University of Cape Town - Department of Physics
Honours Computational Physics

TOPIC 1 - BASIC MONTE CARLO : SAMPLING - COMBINATION ANALYTIC-REJECTION METHOD

This worksheet accompanies the EJS simulation `BasicMC_No6_ComboAnalyticRejection.jar`

The challenge: We need to sample according to a distribution $p(y)$ with $y \in [a, b]$ but have access just to a uniform random generator.

As already seen, the rejection and transformation methods provide two possibilities. However, the rejection method is often incredibly wasteful. Furthermore, some distributions do not lend themselves to sampling by the transformation method (perhaps the integral $\int_a^y p(y')\,dy'$ cannot be performed analytically). However, if one is able to identify a distribution function $f(y)$ that is easily sampled by the transformation method and that matches $p(y)$ in its main features, then a combination of the rejection and transformation methods offers a solution. The only other requirement to be met is that a constant C must exist such that $p(y) \leq Cf(y)$ for all $y \in [a, b]$.

The so-called **'combination analytic-rejection method'** then proceeds as follows:

- Use the transformation method to generate a random y_i according to the distribution $f(y)$
- Generate a z_i uniformly drawn from the interval $[0, Cf(y_i)]$
- If $z_i \leq p(y_i)$ then y_i is accepted, else it is rejected
- Repeat until enough samples are collected
- The set $\{y_i\}$ then follows the distribution $p(y)$

Although this method does involve some wasted effort (some generated numbers are rejected), the acceptance rate is far higher than in the pure rejection method.

Questions:

1. Generate numbers according to the distribution $p(y) = 1/(3.037908621) \times \left(\sin^2(y) + 1\right)/y^{4/3}$ with $1 \leq y \leq 20$. How many times faster is the combination analytic-rejection method than the pure rejection method?

2. Use the combination analytic-rejection method to sample the following (unnormalised) distribution:
$$p(y) = \exp(\cos y) \qquad \text{with } -\pi \leq y \leq \pi.$$