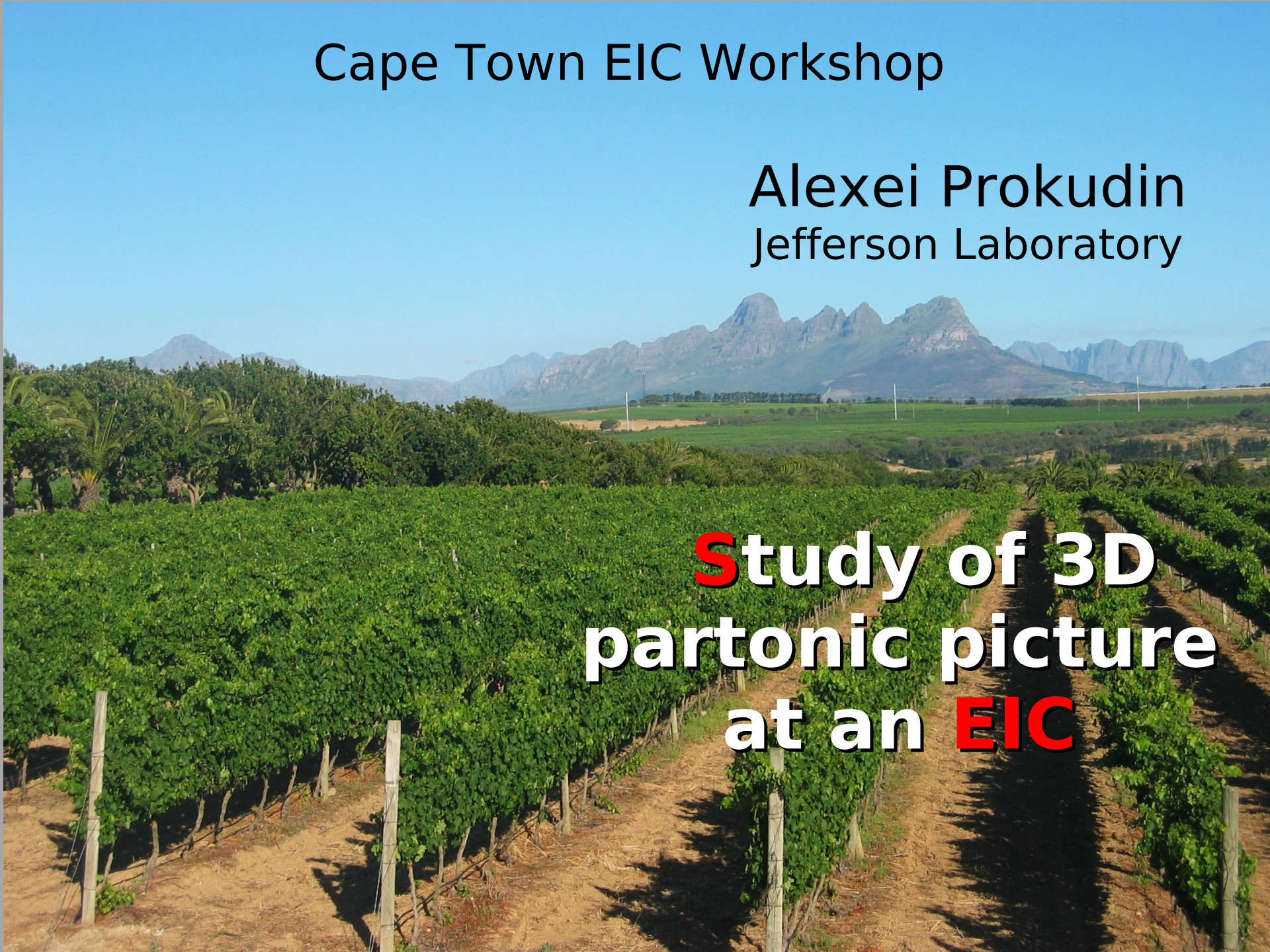


# Cape Town EIC Workshop

Alexei Prokudin  
Jefferson Laboratory

**Study of 3D  
partonic picture  
at an EIC**

A photograph of a vineyard in Cape Town, South Africa. The foreground shows rows of green grapevines supported by wooden posts, with a dirt path running between them. In the middle ground, there is a dense line of trees, including palm trees. The background features the iconic Table Mountain range under a clear blue sky.

# Nucleon landscape

Nucleon is a many body dynamical system of quarks and gluons

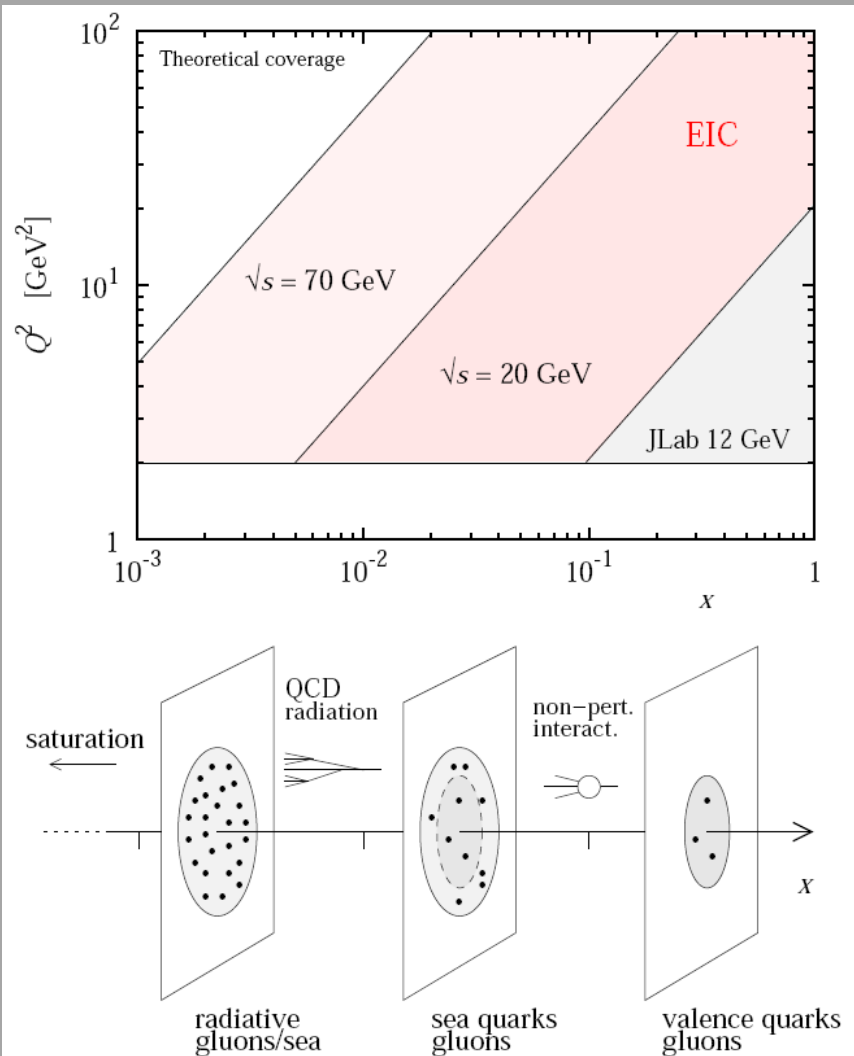
Changing  $x$  we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions

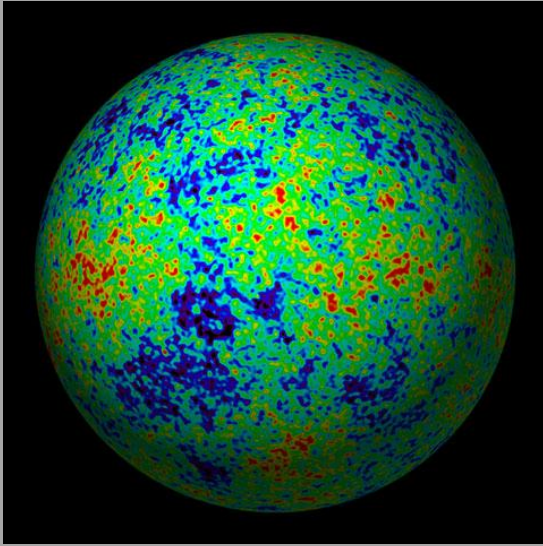
See talks by Markus Deihl and Matthias Burkardt

and Transverse Momentum Dependent distributions



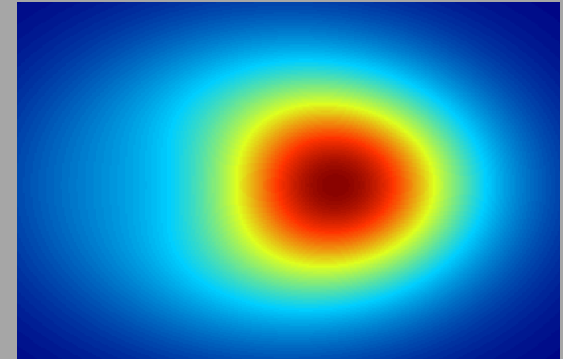
Plot courtesy of Christian Weiss

# Fundamental knowledge from 3D distributions



## **Cosmic Microwave Background**

is the source of information on history of our universe, inflation, distribution of matter, dark matter etc



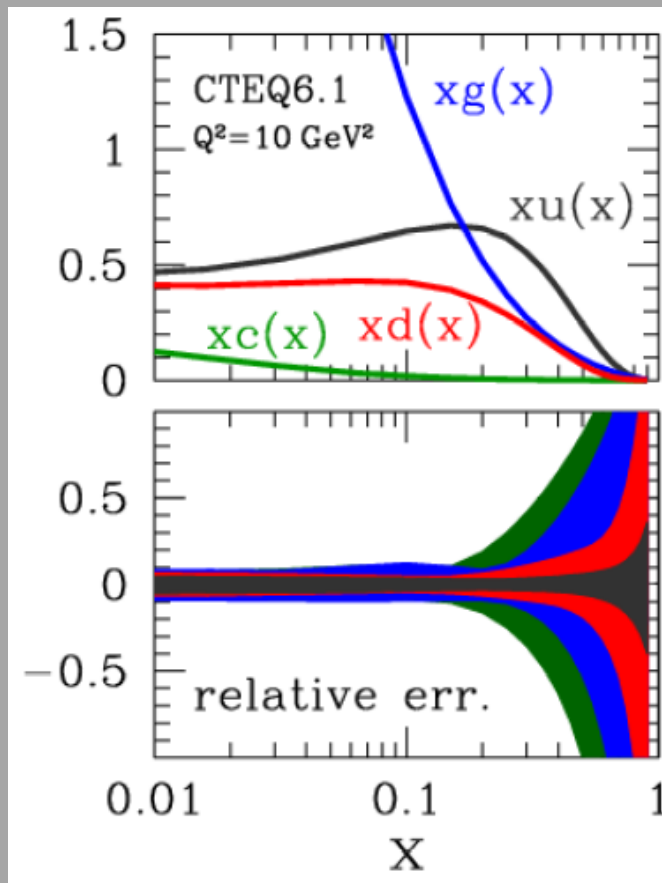
**3 Dimensional partonic picture** gives us insights on the dynamics of the confined system of quarks and gluons.

It also gives information on fundamental properties of the nucleon

Spin is one of these properties

# Hadron tomography

Conventional inclusive processes are sensitive to longitudinal momentum fraction of hadron momenta, they give no information on spatial or momentum 3D distribution of partons



Good knowledge of  
Parton  
Distribution  
Functions (PDFs)  
is acquired at HERA

[See talk by Voica Radescu](#)

However large- $x$  behavior  
has still large uncertainties  
Data from Jlab 12 will be  
important

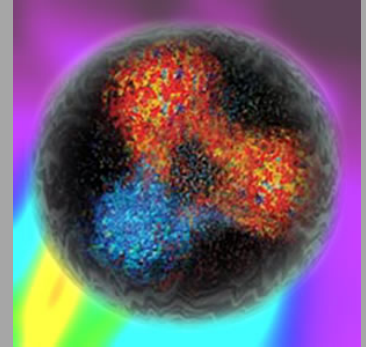
# Wigner distribution

Our goal is to understand 3 dimensional distributions of partons,  
How they move, where they are located inside a nucleon

Wigner distribution (1933) is a possibility

$$W(\mathbf{p}, \mathbf{r}) = \int d^3\eta e^{i\mathbf{p}\eta} \psi^*(\mathbf{r} + \eta/2) \psi(\mathbf{r} - \eta/2)$$

It gives both position and momenta



# Wigner distribution

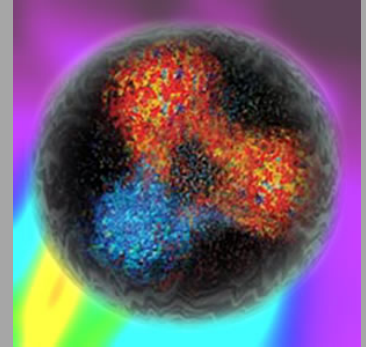
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It gives both position and momenta

**Can it be measured?**

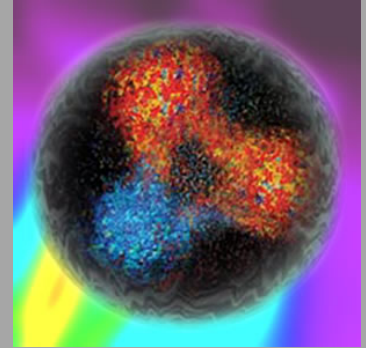


# Wigner distribution

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It gives both position and momenta

**Can it be measured?**

**PROBABLY NOT!**

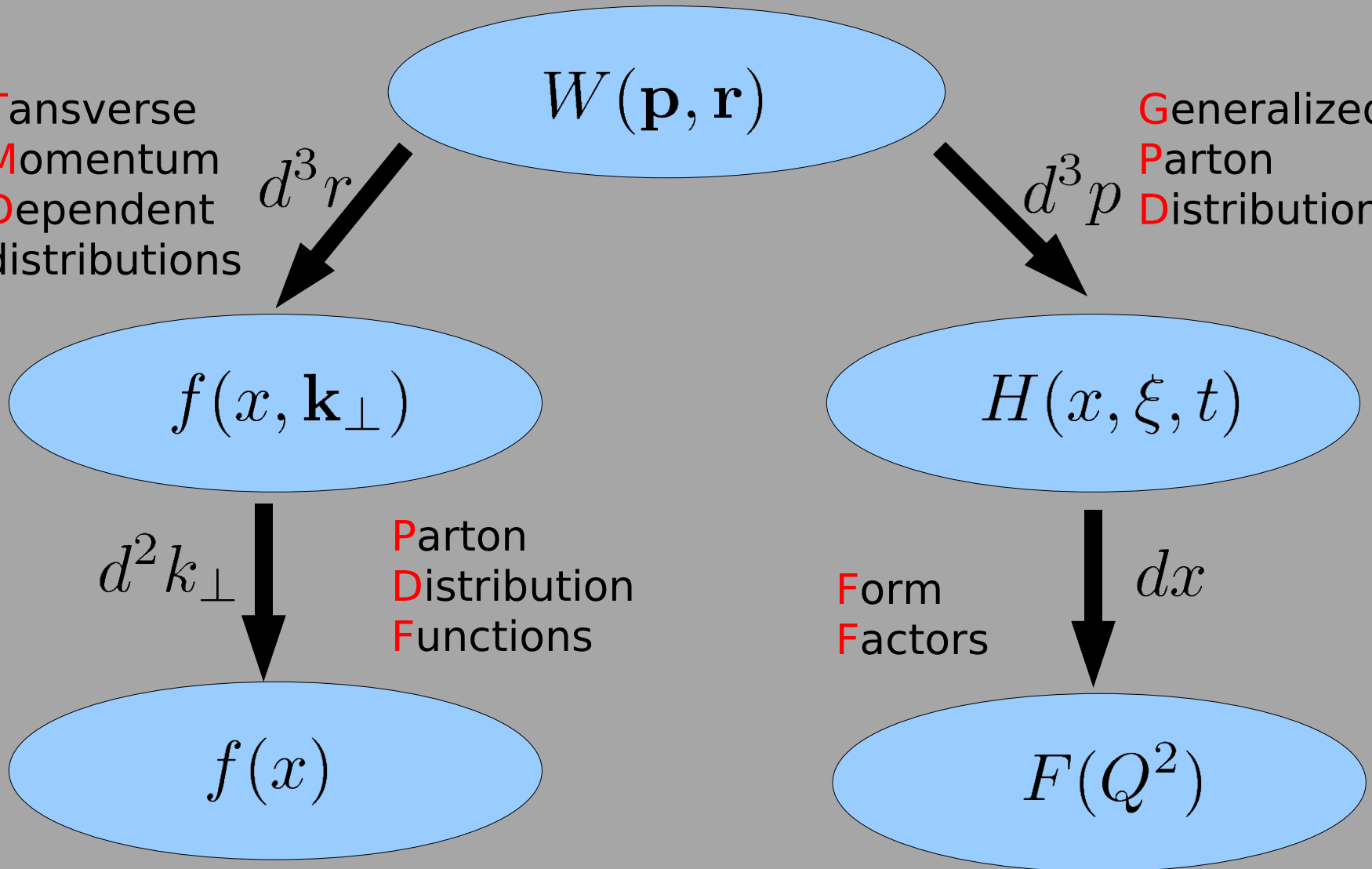
$$\Delta p \Delta r \geq \hbar/2$$

**No simultaneous knowledge on position  
and momenta**

# Wigner distribution

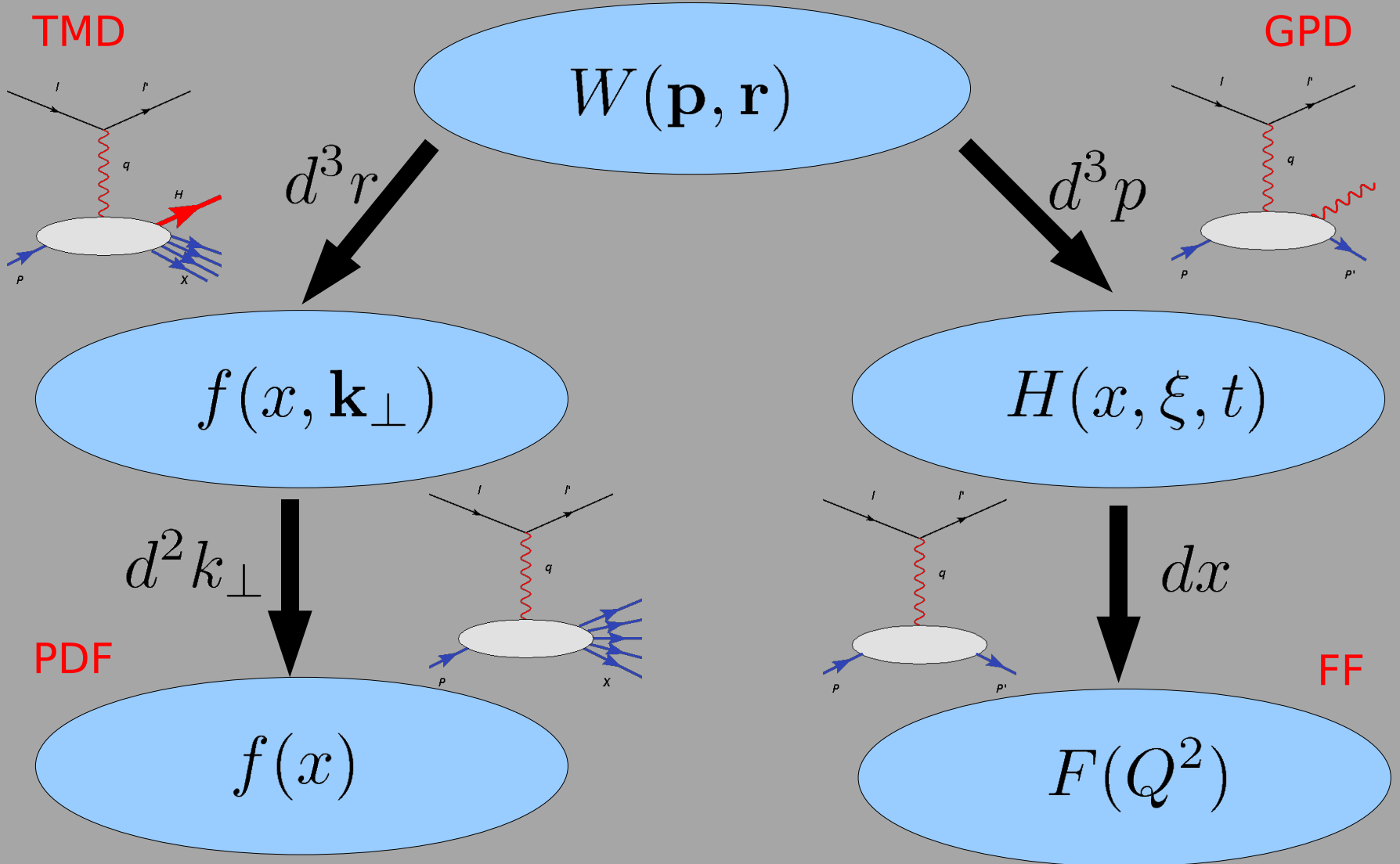
**T**ransverse  
**M**omentum  
**D**ependent  
distributions

**G**eneralized  
**P**arton  
**D**istributions



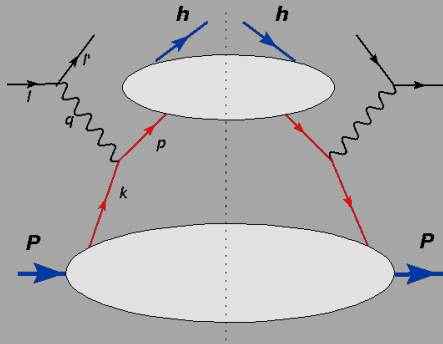


# Wigner distribution



# Transverse Momentum Dependent distributions

## SIDIS



If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} \ll Q$$

it will be sensitive to quark transverse momentum  $k_{\perp}$

$$\mathbf{l} + \mathbf{P} \rightarrow \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

TMD factorization

Ji, Ma, Yuan (2002)

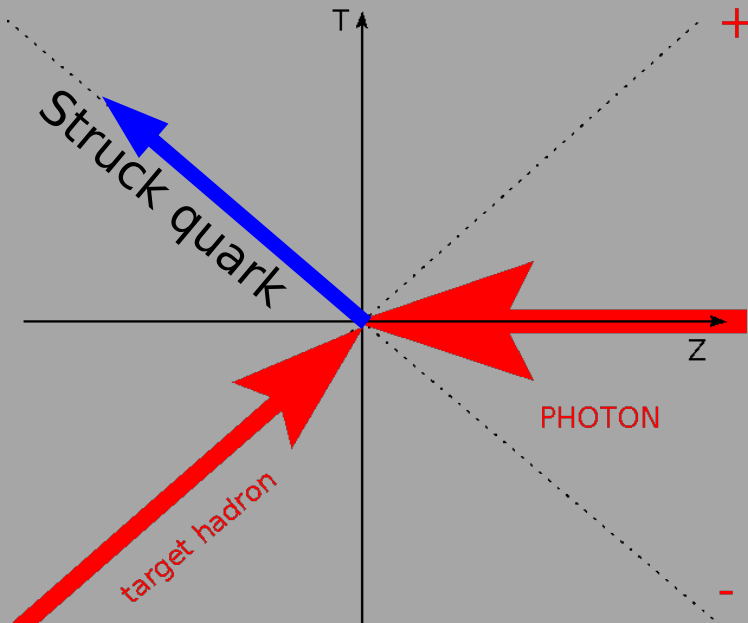
**GAUGE INVARIANT**

$$\Phi_{ij}(x, \mathbf{k}_{\perp}) = \int \frac{d\xi^{-}}{(2\pi)} \frac{d^2\xi_{\perp}}{(2\pi)^2} e^{ixP^{+}\xi^{-} - i\mathbf{k}_{\perp}\xi_{\perp}} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | P, S_P \rangle$$

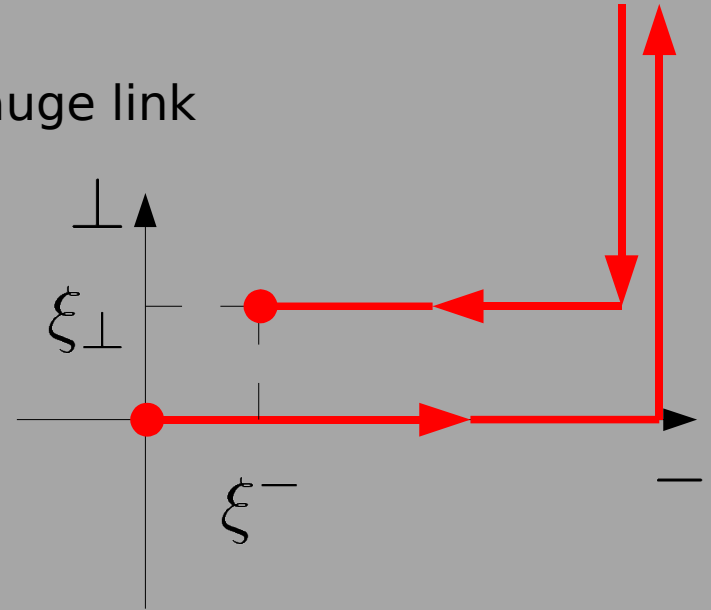
# Transverse Momentum Dependent distributions

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | P, S_P \rangle |_{\xi^+=0}$$

SIDIS in IMF:



Gauge link



# Factorization theorems

## • Related: Factorization Theorems:

- Semi-Inclusive deep inelastic scattering. ✓
- Drell-Yan. ✓
- $e^+/e^-$  annihilation. ✓
- ~~$p + p \rightarrow h_1 + h_2 + X$  !!~~

## • Related: Factorization Theorems:

- Semi-Inclusive deep inelastic scattering. ✓
- Drell-Yan. ✓
- $e^+/e^-$  annihilation. ✓
- $p + p \rightarrow h_1 + h_2 + X$  ✓

## • **TMD** factorization

$$\Lambda_{\text{QCD}}^2 < P_{h\perp}^2 \ll Q^2$$

Sensitive to parton transverse motion.

Ji, Ma, Yuan, Collins, Metz, Rogers, Mulders, etc

## • **Collinear** factorization

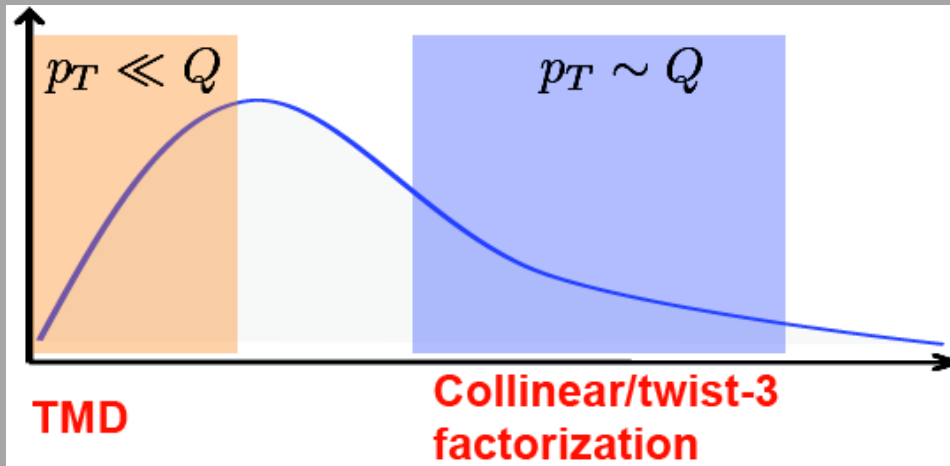
$$\Lambda_{\text{QCD}}^2 \ll P_{h\perp}^2, Q^2$$

Sensitive to multy parton correlations.

Qui, Sterman, Efremov, Teryaev, Kanazava, Koike, etc

# TMD and Collinear factorizations

Both factorizations are consistent in the overlap region

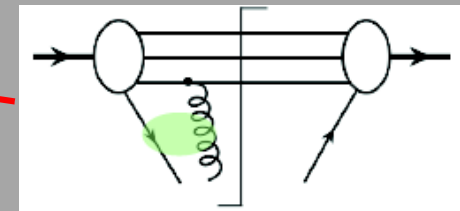


Collins, Mulders, Ji, Qui, Yuan, Bacchetta, Kang, Boer, Koike, Vogelsang, Yuan etc





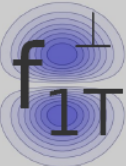

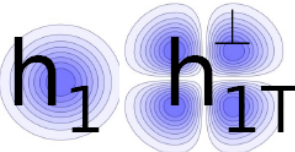
Relation of multiparton correlations and moments of TMDs

$$\int d^2k_T \frac{k_T^2}{M} f_{1T}^\perp(x, k_T^2) + \text{UVCT}(\mu^2) = \mathbf{T}_F(x, x, \mu^2) \quad f_{1T}^{\perp(1)} \equiv \int d^2k_T \frac{p_T^2}{2M^2} f_{1T}^\perp(x, k_T^2)$$

Sivers function



# TMDs

| $N \backslash q$ | U   | L   | T   |
|------------------|---|---|---|
| U                |    |   |    |
| L                |   |    |    |
| T                |  |  |  |

8 functions in total (at leading Twist)

Each represents different aspects of partonic structure

Each function is to be studied

Mulders, Tangerman (1995), Boer, Mulders (1998)

# Sivers function

Let's consider unpolarised quarks inside transversely polarised nucleon

## DISTRIBUTION

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^\perp(x, \mathbf{k}_T^2)$$

Usual unpolarised distribution



This one is called **SIVERS** function  
Correlation of transverse motion and transverse spin  
Sivers (1990)

# Sivers function

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^\perp(x, \mathbf{k}_T^2)$$

This function gives access to 3D imaging

Spin-orbit correlation

Physics of gauge links is represented

Requires Orbital Angular Momentum

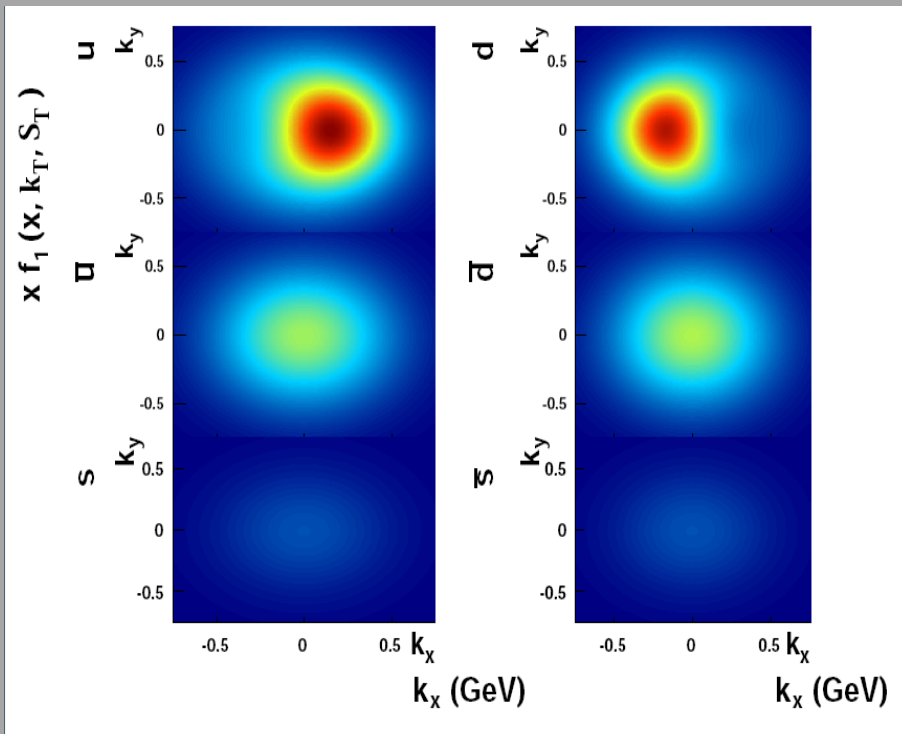
EIC report, Boer, Diehl, Milner, Venugopalan,  
Vogelsang et al , 2011;

Duke workshop report: Anselmino et al Eur.Phys.J.A47:35,2011



# Access to 3D imaging

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^\perp(x, \mathbf{k}_T^2)$$



**Dipole deformation**

Sivers function from  
experimental data  
HERMES and COMPASS

Anselmino et al 2005

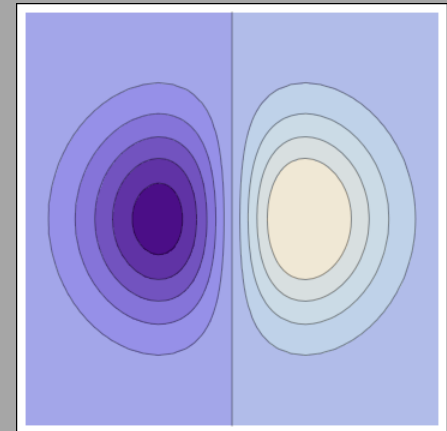
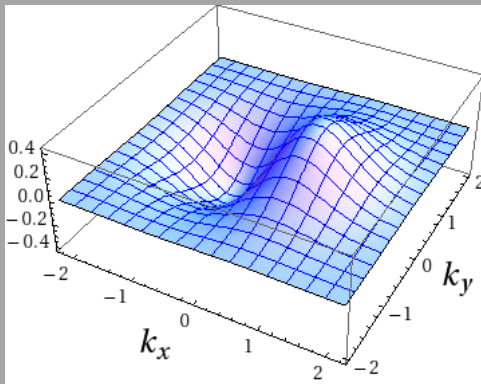
# What do we learn from 3D distributions?

$$f(x, \mathbf{k}_T, \mathbf{S}_T) = f_1(x, \mathbf{k}_T^2) - f_{1T}^\perp(x, \mathbf{k}_T^2) \frac{\mathbf{k}_x}{M}$$

Suppose the spin is along Y direction:  $S_T = (0, 1)$

Deformation in momentum space is:  $k_x \cdot f(k_x^2 + k_y^2)$

This is called “dipole” deformation.



What do we learn from 3D distributions?

$$f(x, \mathbf{k}_T, \mathbf{S}_T) = f_1(x, \mathbf{k}_T^2) - f_{1T}^\perp(x, \mathbf{k}_T^2) \frac{\mathbf{k}_x}{M}$$

We calculate now average shift:  $\langle k_x \rangle$

$$\langle k_x \rangle = \int d^2 k_T \frac{\mathbf{k}_T^2}{2M} f_{1T}^\perp(x, \mathbf{k}_T^2) \equiv f_{1T}^{\perp(1)}(x) M$$

Average momentum shift is equal to the **first moment** of Siviers function

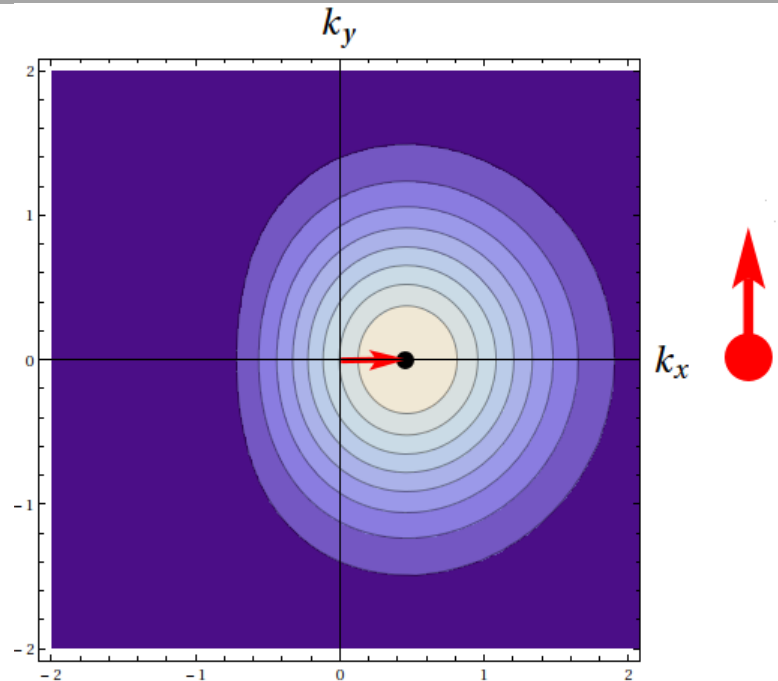
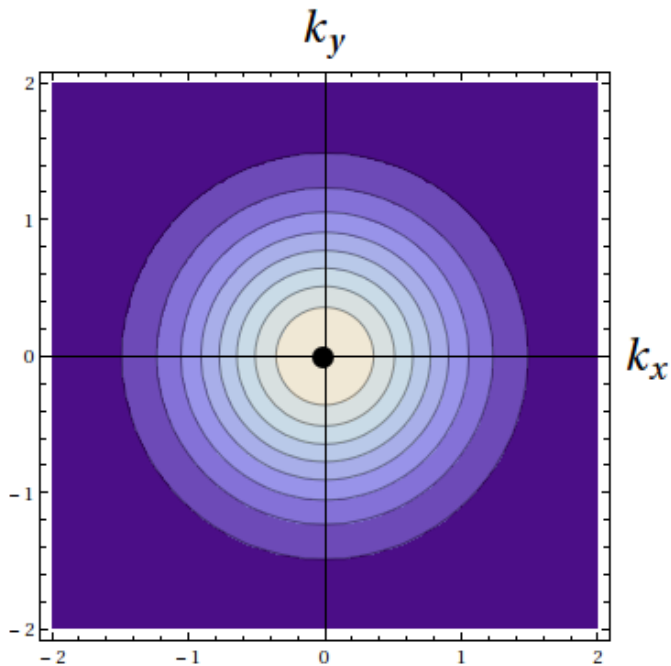
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The same statement in figures:

No polarisation:

Polarisation:  $S_y \Rightarrow \langle k_x \rangle$

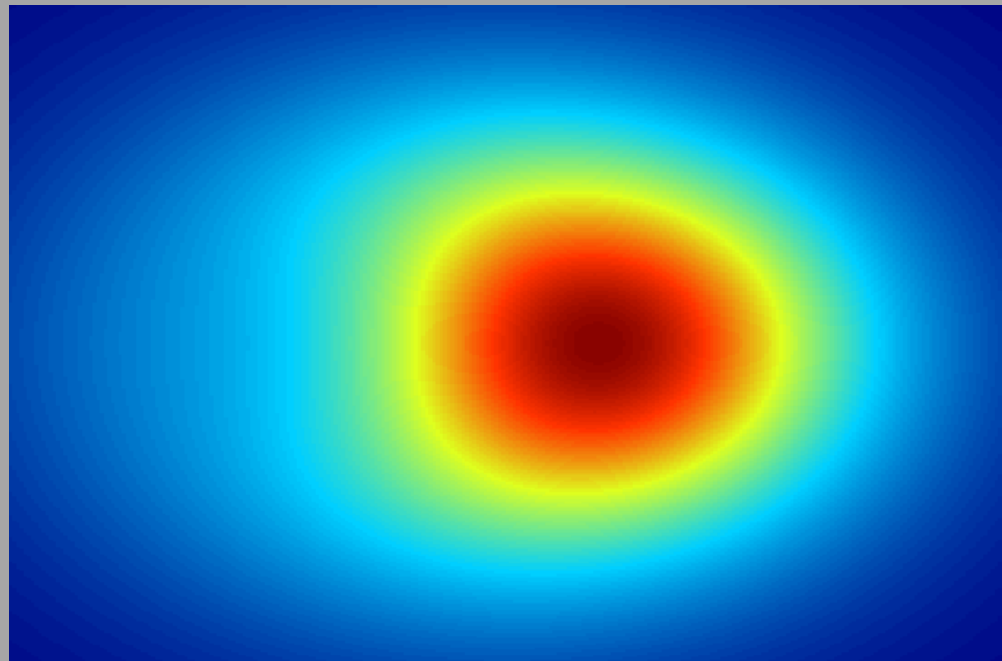


# What do we learn from 3D distributions?

$$f(x, \mathbf{k}_T, \mathbf{S}_T) = f_1(x, \mathbf{k}_T^2) - f_{1T}^\perp(x, \mathbf{k}_T^2) \frac{\mathbf{k}_{T1}}{M}$$

The same statement in figures:

This is what we know from experimental data already:



# How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

$$\sigma^\uparrow - \sigma^\downarrow = -f_{1T}^\perp \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)$$

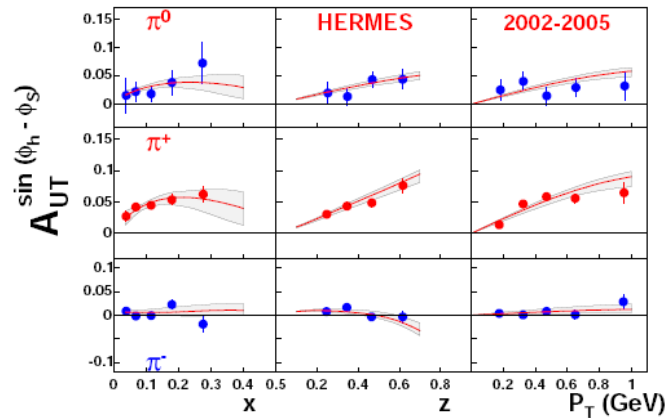
Unpolarised electron beam  
Transversely polarised proton

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = - \frac{\sum_q e_q^2 f_{1T}^\perp \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

See talk by Gunar Schnell

## HERMES

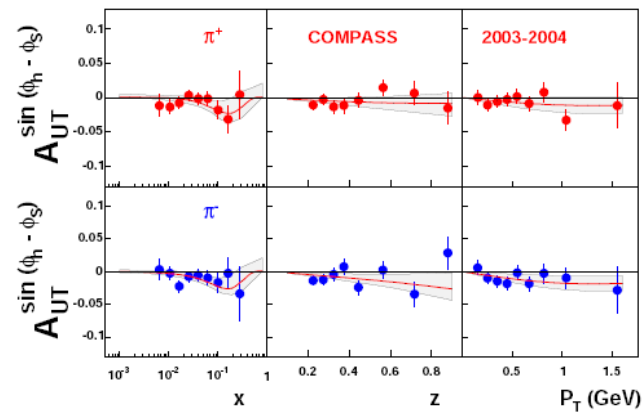
$ep \rightarrow e\pi X$ ,  $p_{lab} = 27.57$  GeV.



Anselmino et al 2010

## COMPASS

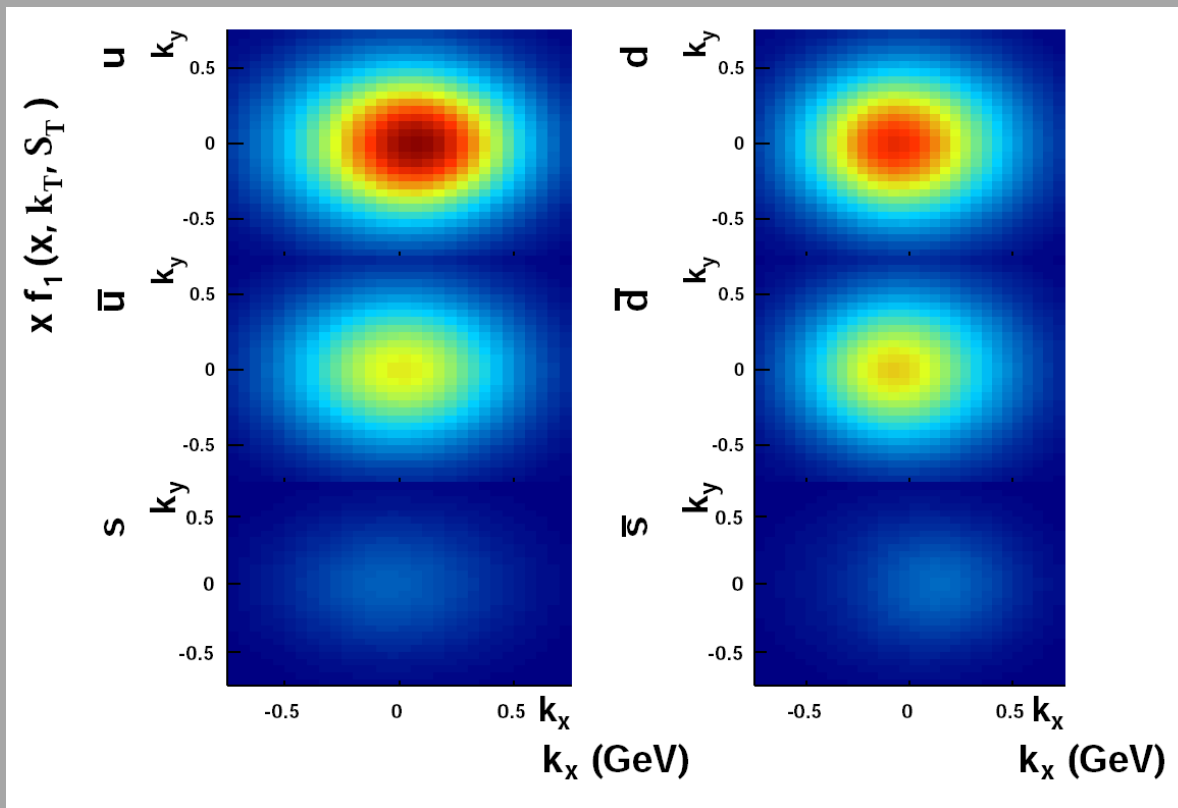
$\mu D \rightarrow \mu\pi X$ ,  $p_{lab} = 160$  GeV.



Anselmino et al 2010

# What do we learn from 3D distributions?

$$f(x, \mathbf{k}_T, \mathbf{S}_T) = f_1(x, \mathbf{k}_T^2) - f_{1T}^\perp(x, \mathbf{k}_T^2) \frac{\mathbf{k}_{T1}}{M}$$



The slice is at:

$$x = 0.1$$

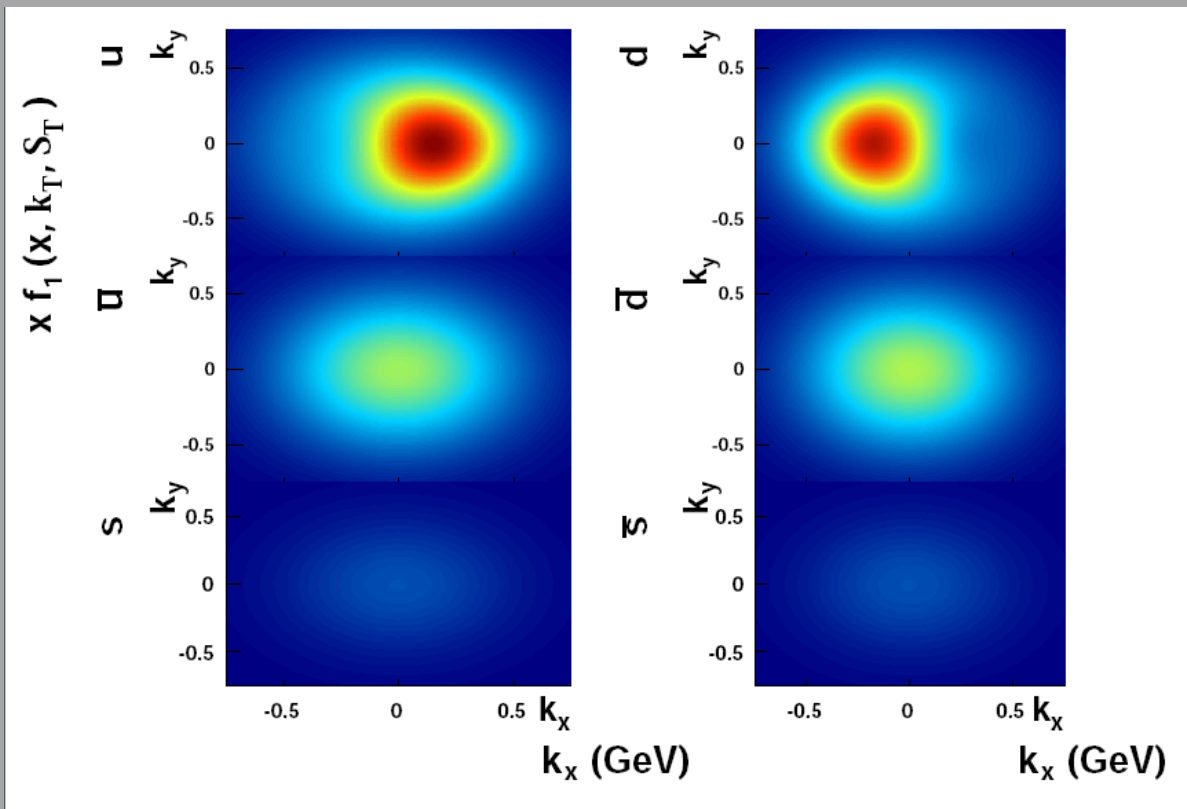
Low-x and high-x region  
is uncertain  
JLab 12 and EIC will  
contribute

No information on sea  
quarks

Picture is still quite  
uncertain

# What do we learn from 3D distributions?

$$f(x, \mathbf{k}_T, \mathbf{S}_T) = f_1(x, \mathbf{k}_T^2) - f_{1T}^\perp(x, \mathbf{k}_T^2) \frac{\mathbf{k}_{T1}}{M}$$



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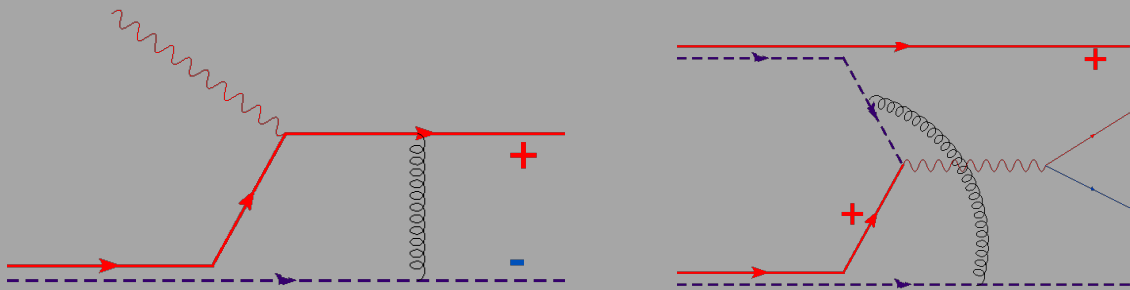
In future we will obtain  
much clearer picture



# Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.

Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky, Hwang,  
Schmidt  
Belitsky, Ji, Yuan  
Collins  
Boer, Mulders, Pijlman,  
etc

$$f_{1T}^{\perp \text{SIDIS}} = -f_{1T}^{\perp \text{DY}}$$

One of the main goals is to verify this relation.  
It goes beyond “just” check of TMD factorization.  
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc

Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou

# TMD theoretical challenges

- Evolution and soft gluon resummation
- Global study at Next-to-Leading order
- Relation to Orbital Angular Momentum

Many more other questions

- What is the  $k_t$  distributions of partons – gaussian, powerlike, sign changing?
- What is the difference of  $k_t$  distributions of quarks and sea quarks?
- How to explore higher twist TMDs?
- How to explore distribution and fragmentation TMDs in a satisfactory way?
- etc

# TMD evolution

Collins-Soper-Sterman factorization can be used

$$\frac{\partial \ln \tilde{F}(x, b_{\perp}, \mu, \zeta)}{\partial \ln \zeta} = \tilde{K}(b_{\perp}, \mu)$$

CS kernel in coordinate space

$$\frac{d\tilde{K}(b_{\perp}, \mu)}{d \ln \mu} = -\gamma_K(\mu)$$

$$\frac{d\tilde{F}(x, b_{\perp}, \mu, \zeta)}{d \ln \mu} = \gamma_F(\mu, \zeta)$$

TMD:

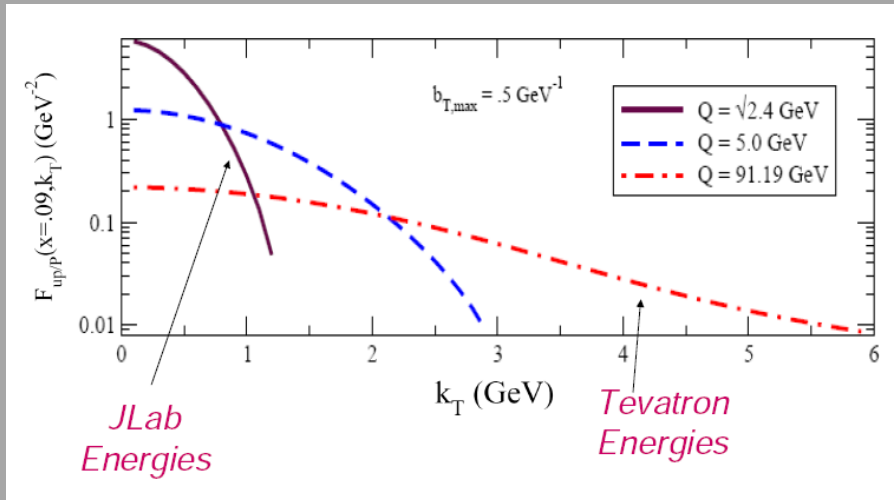
Collins 2011

Rogers, Aybat 2011

Twist-3:

Kang, Xiao, Yuan 2011

Koike, Vogelsang 2011

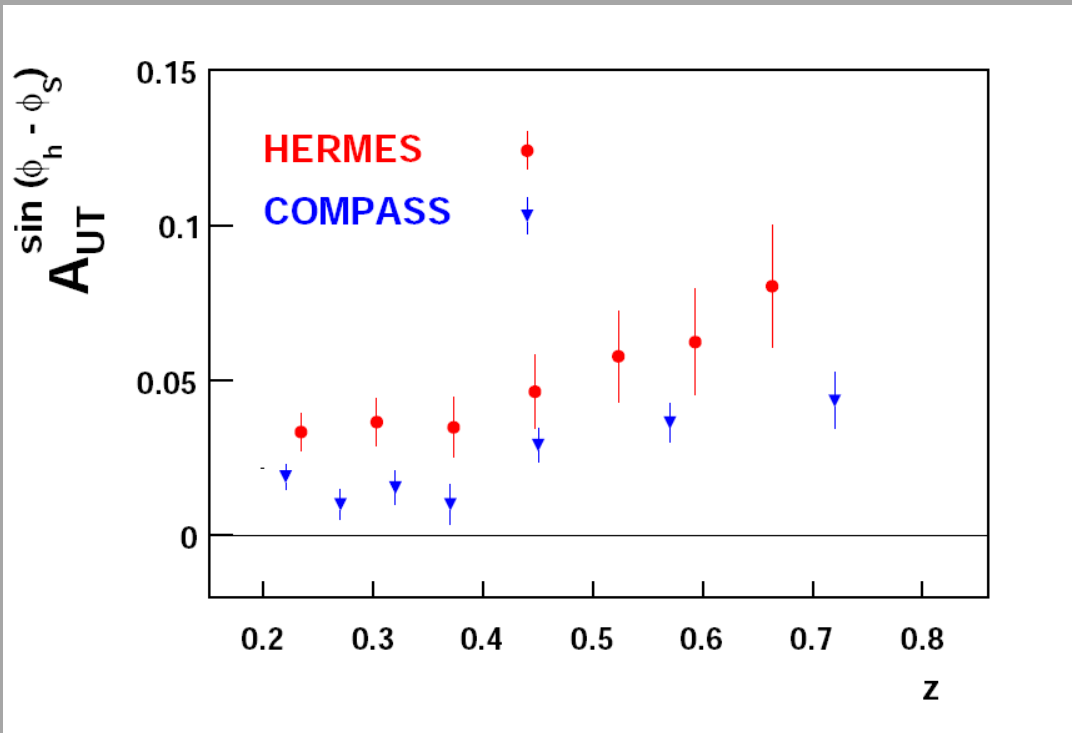


TMDs change with energy and resolution scale

Relevant to EIC

# TMD evolution

Can we see signs of evolution in the experimental data?



Aybat, AP, Rogers 2011

COMPASS data is at

$$\langle Q^2 \rangle \simeq 3.6 \text{ (GeV}^2\text{)}$$

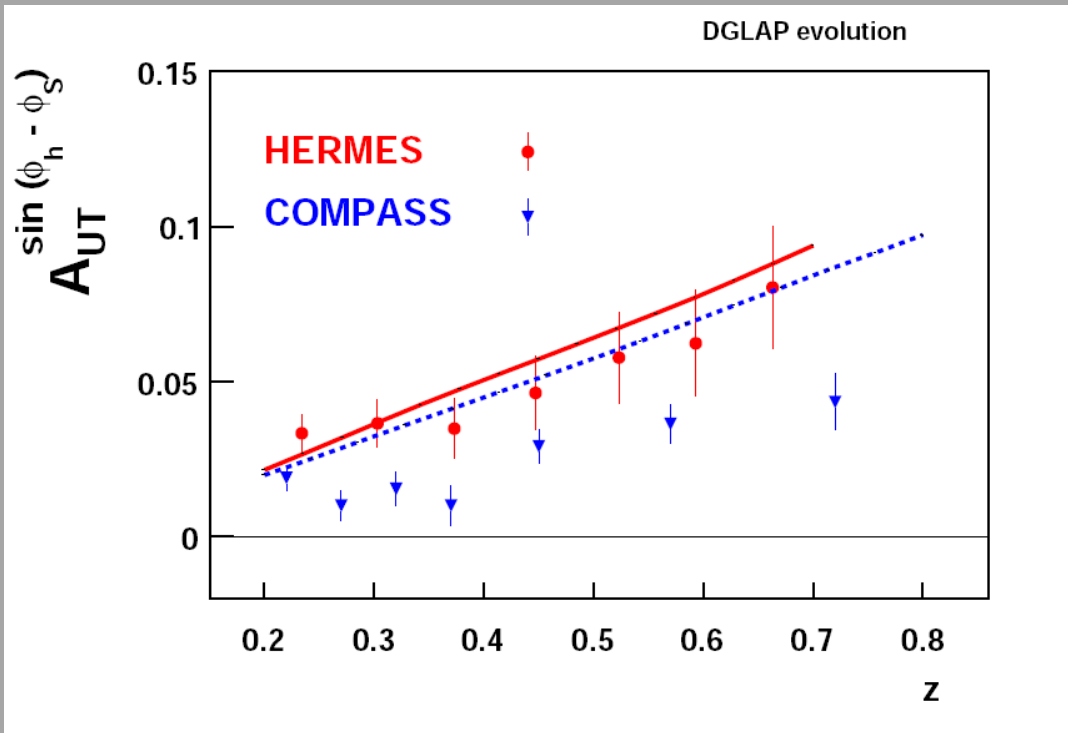
HERMES data is at

$$\langle Q^2 \rangle \simeq 2.4 \text{ (GeV}^2\text{)}$$

# TMD evolution

Can we **explain** the experimental data?

Convention method is to apply DGLAP evolution only



Aybat, AP, Rogers 2011

COMPASS dashed line

$$\langle Q^2 \rangle \simeq 3.6 \text{ (GeV}^2\text{)}$$

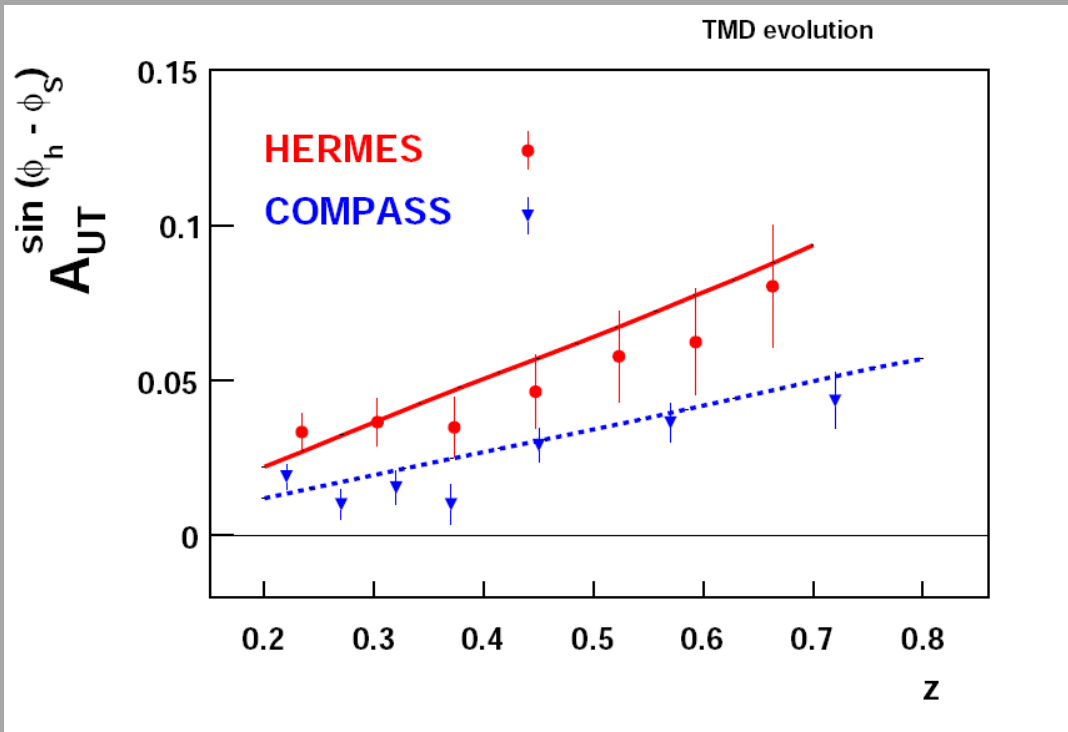
HERMES solid line

$$\langle Q^2 \rangle \simeq 2.4 \text{ (GeV}^2\text{)}$$

# TMD evolution

Can we **explain** the experimental data?

Full TMD evolution is needed!



Aybat, AP, Rogers 2011

COMPASS dashed line

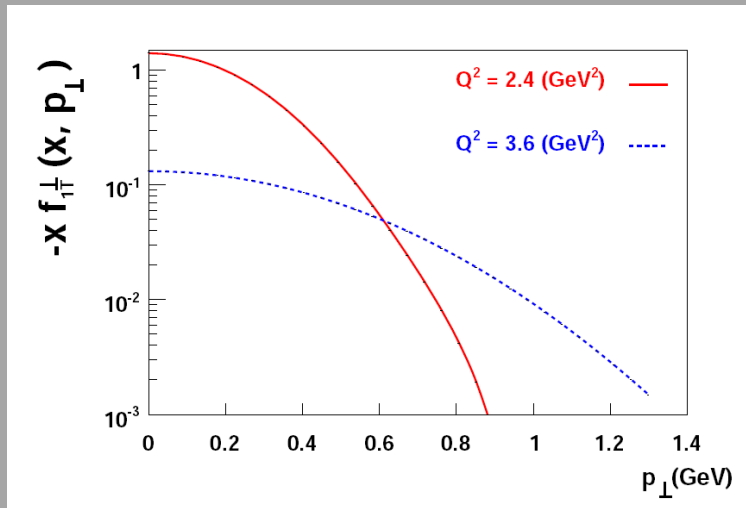
$$\langle Q^2 \rangle \simeq 3.6 \text{ (GeV}^2\text{)}$$

HERMES solid line

$$\langle Q^2 \rangle \simeq 2.4 \text{ (GeV}^2\text{)}$$

# TMD evolution

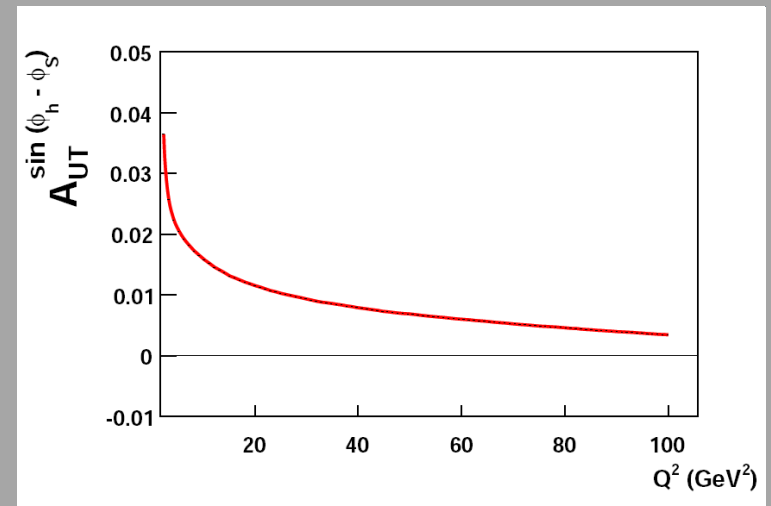
This is the first implementation of TMD evolution for observables



Functions change with energy

Aybat, AP, Rogers 2011

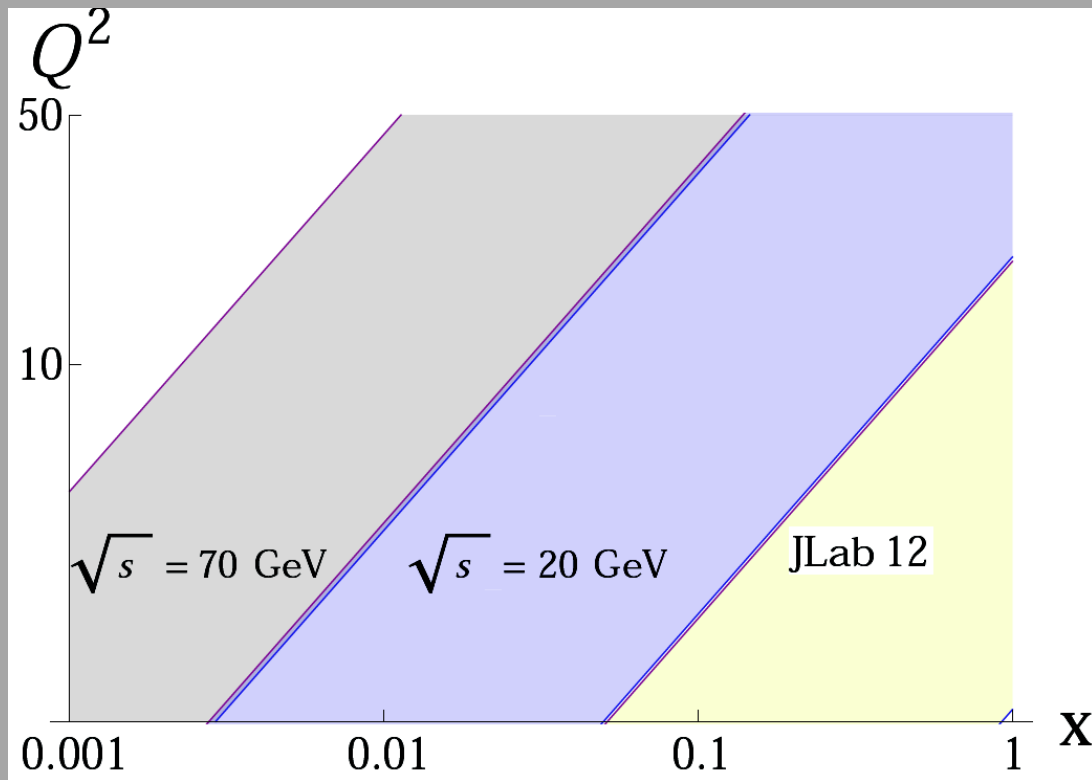
Asymmetry changes with  $Q^2$



Phenomenological analysis with evolution is now possible

# Kinematics

Kinematics  $Q^2 \simeq sxy$

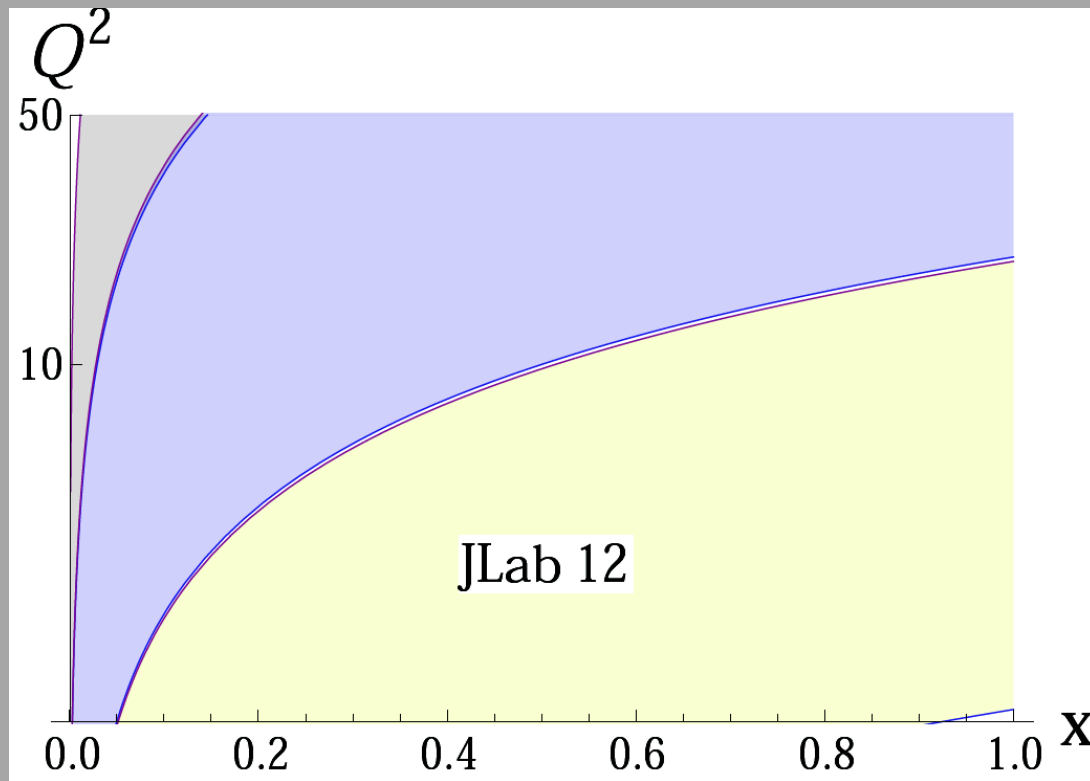


Jlab 12 and future  
Electron Ion Collider  
are complimentary



# Kinematics

Kinematics  $Q^2 \simeq sxy$

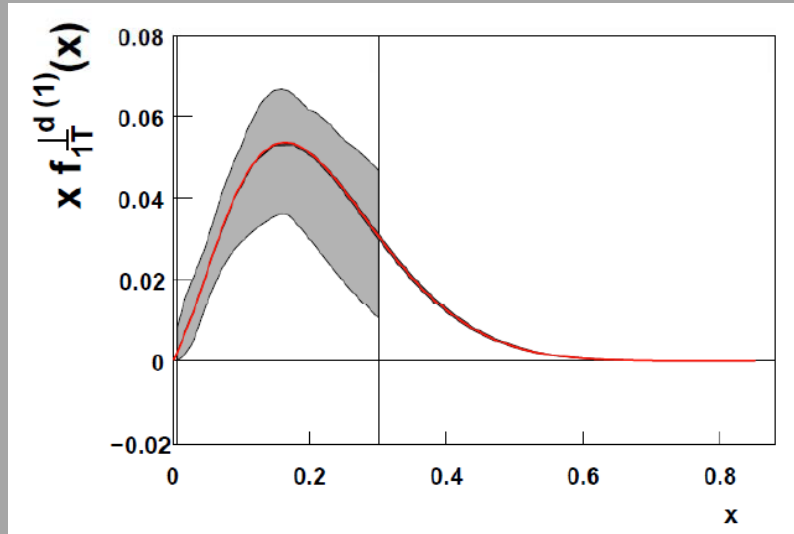


Jlab 12 and future  
Electron Ion Collider  
are complimentary

Jlab and EIC are going to  
provide fine 4D binning  
of the data.  
Exact knowledge of  
evolution is crucial

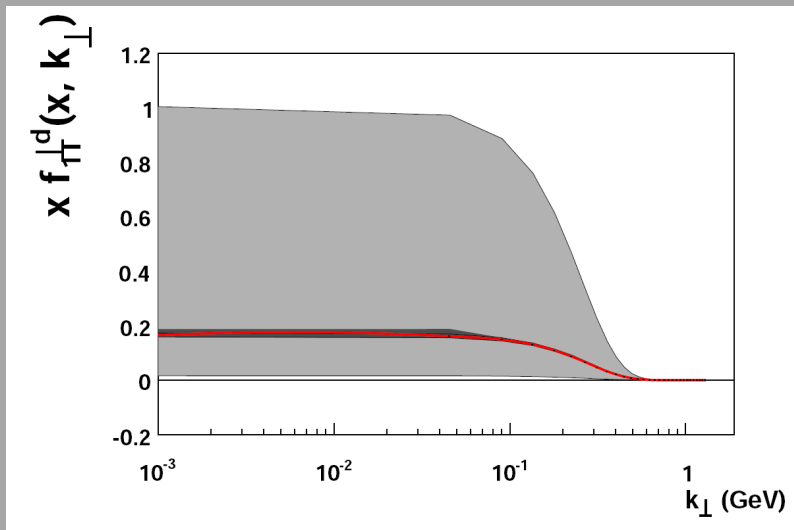
# Future improvement

What do we expect at JLab?



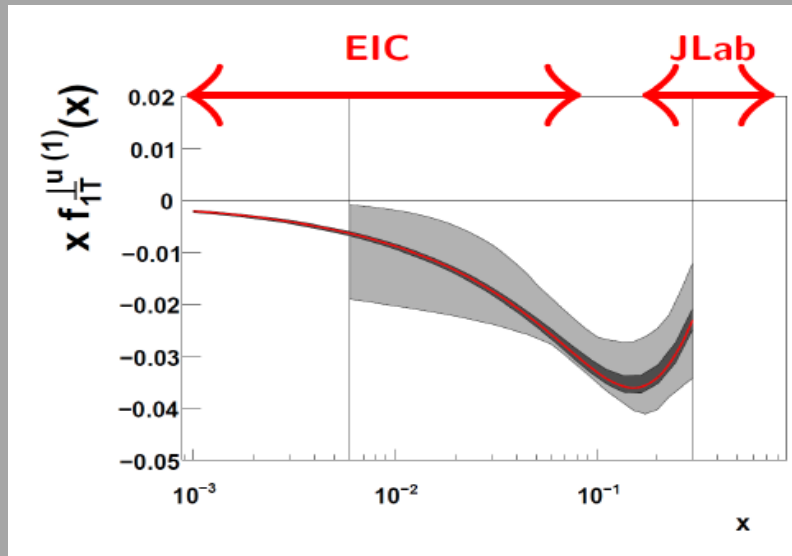
One example  
TMD from Jlab  
future data:  
JLab 12 on 3HE target.

Very big improvement  
in terms of our  
knowledge



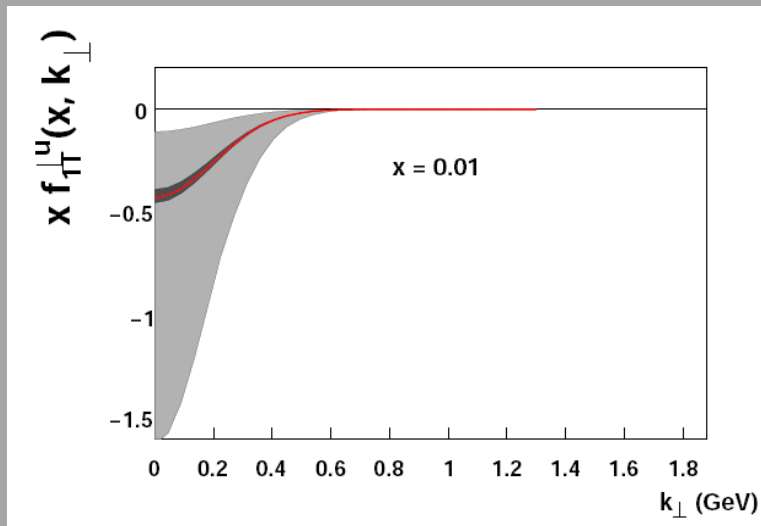
# Future improvement

What do we expect at EIC?



One example  
TMD from EIC  
future data

Very big improvement  
in terms of our  
knowledge



# Possible relations of TMDs and OAM

Bacchetta, Radici 2011 argue that

Inspired by model  
Relations, not full QCD

$$f_{1T}^{\perp(0)}(x) \simeq I(x)E(x, 0, 0)$$

So called “lensing” function  
Burkardt, Metz, etc

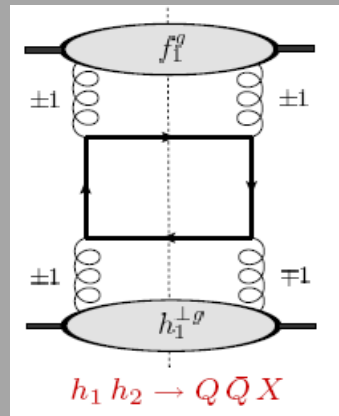
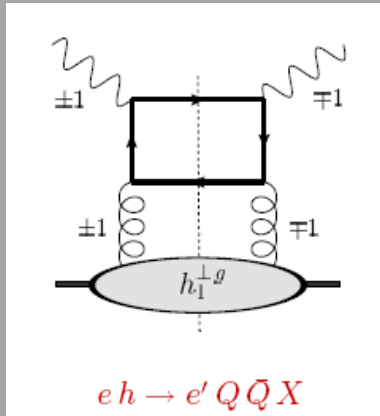
Making direct connection to total OAM from Ji's sum rule:

$$J_q = \frac{1}{2} \int_0^1 dx x (H_q(x, 0, 0) + E_q(x, 0, 0))$$

$$J_u = 0.266, J_d = -0.012 \quad \text{at} \quad Q^2 = 1(\text{GeV}^2)$$

Assumptions based on model calculations of course, but might be interesting.

# Gluons distributions



- Gluon TMDs
- Linearly polarized gluons can be accessed in various channels
- Opportunities of studies at **EIC**, **LHeC**
- $h_1^{\perp,g}$  may contribute to Higgs production, resolve its parity

Boer, Brodsky, Mulders, Pisano 2011

Qiu, Vogelsang, Schlegel 2011

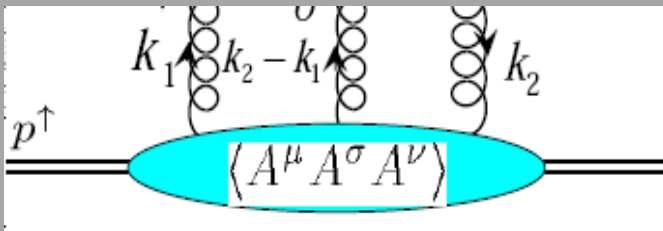
Boer, den Dunnen, Pisano, Schlegel, Vogelsang 2011

- Tri-gluon correlations, Qiu-Sterman matrix elements, complete classification

Koike, Tanaka

- TMD SSA in open charm

Godbole, Misra, Mukherjee, Rawoof 2011



# TMD experimental challenges

- 4D binning of observables  $x, z, Q^2, P_{h\perp}$
- Different targets: proton, neutron, deuteron
- Different final state hadrons  $\pi, K$   
open charm

All this helps to do correct flavour decomposition and correct analysis.

# CONCLUSIONS

- Studies of 3D distributions represent big part of future of nuclear physics
- EIC is an ideal place to explore GPDs and TMDs
- Theory and phenomenology have made a lot of progress in recent years
- We are going to see more progress in future

# CONCLUSIONS

- Studies of 3D distributions represent big part of future of nuclear physics
- EIC is an ideal place to explore GPDs and TMDs
- Theory and phenomenology have made a lot of progress in recent years
- We are going to see more progress in future
- **We are looking forward to EIC!**



# QCD EVOLUTION 2012

<http://www.jlab.org/conferences/qcd2012/>

May 14 - 17, 2012

Jefferson Lab  
Newport News,  
Virginia, USA



## **Organizing committee:**

Alexei Prokudin, Chair  
Anatoly Radyushkin  
Ian Balitsky  
Leonard Gamberg  
Harut Avakian

# QCD EVOLUTION 2012

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Alexei Prokudin

# HUGS 2012

**<http://www.jlab.org/hugs/>**

Summer school: 27th Annual Hampton University Graduate Studies Program. Covers theoretical and experimental aspects of nuclear physics.

**Jefferson Lab, Newport News, Virginia  
June 4 - June 22, 2012**

Fellowships are available and will cover tuition, fees, room and board

The deadline for application submittal is April 2, 2012

HUGS 2012

**WELCOME TO JLAB!**



Alexei Prokudin