.8	7.9
Se	34.0	30.6	25.8	21.5	18.4	16.2	14.3	12.6	11.2	10.1	9.1	8.2
Br	35.0	31.5	26.6	22.2	19.0	16.7	14.8	13.0	11.6	10.4	9.5	8.6
Rb	37.0	33.4	28.2	23.6	20.4	17.9	15.9	14.0	12.4	11.2	10.2	9.3
Sr	38.0	34.4	29.2	24.5	20.9	18.5	16.4	14.5	12.9	11.5	10.5	9.6
Y	39.0	35.4	30.0	25.3	21.6	19.1	17.0	15.0	13.3	12.0	10.9	9.9
Zr	40.0	36.4	30.9	26.1	22.3	19.7	17.5	15.5	13.9	12.4	11.2	10.3
Mo	42.0	38.2	32.6	27.5	23.5	20.8	18.6	16.5	14.8	13.2	12.0	11.0
Rh	45.0	41.1	35.1	29.8	25.4	22.6	20.2	18.0	16.1	14.5	13.1	12.0
Pd	46.0	42.0	36.0	30.6	26.2	23.1	20.7	18.6	16.6	14.9	13.5	12.4
Ag	47.0	43.0	36.9	31.3	26.8	23.7	21.3	19.0	17.0	15.3	13.9	12.8
Cd	48.0	43.9	37.8	32.1	27.5	24.3	21.8	19.5	17.4	15.7	14.3	13.2
Sn	50.0	45.9	39.4	33.6	28.9	25.5	23.0	20.5	18.5	16.6	15.0	13.8
Sb	51.0	46.8	40.3	34.5	29.6	26.1	23.5	21.1	18.9	17.0	15.4	14.1
Te	52.0	47.7	41.1	35.2	30.2	26.7	24.0	21.6	19.3	17.4	15.8	14.4
I	53.0	48.7	42.0	36.0	30.9	27.4	24.6	22.1	19.8	17.9	16.2	14.8
Cs	55.0	50.7	43.8	37.6	32.4	28.7	25.8	23.2	20.8	18.8	17.0	15.6
Ba	56.0	51.7	44.8	38.4	33.1	29.3	26.4	23.7	21.3	19.2	17.4	16.0
Ta	73.0	68.1	59.7	52.1	45.2	40.1	36.2	32.9	29.9	27.1	24.7	22.6
W	74.0	69.0	60.5	53.0	46.0	40.6	36.8	33.5	30.4	27.6	25.1	23.0
Pt	78.0	72.9	64.2	56.3	48.9	43.3	39.1	35.7	32.5	29.5	26.9	24.7
Au	79.0	74.0	65.0	57.0	49.7	43.9	39.8	36.3	33.0	30.0	27.4	25.1
Hg	80.0	74.9	65.9	57.8	50.3	44.6	40.3	36.9	33.5	30.5	27.8	25.4
Tl	81.0	75.8	66.8	58.6	51.2	45.2	41.0	37.5	34.0	30.9	28.3	25.8
Pb	82.0	76.9	67.8	59.5	51.9	45.9	41.6	38.0	34.6	31.4	28.7	26.2
Bi	83.0	77.9	68.7	60.3	52.7	46.6	42.2	38.6	35.2	31.9	29.2	26.8

Table II.

Values of f_0 calculated from the Thomas field.

$\frac{\sin\Theta}{\lambda}\cdot 10^{-8}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
Va	23.0	20.3	16.6	13.5	11.6	10.1	8.7	7.6	6.7	6.0	5.4	4.8
Cr	24.0	21.4	17.4	14.3	12.2	10.6	9.2	8.1	7.2	6.4	5.7	5.2
Mn	25.0	22.1	18.2	14.9	12.8	11.2	9.6	8.5	7.5	6.7	6.0	5.5
Fe	26.0	23.1	19.1	15.6	13.4	11.7	10.2	8.9	7.9	7.1	6.4	5.8
Co	27.0	24.0	19.9	16.3	13.9	12.2	10.7	9.3	8.2	7.4	6.7	6.0
Ni	28.0	24.9	20.7	17.0	14.6	12.8	11,2	9.8	8.7	7.8	7.0	6.4
Cu	29.0	25.8	21.4	17.8	15.2	13.3	11.7	10.2	9.1	8.1	7.3	6.7
Zn	30.0	26.9	22.4	18.6	15.8	14.0	12.2	10.6	9.5	8.5	7.7	7.0
Ga	31.0	27.8	23.3	19.3	16.5	14.5	12.7	11.2	9.9	8.9	8.1	7.3
Ge	32.0	28.7	24.1	20.0	17.1	15.1	13.3	11.7	10.3	9.3	8.4	7.6
As	33.0	29.6	24.9	20.7	17.7	15.6	13.8	12.1	10.7	9.7	8.8	7.9
Se	34.0	30.6	25.8	21.5	18.4	16.2	14.3	12.6	11.2	10.1	9.1	8.2
Br	35.0	31.5	26.6	22.2	19.0	16.7	14.8	13.0	11.6	10.4	9.5	8.6
Rb	37.0	33.4	28.2	23.6	20.4	17.9	15.9	14.0	12.4	11.2	10.2	9.3
Sr	38.0	34.4	29.2	24.5	20.9	18.5	16.4	14.5	12.9	11.5	10.5	9.6
Y	39.0	35.4	30.0	25.3	21.6	19.1	17.0	15.0	13.3	12.0	10.9	9.9
Zr	40.0	36.4	30.9	26.1	22.3	19.7	17.5	15.5	13.9	12.4	11.2	10.3
Mo	42.0	38.2	32.6	27.5	23.5	20.8	18.6	16.5	14.8	13.2	12.0	11.0
Rh	45.0	41.1	35.1	29.8	25.4	22.6	20.2	18.0	16.1	14.5	13.1	12.0
Pd	46.0	42.0	36.0	30.6	26.2	23.1	20.7	18.6	16.6	14.9	13.5	12.4
Ag	47.0	43.0	36.9	31.3	26.8	23.7	21.3	19.0	17.0	15.3	13.9	12.8
Cd	48.0	43.9	37.8	32.1	27.5	24.3	21.8	19.5	17.4	15.7	14.3	13.2
Sn	50.0	45.9	39.4	33.6	28.9	25.5	23.0	20.5	18.5	16.6	15.0	13.8
Sb	51.0	46.8	40.3	34.5	29.6	26.1	23.5	21.1	18.9	17.0	15.4	14.1
Te	52.0	47.7	41.1	35.2	30.2	26.7	24.0	21.6	19.3	17.4	15.8	14.4
I	53.0	48.7	42.0	36.0	30.9	27.4	24.6	22.1	19.8	17.9	16.2	14.8
Cs	55.0	50.7	43.8	37.6	32.4	28.7	25.8	23.2	20.8	18.8	17.0	15.6
Ba	56.0	51.7	44.8	38.4	33.1	29.3	26.4	23.7	21.3	19.2	17.4	16.0
Ta	73.0	68.1	59.7	52.1	45.2	40.1	36.2	32.9	29.9	27.1	24.7	22.6
W	74.0	69.0	60.5	53.0	46.0	40.6	36.8	33.5	30.4	27.6	25.1	23.0
Pt	78.0	72.9	64.2	56.3	48.9	43.3	39.1	35.7	32.5	29.5	26.9	24.7
Au	79.0	74.0	65.0	57.0	49.7	43.9	39.8	36.3	33.0	30.0	27.4	25.1
Hg	80.0	74.9	65.9	57.8	50.3	44.6	40.3	36.9	33.5	30.5	27.8	25.4
Tl	81.0	75.8	66.8	58.6	51.2	45.2	41.0	37.5	34.0	30.9	28.3	25.8
Pb	82.0	76.9	67.8	59.5	51.9	45.9	41.6	38.0	34.6	31.4	28.7	26.2
Bi	83.0	77.9	68.7	60.3	52.7	46.6	42.2	38.6	35.2	31.9	29.2	26.8

Table III.

f_0-Values for *Cs* calculated on the Thomas-Fermi-distribution.

$\frac{\sin\Theta}{\lambda}\cdot 10^{-8}$	f	$\frac{\sin\Theta}{\lambda}\cdot 10^{-8}$	f
0	55	1.0	17.0
0.1	50.7	1.1	15.6
0.2	43.8	1.2	14.5
0.3	37.6	1.3	13.2
0.4	32.4	1.4	12.3
0.5	28.7	1.5	11.3
		1.6	10.4
0.6	25.8	1.7	9.6
0.7	23.2	1.8	9.2
0.8	20.8	1.9	8.6
0.9	18.8	2.0	8.1

References.

1. R. W. James and G. W. Brindley, Phil. Mag. July 1931.
2. L. H. Thomas, Camb. Phil. Soc. Proc. **23**, 542. 1927; E. Fermi, Z. Physik **48**, 72. 1928.
3. D. R. Hartree, Camb. Phil. Soc. Proc. **24**, 89, 111. 1928.
4. G. Wentzel, Z. Physik **43**, 1, 779. 1927.
5. I. Waller, Phil. Mag. 4, 1228. 1927; Nature, July 30. 1927.
6. R. W. James and E. M. Firth, Roy. Soc. Proc. A **117**, 62. 1927; I. Waller and R. W. James, ibid. **117**, 214. 1927; R. W. James, I. Waller and D. R. Hartree, ibid. A **118**, 334. 1928.
7. R. W. James and G. W. Brindley, Roy. Soc. Proc. A **121**, 155. 1928.
8. R. W. James, G. W. Brindley and R. G. Wood, Roy. Soc. Proc. A. **125**, 401. 1929.
9. P. Debye, Ann. Physik **43**, 49. 1914; I. Waller, Upsala, Dissert. 1925.
10. W. L. Bragg and J. West, Z. Krist. **69**, 118. 1928.

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