University of Cape Town - Department of Physics
Honours Computational Physics

TOPIC 1 - BASIC MONTE CARLO : SAMPLING - REJECTION METHOD

This worksheet accompanies the EJS simulation `BasicMC_No4_RejectionMethod.jar`

The challenge: We need to sample according to a distribution $p(y)$ with $y \in [a, b]$ but have access just to a uniform random generator.

One way to achieve this task is to sketch the target distribution $p(y)$ and then to construct a bounding box around $p(y)$ such that the height of the box $z_{\max}$ is no smaller than the largest value of $p(y)$ in the interval. The so-called **'rejection method'** then entails:

- Use the uniform random generator to return a random y_i in the interval $[a, b]$
- Use the uniform random generator to return a random z_i in the interval $[0, z_{\max}]$
- If $z_i \leq p(y_i)$ then accept y_i, else reject y_i
- Repeat until enough samples are collected
- The set $\{y_i\}$ then follows the distribution $p(y)$

The downside is that we do redundant work ('miss numbers' are discarded). Therefore, it is essential that the smallest bounding box be chosen. One advantage of this method, however, is that the distribution does not need to be normalised.

Questions:

1. Use the rejection method to sample according to the distribution,
$$p(y) = 1/(4.6997309) \times \left(\sin^2(y) + 1\right)/y^{4/3},$$
defined over the range $1 \leq y < \infty$. Explain all assumptions made and their implications.

2. Consider the following probability distribution functions:

   i) $p(y) = \cos(2y)$ defined on the region $-\pi/4 \leq y \leq \pi/4$;

   ii) $p(y) = \frac{1}{\sqrt{8}-\sqrt{3}} \frac{y}{\sqrt{y^2-1}}$ defined on the region $2 \leq y \leq 3$;

   iii) $p(y) = \frac{1}{\sqrt{8}} \frac{y}{\sqrt{y^2-1}}$ defined on the region $1 < y \leq 3$.

   Where possible, use the rejection method to generate random numbers according to these distributions. In cases where this is not possible, explain the reason(s) for the failure.

3. Consider the (unnormalised) distribution,
$$p(y) = \exp(\cos y),$$
with $y \in [-\pi, \pi]$. Use the rejection method to sample according to this distribution. Determine the acceptance rate.