



## **COURSE I LABORATORY**

### **PRACTICALS - PART I**

(version: 2018)

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*(Cut these out and attach to the front of your report when you submit)*

**Name:** \_\_\_\_\_

# 1 Investigating Hooke's Law

## Learning objectives:

- Prepare and structure a laboratory report.
- Take a series of measurements.
- Analyse the data by tabulating the readings.
- Analyse the data by drawing a graph and fitting a trendline.

## Instructions:

*Pre-reading:* read through the experiment notes as below and Section A of the "Guide to Reporting and Measurement".

*Practical:* spend 10 minutes discussing what to do with your lab partners, and make any rough work on the back pages of your report book. You can work together during the practical, however you must write your own and unique full write-up of the experiment to be submitted by 17h00 on the day of the practical. Cut out the corresponding assessment scheme at the end of this book and staple it to the front of your report when you submit the report for assessment.

### 1.1 Introduction

You are supplied with a spiral spring suspended from a retort stand, a small bucket, a number of steel ball bearings, and a metre stick. Spend several minutes exploring the function of the equipment. What happens if you add the bucket and some of the ball bearings to the end of the spring?

You should have observed that the application of a force on the spring due to the mass of the bucket and ball bearings will cause the extension of the spring. What is the relationship between the applied force and extension of the spring - is it linear, where doubling the applied force will double the extension? Hooke's Law states that the force,  $F$ , required to stretch a spiral spring is directly proportional to the extension,  $x$ , of the spring, as described mathematically by,

$$F = kx, \quad \dots (1-1)$$

where the spring constant,  $k$ , is a measure of the stiffness of the spring.

Your task is to design an experiment to determine the relationship between the *applied force* and *spring extension*, and if the relationship is linear as suggested by Hooke's Law, determine a value for the spring constant. Additional instructions are included below to guide you through the experiment, and you can add your own steps. After completing the experiment, prepare a full report on the experiment and its results.

### 1.2 Collecting data

Use a triple-beam balance to measure the mass of any one of the ball bearings.

Measure the spring extension relative to a reference point: attach the empty bucket to the end of the spring and use a metre stick to determine the height of the pointer above the table top. Now add the ball bearings to the bucket, one at a time, and record the new position of the pointer as each ball is added. Take as many readings as you can.

### 1.3 Present the data in a table

Record the raw data (*e.g.* Table 1.1), which are the number of balls in the bucket and the values of pointer position you read directly from the metre stick (without performing any calculations).

Use  $g = 9.80 \text{ m/s}^2$  and the mass of the balls to calculate the additional force being applied to the spring as each ball is placed in the bucket and the reading of the pointer position to determine the extension of the spring (*e.g.* Table 1.2).

Pay attention to the number of significant figures and units from which you read values from the ruler, and in your calculations.

**TABLE 1.1: Readings of the pointer position as balls were added.**

Number of balls in the bucket	Reading of pointer position (cm)
0	23.7
1	29.3
2	35.2
3	...
4	....
...	...

**TABLE 1.2: Calculated values of the applied force and spring extension.**

Applied force (N)	Extension $x$ (m)
0.000	0.000
0.134	0.056
...	...
...	...
...	...
...	...

#### 1.4 Present the data in a graph

Plot the force and extension data on a graph, and draw a best-fit line by eye. Consider whether the data point (0,0) should be plotted on your graph.

If the best-fit line is linear, then the spring extension is directly proportional to the applied force, and the constant of that proportionality can be obtained from the slope of the straight-line graph since the equation for a straight line is  $y = mx + c$ .

When you determine the slope of the line of best fit, choose two convenient points on the fitted line that are as far apart as possible. Avoid using the data points when calculating the slope  $m$ , as this finds the slope of a line between two arbitrary data points rather than the best fit line to *all* the data.

#### 1.5 State your conclusion

Quote the findings of this experiment in the conclusion of your report, and use your results to justify your conclusions.

Compare your results to nearby groups – did you get the same or a different result? Include a list of all the factors that you believe could have contributed to any variations between the results of nearby groups. In the next practical, you will learn that these factors are sources of uncertainty in your measurement. Describe several ways in which you could improve the experiment.

## 2 Introduction to Uncertainty



Acknowledgement: this exercise is based on *Introduction to Measurement in the Physics Laboratory: A Probabilistic Approach* by Buffler, A., Allie, S., Lubben, F. & Campbell, B. (2010).

### Learning objectives:

- Develop an understanding of the concepts “measurement” and “uncertainty”.
- Perform Type B evaluations of uncertainty of reading digital and analogue displays.
- Report the final result of a measurement.
- Perform calculations with uncertainty.

### Instructions:

*Pre-practical Vula quiz:* “What do we mean by a measurement?”

*Pre-reading:* read through the exercise notes as below and Sections B1, B3, C1-2 and D1 of the “Guide to Reporting and Measurement”.

*Practical:* work through the series of exercises given below, and use the report book provided for any rough working. This is a “practice” exercise intended to give you lots of experience of towards the learning goals given above. Use your time wisely and try to get through as many questions as possible. Talk the answers through with your lab partners, and you don’t need to show all your working in the report book.

*Assessed exercise:* Once you’ve completed all the questions or by 16h00, whichever occurs sooner, you can ask a demonstrator to check you have achieved all the learning goals as above. You can then proceed to the assessment sheet which you will hand in for marking. Make sure you *show all your working* in the assessment sheet for full marks.

### 2.1 Probability

We will now begin to develop procedures for making and recording scientific measurements for which we need some mathematical tools. The first concept that we need is that of probability, after which we will look at what it means to read both digital and analogue scales.

You can think of probability as the chance that an event will occur. Probability is usually expressed as a number between 0 and 1 (inclusive):

- A probability of 1 means that you are 100 % sure that the event will occur.
- A probability of 0 means that you are 100 % sure that the event will not occur.

For example, say that you have a black bag which contains 4 balls of different colour (red, green, blue and yellow). Without looking into the bag you wonder what the probability is of choosing the green ball from the bag.

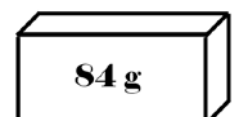
Clearly the **probability** of choosing the green ball is one in four, or 0.25, or 25 %.

Say that you have 10 pairs of socks, each pair having a different pattern. However, they are all loose in a box under your bed. You need a matching pair and grab a single sock. As you put your hand into the box to grab a second sock, you wonder what is the probability of choosing the matching sock.

What is the probability of grabbing the matching sock?

### 2.2 Reading a digital scale

Say that you are given an old, worn block of metal that is marked 84 g.



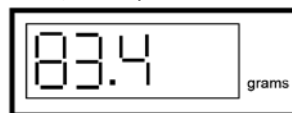
If you are not allowed to weigh the block, how certain will you be that the mass of the block is really 84 g?

Within which of the following ranges is it most likely for the mass to be: 83.95 g – 84.05 g, 83.5 g - 84.5 g, 83.0 g - 85.0 g, 80.0 g - 90.0 g? Explain.

Say now that the physics professor comes along and gives you a new shiny block that is also marked 84 g.

Within which of the following ranges is it most likely for the mass to be: 83.95 g – 84.05 g, 83.5 g - 84.5 g, 83.0 g - 85.0 g, 80.0 g - 90.0 g? Explain.

You now take a digital balance that is set to display one digit after the decimal point (in grams). You put the block on the balance and look at the display. You will probably agree that it is reasonable to record the reading as 83.4 g.



You now set the sensitivity of the digital balance to display **two** digits after the decimal point (in grams). This means that the balance is displaying readings to the nearest 0.01 g or one hundredth of a gram. It is clear that the second digit after the decimal point will either be a 0 or 1 or 2 or 3 or 4 or 5 or 6 or 7 or 8 or 9.

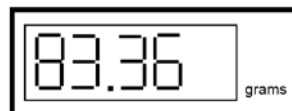
Can you predict for sure what the display will show as the last digit?

Of course not. However, can you say what the probability is of the last digit being a 6?

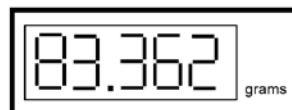
There were ten possibilities for the next digit, therefore there was a one out of ten chance of getting a particular digit. So we can say that the probability of the next digit being a 6, was 0.1 (or 10 %).

Before you looked at the display, there was no way of predicting with a greater certainty than 10% that the last digit would be a 6.

You now see the following on the display. What will you now record as the reading on the display?



We now set the digital balance to display **three** digits after the decimal point (in grams); i.e. the balance will now display the reading to the nearest 0.001 g. What will you now record as the reading on the display?

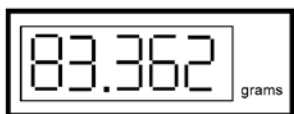
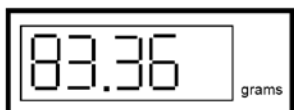
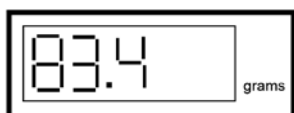


We are now at the end of the range of the display of the digital balance. What can we do if we want a reading with more decimal places?

Would it be possible to design and build an electronic balance that could display a reading with an **infinite** number of decimal places? Explain your answer.

Let us now consider what we know about the mass of the block in each case, **based only on the reading on the digital balance**. Complete the exercise for the third display below.

The display shows:



Inference about the mass of the block, based only on the reading:

The mass of the block lies between 83.35 g and 83.45 g.

The mass of the block lies between 83.355 g and 83.365 g.

The mass of the block lies between \_\_\_\_\_ g and \_\_\_\_\_ g.

In each case above, we can say that the mass of the block lies somewhere on an interval, the width of which reduces in size as the sensitivity of the electronic balance increases.

Do you see that it is impossible in practice to reduce the width of this interval to zero? That is one practical reason why the “true value” of a quantity can never be known.

Which one of the three readings in the table would you regard as the “best” and why?

### 2.3 Reading an analogue scale

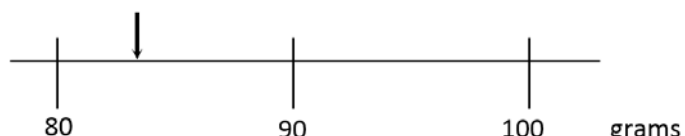
We can use an analogue balance instead of a digital balance to measure the mass of the block. (An analogue balance will have a needle that is displaced in proportion to the weight of the object placed on the balance.) We now need to use our judgement to read the scale after the needle has come to rest.

Let us imagine that when we put the mass on the balance, we see the following on the display:

Was is the reading on the scale?

How certain are you about this reading (“very certain”, “reasonably certain” or “not very certain”)?

Explain your answer.



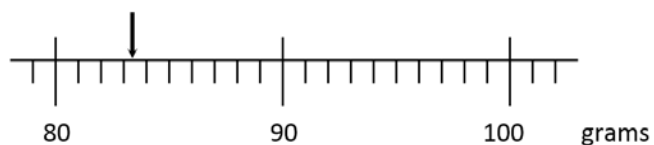
You might have thought that the reading was “between 80 g and 90 g.” However, it is possible to use your judgement and record the reading to the nearest gram.

Try it again. What is the reading on the scale to the nearest gram?

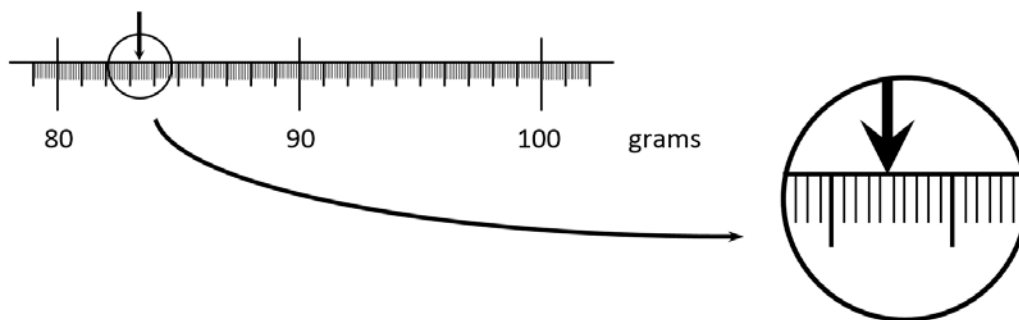
You now decide, in order to improve your measurement, to choose a scale that has markings (called “graduations”) every 1 gram. You now observe the following on the display:

What is the reading on the scale?

You can now make a judgement to the nearest 0.1 gram.



If you want to subdivide each graduation even further, you now have a division marker every 0.1 g. You might need a magnifying glass to read the scale!



What is the reading on the scale?

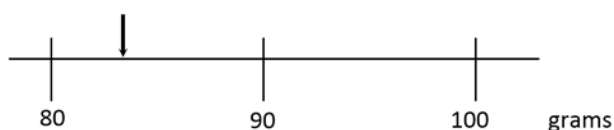
It becomes impractical to continue to add more and more subdivisions. Eventually the scale becomes too small to read. No matter what analogue scale you are reading, you will always need to make a judgement about what the last digit is.

Will you ever be able to find an instrument that gives you a reading of the mass of the block to an **infinite** number of decimal places?

No, of course not. It will never be possible to manufacture such an instrument! It is then clear that the “true” value of the mass can never be known. This is the case for **all** measurements, no matter what you are wanting to measure.

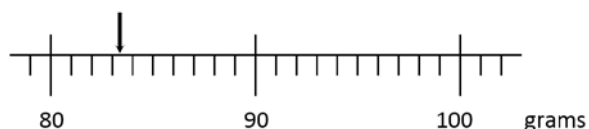
Let us now once again consider what can we know about the mass of the block in each case, if all we have is the reading on the analogue scale. Complete the exercise overleaf.

The analogue scale shows:

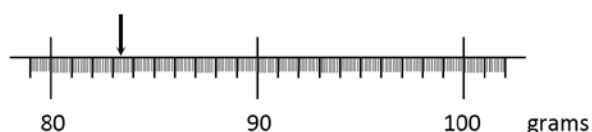


*Inference about the mass of the block, based only on the reading:*

The mass of the block lies between \_\_\_\_\_ g and \_\_\_\_\_ g.



The mass of the block lies between \_\_\_\_\_ g and \_\_\_\_\_ g.



The mass of the block lies between \_\_\_\_\_ g and \_\_\_\_\_ g.

Once again, in each case above, we can say that the mass of the block lies somewhere on an interval, the width of which reduces in size with more markings on the scale. It is important to note that when you decided upon the left and right hand of the intervals in the table above, you could do better than simply taking the nearest markings on the scale in each case. For example, in the first case above, although it is a true statement to say that “the mass of the block lies between 80 g and 90 g”, you can do much better than that.

You might think that the best reading of the mass in the first case is 83 g. Then you should ask yourself, “what are the closest values to the best reading that you think are definitely not possible?” You might therefore decide that the reading is probably not less than 82 g and probably not more than 84 g, and therefore infer that the mass of the block lies between 82 g and 84 g.

Go back and change your answers above if you need to.

Also note that since there is a degree of judgement involved in each case, your friends might have come up with slightly different readings and intervals to yours.

## 2.4 Measurement and uncertainty

The broad aim of performing measurements in science is to increase our knowledge about some physical quantity which is referred to as the **measurand**. We should not think about the measurand as possessing some “true value” that has to be uncovered but rather that the value of the measurand is based on the amount of information we have at hand. If you really wanted to know the ultimate or true value of the measurand you would need an infinite amount of information!

Thus, the information we have about a measurand can be never 100% complete. For example, we saw earlier that whether we are dealing with a digital or an analogue instrument, the information about the measurand from the reading is in fact an interval which cannot be reduced to a point. So, even if there are no other factors influencing the measurement, the scale would limit what we know and **the final result of a measurement will always be an interval**. However, there are usually several factors that will influence the measurement. Each of these factors makes the interval associated with the final result bigger. We call this interval the **uncertainty**. Thus, the larger the uncertainty, the less we know and the more we know, the smaller the uncertainty. When designing an experiment, the aim is try and make the uncertainty as small as possible, but with knowledge that the uncertainty interval cannot be reduced to zero.

A measurement result in science is meaningless without a quantitative statement of the uncertainty.



One of the goals of measurement is to try and minimise the uncertainty when we perform an experiment. This can be achieved by good experimental design as well as by collecting as much data as possible. At the end of each practical you will write down all the factors that could influence the result of your measurement. Each of these factors can be thought of as working against our having perfect knowledge about a measurand and adds to the overall uncertainty. A crucial aspect of experimentation is to identify all such **sources of uncertainty** and to numerically estimate their effect on your measurement result.

Common sources of uncertainty include:

- (a) the effects of environmental conditions on the measurement;
- (b) your judgement in reading analogue instruments;
- (c) the sensitivity of your instruments (e.g. the digital scale);
- (d) the rating or stated calibration of the instrument;
- (e) approximations and assumptions that you make while doing the experiment; and
- (f) variations in repeated readings made under apparently identical conditions.

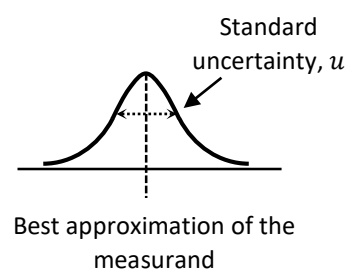
A measurement uncertainty is not meant to be an indication of “mistakes” that you might make in an experiment. If you are aware that you have made a mistake, then you should repeat your experiment. “Human error” is not a valid source of uncertainty. If you know that you did something “wrong”, then why don’t you do it correctly?

## 2.5 Overview of the standard uncertainty

**Uncertainty** in a measurement is a quantitative measure of the factors that decrease your knowledge about the measurand. There are, broadly speaking, two ways of evaluating uncertainties.

If you have a **set of repeated readings** of the same measurand which are dispersed (scattered) then you will evaluate the uncertainty associated with the scatter using statistical methods. This is called a **Type A evaluation of uncertainty**, for which you are fitting a Gaussian distribution to a histogram of the data. More on this in the next practical.

The position of the **centre** of the distribution gives the most probable value of the measurand (called the “**best approximation**” of the measurand), and the average width of the distribution is a measure of the quality of our knowledge about the measurand. The thinner it is, the better knowledge we have. This width is referred to as the **standard uncertainty** (symbol  $u$ ). The more spread out the distribution, the greater the uncertainty. You will learn more about this type of uncertainty in the next practical.



For other types of uncertainty, such as those associated with a single reading or multiple repeated readings that are identical (e.g., 0.1, 0.1, 0.1), you will use the knowledge that you have about the measurement process and the instrument that you are using. This is called a **Type B evaluation of uncertainty**, and there are two ways of calculating it depending on whether the reading is from a digital or analogue display.

## 2.6 Type B evaluation of uncertainty

Type B evaluations also involve fitting a distribution to the data, such as a **triangular or square distribution**. The average width of the distribution also tells us how good our knowledge is about the measurand; which is the standard uncertainty. For this course, you don’t need to understand the role of the triangular or rectangular distributions. If you want to know more about the distributions (which are “probability



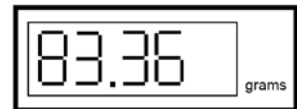
distribution functions”), you can consult a **demonstrator**, or the full version of this guide at the UCT Physics website, “**Introduction to Measurement in the Physics Laboratory: a probabilistic approach**”.

As an illustration of a Type B evaluation of uncertainty, we will consider the uncertainty associated with reading the scale of an instrument.

We will look at two cases: (a) a single digital reading; and (b) a single analogue reading. We would like to emphasise at this stage that the uncertainty we are considering here is from reading the scale only, and that when carrying out an experiment there will several other sources of uncertainty that must also be evaluated and combined to obtain the total uncertainty.

### (a) A single digital reading.

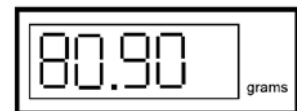
Consider the situation where you want to determine the mass of a block and you see the following on the display of your digital balance.



Clearly the best approximation of the mass is 83.36 g. What about the **standard uncertainty** associated with reading the scale on the display? Well, we saw previously that the 6 is representing the interval 83.355 g to 83.365 g, *i.e.* less than 83.355 g the digit would change the final digit to 5 and greater than 83.365 g it would show 7. All that we can assume is that the value of the mass is distributed between the interval 83.355 g to 83.365 g.

For a digital reading, the standard uncertainty is given by  $\frac{\text{the width of the interval}}{2\sqrt{3}}$ , so in this example the standard uncertainty is:  $u(m_{\text{read}}) = \frac{(83.365 - 83.355)}{2\sqrt{3}} = 0.0029 \text{ g}$ .

What is the best approximation of the mass and the standard uncertainty for this reading if the meter shows:



### (b) A single analogue reading.

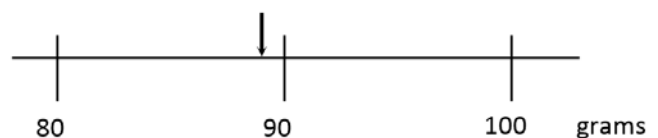
This case is slightly more complicated because it relies on your judgement. Assume that you are using an analogue meter and observe the following:



You might decide that the best approximation of the mass is 83 g. Of course, it could possibly be a bit larger or a bit smaller. So, you now need to ask yourself, “what are the closest values to the best approximation that you think are definitely not possible?” You might therefore decide that the probability of the value being 82 g is zero and that the probability of the value being 84 g is also zero. So, as you go from your best approximation towards these “impossible” values you become less certain about the measurand.

For an analogue reading, the standard uncertainty is  $\frac{\text{the width of the interval}}{2\sqrt{6}}$ , so in this example the standard uncertainty,  $u(m_{\text{read}}) = \frac{(84 - 82)}{2\sqrt{6}} = 0.41 \text{ g}$ .

Determine the best approximation of the mass and the standard uncertainty associated with reading the scale, if the meter shows:



## 2.7 Reporting the result of your measurement

When reporting the result of a measurement, it is better to provide too much information rather than too little. For example, you should describe clearly the methods used to calculate your uncertainties, and present the data analysis in such a way that each of the important steps can be easily followed by the reader of your report.

When reporting the result of a measurement, you should therefore give:

- (i) a clear statement of the measurand; and
- (ii) the **best approximation** of the measurand and its **standard uncertainty** (remember to give the units).

For example, the result of the measurement may be reported as: "...the best approximation of the mass was determined to be 83.45 g with a standard uncertainty of 0.34 g" or " $m \pm u(m) = 83.45 \pm 0.34 \text{ g}$ ."

You can now report your final results for the two examples given on the previous page. This is how you should always the result of a measurement.

## 2.8 Significant digits

If we determine a particular measurement result (after a series of calculations) to be  $m = 35.82134 \pm 0.061352 \text{ kg}$ , how many digits should we quote in our result ?

The uncertainty of 0.061352 kg tells us that we are uncertain about the second decimal place in 35.82134 kg. Our final result is then written as  $m = 35.821 \pm 0.061 \text{ kg}$ .

You should generally quote your uncertainty giving **two** significant figures, and then round off your best approximation of the measurand to the same digit as the second digit in your uncertainty.

Another example:  $T = 0.00345474 \pm 0.00069780 \text{ s}$  should be reported as  $T = 0.00345 \pm 0.00070 \text{ s}$  or  $T = (3.45 \pm 0.70) \times 10^{-3} \text{ s}$ .

You will thus often need to **round off** your calculations to an appropriate number of significant digits. The general rules for rounding off are:

- (a) The last significant figure to be retained remains unaltered if the next digit is less than 5. For example, 3.434 rounds off to 3.43.
- (b) The last significant figure to be retained is increased by one if the next digit is greater than or equal to 5. For example, 3.436 rounds off to 3.44.
- (c) Do not do a double round off: 3.4348 rounded off to three significant figures becomes 3.43. Do not round off 3.4348 to 3.435 to 3.44!

Now try the following, with an acceptable number of significant figures:

$l = 34.47 \pm 0.4572 \text{ m}$ .  $f = 41074 \pm 25.9 \text{ Hz}$ .  $k = 1.3743 \times 10^5 \pm 216 \text{ N m}^{-1}$ .  $I = 23274.64746 \pm 5.566 \text{ A}$ .

## 2.9 Calculations with uncertainty

Very often you will need to calculate a quantity  $R$  from a set of measurements of  $N$  other quantities, such as  $A$  and  $B$ . The question then is how to estimate the standard uncertainty  $u(R)$  from your estimates of the standard uncertainties of the  $N$  measured quantities.

The uncertainty  $u(R)$  is obtained by combining the individual standard uncertainties, whether arising from a Type A evaluation or a Type B evaluation, according to the following general formulae overleaf. The formula for  $u(R)$  depends on the form of the equation used to calculate  $R$ , where  $R = f(A, B)$ , and  $a$ ,  $b$  and  $c$  are constants (numbers).

## Equations for the propagation of uncertainties through calculations.

Form of equation from which result $R$ is calculated	Formula for calculating the standard uncertainty $u(R)$
<p><b>Sum of variables</b></p> $R = a A \pm b B \pm c$ <p>Coefficients <math>a</math>, <math>b</math> &amp; <math>c</math> are constants (numbers with zero uncertainty)</p>	$[u(R)]^2 = [a u(A)]^2 + [b u(B)]^2$ <p>or</p> $u(R) = \sqrt{[a u(A)]^2 + [b u(B)]^2}$
<p><b>Product of variables</b></p> $R = c A^a B^b$ <p>Coefficients <math>a</math>, <math>b</math> &amp; <math>c</math> are constants (numbers with zero uncertainty)</p>	$\left[\frac{u(R)}{R}\right]^2 = \left[a \frac{u(A)}{A}\right]^2 + \left[b \frac{u(B)}{B}\right]^2$ <p>or</p> $u(R) = R \sqrt{\left[a \frac{u(A)}{A}\right]^2 + \left[b \frac{u(B)}{B}\right]^2}$
<p><b>Correlated variables</b></p> <p>(consider co-variance if the instrument used more than once in the same experiment)</p> $R = a A$ <p>Coefficient <math>a</math> is a constant (number with zero uncertainty).</p>	$u(R) = \sqrt{a} u(A)$

Note: these equations are three results from a general function for the propagation of uncertainties. To find out more, consult the full version of this guide at the UCT Physics website, "Measurement Manual".

**(a) Example 1**

To determine the activity of a radioactive sample, a series of observations were made with a gamma ray detector. The count rate with the radioactive sample  $N$  was measured to be 145 counts per minute with a standard uncertainty  $u(N)$  of 12 counts per minute. The background radioactivity  $B$  was measured to be 26 counts per minute with a standard uncertainty  $u(B)$  of 6 counts per minute.

Find the count rate and standard uncertainty associated with the radioactive sample  $N_0$ , given by  $N_0 = N - B$ .

**(b) Example 2**

Now let us presume that we are trying to measure the acceleration due to gravity  $g$  by observing the period  $T$  of a pendulum of length  $l$ . Say that we determine that:

$T = 0.763 \pm 0.021$  s (where  $u(T)$  results from a Type A evaluation of uncertainty); and

$l = 0.1430 \pm 0.0029$  m (where  $u(l)$  results from a Type B evaluation of uncertainty).

Determine  $g$ , and its standard uncertainty, using the formula:  $T = 2\pi \sqrt{\frac{l}{g}}$ .

### 3 The Simple Pendulum

#### Learning objectives:

- Linearise equations to determine relationships.
- Perform Type A evaluations of uncertainty.
- Use Excel to analyse data and plot graphs with trendlines.

#### Instructions:

*Pre-practical Vula quiz:* “Dealing with dispersion in data; an introduction to Type A evaluations of uncertainty”.

*Pre-reading:* read through the experiment notes as below and Sections B2 and E1, 2, 4 of the “Guide to Reporting and Measurement”.

*Practical:* spend 10 minutes discussing what to do with your lab partners, and make any rough work on the back pages of your report book. You can work together during the practical, however you must write your own and unique full write-up of the experiment to be submitted by 17h00 on the day of the practical.

#### 3.1 Introduction

You are supplied with a pendulum consisting of a bob (a mass) attached to a string, which is set up so that the pendulum can swing freely in a vertical plane suspended from a retort stand, a metre rule or measuring tape and a timer. Spend several minutes exploring the function of the equipment.

Your task is to design an experiment to determine the relationship between the *period of oscillation* and *length of the pendulum*, and to use your relationship to determine a value for gravitational acceleration. Additional instructions are included below to guide you through the experiment, and you can add your own steps. After completing the experiment, prepare a full report on the experiment and its results.

If we make two assumptions, that the total mass of the pendulum is concentrated at the centre of the bob, and the angle through which the bob swings is relatively small, *i.e.*,  $\vartheta_{max} \leq 15^\circ$  (to use the “small angle approximation”), we can derive a theoretical value for the period of oscillation of the pendulum which is independent of the mass of the bob and depends only on the length of the pendulum:

$$T = 2\pi \sqrt{\frac{L}{g}}. \quad \dots (3-1)$$

In the first practical, we plotted the two main experimental values on a graph, force against extension, and used the gradient to find the spring constant. If we follow a similar method here, by plotting  $T$  against  $L$ , it will be difficult to fit a line of best fit by hand and determine the gradient to find  $g$ . As an alternative, we can “linearise” equation (3-1) by plotting  $T$  vs  $\sqrt{L}$  which should be a straight-line graph, or squaring both sides to get

$$T^2 = \frac{4\pi^2}{g} L, \quad \dots (3-2)$$

and plotting  $T^2$  against  $L$ . By comparing equation (3-2) to the equation of a straight line,  $y = mx + c$ , we can determine the gravitational acceleration,  $g$ , from the gradient,  $m$ , of the line of best fit, because the gradient is

$$m = \frac{4\pi^2}{g}. \quad \dots (3-3)$$

### 3.2 Method

It is difficult to accurately measure the period of one oscillation,  $T$ , with the provided timer, therefore to reduce the uncertainty in  $T$ , we determine the time taken for several successive oscillations and then divide the total time by the number of oscillations completed. Typically, recording the total time over 20 oscillations will be suitable. You need to record the time,  $20T$ , and then calculate  $T$ .

For various pendulum lengths (between 0.5 m and 1.5 m), record ten (10) data pairs,  $(L_i, 20T_i)$ , for  $i = 1, 2, 3, \dots, 10$ . You can then use this data to determine  $g$  in two ways.

#### a) Perform a Type A evaluation of the uncertainty in the mean value of $g$ .

Calculate  $g_i$  for each data pair.

Use an EXCEL spreadsheet to calculate the mean,  $\bar{g}$ , the standard deviation  $\sigma$ , and the standard uncertainty,  $u(g)$ . We use the “column-wise” approach to calculating these values to show your working, which can be checked with the built in Excel functions “AVERAGE” and “STDEV”.

*(For your convenience, an Excel file called pendulum.xls has been placed on the laboratory PCs. You may set up your own Excel tables, or you may make use of the template provided.)*

#### b) Use a graphical method to find $g$ .

Use Excel’s graphing function to plot  $T^2$  vs  $L$ . Show the equation of the best fit line on the graph.

Save your work, print out the table and the graph, and staple the printed Excel spread sheet in the RESULTS section of your report.

### 3.3 Discussion and conclusion

Consider the following questions:

- Can you compare your results from parts (a) and (b)?
- How do your results compare to a given value of  $g = 9.80 \text{ m/s}^2$ ?
- Comment on the  $y$ -intercept that you get from the fitted line equation of the graph. What do you expect it to be?
- Propose ways to improve the experiment.

## 4 Simple Harmonic Motion (SHM)

### Learning objectives:

- Perform a Type A evaluation of uncertainty with the method of least squares (LinearFit).
- Calculate uncertainties from a sum of variables.
- Calculate uncertainties from correlated variables.
- Calculate uncertainties from a product of variables.

### Instructions:

*Pre-practical Vula quiz:* “Introduction to least squares fitting”, and “Calculations with uncertainties”.

*Pre-reading:* read through the experiment notes as below and Sections C and E3-5 of the “Guide to Reporting and Measurement”.

*Practical:* spend 10 minutes discussing what to do with your lab partners, and make any rough work on the back pages of your report book. You can work together during the practical, however you must write your own and unique full write-up of the experiment to be submitted by 17h00 on the day of the practical.

### 4.1 Introduction

You are supplied with a spring hanging from a retort stand, a series of masses, a metre rule or tape measure and a timer. Spend several minutes exploring the function of the equipment. What happens as you add the masses to the end of the spring?

Your task is to design an experiment to determine the relationship between the *period of oscillation* of a mass on a spring and the magnitude of *the mass*, and to use your relationship to determine a value for the spring constant. Additional instructions are included below to guide you through the experiment, and you can add your own steps. After completing the experiment, prepare a full report on the experiment and its results.

Recalling equation 1-1 for Hooke’s Law, if a mass hangs at rest at the end of a spiral spring, the upward force exerted by the spring on the mass exactly balances the downward gravitational force acting on the mass. The mass is said to be in ‘equilibrium’ and we refer to this position as the ‘equilibrium position’. If the mass is pulled down slightly from its equilibrium position and released, it will oscillate about its equilibrium position as the net force always acts to try to restore the mass to its equilibrium position.

This oscillation is called simple harmonic motion (SHM), and the period  $T$  is given by:

$$T = 2\pi \sqrt{\frac{p}{k}}. \quad \dots (4-1)$$

A part of the spring is also undergoing oscillations, and if it has a non-negligible mass  $p'$ , this will also affect the period of oscillation. We can correct for this effect by replacing  $p$  in equation 4-1 by the effective mass  $P$ , given by:

$$P = p + \frac{1}{3}p'. \quad \dots (4-2)$$

Then the period of oscillation is then given by:

$$T = 2\pi \sqrt{\frac{P}{k}}. \quad \dots (4-3)$$

## 4.2 Method

Attach one, or a combination, of the cylinders to the spring, pull it down slightly and set it oscillating. Record the time taken for 20 (small amplitude) oscillations. Tabulate at least six pairs of readings ( $i = 1, 2, \dots, 6, \dots$ ) of period  $20T_i$  and mass  $P_i$  by using combinations of the four given masses and the non-negligible mass of the spring.

The mass of each of the four metal cylinders you will use has been printed on the cylinder, but take it that the standard uncertainty in the given value of each cylinder is  $\pm 0.61$  g. Use the triple beam balance to determine the mass of the spring  $p' \pm u(p')$ .

Linearise equation 4-3 and decide which variables to calculate to plot a graph of  $y$  vs.  $x$  to determine the spring constant,  $k$ , from the gradient of the graph. Determine the uncertainty for each variable, for each of the readings you took;  $u(x_i)$  and  $u(y_i)$ .

Use **LinearFit** to determine the slope  $m \pm u(m)$  and the intercept  $c \pm u(c)$  of the best fit line for the graph, and include the uncertainty for each variable as a weighting. Print the LinearFit results and staple these into your report book. Make sure to label the graph appropriately with a title and axes titles.

Use the slope of the graph slope,  $m \pm u(m)$ , that you got from LinearFit to calculate the spring constant  $k \pm u(k)$ .

## 4.3 Discussion and conclusion

Report the final result of your measurement.

Consider the following questions:

- How does this result for  $k$  compare to the value you got when you did the Hooke's Law experiment?
- Was the  $y$ -intercept as expected?
- How you could improve the result of the experiment?

## 5 Speed of a wave on a stretched string

### Learning goals:

- Compare different measurements and determine relationships within experimental uncertainty.
- Use an uncertainty budget to summarise the sources of uncertainty in an experiment.
- Use the reductionist approach to simplify uncertainty calculations.

### Instructions:

*Pre-practical Vula quiz:* “Determining relationships within experimental uncertainty” and “The uncertainty budget”.

*Pre-reading:* read through the experiment notes as below and Sections C1-2 and D2-3 of the “Guide to Reporting and Measurement”.

*Practical:* spend 10 minutes discussing what to do with your lab partners, and make any rough work on the back pages of your report book. You can work together during the practical, however you must write your own and unique *abridged\** write-up of the experiment to be submitted by 17h00 on the day of the practical.

### 5.1 Introduction

You are supplied with a stretched string that is attached to a mass hanging from a pulley at one end and driven by a function generator and actuator at the other. Spend several minutes exploring the function of the equipment. What happens to the appearance of the string as you change the frequency of the vibration?

Your task is to design an experiment to determine the speed at which a travelling wave moves along a stretched string *by two different methods and compare the results*. Additional instructions are included below to guide you through the experiment, and you can add your own steps. After completing the experiment, prepare an *abridged\** report on the experiment and its results. (\*An “abridged” report does not include the introduction or method, but does still include the aim.)

The frequency and wavelength of a travelling wave ( $f$  and  $\lambda$  respectively) are related to the speed,  $v$ , of the wave by the equation:

$$v = \lambda f \quad \dots (5-1)$$

The string is fixed at the “pulley end” so the waves that are introduced onto the string by the vibrator are reflected back at the pulley along the string. These reflected waves interfere with the waves that are moving towards the pulley. At certain frequencies called “modes”, standing waves are produced and several “nodes” will be clearly visible on the string as places where the string has no vertical oscillation (see Figure 5-1). In between the nodes are “anti-nodes” where the string undergoes maximum vertical oscillation. The distance between adjacent nodes is half a wavelength (i.e.  $\lambda/2$ ).

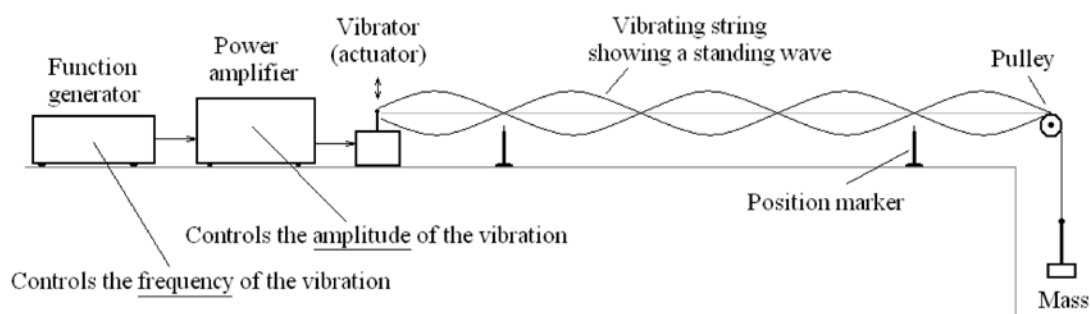


Figure 5-1. Experimental set-up used to produce standing waves in a string.



The speed of a wave on a string is dependent upon the tension,  $T$ , in the string as well as the mass per unit length,  $\mu$ , of the string:

$$v = \sqrt{\frac{T}{\mu}} \quad \dots (5-2)$$

## 5.2 Method

### 1) Use a graphical method to determine the speed of sound on the string, $v_1 \pm u(v_1)$ .

Considering equation 5-1, design an experiment that uses a graphical method to determine the speed of a wave on the string from a series of standing waves,  $v_1 \pm u(v_1)$ . It's not necessary to evaluate the uncertainties  $u(x)$  and  $u(y)$  in the variables you choose for  $x$  and  $y$  in your graph, but have a go if you want to! It's up to you what and how many measurements to perform, and to choose tools from earlier in the course to help you achieve this goal. Remember you can always ask a demonstrator for help.

Use an uncertainty budget to present and evaluate the sources of uncertainty in your experiment.

Adjust the function generator and the power amplifier to find frequencies at which there are standing waves on the string, using settings of a sine wave with amplitude 1 to 4 and frequencies 1 to 100 Hz. Start with the mode at a frequency between 8 to 11 Hz. What happens as you increase the frequency, and can you find higher modes? Note the amplitude adjustment should be as small as possible. If the vibrator makes a harsh *brrrr...* sound, you are overdriving it.

You can measure the distance between nodes by placing pin holders at the position of the nodes and measuring the distance between them with a metre ruler or measuring tape. Note there is not a node at the actuator end of the string. Consider whether you will achieve more accurate data from measuring over multiple combinations of nodes, rather than between two consecutive nodes.

### 2) Use a calculation to determine the speed of sound on the string, $v_2 \pm u(v_2)$ .

Considering equation 5-2, design an experiment that uses a calculation and several supporting measurements to determine the speed of a wave on the string,  $v_2 \pm u(v_2)$ . It's up to you what and how many measurements to perform, and to choose tools from earlier in the course to help you achieve this goal. Remember you can always ask a demonstrator for help.

Use a second uncertainty budget to summarise and evaluate the sources of uncertainty in your experiment. Try to use the reductionist approach in your calculations.

Please don't remove the string from the equipment as it is difficult to replace it. Instead we have provided a test piece of string that you can use for measurements. You can carefully remove the mass piece.

If you need a value for gravitational acceleration, use  $g = (9.800 \pm 0.020) \text{ ms}^{-2}$ .

## 5.3 Discussion and conclusion

Compare the results of the two methods within experimental uncertainty by using a number line.

Revision: January 2018



Final score: / 25

25

Assessment scheme for "INTRODUCTION TO UNCERTAINTY" practical

Student surname: \_\_\_\_\_ Student number: \_\_\_\_\_

The purpose of your laboratory practicals is to teach you skills in measurement science. Please reflect on what skills you learned today and complete the table below, then include this with your script for marking.

Note this assessment scheme corresponds to the pre-practical quiz on Vula and an exercise that will be handed out in the second half of the practical.

Main skills covered in this practical:	Self-Assess	Marker Assess	Max Score
<b>Measurement concepts (pre-prac worksheet)</b> <ul style="list-style-type: none"> <li>Understand what is meant by the "measurand".</li> <li>Understand the difference between "approximately" and "exactly".</li> <li>Evaluate factors influencing a measurement.</li> </ul>			(5)
<b>Evaluate uncertainty</b> <ul style="list-style-type: none"> <li>Perform Type B evaluations of uncertainty of reading digital displays.</li> <li>Perform Type B evaluations of uncertainty of reading analogue displays.</li> <li>Combine uncertainties from a sum of variables.</li> <li>Calculate uncertainties from correlated variables.</li> <li>Propagate uncertainties from a product of variables.</li> <li>Identify additional sources of uncertainty.</li> </ul>			(3) (6)
<b>Reporting the result of a measurement</b> <ul style="list-style-type: none"> <li>The final result of a measurement includes an uncertainty.</li> <li>The uncertainty is rounded to 2 significant figures.</li> <li>The reading is rounded to the same number of decimal places as the uncertainty.</li> <li>The reading and the uncertainty are both in the same order of magnitude or scientific exponent.</li> <li>Units are given and appropriate.</li> </ul>			(5)
<b>In the next practical session: follow up with the marker directly for feedback.</b> <ul style="list-style-type: none"> <li>Check sections where marks were lost.</li> <li>Know what to do to improve next time.</li> </ul>			<b>Marker name (print):</b>

Revision: January 2018



Final score: / 20

20

Assessment scheme for "INVESTIGATING HOOKE'S LAW" practical

Student surname: \_\_\_\_\_ Student number: \_\_\_\_\_

The purpose of your laboratory practicals is to teach you skills in measurement science. Please reflect on what skills you learned today and complete the table below, then include this with your script for marking.

Main skills covered in this practical:	Self-Assess	Marker Assess	Max Score
<b>Perform an experiment</b> <ul style="list-style-type: none"> <li>Plan the experiment with your lab partners.</li> <li>Collect sufficient raw data and record by hand in the report book.</li> </ul>			(2)
<b>Introduce a report</b> <ul style="list-style-type: none"> <li>Include a brief aim and introduction of the experiment.</li> <li>Draw a clear and simple schematic or set-up of the experiment.</li> <li>Describe the method concisely.</li> <li>The aim/introduction/method is no longer than 1 page of A4.</li> </ul>			(4)
<b>Data analysis</b> <ul style="list-style-type: none"> <li>Calculate the extension data from the metre stick readings.</li> <li>Calculate the force data from the number of balls.</li> <li>Present the data in table form per the general rules.</li> <li>Draw a graph by hand to relate the force and extension of the spring.</li> <li>Present the graph per the general rules.</li> <li>Draw a line of best fit on the graph.</li> <li>Use the line of best fit to estimate the slope of the line.</li> <li>Determine the spring constant from the gradient of the graph.</li> </ul>			(8)
<b>Evaluate uncertainty</b> <ul style="list-style-type: none"> <li>Compare your result to nearby groups.</li> <li>Identify factors influencing the final result.</li> <li>Suggest improvements to the experiment.</li> </ul>			(3)
<b>Conclude a report</b> <ul style="list-style-type: none"> <li>Report the final result of the experiment.</li> <li>Refer back to the aim of the experiment.</li> <li>Quote any final values with appropriate units.</li> </ul>			(3)
<b>In the next practical session: follow up with the marker directly for feedback.</b> <ul style="list-style-type: none"> <li>Check sections where marks were lost.</li> <li>Know what to do to improve next time.</li> </ul>			<b>Marker name (print):</b>

Revision: January 2018



Final score: / 25

Assessment scheme for "SIMPLE PENDULUM" practical

Student surname: \_\_\_\_\_ Student number: \_\_\_\_\_

The purpose of your laboratory practicals is to teach you skills in measurement science. Please reflect on what skills you learned today and complete the table below, then include this with your script for marking.

Main skills covered in this practical:	Self-Assess	Marker Assess	Max Score
<b>Preparation</b> <ul style="list-style-type: none"> <li>Complete the pre-practical reading and preparation quiz.</li> </ul>			(5)
<b>Perform an experiment</b> <ul style="list-style-type: none"> <li>Measure <math>20T</math> for a range of different <math>L</math>, with a timer and ruler/tape.</li> <li>Collect <i>sufficient</i> raw data and record by hand.</li> </ul>			(2)
<b>Introduce a report</b> <ul style="list-style-type: none"> <li>Include a <i>brief</i> aim and introduction of the experiment.</li> <li>Draw a simple set-up diagram and describe the method concisely.</li> </ul>			(2)
<b>Data analysis</b> <ul style="list-style-type: none"> <li>Linearise equation 3-1 and calculate appropriate values of <math>x</math> and <math>y</math>.</li> <li>Present the data in table form according to the general rules.</li> <li>Use a spreadsheet to analyse the data. Print your results.</li> <li>Determine the average value of <math>g</math> with Excel.</li> <li>Use Excel to plot a linear graph and add a trendline.</li> <li>Present the graph according to the general rules.</li> <li>Use the gradient to find a second value of <math>g</math>.</li> </ul>			(2) (2) (1) (1) (1) (2) (1)
<b>Evaluate uncertainty</b> <ul style="list-style-type: none"> <li>Use a column-wise approach to find the standard deviation, <math>\sigma</math>.</li> <li>Perform a Type A evaluation of uncertainty to find <math>u(g)</math>.</li> </ul>			(2)
<b>Conclude a report</b> <ul style="list-style-type: none"> <li>Report the final results of the experiment.</li> <li>Compare your results for <math>g</math> with the given value and comment.</li> <li>Comment on the <math>y</math>-intercept.</li> <li>Propose ways to improve the experiment.</li> </ul>			(4)
<b>In the next practical session: follow up with the marker directly for feedback.</b> <ul style="list-style-type: none"> <li>Check sections where marks were lost.</li> <li>Know what to do to improve next time.</li> </ul>			Marker name (print):

Revision: January 2018



Final score: / 25

Assessment scheme for "SIMPLE HARMONIC MOTION" practical

Student surname: \_\_\_\_\_ Student number: \_\_\_\_\_

The purpose of your laboratory practicals is to teach you skills in measurement science. Please reflect on what skills you learned today and complete the table below, then include this with your script for marking.

Main skills covered in this practical:	Self-Assess	Marker Assess	Max Score
<b>Preparation</b> <ul style="list-style-type: none"> <li>Complete the pre-practical reading and preparation quiz.</li> </ul>			(5)
<b>Introduce a report</b> <ul style="list-style-type: none"> <li>Include a <i>brief</i> aim and introduction of the experiment.</li> <li>Draw a simple set-up diagram; describe the method concisely.</li> </ul>			(2)
<b>Perform an experiment</b> <ul style="list-style-type: none"> <li>Measure <math>20T</math> with a timer for at least 6 combinations of masses.</li> <li>Measure the mass of the spring, <math>p</math>, with the triple beam balance.</li> </ul>			(2)
<b>Data analysis</b> <ul style="list-style-type: none"> <li>Present data in table form according to the general rules.</li> <li>Linearise equation 4-3.</li> <li>Calculate appropriate values of <math>x</math> and <math>y</math>.</li> <li>Use Linearfit to plot a graph of <math>y</math> against <math>x</math> with weightings.</li> <li>Present the graph according to the general rules.</li> <li>Determine the value of the spring constant, <math>k</math>.</li> </ul>			(6)
<b>Evaluate uncertainty</b> <ul style="list-style-type: none"> <li>Perform a Type B evaluation of uncertainty to find <math>u(20T)</math>.</li> <li>Perform a Type A evaluation of uncertainty to find <math>u(m)</math>.</li> <li>Calculate the uncertainties <math>u(x_i)</math>.</li> <li>Calculate the uncertainties <math>u(y_i)</math>.</li> <li>Calculate the uncertainty <math>u(k)</math>.</li> </ul>			(5)
<b>Conclude a report</b> <ul style="list-style-type: none"> <li>Report your final result; <math>k \pm u(k)</math>.</li> <li>Refer back to the aim of the experiment.</li> <li>Compare your result for <math>k</math> to that from Prac 1, Hooke's Law.</li> <li>Comment on the value of the <math>y</math>-intercept.</li> <li>Propose ways to improve the experiment.</li> </ul>			(5)
<b>In the next practical session: follow up with the marker directly for feedback.</b> <ul style="list-style-type: none"> <li>Check sections where marks were lost.</li> <li>Know what to do to improve next time.</li> </ul>			Marker name (print):

Revision: January 2018



Final score: \_\_\_\_\_

25

Assessment scheme for “SPEED OF A WAVE ON A STRETCHED STRING” practical

Student surname: \_\_\_\_\_ Marker name (print): \_\_\_\_\_

The purpose of your laboratory practicals is to teach you skills in measurement science. Please reflect on what skills you learned today and complete the table below, then include this with your script for marking.

Main skills covered in this practical:	Self-Assess	Marker Assess	Max Score
<b>Preparation</b> · Complete the pre-practical reading and preparation quiz.			(5)
<b>Prepare an abridged report</b> · Include a brief aim of the experiment.			(1)
<b>Perform an experiment</b> · <i>Method 1</i> – Linearise equation 5-1 and make appropriate measurements to determine $v_1$ . · <i>Method 2</i> – Make appropriate measurements to determine $v_2$ .			(2) (2)
<b>Data analysis</b> · <i>Method 1</i> - Calculate appropriate values of $x$ and $y$ and present data in table form according to the general rules. · Plot a graph of $x$ and $y$ , find the equation of the line of best fit, and present the graph according to the general rules. · Determine a value for $v_1$ . · <i>Method 2</i> – Determine a value for $v_2$ .			(4)
<b>Evaluate uncertainty</b> · <i>Method 1</i> - Evaluate the uncertainty $u(v_1)$ in an uncertainty budget. · <i>Method 2</i> - Evaluate the uncertainty $u(v_2)$ in an uncertainty budget. · Use the reductionist approach to evaluate the uncertainty $u(v_2)$ . · Present both budgets in table form according to the general rules.			(1) (3) (2) (1)
<b>Conclude a report</b> · Report the two final results for $v_1$ and $v_2$ . · Plot the results on a number line and compare the two results for $v_1$ and $v_2$ , within experimental uncertainty.			(2) (2)
<b>In the next practical session: follow up with the marker directly for feedback.</b> · Check sections where marks were lost. · Know what to do to improve next time.		Marker name (print):	