University of Cape Town - Department of Physics
Honours Computational Physics

TOPIC 1 - BASIC MONTE CARLO : 2D RANDOM WALK

This worksheet accompanies the EJS simulation `BasicMC_No3_RandomWalk.jar`

Consider the case of a 2D random walk from the origin $(0, 0)$ where the components of the displacement in a particular step, Δx_i and Δy_i, are each drawn from flat distributions in the interval $\left[-\frac{1}{2}, +\frac{1}{2}\right]$. The distance travelled in step i is given by,

$$r_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2},$$

while the magnitude of the displacement from the origin after N steps is:

$$R = \sqrt{\left(\sum_{i=1}^{N} \Delta x_i\right)^2 + \left(\sum_{i=1}^{N} \Delta y_i\right)^2}.$$

Questions:

1. Given the distributions for Δx and Δy, complete the following tasks.
   (a) Draw a simple sketch of the distribution of step lengths r. Justify your answer and be sure to include the correct range of r values.
   (b) Calculate the root-mean-square step length r_{rms}.
   (c) What is the average vector displacement from the origin after N steps?
   (d) Derive the relationship between R_{rms} (the root-mean-square distance from the origin on completion of the walk) and the number of steps N. Make sure that you understand the nature of the averaging process.

2. Now explore the associated EJS model.
   (a) Does the 'Step Length Distribution' agree with your earlier prediction?
   (b) Can you make sense of the 'Displacement Histogram' and the plot of endpoints? Investigate the effect of the number of steps taken in each walk and the number of trials generated.
   (c) Use the simulation to check your prediction for the dependence of R_{rms} on N and r_{rms}.

   (Right-click inside any of the frames and select
   `Elements options→Root-Mean-Square Displacement Dependence→Analyze data`
   to fit a curve to the R_{rms} vs. $\sqrt{N}$ graph)