University of Cape Town - Department of Physics
Honours Computational Physics

TOPIC 1 - BASIC MONTE CARLO : RADIOACTIVE DECAY POISSON STATS

This worksheet accompanies the EJS simulation `BasicMC_No2_RadioactiveDecayPoisson.jar`

Radioactive decay is a random process. Therefore, it is impossible to say when a particular nucleus will decay. However, one can define a sample decay rate Λ, such that the average number of decays of the sample in a time interval dt is given by $\Lambda\,dt$. The sample decay rate Λ depends on the decay constant λ for the nuclide concerned, as well as the number of nuclei $N(t)$ present at time t. As such, it is not a constant, but will show little change for large samples over time scales much shorter than the lifetime of the nuclide.

Questions:

1. Write pseudo-code to simulate the decay of a very large number of nuclei and determine the distribution of decays measured in a counting time interval T.

2. The probability of n decays being observed in a certain counting interval, when the average number of decays per counting interval is μ, is given by:

$$P(n;\mu) = \frac{\mu^n e^{-\mu}}{n!},$$

   (i.e. Poisson statistics apply).

   (a) Present a clear derivation of the probability of counting n random events in a time interval, when the average number is μ (i.e. derive the Poisson distribution).

   (b) For a Poisson distribution, prove each of the following:

   i) $\langle n \rangle = \mu$,
   ii) $\langle n^2 \rangle = \mu(\mu + 1)$,
   iii) $\langle (n - \mu)^2 \rangle = \mu$,

   where $\langle f \rangle$ signifies the mean value of f and μ is the average number of counts in the counting interval.

   (c) For suitably large μ, show that the Poisson distribution is well approximated by a Gaussian distribution.

3. Run the associated simulation for the specific case of $\Lambda = 1.2$ s^{-1} and counting times of 10 s and 1 s. How are the results similar? How are they different? How well do the results in each case match the Poisson and Gaussian predictions?

4. Write pseudo-code to generate the distribution of times between successive decays of radioactive nuclei from a sample with sample decay rate Λ.

5. Use the EJS simulation to investigate the distribution of waiting times of nuclei for the specific choice of $\Lambda = 1.2$ s^{-1}. Investigate the dependence of the results on the sample decay rate and counting time interval.

6. Suppose that the mean counting rate of a certain detector of random events is 4 counts per second. What is the probability of obtaining zero counts in a one second counting interval? What is the most likely interval between successive counts?

7. Some experiments have such painfully slow counting rates that the experimenter may begin to question the performance of even the most reliable equipment. Suppose you are running an experiment that yields no counts in 8 hours and two counts in the 9th hour. What is the likelihood that the equipment is malfunctioning?