DEPARTMENT OF PHYSICS
UNIVERSITY OF CAPE TOWN
IYUNIVESITHI YASEKAPA • UNIVERSITEIT VAN KAAPSTAD

COURSE I LABORATORY

GUIDE TO

REPORTING AND MEASUREMENT

(version: 2018)

Please keep this guide safe and in good condition as you will use it throughout your 1st and 2nd semester laboratory activities.

Name: ______________________________

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FOREWORD

Welcome to the Physics Course I Laboratory

As a science or engineering graduate, it will be assumed by your future employer that you have certain skills. These will include:

- **problem solving skills;**
- **the ability to engage with apparatus that you may not have seen before;**
- **the ability to plan and execute an experiment or some type of investigation;**
- **the ability to collect, analyse and interpret data, and**
- **the ability to communicate and present your findings either orally or in the form of a written report.**

Developing skills in scientific measurement

Physics is about modelling and understanding the phenomena of nature, and physics experiments are concerned with the creation of new knowledge through measurement. These measurements are used to formulate new physics theories, or test existing theories. However, there are many factors that influence the result of an experiment, for example factors associated with the calibration of the apparatus you are using, how you used the apparatus, environmental effects, etc. Each of these factors work against you getting a "perfect" result in your experiment.

During this course you will learn how to report a scientific measurement result. In the same way that there are mathematical tools of physics such as coordinate systems, vectors, force diagrams, etc. which are universally accepted ways of communicating physics models, there are clear guidelines regarding the way in which scientific measurement results should be reported. These have been agreed upon by all the international physics, chemistry and other science organizations, and are methods you will be taught in this course. The way that you learn to analyze your experiments in this course is the same way in which scientists all around the world do so. For example, you will learn the scientific methods of numerically estimating uncertainties in an experiment in a realistic way. "Uncertainty" in a science measurement is a parameter (a number) that comes from analyzing all the factors that you think have influenced your experiment. This parameter, together with the best approximation of the measured, form part of the measurement result. You will see in this course that the measurement result is a compact way of summarizing all the knowledge that you have about a measurand. A measurement result is a statement of probabilities. We can never know the true value of a measurand: we can only make statements about our knowledge about the interval in which the measurand exists.

Furthermore, since the nature of the enterprise of physics is about the creation of new knowledge, the more "careful" you are in a physics experiment, the smaller your uncertainty should be. Therefore, scientists design and carry out experiments so as to have as small an uncertainty as is realistically possible. The smaller your uncertainty is, the better knowledge you have about a particular measurand. Of course, it is impossible to make a measurement which has zero uncertainty.

Having an interval as the result of an experiment, and not a single point, also allows us to compare two measurement results with each other, or with a theoretical (calculated) value. For example, we can say that the two measurement results 3.4 ± 0.6 J and 3.7 ± 0.4 J agree with each other, while the measurement 10.41 ± 0.07 m s^{-2} does not agree with the accepted value for Cape Town of 9.79 ± 0.01 m s^{-2}.

Safe laboratory practice

The Physics Course I Laboratory is a safe teaching laboratory and we expect you to approach all activities in the laboratory with a "safety first" attitude – this means that the first thing you consider upon entering the laboratory is the safety of yourself, the other people in your class, the staff and equipment, and also safety is a priority and important to you.

You can take these safety skills to whichever scientific field you may venture. Here are three basic rules of the laboratory:

- **You must wear enclosed and sturdy shoes, and bare feet and/or open shoes (e.g. flip-flops) are not allowed in the laboratory.** This rule minimises the risk of you injuring your feet, such as due to broken glass or equipment dropped from the benches.
- **No eating is allowed in the laboratory and drinking is only allowed from closed containers.** It is safer to take a short break from the laboratory if you feel the need for refreshment. Occasionally hazardous substances such as radioactive sources are used in the laboratory, therefore this rule minimises the risk of you injuring yourself due to an exposure to a hazardous substance. It also minimises damage to equipment, such as the damage of electrical equipment due to water contact, and maintains the laboratory as a clean space to work in.
- **Behave professionally in the laboratory and be considerate and respectful of students in nearby groups.** More workplace accidents happen when people fool around than when they carry out their normal work.

A useful reference to read more about laboratory practice is *Etiquette in the Laboratory* (1976), American Journal of Physics, 7 (44). See http://scitation.aip.org/content/aapt/journal/ajp/44/7/10.1119/1.10320

Laboratory staff

During practical sessions, the laboratory space is managed by the following people:

- **The academic in charge;**
- **The chief scientific officer and scientific officer;**
- **The lab manager, and**
- **The demonstrators.**

You can ask any of the people above for help during the course of a practical, for issues such as conducting an experiment, administration of the laboratory course, concerns about safety and faulty or damaged equipment. If you are **absent from a practical or tutorial** or want to **query a tutorial or practical mark** entered on WebApp or Vula, get in touch with the lab manager; Mr. Mark Christians.

Pre-practical preparation

Before you attend each practical session, read through the material on the practical that you are about to perform and take a look at the relevant instructional videos in the link given below. You will use the practical time the most efficiently if you come prepared and ready to ask questions.

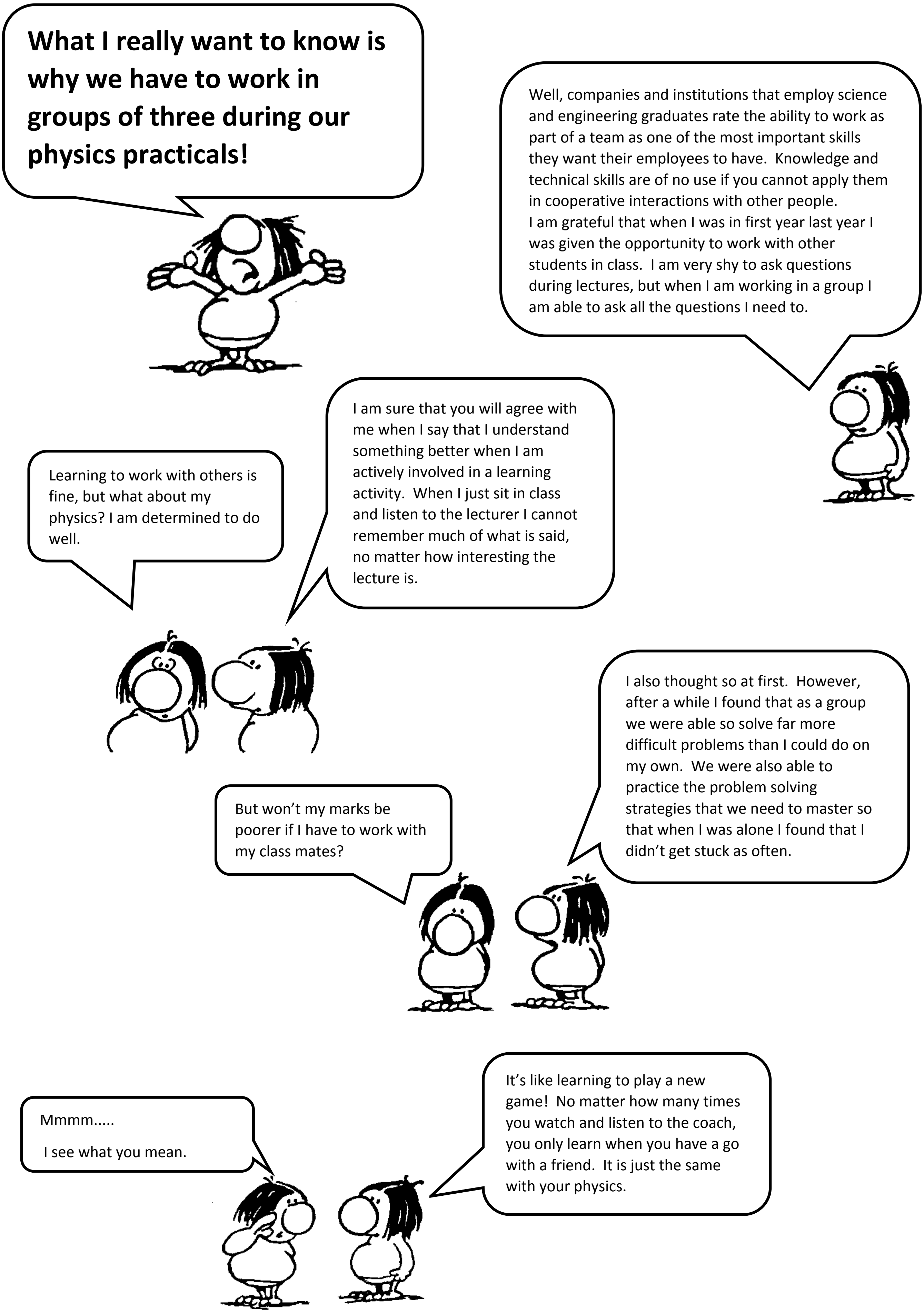
http://www.phy.uct.ac.za/phy/courses/PHYLAB1.

Practical groups

It is intended this course is undertaken in cooperative learning environment. We believe that working in **groups of 3** is best for these activities. Compare your responses to those of your work partners. Help each other to figure out what is going on. Resolve any difficulties that you might have and call a demonstrator if you need to.

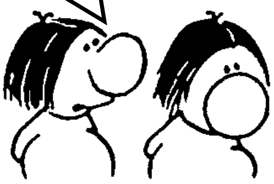
You can choose your own group at the beginning of the semester. We use these groups to allocate and schedule the marking of laboratory reports, so please submit a registration form for your group to the lab manager in the first session of the semester. **This will remain your group, and your laboratory day, for the rest of the semester.** However, if you want to change your group at any time and for any reason, please talk to the lab manager who can assist you in arranging a different group and change in registration, as well as making sure your marked lab reports are returned to the correct session.

Some guidance on how to work in groups is given in the following pages:



You know, sometimes I don't want to explain something to someone else because I am afraid that I might confuse myself.

Don't worry. Resolving that type of confusion is an important part of the learning process. When you talk about the physics in a group you will come away with a better understanding of both the abstract concepts and the problem solving techniques.

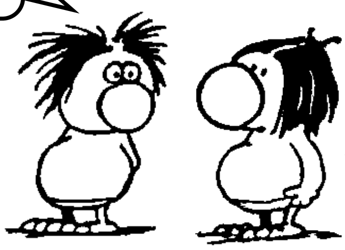


What if I am put in a group with people that I don't get on with too well?

I have found that even when I think that I don't get on with someone, after a while when we have focused on what we have to do, things actually turn out fine. I have rarely met anyone that I can absolutely not work with.

But anyway, the groups don't stay together permanently, so you won't be with the same people forever!

What can I do so that our group works well together?



There are a number of things that you can do in a group that will help you to work together better. The most important is that everyone has a turn to participate and explain their understanding and reasoning. You therefore need to contribute and listen carefully to what your group members are saying.

Check that everyone understands before moving on. Don't exclude people by using a language that one person doesn't understand.

When you are stuck on a question, there are a number of questions that you should ask yourselves <u>before</u> you call a demonstrator:

- Do we understand the question?
- Have we consulted the Guide to Reporting and Measurement?
- Would it be useful to draw a diagram?
- <u>What question are we going to ask the demonstrator</u>?

... this is very important ... it's no use saying to the demonstrator,

"We're stuck" ... or ... "We don't know what to do."

SECTION A – PERFORMING AN EXPERIMENT

A1 Planning an experiment

There are a number of stages to an experiment, each requiring you to think about different things.

Stage 1. Identify the problem that needs solving, or the question that needs answering. What is the aim of the experiment? Think about what results you need in order to make a valid conclusion.

Stage 2. Plan your experiment carefully (see notes below).

Stage 3. Do the experiment and complete the analysis of the data.

Stage 4. Formulate your conclusions to the experiment based your results.

Stage 5. Communicate your experiment in the form of a report.

Never rush into taking readings without careful planning. Of course, it is a good thing to familiarise yourself with the apparatus before starting, but before doing serious measurements sit down and plan carefully exactly what you are going to do. In most cases you will be working with one or two other students in the laboratory. It is important that you discuss things with them. Listen to each other's opinions and talk to a demonstrator, if necessary.

Some important considerations are:

- What are the main steps that I need to carry out? A flow chart is often useful.
- What apparatus do I need and how does it work?
- Are there any precautions that I need to take?
- How much time do I have for the experiment?
- What variables are involved and what exactly must I measure?
- How many measurements do I need to make?
- What tables must I draw up? Detailed tables should be drawn up <u>before</u> taking readings as the tables will serve as a guide.
- What are the main steps in the analysis? Again, a flow chart is often useful.
- Which graphs (if any) will I need to plot?
- What influences will affect the measurements? Make a list beforehand and add to it as you proceed.

After the experiment, **reflect** on how well your plan worked, noting both successful aspects and failures of the plan. Keep in mind the structure of your **report** that you will have to write. Remember that if, while writing the report, you suddenly remember that you did not record something important while doing the experiment, *it will be too late*!

A2 Writing a laboratory report

The purpose of a laboratory report is to communicate the aim, process and outcome of an investigation to an outside audience. It is a record of your direct ("hands-on") experience in the laboratory. In most cases, a scientific investigation is considered to be incomplete without a report. By synthesising (putting together) the different aspects of your laboratory experience in a structured and coherent report, the essence of your investigation becomes clearer in your own mind. In the process, you develop your skills of reasoning and ability to communicate in writing.

There are many acceptable ways of presenting a scientific laboratory report, but, almost all reports will include the components outlined here.

Components of a laboratory report

Title:

This will include the author's name (your name), your partner's names, the course code, the date of the experiment and a suitable title of the report.

The title must be short but factual and descriptive. It must summarise the major aspects to be dealt with in the report. The key words will often come from the laboratory task that has been set and you need to identify these. These words help clarify the requirements of the task and also alert the reader as to what the report is about.

Introduction and aim:

The introduction puts the report into perspective by giving the reader relevant background information about the phenomenon being investigated. This is in order to prepare the reader for what he or she is about to read. This background may include some historical information or developments, earlier experimental work, the theory or law governing the phenomenon being investigated. These introductory remarks must be kept brief to avoid obscuring the main point of the investigation. Sometimes, you will be asked to carry out an investigation in order to dispute or challenge a claim that has been made against a generally accepted scientific phenomenon. If that is the case, state the currently held theory and the claim that has been made. Your introduction must state the aim of your investigation. The integration of the aim into the introduction allows for the smooth transition from general information to the specific goal of the investigation.

Method:

Present a clear, concise and step-by-step description of the apparatus, techniques and procedures used. List and name the apparatus with brief descriptions of the main parts as well as the functions thereof. A neatly sketched and labelled **diagram** of the experimental set-up is essential and can save you paragraphs of tedious written descriptions. Briefly describe the procedures that you followed in the investigation. Wherever appropriate, give a reason for each step you took in a procedure. Sometimes more than one section is required for this material. For example, if two techniques were used, i.e. Technique No.1, Technique No.2, etc. then a brief explanation of each of these techniques must be given. Do not omit any significant steps. A description of method helps you to recall the problems that were associated with experimental procedures such as precision of instruments, strengths and weaknesses of certain techniques, recording of observations etc. This will help you when you have to summarise your conclusions and recommendations at the end of your report.

Results and discussion

The main part of this section are your tables of results and graphs. You must have a consistent way of recording your observations and calculations. Data are normally summarised and displayed in tables and graphs. Each table and graph is usually referenced by a number and should be numbered in sequence, e.g. Table 1, Table 2, Figure 1, Figure 2 etc. Each table is accompanied by a title and each graph by a caption which describes the purpose for which it has been presented. (e.g. "Table 1: Measurements of the width of the cylinder" and "Graph 3: To determine the viscosity of the sample of oil").

Tables and figures must be referred to in the text, e.g. “The apparatus was arranged as shown in Figure 1”; “The data gathered were recorded as shown in Table 1 below” or “The data in Table 1 were used to plot the graphs in Figure 1, 2 and 3.” These brief statements help to link the different parts of the report.

Then examine and extract important aspects of the data and use these to explain various relationships or determine your measurement results. For example, what is the shape of your graph and what does it suggest in terms of the relationship between the variables? If it is a straight line, what is the value of the slope? Remember to quote your results with the appropriate significant figures and the corresponding uncertainties. How does an expected value compare with your own and what reasons can you give for this? You may give tentative explanations for your data but be careful not to mix facts with opinions.

Conclusion and recommendations:

This is normally a section in which you say what the investigation has shown and to what extent the problem or claim stated in the introduction has been resolved. Remember that any conclusions must be supported by evidence from your data. Always quote any final results, together with their uncertainties in your conclusion. Avoid making vague statements such as “This was a successful experiment.” You may also need to discuss sources of uncertainty and any improvements that could be made to the apparatus (and measurements). Again, avoid meaningless phrases such as “it was caused by human error.”

Writing a report allows you to reflect critically on the whole experiment and check your understanding of the purpose of the investigation as well as produce an accurate record of it. Note that an physics practical is not a set of procedures designed to reproduce some “correct” answer. It is a problem that has been posed that requires an experimental solution which may include making measurements, implementing different procedures and techniques and then the formulation of a suitable report.

Whether you graduate and leave with a B.Sc. to work in industry or whether you stay on at university to become a post-graduate student, you will find that report writing will remain as one of the most important activities in your career.

Scientific style

Very often, in reports of this kind, writers prefer to use the passive construction or impersonal style to report procedures followed in conducting experiments, by writing, for example:

“Five measurements were taken,” instead of “I / We took five measurements.”

Both styles are acceptable. As you do most of your practicals in groups, you are likely to visualise what you did as a group and report it as a group activity in which case the personal pronoun “we” is appropriate. However, once you have chosen a style of writing, then you must use it throughout your report and not switch back and forth between the two.

Marking the report

When marking your reports, feedback will be given that is relevant to your individual report. All the feedback you get is intended to help you consider ways of improving your report. Marks for the report will be given for your data collection and processing (which includes your method, tables, graphs and calculations) as well as the overall coherence of the report as a piece of writing. The important thing is that you understand the mark that you receive and speak to someone about it if not. The value of this process cannot be underestimated. Make sure you follow up with the demonstrator who marked your report in the following practical so you know how to improve for next time.

The laboratory report checklist

When preparing your report, use the following checklist to see whether or not you have included everything correctly.

Introduction and aim:

- ✓ Have I written my name, my partners' names and my course on the front of my report book?
- ✓ Does my report have a title and a date?
- ✓ Does my aim clearly outline the purpose(s) of my experiment (in one or two sentences)?
- ✓ Have I clearly outlined the theory involved in my investigation?

Apparatus:

- ✓ Have I drawn the experimental set-up (with a diagram)?
- ✓ Is my diagram labelled and does it have a heading?

Method:

- ✓ Have I described how I went about my experiment (i.e. what I <u>did</u>)?
- ✓ Have I clearly described the significant steps in the procedure (including the analysis on my data)?
- ✓ Have I explained why certain aspects of the procedure were undertaken?

Data / Results:

- ✓ Have I recorded all relevant measurements?
- ✓ Do all my tables have titles?
- ✓ Do all the columns in my tables have headings with units?
- ✓ Are all my data recorded correctly?

Graphs:

- ✓ Does my graph have a title which should state <u>why</u> I plotted my graph?
- ✓ Have I used a decent scale so my graph fills the whole page?
- ✓ Have I labelled the axes, including the correct units?
- ✓ Have I used a ⊙ for the data points and not a ● (blob)
- ✓ Have I used pencil for my graph (and not pen)?
- ✓ Have I drawn the best straight line through my data points? (if necessary)
- ✓ Have I shown how I got the slope from my line? (if necessary)
- ✓ Have I recorded the slope and intercept (with uncertainties) correctly?

Analysis and discussion:

- ✓ Have I described and explained important relationships revealed by the data (and graphs)?
- ✓ Have I <u>clearly</u> set out my calculations?
- ✓ Are my calculations in the correct section of the report?
- ✓ Have I used SI units throughout?
- ✓ Have I shown how I calculated the uncertainties in the results?
- ✓ Is my final result presented correctly with the appropriate significant figures?
- ✓ Have I described and explained the results of my calculations?

Conclusion and recommendations:

- ✓ Does my conclusion refer back to my aims for doing the experiment?
- ✓ Have I fulfilled all my aims for doing the experiment?
- ✓ Have I quoted all my results with their uncertainties?
- ✓ Have I recommended how this experiment may be improved?

A3 Drawing tables and graphs

Tables are a way of organising your data in a structured way while you are working in the laboratory as well as a way of presenting your data in a clear way when you write your report. A graph is an extremely useful way to both present readings of two variables that vary as a function of each other, as well as to analyse the data in order to extract further information. This section describes what we call the "**general rules**" for preparing tables and graphs.

Deciding how many readings to take

Say that you are recording readings of a quantity in an experiment that varies as a function of some other quantity. Since it is impossible to take an infinite number of readings, you will need to usually consider the following:

i. Over what range do you want to record data? The range is the difference between the largest and smallest reading in the series.
ii. How many readings between the smallest and largest reading do you want to record, and how will these readings be spaced apart?

Answering these questions before you start to take readings allows to plan your experiment properly and therefore use your time most effectively.

Guidelines for drawing up tables of data

Here are a few guidelines when drawing up a table.

i. Plan your table, especially how many columns and rows you will need. Try to anticipate what readings you will take. Remember to include columns for the results of any calculations. For example, you may sometimes need to calculate the average of a number of repeated readings.
ii. Each table must have a title which should reflect your reason for recording the data. If you use more than one table in your report, then number them Table 1, Table 2, etc.
iii. Each column must have a heading. Units should be included with the heading and not written alongside each reading in the table.
iv. Record your data carefully. Remember, for example, that writing 36.0 cm and 36 cm is not the same. In the first case you are recording a reading to the nearest millimetre and in the second case you only measured to the nearest centimetre.

An example of a table:

Table 1: Data of the extension of a spring due to an applied force ← Title

Headings with units →

Applied force, F (N)	Extension of spring, x (m)	$\bar{x}$ (m)
0.135	0.039 0.038 0.039	0.030
0.270	0.076 0.075 0.077	0.076
0.406	0.114 0.112 0.113	0.113
0.541	0.150 0.149 0.152	0.150
0.676	0.185 0.186 0.186	0.186

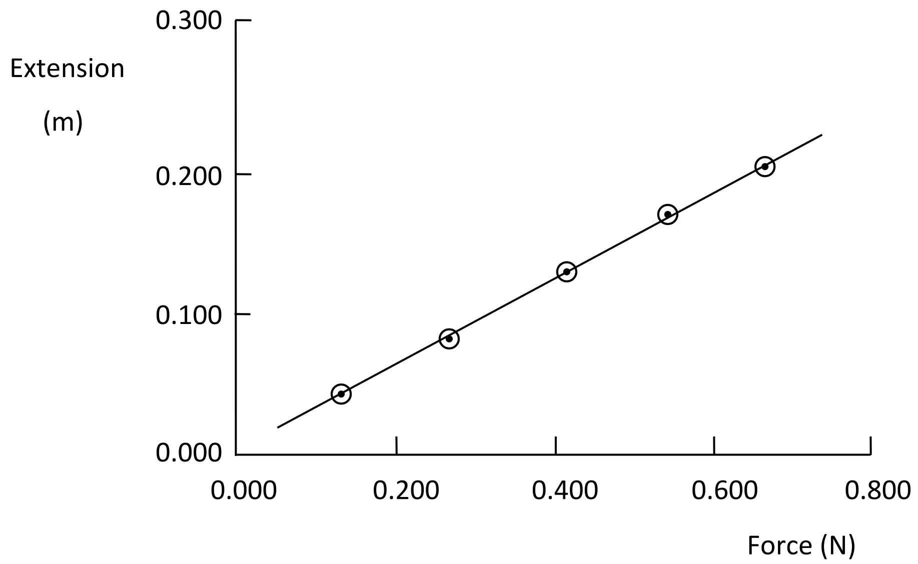
Data Calculations

Guidelines for plotting a graph

i. Use a pencil.
ii. Your graph must have a caption (or title) which should describe why you plotted the graph.
iii. Each axis should be labelled with the name of the variable and the units.
iv. Choose appropriate scales on the axes so that the graph will not be too small on the page, but will cover a fair portion of the page in each direction.
v. Use a ⊙ or × for the data points and not a ● (blob).
vi. Decide whether or not each axis should start from zero (it is not always necessary to show the origin). When do you need to draw the axes from zero?
vii. Axes should be marked in factors of 1, 2, 5, or these times a power of ten. Other factors such as 3 or 4 usually make your scales difficult to read between divisions.
viii. The line or curve that you draw through your data should be a reasonable "best fit" so as to model the trend of the experimental points. Your graph should not join the points.

See below for an example of a straight line graph:

Figure 1: Graph of extension of the spring versus applied force in order to determine the spring constant of a spring.



Interpretation of graphs

One of the aims in investigating physical phenomena is to establish the relationship between the variables that are being measured. For example, if we are investigating an object that is experiencing uniform acceleration, a, then we will find the function describing the relationship between velocity, v, and time, t, to be a straight line of the form,

$$y = mx + c;$$

an example of which is shown in graph (a) of Figure 2. In this case,

$$v = u + at\ ,$$

where u is the initial velocity.

On the other hand, the relationship between the position, r, and the time, t, will be of the form,

$$y = ax^2 + bx + c,$$

i.e. a parabola as shown in graph (b) of Figure 2. In this example we will find,

$$r(t) = r_0 + ut + \frac{1}{2at^2},$$

where r_0 is the initial position. Some other functions that you will encounter are the hyperbola, which is shown in graph (c) of Figure 2 and of the form,

$$xy = c,$$

and the exponential function which is shown in graph (d) of Figure 2 and of the form,

$$y = ae^{bx}.$$

Note that we say that "y is proportional to x" if a graph of y versus x yields a straight line through the origin. i.e. $y = mx + c$ with $c = 0$. Sometimes "directly proportional to" is used which means the same thing. However, if $c \neq 0$, then we cannot say that "y is proportional to x", but can only say that "y is linearly related to x".

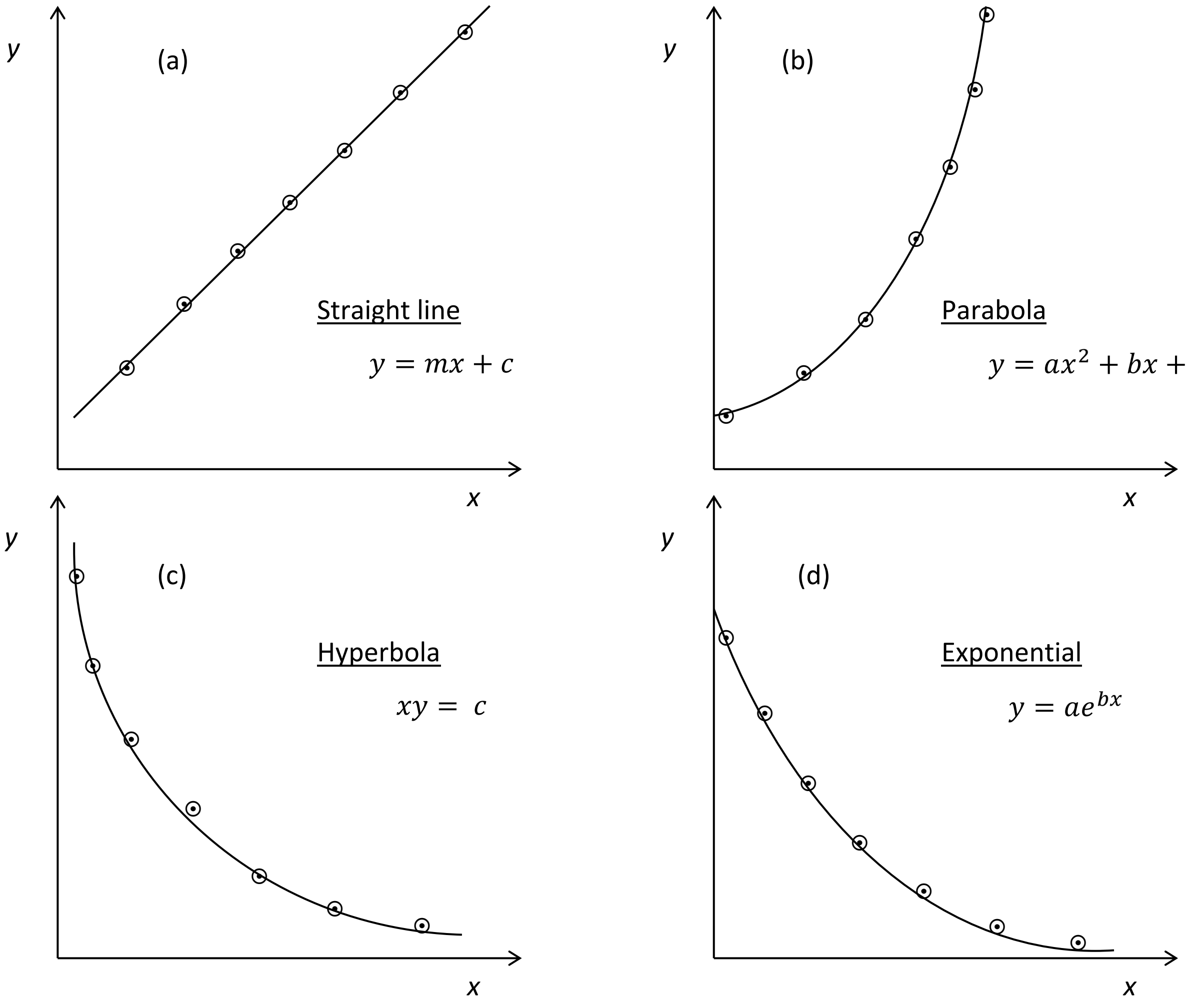


Figure 2: Different types of graphs that you may encounter.

A4 Significant figures

What are significant figures?

The number of figures that convey meaningful information in recorded data are known as the “number of significant figures”.

Significant figures in a number are:

- All non-zero digits (e.g., 54 has two significant digits while 456.78 has five significant digits).
- All zeros appearing anywhere between two non-zero digits (e.g., 302.05 has five significant digits).
- All trailing zeros, where the number has a decimal point (e.g., 36.65000 has seven significant digits).
- Leading zeros are <u>not</u> significant (e.g., 0.0003 has only one significant figure).

Do not confuse significant figures with decimal places. The number of decimal places refers to the number of figures after the decimal point, e.g., 46.320 mm has five significant figures, but three decimal places; while 0.0040 mm has two significant figures, but four decimal places.

<u>Recording significant figures in readings and calculations</u>

When taking readings, the number of significant figures recorded is determined by the precision of the instrument. For example, if the scale on a particular instrument makes it possible to read a mass to 1/10th of a gram, then it is incorrect to record the reading of some mass to 1/100th of a gram, i.e., in such a case the recording of a mass of say 12.4 g would be correct, but recording that same mass as 12.40 g using the same instrument would be incorrect.

Similarly, when using a typical wooden laboratory metre rule to measure distance, you can only take readings to the nearest millimetre. For example, you may record the reading of a distance as say 127 mm, but to record a reading of 127.0 mm will be unreasonable because that degree of precision (to 0.1 mm) cannot be achieved when using a typical laboratory meter rule. (Consider the problem of parallax.)

SECTION B – MEASUREMENT UNCERTAINTY

B1 Overview

The purpose of undertaking a measurement in science is to provide knowledge about some physical quantity. The physical quantity that we may wish to investigate is called the **measurand**, e.g., the measurement may be the air pressure in a soccer ball, the temperature of the air in a room, or the voltage across a resistor.

It is important to realise that the value of a measurand can never be determined with absolute certainty. The best that can be achieved through a process of measurement is to improve our knowledge about some specified measurand. We cannot think of a measurand as having some "true" or "exact" value that can actually be found; we have to think of the knowledge that we have of any measurand always as being limited and incomplete.

The following framework for measurement uncertainty is based on a *probabilistic approach* and the relevant standards are expressed in ISO/IEC 17025. These standards are clarified in the associated Guide to the Expression of Uncertainty in Measurement (GUM) as well as in the ISO International Vocabulary of Basic and General Terms in Metrology (VIM). They are also documented in the NIST Technical Note 1297 of 1994.

Note that the term "error" is often used quite loosely in conversation in laboratory work but it is important to realise that the word "error" is not synonymous with "uncertainty". The term "uncertainty" is clearly defined, while use of "error" may be misleading, so the use of the word "error" in the context of laboratory work is discouraged.

Decide on the best approximation of the measurand

Having taken the readings, x_1 or (x_1, x_2, x_3, …. x_n for n readings) as the case may be, the next step is to decide on the **best approximation** of the value of the measurand.

In the case where only **one reading** has been taken, then that reading is the best approximation of the value of the measurand, e.g., x. However, where **more than one reading** has been taken, then the best approximation of the value of the measurand is the mean (average) of those readings, e.g., $\bar{x}$.

List all the possible sources of uncertainty

There may be many sources of uncertainty to be taken into account in any one measurement. These sources include, for example: a) the manufacturer's rating of the instrument used, b) the reading of the scale of the instrument, c) corrections that might need to be included as a result of ambient conditions, and so on…

Which of these sources of uncertainty are to be taken into account in any particular measurement is based mainly on experience and skill in measurement, but the important first step is to identify and list them.

Evaluate the uncertainties that have been identified.

Having identified the relevant sources of uncertainty in the measurement of x, we can evaluate the value of each uncertainty. The key value of the uncertainty in the measurement is known as the **standard uncertainty** and the symbol is $u(x)$. Subscripts are used to indicate difference sources; for example, $u(x_{\text{rating}})$ and $u(x_{\text{reading}})$.

The way in which we evaluate the uncertainty in our knowledge of a measurand has its roots in a method of statistical inference based on the **Bayesian theorem**; and the functions used to describe how the uncertainty in our knowledge of a particular measurand is formulated are known as probability density functions (pdfs). However, in this course we will not delve into the statistical underpinnings of uncertainty evaluation, nor will we be using pdfs in a rigorous way, but we will refer to the relevant pdf as appropriate and will simply use the method as required.

There are two ways to evaluate the **standard uncertainty**, a TYPE A evaluation and a TYPE B evaluation; and the method you will use depends on the nature of the measurement.

- If a **number of readings** are available, in which there is **scatter**, then a statistical, TYPE A, evaluation of the uncertainty should be used.
- All other evaluations of uncertainty are TYPE B evaluations. TYPE B evaluations may be applied to many sources of uncertainty, e.g., the rating of the instrument, the reading of a scale, the correction factors that may need to be applied to the measurement, etc.

The TYPE A and TYPE B evaluation methods are discussed in detail in the sections that follow.

<u>Combine the standard uncertainties to find the combined standard uncertainty.</u>

Depending on the nature of the measurement it may be necessary to combine the standard uncertainties, (x_{rating}), $u(x_{\text{reading}})$, etc., to give a **combined standard uncertainty**.

<u>Quote the result of the measurement</u>

The final step is to quote the result of the measurement: e.g., $x \pm k.u_c(x)$, stating the relevant coverage probability (also referred to as the 'level of confidence').

See D1 Quoting a result for more information on reporting the result of the measurement.

B2 TYPE A evaluation of uncertainty

In a TYPE A evaluation of the uncertainty, a statistical method is used to infer the value of the uncertainty associated with a measurement by quantifying the "scatter", or the "spread" in the values of a **set of data**. There are two commonly used cases:

1) The data are a set of readings of the form: $x_1, x_2 \ldots . x_n$

A histogram of the readings shows a distribution around some mean value $\bar{x}$ that can be represented by a symmetrical bell-shaped curve (referred to as a "Gaussian") where:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\bar{x})^2}{2\sigma^2}\right]}$$

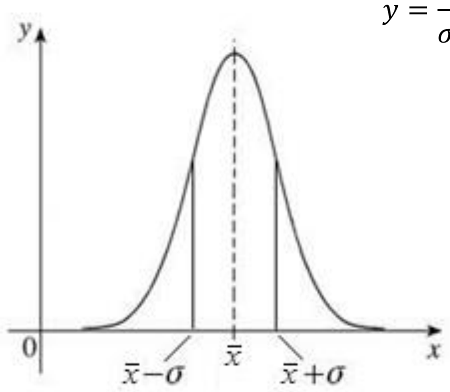
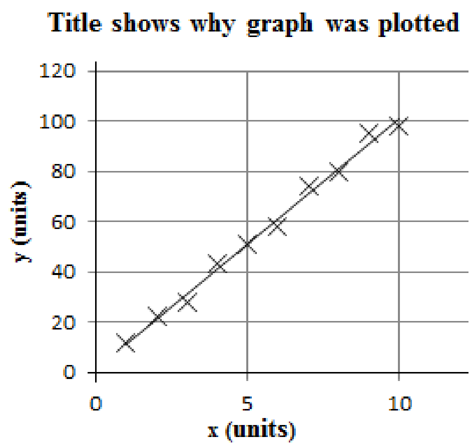


Figure 3: Normal or 'Gaussian' distribution function.

When the relevant set of readings, $x_1, x_2 \ldots . x_n$, for n readings are plotted as a histogram, and the histogram allows one to fit a curve similar in shape to a normal distribution, then the best approximation of x is $\bar{x}$, and the standard uncertainty $u(x)$ is given by the **experimental standard deviation of the mean,** $\frac{\sigma}{\sqrt{n}}$.

2) The data are a set of readings of the form: $(x_1, y_1), (x_2, y_2) \ldots . (x_n, y_n)$



In this case the function that is used to analyse the data is 'linearised' (details to be covered in E2 Linearising equations) and the data pairs, (x_i, y_i) i = 1, 2, …n are plotted as a straight line in the form $y = mx + c$.

A technique known as a '**least squares fit**' is used to fit a line to the data and then the best approximation and the standard uncertainty may be derived from the gradient $m \pm u(m)$ and the intercept $c \pm u(c)$of the fitted line.

Formulae for a TYPE A evaluation of uncertainty

Consider an experiment in which a set of readings has been taken, where $x_1, x_2 \ldots . x_n$, and n is the number of readings and x_i is the i^{th} reading. If there is a scatter in the values of these readings, then it can be reasonably assumed that the distribution of the readings 'fits' a Gaussian probability density function.

Using this assumption*, there are three values required to calculate the uncertainty in the measurement:

- the **mean** $\bar{x}$ (sometimes the symbol μ is also used for the mean)
- the **standard deviation** σ, and
- the experimental standard deviation of the mean which, when the distribution of the values of the mean of the readings is normal, is the **standard uncertainty** $u(x)$.

The following equations are used to calculate the parameters used to determine the uncertainty associated with the measurement:

Mean of the data: $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$ … (B2-1)

Standard deviation: $$\sigma = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$ … (B2-2)

Standard uncertainty: $$u(x) = \frac{\sigma}{\sqrt{n}}$$ … (B2-3)

Note:

1) *The assumption that the distribution of the values of the readings is normal validates the statement that "the **experimental standard deviation of the mean** is the **standard uncertainty** $u(x)$" for the majority of cases. However, it is important to realise that a different approach may be required where the distribution of values is not 'normal'.
2) Because a Gaussian distribution was assumed, the probability that the measurand lies **within one standard uncertainty** of the best approximation, is **68%**.

Example: TYPE A evaluation of uncertainty associated with a set of data

Consider an example in which the measurand is the period T of an oscillating pendulum. A stopwatch has been used to take ten readings (n = 10) of the time taken for twenty oscillations of the pendulum. It is assumed that the distribution of the values in this set of data can be modelled by a Gaussian distribution.

Table B2.1: Data recorded using digital stopwatch.

Time for 20 oscillations, T_{20} (s)		Period, $T = T_{20}/20$ (s)	
19.43	21.65	0.972	1.083
20.49	20.82	1.025	1.041
20.76	19.77	1.038	0.989
20.63	20.39	1.032	1.020
20.56	19.02	1.028	0.951

Using Eq. (1), the **mean**, $\bar{T}$ = 1.0176 s. This is the best approximation of the measurand.

Using Eq. (2), the **standard deviation** σ is 0.0379 s. However, this value is <u>not</u> the standard uncertainty. For that we still need to calculate the experimental standard deviation of the mean, which is the standard uncertainty in this case.

Using Eq. (3), the **standard uncertainty** $u(T)$ = 0.0119 s.

The result of this measurement, the period T of the pendulum, may be quoted as:

T = (1.018 ± 0.012) s, and since this is to one standard uncertainty - and a Gaussian pdf was used - the coverage probability is 68%.

Note: The result may be quoted to a higher coverage probability (a greater level of confidence) by using an expanded uncertainty, see D1 Quoting a result.

B3 TYPE B evaluation of uncertainty

A TYPE B evaluation of uncertainty is any evaluation of an uncertainty that is not a TYPE A evaluation. So, for example, if the uncertainty as a result of the rating of the instrument has to be evaluated, or if only one reading is available, then a non-statistical TYPE B evaluation of uncertainty is performed.

The methods of TYPE B evaluation of uncertainty will be presented by way of examples.

Example 1: Uncertainty associated with the **rating** of an instrument

Manufacturers of measuring instruments will state the information required to evaluate the uncertainty associated with the rating of the instrument, e.g. $u(x_{\text{rating}})$, by specifying the rating as:

a) a percentage of the maximum value that can be displayed, or
b) a percentage of the full scale deflection (FSD), or
c) simply by stating that the rating is a percentage of whatever is being displayed on the instrument.

In the first example we consider that the measurand is a pressure p and the instrument being used to take the readings of the pressure in this case is a typical bourdon tube gauge.

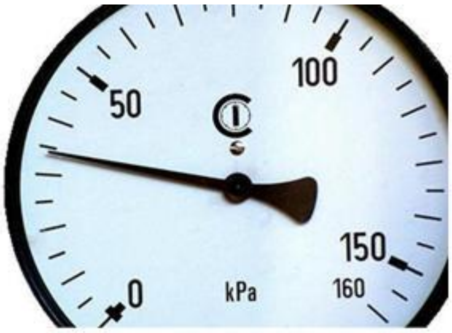


Figure 4: Reading on a bourdon tube pressure gauge.

Just by looking at the position of the needle on the gauge we can see that the best approximation of the measurand is: p = 34 kPa. (There is only one reading.)

Now we want to evaluate the uncertainty associated with the **manufacturer's rating** of the instrument, $u(p_{rating})$. The reason for selecting this source is because the bourdon gauge is a simple instrument and so the uncertainty due to the rating of the instrument far outweighs the contribution of all the other applicable sources of uncertainty. Typically, for a bourdon tube gauge, the uncertainty in the rating of an instrument of the type shown here is ±2.5% of the full scale deflection (FSD) of the instrument; and in this case, FSD is 160 kPa. The standard uncertainty (due to the **rating**) associated with this instrument will be:

$$u(p_{rating}) = 160 \text{ kPa} \times \frac{2.5}{100} = 4.0 \text{ kPa} \qquad \text{… (B3-1)}$$

And so the result of the measurement of the pressure shown on this instrument will be quoted as:

$$p \pm u(p) = (34.0 \pm 4.0) \text{ kPa.}$$

In every measurement, the instrument manufacturer's specification needs to be checked to determine the instrument rating. Caution: check whether the manufacturer's rating has been given **to one or more standard deviations**; as this has to be taken into account when you combine the uncertainties, see D1 Quoting a result.

Example 2: Uncertainty associated with **reading** a digital display

In this example, the measurand is a temperature T and the instrument chosen to take readings of the temperature is an electronic thermometer that has a 3-digit digital display. The reading you see on the thermometer is shown in Figure 5.

Figure 5: Reading on a digital thermometer.

By inspection we can see that the best approximation of the measurand is: T_1 = 25.3 °C. (There is only one reading.)

Now we want to evaluate the uncertainty associated with the reading of the display, $u(T_{\text{read}})$. This uncertainty arises because even though the display shows 25.3 °C, any actual temperature between 25.25 °C and 25.35 °C would result in the same display on the instrument when rounded to 3 digits.

The evaluation of $u(T_{\text{read}})$ proceeds as follows:

1) The only information we have about the temperature is that it is between a lower bound, T_{lower}= 25.25 °C and an upper bound, T_{upper}= 25.35 °C.
2) We make the assumption that it is equally probable for the temperature T to be <u>any</u> value between T_{lower} and T_{upper}.
3) Having assumed that it is *equally probable* that the value of T lies anywhere in a symmetric interval around 25.30 °C, this uncertainty evaluation can be modelled on a uniform or rectangular probability density function, see Figure 6, and so the standard uncertainty $u(T_{\text{read}})$ can be evaluated as:

$$u(T_{read}) = \frac{(T_{\text{upper}} - T_{\text{lower}})}{2\sqrt{3}} = \frac{(25.35 - 25.25)\ °\text{C}}{2\sqrt{3}} = 0.029\ °\text{C} \qquad \text{... (B3-2)}$$

The reason for dividing by $\sqrt{2}$ in equation B3-2 is because this is the factor applicable to the assumptions regarding a rectangular pdf.

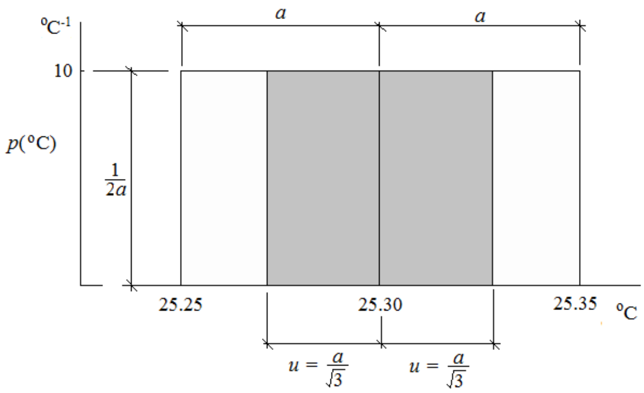


Figure 6: Rectangular probability density function.

Note that the total area under the pdf is equal to 1, meaning that there is a 100% probability of the measurand being in the ± a interval. Moreover, the grey shaded area under the rectangular pdf has an area of 0.58, so there is a 58% probability that the measurand is in the ± u interval. Therefore, because a rectangular distribution was assumed, the *coverage probability* that the measurand lies **within one standard uncertainty,** i.e., within ± $u(T)$ of the best approximation T, is **58%**.

Example 3: Uncertainty associated with **reading** an analogue display

In this example, the measurand is a temperature T but now the instrument chosen to take readings of the temperature has an analogue display. The reading you see on the thermometer is shown in Figure 7.

The best approximation of the measurand is T_1 = 75 °C (there is only one reading).

In this example the only source of uncertainty that we will take into account is the uncertainty associated with the reading of the dial of the instrument, $u(T_{\text{read}})$. This uncertainty arises because the instrument really tells us is that the temperature is between 70 °C and 80 °C, and is most probably 75 °C.

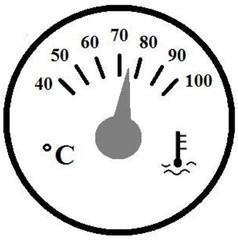


Figure 7: Reading on an analogue thermometer.

The evaluation of $u(T_{\text{read}})$ proceeds as follows:

1) As was the case in Example 2, we judge that the temperature is between a lower bound T_{lower} = 70 °C and an upper bound T_{upper} = 80 °C.
2) However, in this case it is not equally probable that temperature T will be any value between T_{lower} and T_{upper}. The temperature T is most likely to be a value 'close to' 75 °C, which we can say as the needle is approximately half way between 70 and 80 °C. The choice of the upper and lower limits is arbitrary, and tends to be a pair positioned symmetrically around the most likely value, and aligned with tick marks of the instrument.
3) Having assumed that it is *most probable* that the value of T lies at the centre of a symmetric interval around 75 °C, this uncertainty evaluation can be modelled on a triangular probability density function, see Figure 8, and so the standard uncertainty $u(T_{\text{read}})$ can be evaluated as:

$$u(T_{\text{read}}) = \frac{\left(T_{\text{upper}} - T_{\text{lower}}\right)}{2\sqrt{6}} = \frac{(80-70)\ {}^\circ\text{C}}{2\sqrt{6}} = 2.1\ {}^\circ\text{C} \qquad \text{... (B3-3)}$$

The reason for dividing by $\sqrt{6}$ in equation B3-3 is because this is the factor applicable to the assumptions made regarding a triangular pdf.

Note that the total area under the pdf is equal to 1, meaning that there is a 100% probability of the measurand being in the ± a interval. Moreover, the grey shaded area under the triangular pdf has an area of 0.65, so there is a 65% probability that the measurand is in the ± u interval. Therefore, because a triangular distribution was assumed, the *coverage probability* that the measurand lies **within one standard uncertainty,** i.e., within ± $u(T)$ of the best approximation T, is **65%**.

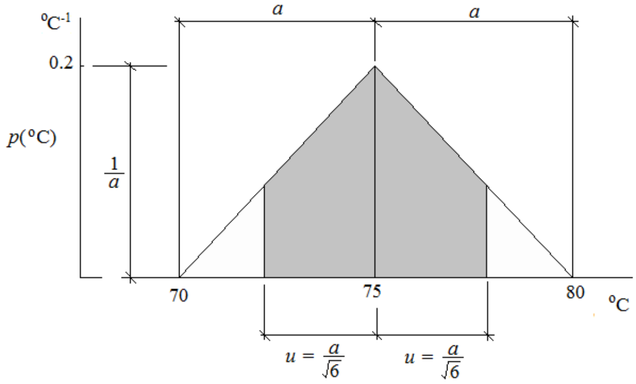


Figure 8: Triangular probability density function.

B4 Combination of uncertainty in one measurement

In the previous three examples the uncertainty associated with only one possible source of uncertainty was evaluated in each example. However, there are many possible sources of uncertainty that may have to be taken into account in any one measurement and once these have been identified and evaluated the **combined standard uncertainty** in the measurement has to be calculated.

General formula for combining uncertainties

Assume that the measurand is some quantity *k* and that some number of sources of uncertainty have been identified and designated as: $u(k_1), u(k_2), u(k_3), u(k_4)$ …

Once the uncertainties associated with each of these sources has been evaluated, the values of the standard uncertainties may be combined, in quadrature, to find the **combined standard uncertainty** $u(k)$, as follows:

$$u(k) = \sqrt{[u(k_1)]^2 + [u(k_2)]^2 + [u(k_3)]^2 + [u(k_4)]^2 + \cdots} \qquad \text{… (B4-1)}$$

It does not matter whether the uncertainty associated with the individual sources has been determined by means of a TYPE A or a TYPE B evaluation, they can all be combined using equation B4-1. Before you combine uncertainties, make sure that they have all been adjusted to relate to the one (or the same) standard uncertainty.

Example 1: uncertainty associated with using an analogue voltmeter

In this example, the measurand is the voltage V across a torch battery. An analogue voltmeter is chosen to determine the voltage and when the leads of the voltmeter are connected to the battery, the reading shown in Figure 9 appears on the display of the voltmeter:

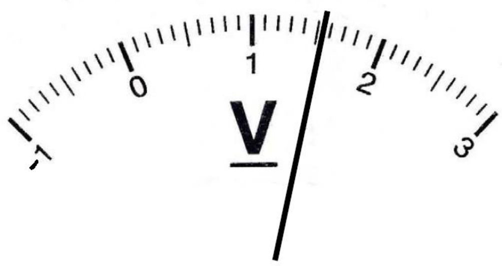


Figure 9: Reading on an analogue voltmeter display.

The displayed reading is the best approximation of the measurand, in this case 1.55 V.

Consider that in this measurement there are two sources of uncertainty to be evaluated:

1) the uncertainty associated with the rating of the instrument, $u(V_{\text{rating}})$ and
2) the uncertainty associated with reading the display, $u(V_{\text{read}})$.

The typical rating for a moving coil instrument of this kind is 2% of the full scale deflection (FSD).

So in this case the standard uncertainty, $u(V_{\text{rating}}) = 3.0\text{ V} \times \frac{2}{100} = 0.060\text{ V}.$

To evaluate the uncertainty in the reading, consider that the lower and upper bounds of the value of the measurand are 1.5 V and 1.6 V and that the evaluation of this uncertainty may be modelled on a triangular pdf, so

$$u(V_{\text{read}}) = \frac{(V_{\text{upper}} - V_{\text{lower}})}{2\sqrt{6}} = \frac{(1.6 - 1.5)\text{ V}}{2\sqrt{6}} = 0.021\text{ V}.$$

The standard uncertainties are combined:

$$u(V) = \sqrt{\left[u(V_{\text{rating}})\right]^2 + [u(V_{\text{read}})]^2} = \sqrt{(0.060\ \text{V})^2 + (0.021\ \text{V})^2} = 0.064\ \text{V}$$

Finally, the battery voltage can be quoted as V = (1.550 ± 0.064) V, to one standard uncertainty. Given that the standard uncertainty was obtained by using a triangular probability density function, the coverage probability (level of confidence) in this result is 65%.

The **oscilloscopes in PHYLAB1** have the following ratings, to one standard uncertainty:

- **±5% on the x –axis** (for time-base between 0.1 µs and 50 ms per division),
- **±3% on the y –axis** (for voltage between 5 mV and 5 V per division).

Example 2: measurement of VOLTAGE using an oscilloscope

Consider an example in which the measurand is the peak-to-peak voltage V of a sine wave produced by a signal generator. You connect the signal generator to an oscilloscope and you see the wave shown in Figure 10:

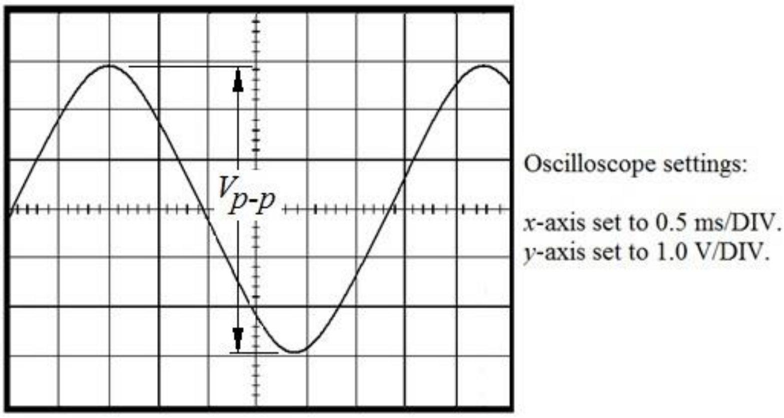


Figure 10: Reading a voltage on an oscilloscope.

After careful consideration, you decide that the peak-to-peak voltage spans 5.8 divisions and so the best estimate of V is 5.8 V (5.8 Divisions × 1.0 Volt/Division).

Now there are three sources of uncertainty to be evaluated:

1) the uncertainty associated with the instrument rating,
2) the uncertainty associated with the reading at the top peak on the display, and
3) the uncertainty associated with the reading at the bottom peak on the display.

The **oscilloscopes in PHYLAB1** have a rating of **±3% on the y –axis** to one standard uncertainty (for voltage between 5 mV and 5 V per division). Therefore the standard uncertainty associated with the rating is,

$$u(V_{\text{rating}}) = 5.8\ \text{V} \times \frac{3}{100} = 0.17\ \text{V}.$$

To evaluate the uncertainty associated with the **reading** of the display, consider that the lower and upper bounds of every reading will be 0.1 of a division on either side of the best approximation anywhere on the display and that the evaluation of this uncertainty may be modelled on a triangular pdf. We need to multiply the uncertainty in divisions by the sensitivity scale of the oscilloscope settings to achieve an uncertainty in volts:

$$u(V_{\text{read}}) = \frac{(V_{\text{upper}} - V_{\text{lower}})}{2\sqrt{6}} = \frac{(0.1 - (-0.1))\ \text{DIV}}{2\sqrt{6}} \times 1.0 \frac{\text{V}}{\text{DIV}} = 0.041\ \text{V}.$$

Finally, these three standard uncertainties are combined to give the combined standard uncertainty associated with the measurement of V. Note the uncertainty in reading the top and bottom peaks on the display are equivalent:

$$u(V) = \sqrt{[u(V_{\text{rating}})]^2 + [u(V_{\text{read,top}})]^2 + [u(V_{\text{read,bottom}})]^2} = \sqrt{(0.17\ \text{V})^2 + 2(0.041\ \text{V})^2} = 0.18\ \text{V}.$$

The result of this measurement can be quoted as:

Voltage, V = (5.80 ± 0.18) V, to one standard uncertainty.

Given that the standard uncertainty was obtained by using a triangular probability density function, the coverage probability (level of confidence) in this result is 65%.

Example 3: measurement of PERIOD using an oscilloscope

Next consider an example in which the measurand is the period T of a sine wave produced by a signal generator. You connect the signal generator to an oscilloscope and you see on the screen the wave shown in Figure 11:

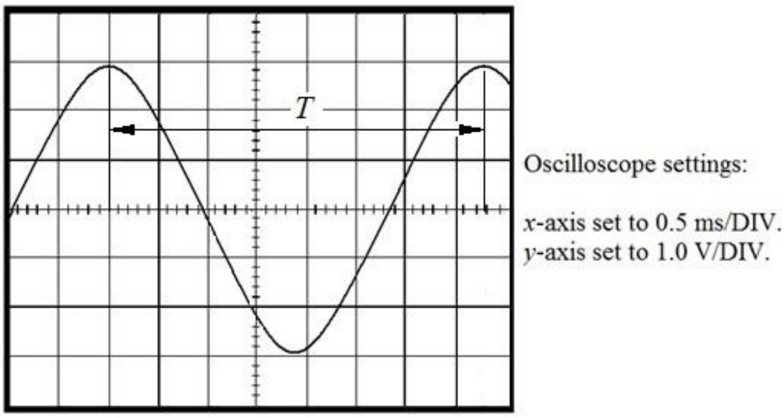


Figure 11: Reading a period on an oscilloscope.

After careful consideration, you decide that the period spans 7.6 divisions, and so the best approximation of T is 3.8 ms (i.e. 7.6 Divisions × 0.5 ms/Division).

Again there are three sources of uncertainty to be evaluated:

1) the uncertainty associated with the instrument rating,
2) the uncertainty associated with the reading on the left of the display, and
3) the uncertainty associated with the reading at the right of the display.

The **oscilloscopes in PHYLAB1** have a rating of **±5% on the x –axis** to one standard uncertainty (for time-base between 0.1 µs and 50 ms per division). Therefore, the standard uncertainty associated with the rating is,

$$u(T_{\text{rating}}) = 3.8\ \text{ms} \times \frac{5}{100} = 0.19\ \text{ms}.$$

To evaluate the uncertainty associated with the **reading** of the display, consider that the lower and upper bounds of every reading will be 0.1 of a division on either side of the best approximation anywhere on the display and that the evaluation of this uncertainty may be modelled on a triangular pdf. We need to multiply the uncertainty in divisions by the sensitivity scale of the oscilloscope settings to achieve an uncertainty in volts:

$$u(T_{read}) = \frac{(T_{\text{upper}} - T_{\text{lower}})}{2\sqrt{6}} = \frac{(0.1 - (-0.1))\ \text{DIV}}{2\sqrt{6}} \times 0.5\,\frac{\text{ms}}{\text{DIV}} = 0.021\ \text{ms}.$$

As with the measurement of the voltage, these standard uncertainties are combined to give the combined standard uncertainty associated with the measurement of T:

$$u(T) = \sqrt{\left[u(T_{\text{rating}})\right]^2 + \left[u(T_{\text{read,left}})\right]^2 + \left[u(T_{\text{read,right}})\right]^2} = \sqrt{(0.19\ \text{ms})^2 + 2(0.021\ \text{ms})^2} = 0.19\ \text{ms}.$$

Note that as $u(T_{\text{rating}})$ is considerably larger than $u(T_{\text{read}})$ by an order of magnitude larger, it dominates the combined uncertainty, $u(T)$.

The result of this measurement can be quoted as:

Period T = (3.80 ± 0.19) ms, to one standard uncertainty.

And given that the standard uncertainty was obtained by using a triangular probability density function, the coverage probability (level of confidence) in this result is 65%.

SECTION C – CALCULATIONS WITH UNCERTAINTIES

C1 Calculation of uncertainty across multiple measurements

Often, having completed two or more measurements where the results of those measurements are say $x \pm u(x)$ and $y \pm u(y)$, you need to use these results to **calculate** some other value, say z and its associated uncertainty $u(z)$. **Caution:** you cannot perform the same operation on the uncertainties as you do on the variables (e.g. in the case of $z = x + y,\ u(z) \neq u(x) + u(y)$).

The formulae required to calculate or propagate uncertainties are given in Table C1-1. We will not derive these equations, as this requires partial derivatives which you may not have met yet in your mathematics course.

Table C1-1: Formulae used in the calculation of uncertainties.

Form of equation from which result R is calculated	*Formula for calculating the standard uncertainty $u(R)$*
Sum of variables $R = a\,A \pm b\,B \pm c$ Coefficients a, b & c are constants (numbers with zero uncertainty)	$[u(R)]^2 = [a\ u(A)]^2 + [b\,u(B)]^2$ or $u(R) = \sqrt{[a\ u(A)]^2 + [b\,u(B)]^2}$
Product of variables $R = c\,A^a\,B^b$ Coefficients a, b & c are constants (numbers with zero uncertainty)	$\left[\frac{u(R)}{R}\right]^2 = \left[a\frac{u(A)}{A}\right]^2 + \left[b\frac{u(B)}{B}\right]^2$ or $u(R) = R\sqrt{\left[a\frac{u(A)}{A}\right]^2 + \left[b\frac{u(B)}{B}\right]^2}$
Correlated variables (consider co-variance if the instrument used more than once in the same experiment) $R = a\,A$ Coefficient a is a constant (numbers with zero uncertainty).	$u(R) = \sqrt{a}\ u(A)$

Note: these equations are three results from a general function for the propagation of uncertainties. To find out more, consult the full version of this guide at the UCT Physics website, "Measurement Manual".

C2 Examples of uncertainty calculations

Example 1: Uncertainty associated with reading a metre stick

Consider a pendulum (see Figure 12), where the effective length is the difference between the position, x_{ring}, of the centre of the ring relative to the metre stick at one end, and the position, x_{ball}, of the centre of the mass of the ball relative to the metre stick at the other. So, there are two measurements of position to be made.

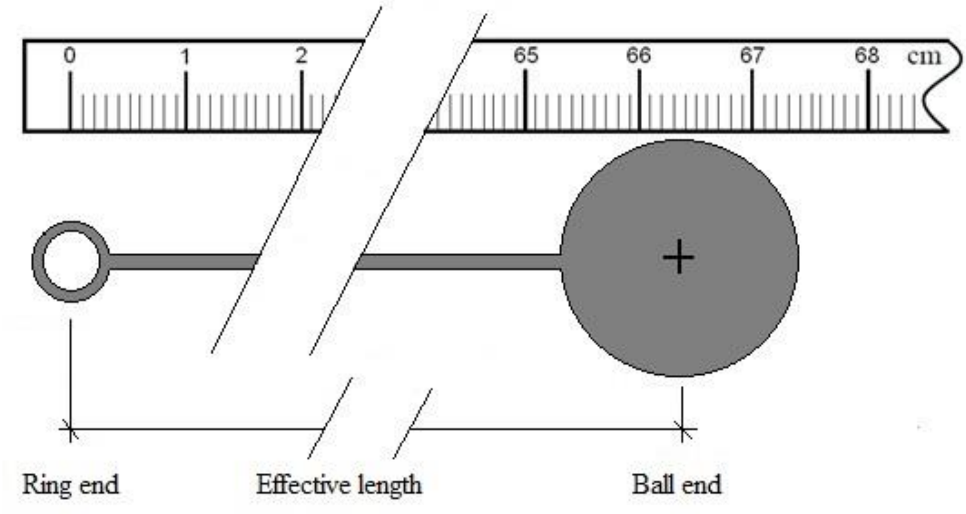


Figure 12: Reading of pendulum length on a metre stick.

In identifying sources of uncertainty, consider that the problem of parallax is a significant contributor to the uncertainty associated with both measurements, while there is also an uncertainty as to where the centre of mass of the ball might actually be.

The best approximation for the position of x_{ring} is 0.0 cm, because the metre stick was lined up that way (we did not have to start at zero but in this case we did and we read in centimetres as the unit of the instrument). Using a triangular pdf, the standard uncertainty, $u(x_{\text{ring}})$ can be evaluated as:

$$u(x_{\text{ring}}) = \frac{(x_{\text{ring,upper}} - x_{\text{ring,lower}})}{2\sqrt{6}} = \frac{(0.1 - (-0.1))\ \text{cm}}{2\sqrt{6}} = 0.041\ \text{cm}.$$

The position of the centre of the ring relative to the metre stick may be quoted as x_{ring} = (0.000 ± 0.041) cm, to one standard uncertainty.

The best approximation of the position of the ball, x_{ball} is 66.4 cm. Using a triangular pdf, the standard uncertainty, $u(x_{\text{ball}})$ can be evaluated as:

$$u(x_{\text{ring}}) = \frac{(x_{\text{ball,upper}} - x_{\text{ball,lower}})}{2\sqrt{6}} = \frac{(66.6 - 66.2)\ \text{cm}}{2\sqrt{6}} = 0.082\ \text{cm}.$$

The position of the centre of the ball relative to the metre stick may be quoted as x_{ball} = (66.400 ± 0.082) cm, to one standard uncertainty.

To find the effective length L of the pendulum we need to subtract the position of one from the position of the other. The best approximation of L is the difference between the two best approximations of the measurements so:

$$L = (x_{\text{ball}} - x_{\text{ring}}) = (66.4\ \text{cm} - 0.00\ \text{cm}) = 66.4\ \text{cm}.$$

Therefore the best approximation of L is 66.4 cm.

We combine the two standard uncertainties, $u(x_{\text{ball}})$ and $u(x_{\text{ring}})$, using the first equation in Table C1-1. The appropriate formula for the calculation of the combined standard uncertainty is:

$$u(L) = \sqrt{[u(x_{\text{ball}})]^2 + [u(x_{\text{ring}})]^2} = \sqrt{(0.082 \text{ cm})^2 + (0.041 \text{ cm})^2} = 0.092 \text{ cm}$$

Finally, the effective length of the pendulum is L = (66.400 ± 0.092) cm, to one standard uncertainty. Given that the standard uncertainty was obtained by using a triangular probability density function, the coverage probability (level of confidence) in this result is 65%.

Example 2: Calculating the uncertainty where measurements are correlated.

Consider the situation where a starter pistol is fired and the sound wave reflects off a nearby wall to be heard by an observer. The total distance, x, travelled by the sound to return to can be seen as either:

$x = d_1 + d_2$, where d_1 is the distance from the source to the wall, and d_2 is the distance from the wall to the listener - and you treat the two distances **as if these were two independent measurements -**

or

$x = 2d$, where the same measurement, d, is used twice. In this case the measurement 'there' correlates with the measurement 'back', and so a **covariance factor** of $\sqrt{2}$ has to be included in the calculation of $u(x)$; which means you get the same result as with the other equation.

Using the first method, where $x = d_1 + d_2$. By means of the tape measure you determine d_1 = (56.00 ± 0.25) m, and d_2 = (56.00 ± 0.25) m. Which means that the 'best approximation' of the total distance $x = d_1 + d_2$ = 56.00 + 56.00 = 112.00 m.

Now, with reference to Table C1-1, (measurements are being added) you calculate $u(x)$:

$$[u(x)]^2 = [1\, u(d_1)]^2 + [1\, u(d_2)]^2 = [0.25 \text{ m}]^2 + [0.25 \text{ m}]^2, \text{so } u(x) = 0.3535 \text{ m}.$$

Using the second method, where $x = 2d$. By means of the tape measure you determine $d = (56.00 \pm 0.25 \text{ m})$. Which means that the 'best approximation' of the total distance $x = 2d = 2(56.00) = 112.00 \text{ m}$.

Now, with reference to Table C1-1, (correlated variables) you calculate $u(x)$:

$$u(x) = \sqrt{2}\ u(d) = \sqrt{2}\,(0.25 \text{ m}), \text{so } u(x) = 0.3535 \text{ m}.$$

The result of either calculation will be quoted as: $x \pm u(x) = 112.00 \pm 0.35 \text{ m}$.

Example 3: Calculating the uncertainty where measurements are multiplied together.

Calculate the speed of sound in air having determined the time taken and the distance covered by a sound. The results are:

distance, $x \pm u(x)$ = (112.00 ± 0.35) m, and

time, $t \pm u(t)$ = (0.326 ± 0.018) s.

The calculated 'best approximation' of the speed of sound is

$$v = \frac{x}{t} = \frac{112.0 \text{ m}}{0.326 \text{ s}} = 343.3 \text{ m s}^{-1}.$$

Now, with reference to Table C1-1, (measurements are being multiplied) you calculate $u(x)$:

$$\left[\frac{u(v)}{v}\right]^2 = \left[1\frac{u(x)}{x}\right]^2 + \left[-1\frac{u(t)}{t}\right]^2$$

$$\left[\frac{u(v)}{343.3\text{ m s}^{-1}}\right]^2 = \left[1\frac{0.36\text{ m}}{112.0\text{ m}}\right]^2 + \left[-1\frac{0.018\text{ s}}{0.326\text{ s}}\right]^2$$

so $u(v)$ = 18.99 m s^{-1}.

The result of the calculation will be quoted as: $v \pm u(v)$= (343 ± 19) m s^{-1}.

Example 4: Calculating frequency of an oscillation having measured the period.

Assume the measured period of an oscillation to be $T \pm u(T)$ = (2.600 ± 0.055) ms.

Then the calculated 'best approximation' of the frequency is

$$f = \frac{1}{T} = T^{-1} = (2.600 \times 10^{-3})^{-1} = 384.615\text{ Hz.}$$

The uncertainty in this calculated result is

$$\left[\frac{u(f)}{f}\right]^2 = \left[-\frac{u(T)}{T}\right]^2 = [-1]^2\left[\frac{u(T)}{T}\right]^2 \rightarrow u(f) = \frac{u(T)}{T}f$$ From Table C1.1

So $u(f) = \frac{0.055\times10^{-3}\text{ s}}{2.600\times10^{-3}\text{ s}}(384.615\text{ Hz}) = 8.136\text{ Hz.}$

The result of the calculation will be quoted as: $f \pm u(f)$ = (384.6 ± 8.2) Hz.

Note in all these examples, the uncertainty is quoted to two (2) significant figures, and the best approximation has the same number of decimal places as the uncertainty. See the next section, D1, for more information on quoting.

The "reductionist approach" to simplify the propagation of uncertainties

Consider Example 3 again in terms of the fractional uncertainty, where the uncertainty is expressed as a fraction of the best approximation of the measurand (see Section D1):

$$\left[\frac{u(v)}{v}\right]^2 = \left[1\frac{u(x)}{x}\right]^2 + \left[-1\frac{u(t)}{t}\right]^2$$

$$\frac{u(v)}{v} = \sqrt{[0.0021]^2 + [0.055]^2} = 0.055$$

Note how the fractional uncertainty in t is an order of magnitude larger than the fractional uncertainty in x. As we are combining the fractional uncertainties in quadrature (sum of squares), the fractional uncertainty in v is dominated by the larger term; the fractional uncertainty in t. Effectively you only need to consider the larger component so can approximate the fractional uncertainty in v,

$$\left[\frac{u(v)}{v}\right]^2 \approx \left[-1\frac{u(t)}{t}\right]^2$$

So you can find, $u(v) = 0.036\ v = 0.055(343.3\text{ ms}^{-1}) = 18.99\text{ m/s}$, as before.

Often you can **substitute** or combine equations to use the reductionist approach to great advantage. You will have an opportunity to try this out in Practical 5 in Part I, Vibrating String.

Note the reductionist approach is only an appropriate approximation when you have at least one order of magnitude in difference between different fractional uncertainty components.

SECTION D – REPORTING RESULTS

D1 Quoting a result

The outcome of a measurement and/or an experiment is the best approximation of the measurand as well as the standard uncertainty, for example, the result of a measurement of gravitational acceleration g may be:

$$g = (9.790 \pm 0.052)\ \mathrm{ms}^{-2}.$$

(best approximation) ± (standard uncertainty) (68% coverage probability)

Note that:

- The result of a measurement is **not a single value** (not a point), it is **an interval of values**.
- The greater the uncertainty, the 'wider' the interval that quantifies the result of the uncertainty in the measurement.
- Because there is always some uncertainty in measurement, the width of the interval is never zero!

Quoting the number of significant figures in the uncertainty

- As a general rule, where a rigorous method to determine the uncertainty has been used, the standard uncertainty $u(z)$ should be quoted to **two significant figures**,
- for example: ± 0.032, ± 5.4, ± 72 x 10^3
- However, where a more casual approach (guessing) has been used, it is necessary to make a 'common sense' decision and to quote the standard uncertainty $u(z)$ to **one significant figure**:
- for example: ± 0.04, ± 6, ± 8 x 10^4

Quoting the number of decimal places in the best approximation

Having quoted the uncertainty to two significant figures, count the number of decimal places to which this corresponds, then write the best approximation to the same number of decimal places as the uncertainty.

For example:

$$g = (9.790 \pm 0.052)\ \mathrm{ms}^{-2}.$$

(has three decimal places here) and (two sig. figures and three decimal places here).

Quoting the result using fractional (%) uncertainty

It may be convenient on occasion to express a standard uncertainty $u(x)$ as a fraction of the best approximation of the measurand x. Thus, we may sometimes refer to

the fractional uncertainty given by $\frac{u(x)}{x}$, or

the percentage uncertainty given by $\frac{u(x)}{x} \times 100\ \%$.

For example, if the quoted result of a measurement is (3.60 ± 0.32) units, then the fractional uncertainty is 0.32 / 3.60 = 0.09 and the percentage uncertainty is 9%.

This form of quoting the uncertainty is commonly used by manufacturers of electronic components where the uncertainty in the quoted value is called the "tolerance" and it is quoted as a percentage of the nominal value of the component.

For example, if the value of a resistor is given as 220 Ω with a tolerance of 5%, it means that the resistance of the resistor is (220 ± 11) Ω.

Quoting experimental results as in many textbooks

There are other ways of quoting the results of experiments and a common method used in textbooks is, for example, the quoting of the Universal Gas Constant R to be:

$$R = 8.31451(72)\ \text{J K}^{-1}\ \text{mol}^{-1}.$$

The seven and two in brackets indicate that the last two digits '51' have an uncertainty of '72' associated with them; in other words, the book value is:

$$R = (8.31451 \pm 0.00072)\ \text{J K}^{-1}\ \text{mol}^{-1}.$$

Quoting the result with an expanded uncertainty

It sometimes happens that uncertainties are quoted to a **greater coverage probability** (i.e., with a greater level of confidence) and this is done simply by multiplying the standard uncertainty by what is known as a **coverage factor**, k.

When this is done the uncertainty is referred to as the **expanded uncertainty**, U, where $U = ku$.

The coverage factor, k, will depend on the coverage probability (level of confidence) with which the result is to be quoted, as well as the way in which the standard uncertainty, u, was determined. The coverage factors for the different combination are given below:

If a **Gaussian pdf** was used:

k = 1, and $y \pm ku$ defines a 68% coverage probability;
k = 2, and $y \pm ku$ defines a 95% coverage probability;
k = 3, and $y \pm ku$ defines a 99% coverage probability.

If a **rectangular pdf** was used:

k = 1, and $y \pm ku$ defines a 58% coverage probability;
k = 1.65, and $y \pm ku$ defines a 95% coverage probability;
k = 1.73, and $y \pm ku$ defines a 100% coverage probability.

If a **triangular pdf** was used:

k = 1, and $y \pm ku$ defines a 65% coverage probability;
k = 1.81, and $y \pm ku$ defines a 95% coverage probability;
k = 2.45, and $y \pm ku$ defines a 100% coverage probability.

It is important to note that using a coverage factor does not change the result of the measurement; the standard uncertainty remains unchanged. All that happens is that the result is presented in a different way.

The implication of this is that it is essential to report on the coverage probability (level of confidence) when the result is quoted. So for example the value of g on the previous page may be quoted as:

$$g = (9.79 \pm 0.16)\ \text{m s}^{-2}\ \text{(99\% coverage probability), where } k = 3.$$

Coverage probability should be kept **consistent** when comparing results modelled with different probability distributions, i.e. say you directly compare results of measurements with Type A and Type B evaluations of uncertainty. This could be achieved by increasing the coverage factor to k = 2, which would calculate the standard uncertainty for both evaluations with for a coverage probability of 95%.

Say that a measurement was made in order to determine the mass m of an object. The pdf of the result is shown below in Figure 13(a) in which it may be seen that the best approximation of the mass m is 83.50 g, with a standard uncertainty of 0.12 g. This defines an interval 83.50 $\pm$ 0.12 g within which we are 58% confident that that the mass of the object will lie.

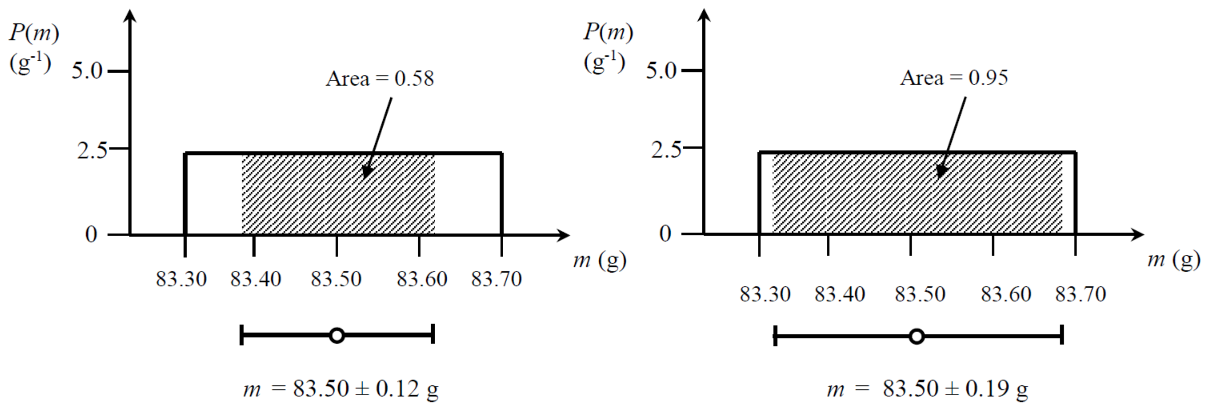


Figure 13: Probability density functions (pdfs) of a measurement corresponding to coverage factors of 1 (a) and 2 (b).

The result in Figure 13(a) may be described as "m = (83.50 $\pm$ 0.12) g, where the number following the symbol $\pm$ is the numerical value of the standard uncertainty u = 0.12 g and defines an interval estimated to have a coverage probability of 58 percent".

The result in in Figure 13(b) may be described as "m = (83.50 $\pm$ 0.19) g, where the number following the symbol $\pm$ is the numerical value of an expanded uncertainty $U = k\,u$ with U determined from a standard uncertainty u = 0.12 g and a coverage factor k = 1.65, and defines an interval estimated to have a coverage probability of 95 percent".

We are 58 % confident that the value of the mass lies between 83.38 g and 83.62 g.
We are 95 % confident that the value of the mass lies between 83.31 g and 83.69 g.
We are 100 % confident that the value of the mass lies between 83.30 g and 83.70 g.

Note that by introducing an expanded uncertainty and coverage factor k > 1, we are not affecting the result of the measurement. We are only changing the way that we present the result.

Note in PHYLAB1 courses, it is rarely required to quote uncertainties to a greater coverage probability than k = 1. We include this section here so you are aware of it for future study.

D2 Comparing results

To say that two results are "close" or "nearly the same" is meaningless in the context of laboratory work.

The results of any two experiments can only be meaningfully compared if the intervals associated with each of the results are known.

More specifically:

- If the intervals that represent the results of two measurements overlap, then we say these two results "**agree within experimental uncertainty**."
- If the intervals that represent the results of two measurements do not overlap, then we say these results "**do not agree within experimental uncertainty**."

For example, say three students measure the period T of a pendulum and they each quote the result as follows:

$$T_1 = (5.73 \pm 0.41)\ \text{s}$$

$$T_2 = (5.62 \pm 0.10)\ \text{s}$$

$$T_3 = (6.28 \pm 0.25)\ \text{s}$$

The three measurements may be presented in the form of intervals on a number line:

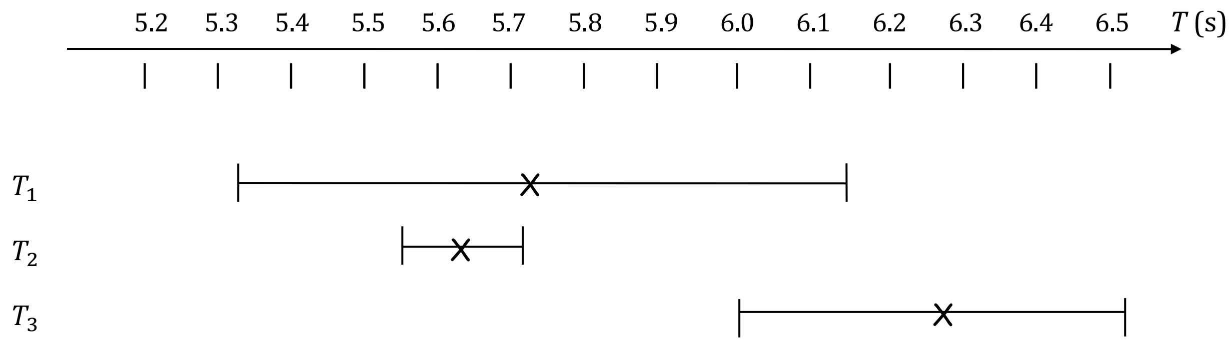


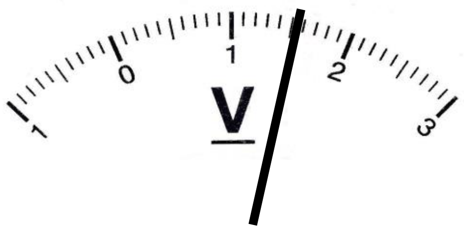
Figure 14: Comparison of results

- Note that the intervals associated with T_1 and T_2 overlap, and therefore "these two results **agree** within experimental uncertainty".
- The intervals associated with T_1 and T_3 also overlap so "these two results **agree** within experimental uncertainty".
- However, the interval associated with T_3 does not overlap with the interval associated with T_2 and therefore "these two results **do not agree** within experimental uncertainty."

D3 The uncertainty budget

An **uncertainty budget** is an evaluation (usually presented in the form of a table) of all the contributions of uncertainty in a particular measurement (together with numerical estimates). These uncertainty components are then used to calculate the **combined standard uncertainty** for the measurement result. An uncertainty budget is a useful way to present how you dealt with all the uncertainties in a measurement.

Example 1: A measurement using an analogue voltmeter



Consider the situation where you are using an analogue voltmeter to measure the voltage across the terminals of a battery. After connecting the voltmeter across the terminals of the battery, what you see on the voltmeter is shown alongside.

Say that you decide that the reading V is 1.54 volts, and using a triangular pdf, you determine the standard uncertainty $u(V_{\text{read}})$ on this scale reading to be:

$$u(V_{\text{read}}) = \frac{(1.57 - 1.51)\ \text{V}}{2\sqrt{6}} = 0.012\ \text{V}.$$

It is usually not the case that you be 100 % confident that the instrument (in this case an analogue voltmeter) gives a perfect reflection of the input (i.e. the voltage across the terminals of the battery). All analogue and digital instruments have some **internal calibration** setting, referred to as the "rating" of the instrument. The uncertainty associated with this rating is usually indicated by the manufacturers of the instrument and is often quoted as a percentage.

Say that we are told that the meter we are using here has a percentage calibration uncertainty of 1%. Then the uncertainty associated with the rating of the instrument is:

$$u(V_{\text{rating}}) = 0.01(1.54\ \text{V}) = 0.015\ \text{V}.$$

There may be other sources of uncertainty associated with the instrument, such as the influence of the temperature of the surroundings, or the contact resistance in the probes. Let us say that these are negligible in this case.

After you have determined all the possible sources of uncertainty and assigned a numerical value to each, then you should draw up a table of the sort shown below, which is called an **uncertainty budget**. Each source of uncertainty is listed together with its standard uncertainty. You should note that these standard uncertainties can result from either Type A or Type B evaluations.

Table D3-1: Uncertainty budget for the single voltmeter reading, V.

Uncertainty component	*Standard uncertainty* (V)	*Type of evaluation*
Reading of the analogue voltmeter display, $u(V_{\text{read}})$	0.012	Type B
Rating of the analogue voltmeter, $u(V_{\text{rating}})$	0.015	Type B
Combined standard uncertainty: $u(V) = \sqrt{(0.012\ \text{V})^2 + (0.015\ \text{V})^2} = 0.023\ \text{V}.$		

The final result in this case may then be recorded as: "the best approximation of the voltage V is 1.520 volts with a combined standard uncertainty of 0.023 volts."

Example 2: A set of dispersed digital readings

Now let us presume that we are trying to measure the acceleration due to gravity g by observing the period T of a pendulum of length l. We determine that l = 0.2619 $\pm$ 0.0058 m (where $u(l)$ results from a Type B evaluation of uncertainty with an analogue metre stick).

We then use a digital stopwatch to measure the period of the pendulum. You cause the pendulum to oscillate and then record the time for 20 complete swings. You repeat this procedure 10 times and observe the data:

Table D3-2: Data recorded using a digital stopwatch.

Time for 20 swings, T_{20} (s)		Period, $T = T_{20}/20$ (s)	
19.56	21.31	0.978	1.066
20.49	20.82	1.025	1.041
20.76	19.78	1.038	0.989
20.63	20.39	1.032	1.020
21.56	20.02	1.078	1.001

Since we observe a **dispersion** (scatter) in the readings, the best estimate of the period T is given by the arithmetic mean $\bar{T}$, which is 1.027 s, and the standard uncertainty associated with the scatter is given by the **experimental standard deviation of the mean**, which is 0.010 s. Then $u(\bar{T})$ = 0.010 s. This is a Type A evaluation of uncertainty. Another source of scatter you might consider is the uncertainty in the gradient of a linear graph ("the scatter in the (x, y) data, $u(m)$").

Let us say that the manufacturers of the stopwatch report that it is accurate to 0.5%. Then the uncertainty associated with the internal **calibration or rating,** $u(T_{\text{rating}})$, will be:

$$u(T_{\text{rating}}) = (0.005)(1.027\text{ s}) = 0.0051\text{ s}.$$

To determine g, we use the formula:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \rightarrow \quad g = \frac{4\pi^2 l}{T^2}.$$

This is our **model equation** for this measurement. The best approximation for g is given by,

$$g = \frac{4\pi^2(0.2619\text{ m})}{(1.027\text{ s})^2} = 9.8028\text{ ms}^{-2}.$$

The uncertainty budget for this measurement is shown below.

Table D3-3: Uncertainty budget for the measurement of gravitational acceleration, g.

Uncertainty component	Standard uncertainty	Type of evaluation
Scatter in the T data, $u(\bar{T})$	0.010 s	Type A
Rating of the stopwatch, $u(T_{\text{rating}})$	0.0051 s	Type B
Reading of l from the analogue metre stick, $u(l)$	0.0058 m	Type B
The combined uncertainty for T is $u(T) = \sqrt{(0.010\text{ s})^2 + (0.0051\text{ s})^2} = 0.011\text{ s}.$		
The standard uncertainty for g is given by $u(g) = g\sqrt{\left(\frac{u(l)}{l}\right)^2 + \left(\frac{u(T)}{T}\right)^2}$ $u(g) = (9.8028\text{ ms}^{-2})\sqrt{\left(\frac{0.0058\text{ m}}{0.262\text{ m}}\right)^2 + \left(2\frac{0.011\text{ s}}{1.027\text{ s}}\right)^2} = 0.302\text{ ms}^{-2}.$		

The final result may then be recorded as: "the best estimate of the acceleration due to gravity g is 9.80 m s^{-2} with a standard uncertainty of 0.30 m s^{-2}."

SECTION E – FITTING A LINEAR FUNCTION TO THE DATA

E1 The process of fitting a function to a set of data

Sometimes you cannot directly measure the information you are seeking, so instead you measure other values and determine the required information indirectly. This process often involves the method of 'fitting' some known mathematical function to a set of data.

The 'fitting' process is shown in Figure 15.

Measurement

Tabulate the data (readings).

↓

Re-organise the data in accordance with the linearised function you plan to use. See E2 Linearising equations.

↓

Plot the graph to confirm that the expected relationship is indeed linear.

↓

Use a least squares fit to fit a straight line to the data, the equation of which is $y = mx + c$. See E3 Method of least squares.

↓

Extract the information you are after from the coefficients m and c of the fitted function.

↓

Determine the uncertainties $u(m)$ and $u(c)$ by determining how well the scattered data fits the mathematical function.

This is a **TYPE A** evaluation.

↓

Quote the results of the measurement; $m \pm u(m)$ with units and $c \pm u(c)$ with units.

↓

Calculation

Calculate the value of the measurand you are after and do the necessary uncertainty calculations to determine the uncertainty in the final result. See C1 Calculation of uncertaint.

Figure 15: Process for extracting information by fitting mathematical functions.

E2 Linearising equations

When presented with a relationship between experimental variables that is not linear, the method of 'linearising' can be used extract information from recorded data.

Effectively, you rearrange the non-linear equation to take the form of a linear equation; $y = mx + c$. After plotting values on a graph for y against x, you can use the gradient, m, and y-intercept, c to calculate useful values.

Example:

Consider an experiment to investigate the relationship between the position s of an object over time t while that object is subject to a uniform (constant) acceleration a. This experiment is underpinned by the following kinematic equation:

$$s = \frac{1}{2}at^2 + ut + s_0, \qquad \text{... E2-1}$$

where u is the initial velocity and s_0 is the initial position of the object.

If a set of data, (t_i, s_i), *i* = 1, 2, 3, ..., *n*, were collected, then it would be difficult to determine the coefficients a, u and s_0 by plotting a graph of the equation E2-1 because the relationship between the position s and the time t is a parabola of the form $y = ax^2 + bx + c$.

Equation E2-1 can be 'linearised' by considering the position relative to the initial position s_0 and by dividing throughout by t (given that $t \neq 0$):

$$\frac{s - s_0}{t} = \frac{1}{2}at + u. \qquad \text{... E2-2}$$

In the rearranged equation, there is a linear relationship between $\frac{s-s_0}{t}$ and t, of the form, $y = mx + c$:

$$\begin{array}{ccccc} \frac{s - s_0}{t} & = & \frac{1}{2}a & t & + u \\ y & = & m & x & + c \end{array}$$

Therefore plotting a graph with calculated values of $\left(t_i, \frac{s_i - s_0}{t_i}\right)$ as the values (x_i, y_i) should yield a linear relationship.

You can now determine the acceleration, a, in equation E2-1 from the gradient of the graph and the initial velocity, u, from the y-intercept.

E3 Method of least squares

An accurate method to find the slope m and the intercept c of the best fit line is to use the method of least squares. This method also allows you to determine the uncertainty in the slope, $u(m)$, and the uncertainty in the intercept $u(c)$.

Given a set of data pairs (x_i, y_i), *i* = 1, 2, 3, …, *n,* and that the data pairs represent a linear relationship, then the slope m and the intercept c of the best fit line can be determined by the following equations:

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum {x_i}^2 - (\sum x_i)^2} \qquad \text{… E3-1}$$

$$c = \frac{\sum {x_i}^2 \sum y_i - \sum x_i y_i \sum x_i}{n\sum {x_i}^2 - (\sum x_i)^2} \qquad \text{… E3-2}$$

Let $d_i = y_i - (mx_i + c)$, the distance between each data point and the line of best fit. The uncertainties associated with m and c can be determined using:

$$u(m) = \sqrt{\frac{\sum {d_i}^2}{n\sum {x_i}^2 - (\sum x_i)^2}\left(\frac{n}{n-2}\right)}, \qquad \text{… E3-3}$$

$$u(c) = \sqrt{\frac{\sum {d_i}^2 \sum {x_i}^2}{n(n\sum {x_i}^2 - (\sum x_i)^2)}\left(\frac{n}{n-2}\right)}. \qquad \text{… E3-4}$$

A computer is generally available to you in PHYLAB1 and the method of least squares can be employed using the following programs:

- Microsoft Excel (by typing in the equations) or
- LinearFit (by entering the data).

If a computer is not available, use your calculator to draw up tables of data as suggested below and solve the equations above. Some calculators will allow you to analyse data pairs.

i	x_i	y_i	${x_i}^2$	$x_i y_i$	${d_i}^2$
…	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
	$\sum x_i$	$\sum y_i$	$\sum {x_i}^2$	$\sum x_i y_i$	$\sum {d_i}^2$

E4 Using EXCEL (2010 version)

Tables with Excel

Initiate Excel by clicking on the Excel shortcut on the computer's desktop. Note the Excel worksheet is divided into a series of columns (A-Z) and rows (1:99999). Each cell has an index corresponding to its column and row, e.g. A2. If you want to get Excel to do a calculation, click on the cell where you want the answer to go, and TYPE **=** followed by the required mathematical expression. For example, if the value in the particular cell is obtained by multiplying the value in cell C3 by that in D3, TYPE **= C3*D3** and press enter (or you can just TYPE **=**, then click on cell C3, TYPE *****, click on D3 and hit the enter key). In Excel, the ***** symbol is used to indicate multiply, the **/** symbol divide, and the **^** symbol "raise to the power". Use fractional powers for roots, *e.g.* to the power of **½** is equivalent to taking the square root of a number.

To apply the same formula to a list of cells in a column; select the cell where you have typed the formula, move the cursor over the bottom right hand corner until the **+** symbol appears, and then drag it down the column. Excel copies the formula to all the cells and also adjusts the cell indices in each formula by incrementing their row numbers in line with where you dragged the cursor. You can also copy horizontally, which will increment the column letters in the cell indices. Try this out and click one of the copied cells to see how the cell index has changed in the equation. If you want to fix the cell index to use the same value in all formulae across different rows or columns, use a $ sign to fix a column and/or a row index (*e.g.* E5 included in a formula is left unchanged when copied to a new cell).

Take care to display the correct number of significant figures in your data. Since Excel calculates the answer to as many places as can physically fit in the cell, you often need to adjust the number of decimal places in columns to reflect the correct number of significant figures. Increasing and decreasing the number of decimal places can be achieved using the buttons marked . To outline the cells or add borders to your table use the button marked . To add headings (and units) to columns, click on the cell and TYPE as you would normally do. To produce superscripts and subscripts in the titles, select the text you want to raise or lower, and use the right click to access the "Format Cells" menu. Then select "**Superscript**" or "**Subscript**". Further options can be accessed in the Excel menus or using the right-click menu.

Plotting with Excel

When you are ready for Excel to plot your data, you need to highlight the two columns of data you wish to plot and use **INSERT**, **SCATTER** to call up the chart (plotting) procedure; ideally in the order x **then** y. Select the chart sub-type option  which allows you to plot the points without lines. Under the **CHART TOOLS** set of menus, you can change the **Design** (colour and style of data points), **Layout** (add titles, axes-titles, gridlines) and the **Format** (chart size). Move your plot into the desired position by clicking just inside the borders and dragging it to where you want it positioned. Other options can be accessed by selecting the chart with a right-click of the mouse. You can change the origin and scale of the plot so that your graph covers most of the page. To change the scale on the x-axis, click on the axis to select it. Next right-click to access the **FORMAT AXIS** option and in the Axis Options sub-menu, adjust the minimum and maximum values as required. You can also change the number of decimal places or to scientific exponent form under the Number sub-menu.

Best fit with Excel

To draw the line of best fit through your points, click somewhere inside your graph to select it, then click on **CHART TOOLS, LAYOUT, ADD TRENDLINE** and select the **LINEAR** option (or alternatively select the data points and access by the right-click mouse menu). If you need to know the slope and intercept of this straight line, click on **OPTIONS** and check the **DISPLAY EQUATION ON CHART** box.

E5 Using LINEARFIT

LinearFit can be used to calculate the slope $m \pm u(m)$ and the intercept $c \pm u(c)$ of the linear line of best fit.

To start the program, click on the LinearFit shortcut on the desktop. The program will open and present the page as shown in Figure 16. Type the x and y values into the appropriate columns, or alternatively import your data in a .csv file (e.g. saved from Excel) in the format x, y, $u(x)$, $u(y)$ and LinearFit will automatically calculate $m \pm u(m)$ and $c \pm u(c)$. When printing, don't forget to add your student number to help you identify your work at the printers.

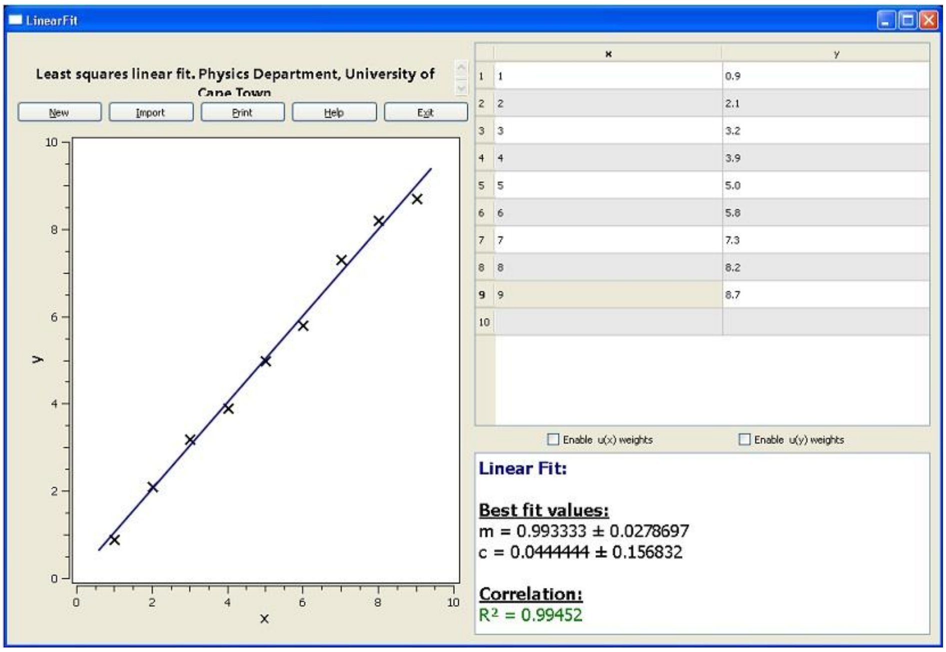


Figure 16: Data input and fitting screen of LinearFit.

LinearFit with weightings

Tick the "Enable $u(x)$ weights" and the "Enable $u(y)$ weights" boxes. This will open two additional columns for you to type in the weightings, *i.e.*, the $u(x_i)$ and $u(y_i)$ values for the respective x_i and y_i readings, *i* = 1, 2, 3, …, n. When these weightings are included, the individual uncertainties are presented as intervals around each data point; see Figure 17.

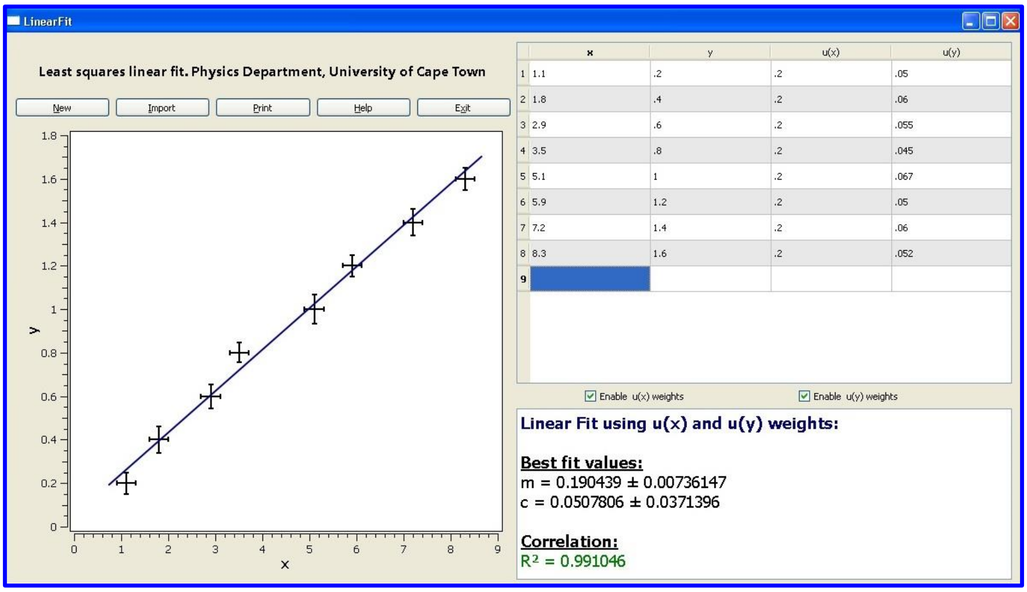


Figure 17: LinearFit with weightings enabled.

SECTION F – EXAMPLE OF FITTING A GAUSSIAN

Consider an example in which the measurand is the time taken for you to travel from wherever you live to the university campus. So, for the next 40 trips to the university you use the stopwatch on your cell phone to determine your travel time in minutes, i.e. you take a set of readings.

The data are tabulated as follows:

Table F-1: Time taken to get from home to campus (in minutes).

17.1	16.9	17.4	16.4	16.8	17.9	16.0	18.2
18.1	19.3	16.9	17.4	17.8	17.1	17.6	17.8
16.4	17.4	15.9	15.8	17.7	17.3	17.4	16.8
16.6	16.8	17.6	17.4	18.9	18.4	17.9	17.8
18.3	17.2	18.1	18.7	16.3	17.4	18.4	16.5

A simple calculation shows that the average time taken is 17.39 minutes.

We wish to present this data in a histogram, so the next step is to draw up a frequency table (or distribution table). Note that in this example a bin width of 0.5 minutes was chosen. The choice of bin width is arbitrary and is normally chosen to suit the data.

Table F-2: Frequency table for data in Table F-1.

Bin (minutes)	Number of readings per bin
15.5 to 15.9	2
16.0 to 16.4	4
16.5 to 16.9	7
17.0 to 17.4	10
17.5 to 17.9	8
18.0 to 18.4	6
18.5 to 18.9	2
19.0 to 19.4	1
	Total : 40

Next we plot a histogram showing how the data are distributed over the values of the readings taken by plotting the number of readings per bin vs. the time interval of bins (see Figure 18). Finally, to show that the general shape of the histogram suggests that the distribution of the data is Gaussian in shape, we superimpose a trend line, shown as the light grey line in Figure 18.

It should be noted that as more and more readings are added to the set of data, and the bin width is narrowed, the distribution of most data of this sort will tend towards the formation of a symmetrical Gaussian distribution.

The Gaussian probability density function (pdf)

Having decided that the Gaussian probability density function (pdf) is a suitable probability density function by which to model our knowledge of the measurand, we need to consider the implications. The probability density function of a continuous random variable is a function that can be used to obtain the probability that the random variable may take a specific value within some given interval.

When the probability density function (pdf) is normalised and portrayed graphically, the probability that the random variable may take a specific value within a given interval is indicated by the area under the graph subtended by that interval. So, if the given interval is all possible values, then the total area under the pdf is "1" (unity), meaning that the probability that the specified value exists is also 1. As the interval is made "narrower", so the probability that the specific number is within that interval becomes less.

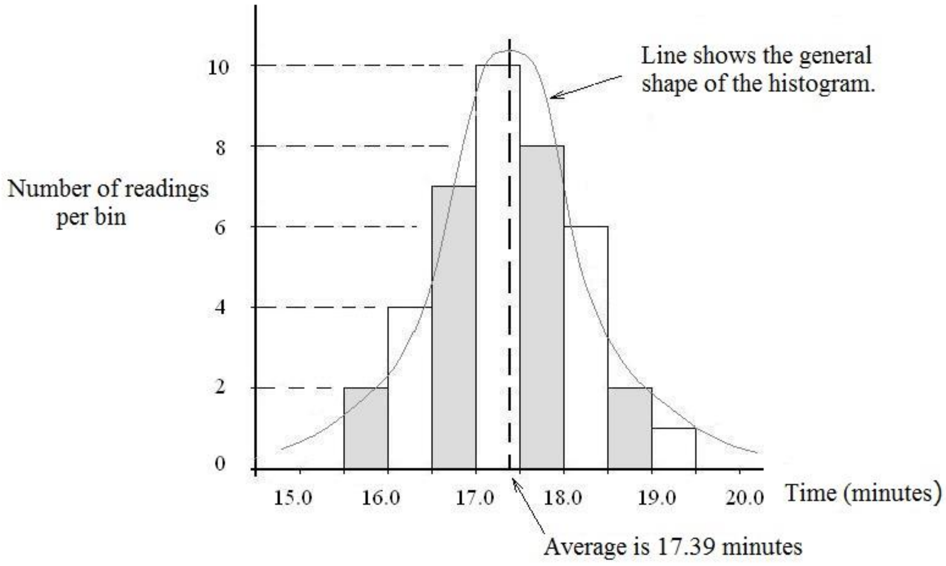


Figure 18: Histogram of travel times.

The important features of the Gaussian pdf are illustrated graphically in Figure 19.

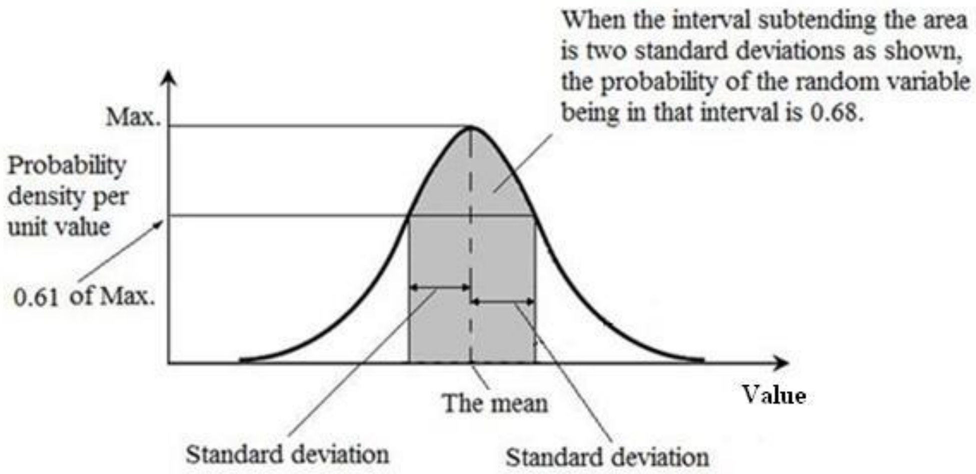


Figure 19: Gaussian pdf showing standard deviation.

The equations used to calculate the standard deviation are given in B2 TYPE A evaluation of uncertainty, and the standard deviation can also be found graphically by drawing a horizontal line at 0.61 of the maximum height of the Gaussian, as shown in Figure 19.

In laboratory work, the use of the standard deviation gives a measure of the consistency of the measurement process that was used in an experiment, and makes it possible to specify the level of confidence with which the result of the measurement is quoted. In this case, the coverage probability (level of confidence) is 68%, as explained in Figure 19.

However, exercise caution as the standard deviation and mean are only appropriate metrics for data that can be modelled with a Gaussian distribution. You may encounter data that is better modelled with a different distribution (*e.g.* radioactive decay is modelled with a Poisson distribution).