Multi-gluon correlations in the Color Glass Condensate

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Stellenbosch EIC workshop

Outline

- The CGC, JIMWLK evolution
- Bulk particle production in dense-dense collisions; the glasma
- Dense-dense correlations
  - Power counting
  - Two particle correlations: the near-side ridge
- Dilute-dense collisions
  - Particle production: k_T-factorization
  - Correlations: the away-side peak
  - Finite-N_c effects from JIMWLK

References:

- Glasma: T.L. and L. McLerran, *Nucl. Phys.* **A772** (2006) 200 [arXiv:hep-ph/0602189].
- Single inclusive: T.L., *Phys.Lett.* **B703** (2011) 325 [arXiv:1105.5511 [hep-ph]].
- Ridge: K. Dusling, F. Gelis, T.L. and R. Venugopalan, *Nucl. Phys.* **A836** (2010) 159 [arXiv:0911.2720 [hep-ph]].
- Ridge: T.L., S. Srednyak and R. Venugopalan, *JHEP* **01** (2010) 066 [arXiv:0911.2068 [hep-ph]].
- CMS ridge: A. Dumitru, K. Dusling, F. Gelis, J. Jalilian-Marian, T.L. and R. Venugopalan, *Phys.Lett.* **B697** (2011) 21 [arXiv:1009.5295 [hep-ph]].
- Multigluon correlations in JIMWLK: A. Dumitru, J. Jalilian-Marian, T.L., B. Schenke and R. Venugopalan, *Phys. Lett.* **B706** (2011) 219 [arXiv:1108.4764 [hep-ph]]

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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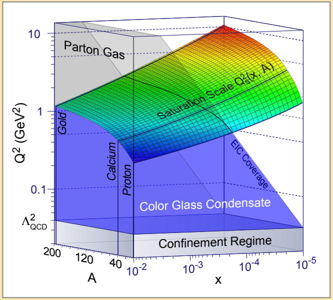
▼

$\mathbf{p}_T \sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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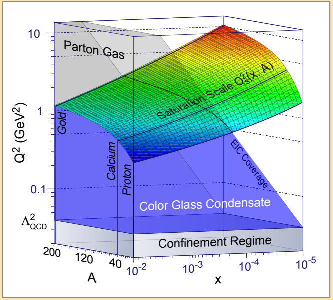

CGC: Effective theory for wavefunction of nucleus

- Large x = source ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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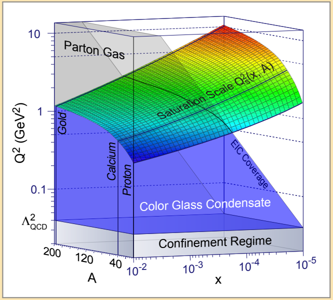

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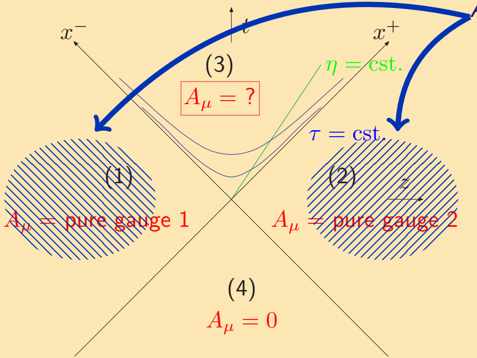
Glasma field configuration of two colliding sheets of CGC.

Gluon fields in AA collision

Classical **Y**ang-**M**ills



2 pure gauges

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}_T)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$

$$U_{(1,2)}(\mathbf{x}_T) = Pe^{ig\int dx^- \frac{\rho(\mathbf{x}_T, x^-)}{\nabla_T^2}}$$

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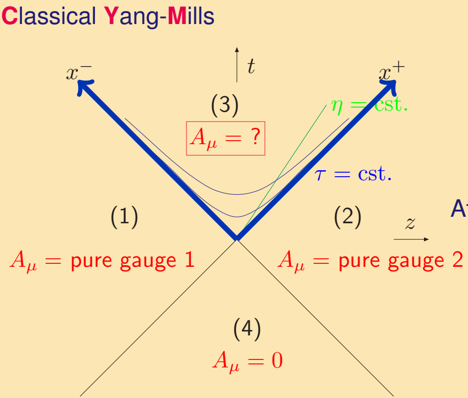

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At $\tau = 0$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta\Big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

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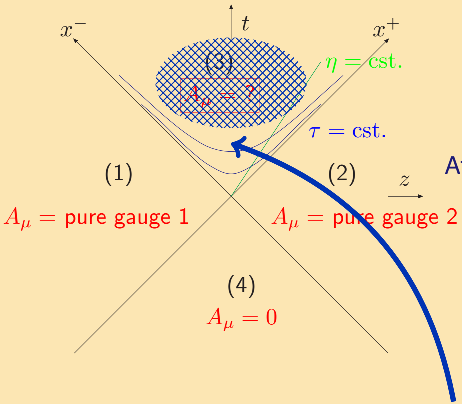

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Solve numerically Yang-Mills equations for $\tau > 0$

This is the **glasma** field $\Longrightarrow$ Then average over ρ.

Gluons with $p_T \sim Q_s$ — strings of size $R \sim 1/Q_s$

In dilute limit reduces to k_T-factorization.

JIMWLK evolution

Need Wilson lines from probability distribution $W_y[U]$.

Energy/rapidity dependence of $W_y[U]$ from JIMWLK RGE:

$$\partial_y W_y[U(\mathbf{x}_T)] = \mathcal{H} W_y[U(\mathbf{x}_T)]$$

JIMWLK Hamiltonian:

$$\mathcal{H} \equiv \frac{1}{2} \int_{\mathbf{x}_T \mathbf{y}_T \mathbf{z}_T} \frac{\delta}{\delta \widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)} \mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) \cdot \mathbf{e}_T^{ca}(\mathbf{y}_T, \mathbf{z}_T) \frac{\delta}{\delta \widetilde{\mathcal{A}}_b^+(\mathbf{x}_T)},$$

$$\mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) = \frac{g}{\sqrt{4\pi^3}} \frac{\mathbf{x}_T - \mathbf{z}_T}{(\mathbf{x}_T - \mathbf{z}_T)^2} \left(1 - U^\dagger(\mathbf{x}_T) U(\mathbf{z}_T)\right)^{ba}$$

($\delta/\delta\widetilde{A}^+$ is Lie derivative on SU(3) group)

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Numerics using Langevin formulation

Mean field approximation: BK Balitsky-Kovchegov

Evolution equation for correlator

$$\partial_y \left[\frac{1}{N_c} \operatorname{Tr} \left\langle 1 - U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) \right\rangle \right]$$

Gluon spectrum in the glasma

Most up to date calculation T.L., *Phys.Lett.* **B703** (2011) 325 ;
First calculation to actually use JIMWLK for gluon production in *AA*

Q_s is only dominant scale

Parametrically $$\frac{dN_g}{dy\, d^2\mathbf{x}_T\, d^2\mathbf{p}_T} = \frac{1}{\alpha_s} f\left(\frac{p_T}{Q_s}\right)$$

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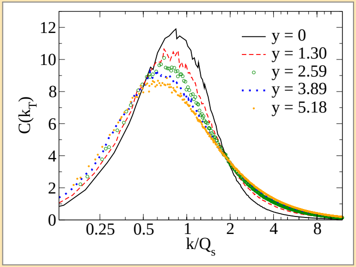

Unintegrated gluon distribution

$$C(\mathbf{k}_T) = \frac{{k_T}^2}{N_c} \operatorname{Tr} \langle U(\mathbf{k}_T) U^\dagger(\mathbf{k}_T) \rangle$$

becomes **harder** due to evolution.

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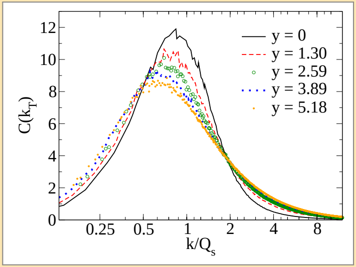

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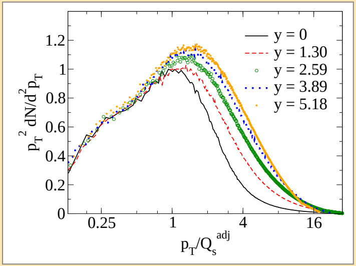

Produced gluon spectrum: harder at higher $\sqrt{s}$ (Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Gluon multiplicity and mean p_T

Q_s is only dominant scale

Parametrically $$\frac{\mathrm{d}N_g}{\mathrm{d}y\,\mathrm{d}^2\mathbf{x}_T} = c_N \frac{C_F}{2\pi^2\alpha_s} Q_s^2 \qquad \langle p_T \rangle \sim Q_s$$

Note: in full CYM total gluon multiplicity, IR finite, no cutoff.

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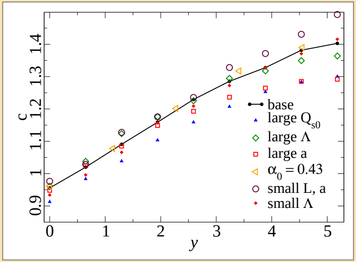

Scaled multiplicity increases with energy (Midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

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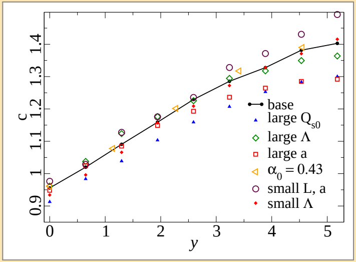

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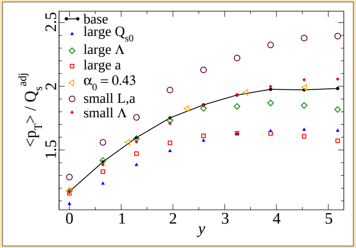
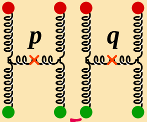
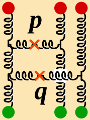
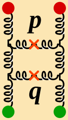


Harder gluon spectrum $\Longrightarrow$ higher $\langle p_T \rangle / Q_s$ as scaling regime sets in.
(Still very large lattice cutoff effects.)

“Classical” and “quantum” correlations







Correlations from

- Sources
  - “disconnected”, “classical”
  - can be near side $\Delta\phi \sim 0$
  - Dominate for dense
- Loops
  - “connected”, “quantum”
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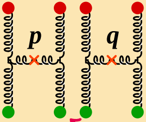

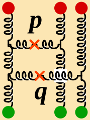

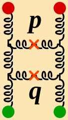

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Power counting:

- “A”: $\rho \sim 1/g$
- “p”: $\rho \sim g$

Dense-dense power counting

Basic power counting: $\frac{dN}{d^3\boldsymbol{p}} \sim \frac{1}{\alpha_s}$

Fixed sources: correlations loop/quantum effects, suppressed by α_s

$$\text{E.g. Poisson}\quad \overbrace{\langle N^2\rangle}^{1/\alpha_s^2+\cdots} - \overbrace{\langle N\rangle^2}^{1/\alpha_s^2+\cdots} = \overbrace{\langle N\rangle}^{1/\alpha_s+\cdots}$$

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But in CGC must average over color charge ρ:

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}_1} \cdots \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}_n} \right\rangle_{\text{conn.}} = \left[\int_{[\rho]} \underbrace{W[\rho_1(y)]\, W[\rho_2(y)]} \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}_1} \cdots \frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}_n} \right]_{\text{conn.}} \sim \frac{1}{\alpha_s^n}$$

LLog corrections factorize into evolution of ρ **distribution**

$$\text{E.g. neg. bin} \; \overbrace{\langle N^2 \rangle}^{1/\alpha_s^2} - \overbrace{\langle N \rangle^2}^{1/\alpha_s^2} = \frac{1}{k} \overbrace{\langle N \rangle^2}^{1/\alpha_s^2} + \overbrace{\langle N \rangle}^{1/\alpha_s}$$

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Dominant correlations come from sources

Quantum correlations, enhanced by $\ln 1/x \sim 1/\alpha_s$

$\Longrightarrow$ appear as "classical" in effective theory.

Full numerical calculation

Numerical result in MV model T.L., Srednyak, Venugopalan, *JHEP* **01** (2010) 066, Note: pre-CMS ridge.

$$C_2(\mathbf{p},\mathbf{q}) \equiv \left\langle \frac{\mathrm{d}^2 N_2}{\mathrm{d}y_p\,\mathrm{d}^2\mathbf{p}_T\,\mathrm{d}y_q\,\mathrm{d}^2\mathbf{q}_T} \right\rangle$$

$$\kappa_2(\mathbf{p}_T,\mathbf{q}_T) = \overbrace{S_\perp Q_s^2}^{\text{(\# of independent regions)}} \times \left(\frac{C_2(\mathbf{p},\mathbf{q})}{\left\langle \frac{\mathrm{d}N}{\mathrm{d}y_p\,\mathrm{d}^2\mathbf{p}_T} \right\rangle \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_q\,d^2\mathbf{q}_T} \right\rangle} - 1 \right)$$

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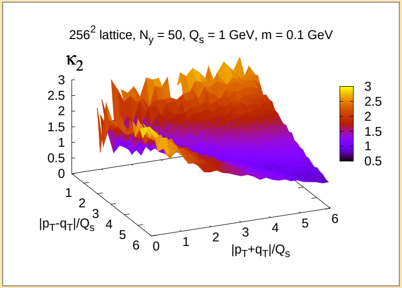
Here plotted vs. $|\mathbf{p}_T - \mathbf{q}_T|, |\mathbf{p}_T + \mathbf{q}_T|$

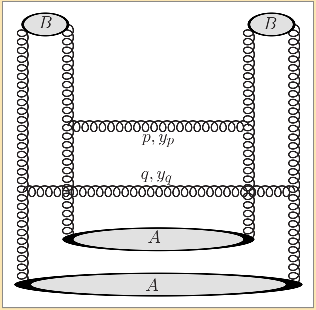
- $\kappa_2 \sim 1$
- Angular structures at
  - $\mathbf{p}_T \approx \mathbf{q}_T \implies$ ridge
  - $\mathbf{p}_T \approx -\mathbf{q}_T \implies$ away side



Where does the near side correlation come from?

$\mathbf{k}_T$-factorized approximation, K. Dusling, F. Gelis, T.L. and R. Venugopalan, *Nucl. Phys.* **A836** (2010) 159

$$C(\mathbf{p},\mathbf{q}) \sim \int_{\mathbf{k}_T} \Big\{ \Phi_B^2(\mathbf{k}_T)\Phi_A(\mathbf{p}_T-\mathbf{k}_T)\left[\Phi_A(\mathbf{q}_T+\mathbf{k}_T)+\Phi_A(\mathbf{q}_T-\mathbf{k}_T)\right] + (\mathbf{k}_T \leftrightarrow -\mathbf{k}_T) + (A \leftrightarrow B) \Big\}$$



(Gaussian approx: only 2-pt. f'n.)

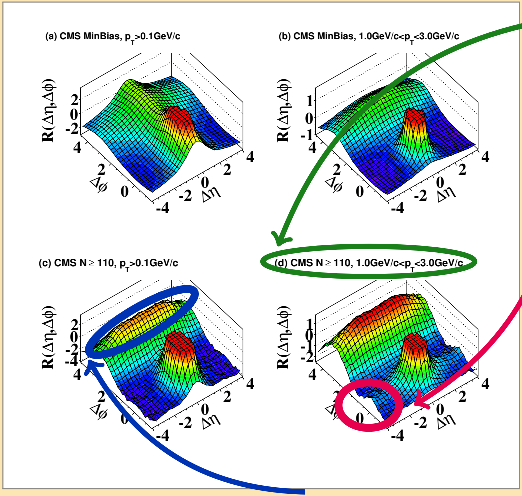
- $|\mathbf{p}_T + \mathbf{k}_T|$ and $|\mathbf{q}_T \pm \mathbf{k}_T|$ like to be $\sim Q_s$ $\Longrightarrow$ Correlation enhanced when $\mathbf{p}_T || \mathbf{q}_T$
- Here assume $\Delta y \ll 1/\alpha_s$, boost invariant approximation for rapidity structure.
  (Going beyond this: work in progress)

Interpretation

Ridge $\approx$ double parton scattering + intrinsic $\mathbf{k}_T$.

Φ is classical field mode $\Longrightarrow$ can be one, two, n gluons $\Longrightarrow$ naturally has multigluon correlations

CMS ridge



High N_{ch} trigger
$\Longrightarrow$ central events.

"Ridge" structure:

- $\Delta\phi \sim 0$, large $\Delta\eta$
- New at high $\sqrt{s}$
- Stronger in AA

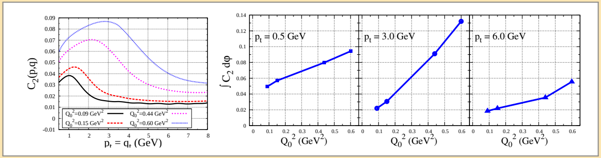
Normal away-side ($\Delta\phi = \pi$) jet peak;

CMS ridge qualitatively understood from CGC

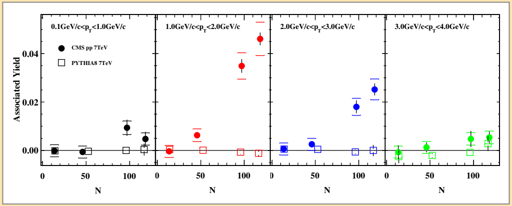
Dumitru, Dusling, Jalilian-Marian, Gelis, T.L., Venugopalan, *Phys.Lett.* **B697** (2011) 21

- Dependence on p_T: rise, then fall for $p_T \gg Q_s$
- Dependence on multiplicity: strongest for central, with largest average Q_s

Theory:



Data:

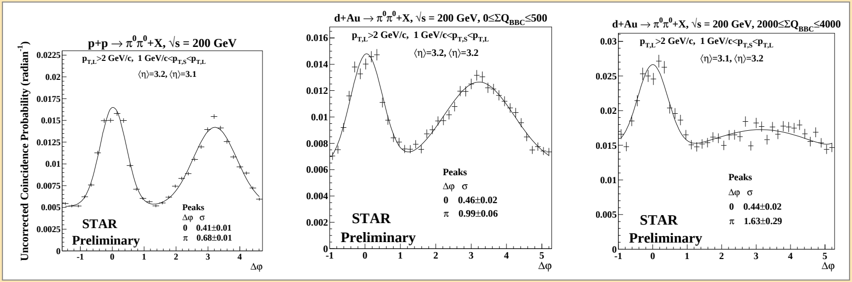


In AA: Azimuthal structure enhanced by flow;
origin of correlation combination of geometry and gluon fields.

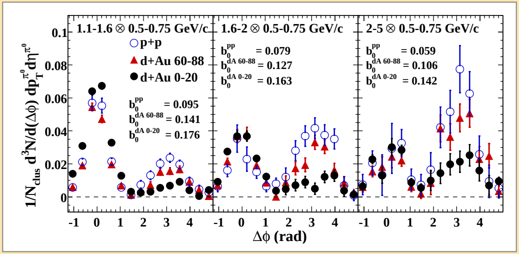
Dilute-dense: Forward dihadron correlations in dAu

Two particle collision vs. $\Delta\varphi$: away-side peak goes away

pp | peripheral *d*Au | central *d*Au



STAR, [arXiv: 1102.0931]



PHENIX, [arXiv: 1105.5112], PRL

Calculating 2-particle correlation in forward pA

- Quark from p (large x) from pdf
- Radiate gluon
- Propagate eikonally in color field of target A $\Longrightarrow$ Wilson lines U

$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3k_1\ \mathrm{d}^3k_2} \propto \int_{\mathbf{x}_T,\bar{\mathbf{x}}_T,\mathbf{y}_T,\bar{\mathbf{y}}_T} e^{-i\mathbf{k}_{T1}\cdot(\mathbf{x}_T-\bar{\mathbf{x}}_T)}\ e^{-i\mathbf{k}_{T2}\cdot(\mathbf{y}_T-\bar{\mathbf{y}}_T)}\ \mathcal{F}(\bar{\mathbf{x}}_T-\bar{\mathbf{y}}_T,\mathbf{x}_T-\mathbf{y}_T)$$

$$\Big\langle \hat{Q}(\mathbf{y}_T,\bar{\mathbf{y}}_T,\bar{\mathbf{x}}_T,\mathbf{x}_T)\ \hat{D}(\mathbf{x}_T,\bar{\mathbf{x}}_T) - \hat{D}(\mathbf{y}_T,\mathbf{x}_T)\hat{D}(\mathbf{x}_T,\bar{\mathbf{z}}_T) - \hat{D}(\mathbf{z}_T,\bar{\mathbf{x}}_T)\hat{D}(\bar{\mathbf{x}}_T,\bar{\mathbf{y}}_T)$$

$$+\frac{C_F}{N_c}\hat{D}(\mathbf{z}_T,\bar{\mathbf{z}}_T) + \frac{1}{N_c{}^2}\Big(\hat{D}(\mathbf{y}_T,\bar{\mathbf{z}}_T)+\hat{D}(\mathbf{z}_T,\bar{\mathbf{y}}_T)-\hat{D}(\mathbf{y}_T,\bar{\mathbf{y}}_T)\Big)\Big\rangle_{\text{target}}$$

$(\mathbf{z}_T = z\mathbf{x}_T + (1-z)\mathbf{y}_T,\ \bar{\mathbf{z}}_T = z\bar{\mathbf{x}}_T + (1-z)\bar{\mathbf{y}}_T.)$

Need target expectation values of Wilson line operators

$$\hat{D}(\mathbf{x}_T-\mathbf{y}_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T)$$

$$\hat{Q}(\mathbf{x}_T,\mathbf{y}_T,\mathbf{u}_T,\mathbf{v}_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T)U(\mathbf{u}_T)U^\dagger(\mathbf{v}_T)$$

Approximations for 4-point function $\langle \hat{Q} \rangle$

Motivation for approximations

Getting the dipole $\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \rangle$ is easy from BK; an approximation using only $\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \rangle$ is much easier for practical work.

- In phenomenology of 2-particle correlations, (Marquet 2007, Tuchin 2009, Albacete & Marquet 2010) only used "naive large N_c" approximation:

$$\left\langle \hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) \right\rangle \underset{N_c \to \infty}{\approx} \frac{1}{2} \left\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \right\rangle \left\langle \hat{D}(\mathbf{u}_T, \mathbf{v}_T) \right\rangle + \left\langle \hat{D}(\mathbf{x}_T, \mathbf{v}_T) \right\rangle \left\langle \hat{D}(\mathbf{u}_T, \mathbf{y}_T) \right\rangle$$

- We also compare to "Gaussian" approximation, where $\left\langle \hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) \right\rangle$ is related to $\left\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \right\rangle$ assuming Gaussian correlators for Wilson lines.
  "Gaussian truncation" of Kuokkanen, Rummukainen, Weigert, see also Iancu, Triantafyllopoulos

Choose two coordinate configurations

A. Dumitru, J. Jalilian-Marian, T.L., B. Schenke and R. Venugopalan, *Phys. Lett.* **B706** (2011) 219

General expression: several Fourier-transforms
Simplify and study 2 special configurations for

$$\hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) = \frac{1}{N_c} \operatorname{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) U(\mathbf{u}_T) U^\dagger(\mathbf{v}_T)$$

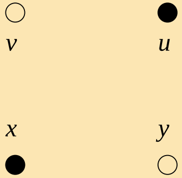
"Line": $\mathbf{u}_T = \mathbf{x}_T; \quad \mathbf{v}_T = \mathbf{y}_T$

"Naive large N_c" $Q_{|}^{\text{naive}}(r) = D(r)^2$

Gaussian

$$Q_{|}(r) \approx \frac{N_c + 1}{2} \left(D(r)\right)^{2\frac{N_c+2}{N_c+1}} - \frac{N_c - 1}{2} \left(D(r)\right)^{2\frac{N_c-2}{N_c-1}}$$

"Square"



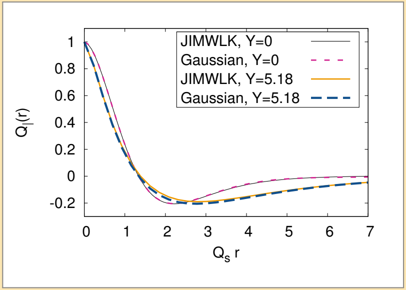
"Naive large N_c" $Q_{\square}^{\text{naive}}(r) = D(r)^2$

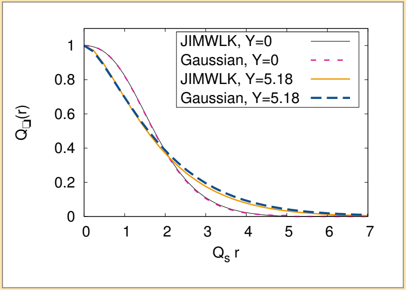
Gaussian:

$$Q_{\square}(r) \approx (D(r))^2 \left[\frac{N_c + 1}{2} \left(\frac{D(r)}{D(\sqrt{2}r)} \right)^{\frac{2}{N_c+1}} - \frac{N_c - 1}{2} \left(\frac{D(\sqrt{2}r)}{D(r)} \right)^{\frac{2}{N_c-1}} \right].$$

Gaussian is good

Initial condition $y = 0$ satisfies Gaussian approximation by construction.
But result stays very close at later rapidities.

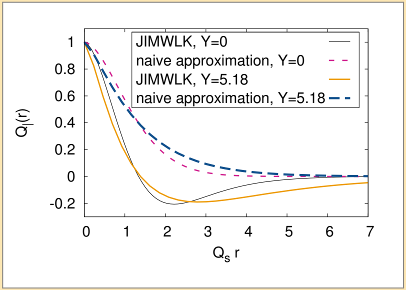


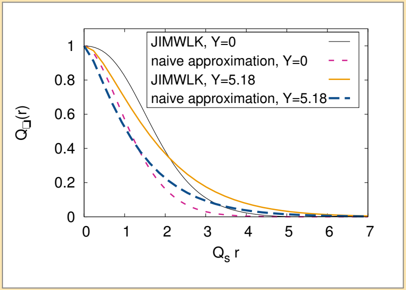


Line

Square

Naive large N_c is not good





Line

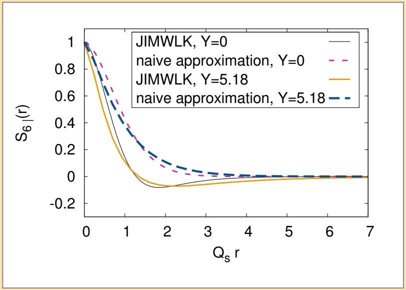
Square

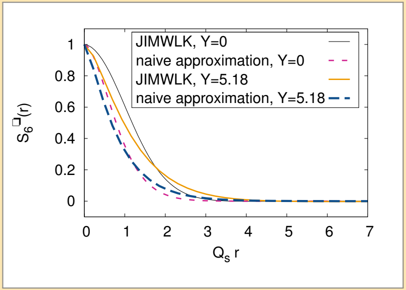
Even characteristic length/momentum scale differs by factor ~ 2.

6pt function

- Actually cross section has not $\langle \hat{Q} \rangle$, but $\langle \hat{Q}\hat{D} \rangle$.

As expected, the "naive large N_c" approximation for the 6pt function is as bad as for the 4pt function.





Line

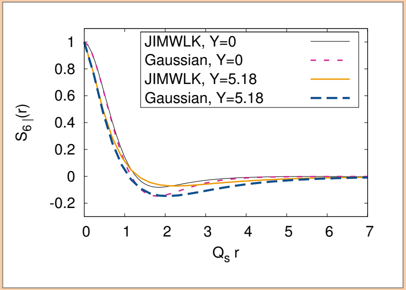
Square

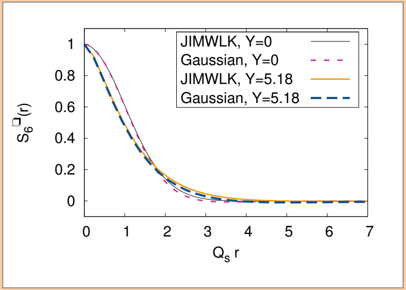
6pt function: “best known” approximation

- Actually cross section has not $\langle \hat{Q} \rangle$, but $\langle \hat{Q}\hat{D} \rangle$.
- Gaussian (MV) calculation for this not known (tedious)
- But: we know Kuokkanen et al that $\langle \hat{D}\hat{D} \rangle \approx \langle \hat{D} \rangle \langle \hat{D} \rangle$ works pretty well
- Compare JIMWLK with “best known” ∼Gaussian approx. $\langle \hat{Q}\hat{D} \rangle \approx \langle \hat{Q} \rangle \langle \hat{D} \rangle$

6pt: “best known” ∼Gaussian works pretty well

At least for small r, which counts for $p_T \gtrsim Q_s$





Line Square

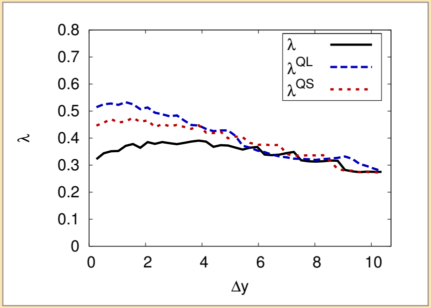
Evolution speeds

Evolution speed: define characteristic momentum scale Q_s for each correlator. (Inverse of characteristic length scale.)

Evolution speed is

$$\lambda \equiv \frac{d \ln Q_s^2}{dy}$$

Result: the higher point functions evolve "faster"



(This is a transient effect specific to MV initial condition; goes away for high rapidity.)

2-particle correlation, the actual spectrum

With H. Mäntysaari, work in progress

To get the actual cross section, e.g.

$$\frac{\mathrm{d}\sigma^{pA\to\pi^0\pi^0 X}}{\mathrm{d}^3k_1\ \mathrm{d}^3k_2}$$

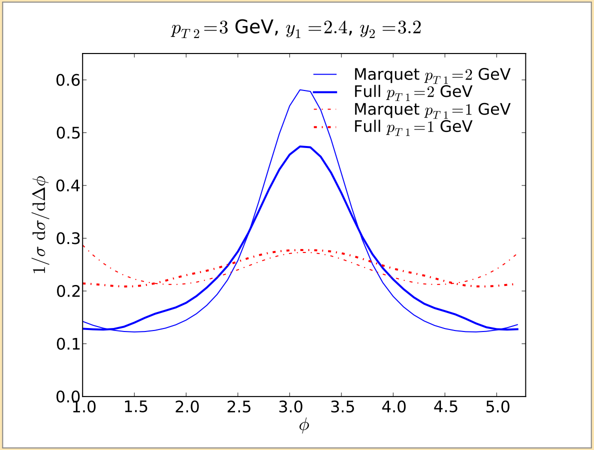
from
$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3k_1\ \mathrm{d}^3k_2} = \int_{\mathbf{x}_T,\bar{\mathbf{x}}_T,\mathbf{y}_T,\bar{\mathbf{y}}_T} e^{-i\mathbf{k}_{T1}\cdot(\mathbf{x}_T-\bar{\mathbf{x}}_T)}\ e^{-i\mathbf{k}_{T2}\cdot(\mathbf{y}_T-\bar{\mathbf{y}}_T)} \underbrace{\mathcal{F}(\bar{\mathbf{x}}_T-\bar{\mathbf{y}}_T,\mathbf{x}_T-\mathbf{y}_T)}_{\text{LC wavef.}}\underbrace{\langle\hat{Q}\hat{D}+\hat{D}\hat{D}+\dots\rangle}_{\text{Wilson line operators}}$$

one needs

1. Calculate Fourier-transform integrals
   - Easy in the approx. of Marquet, Albacete $\Longrightarrow$ only need FT of dipole.
   - Complicated 6-dimensional oscillatory integral in general case
2. Convolute with fragmentation functions
3. Model angular smearing from fragmentation

- Steps 1 and 2 in progress, still looking for a good way to do 3.
- Here: preliminary results at parton level, i.e. only for step 1.

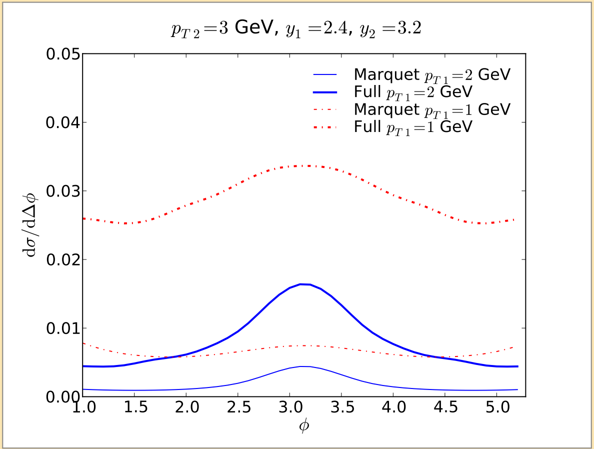
Angular correlation, peak width at parton level



Correlation vs. $\Delta\varphi$, area under curve normalized to 1 to show angular structure.

Including the “best known” approximation for the 6-pt function **broadens the peak** compared to the approximation of Marquet.

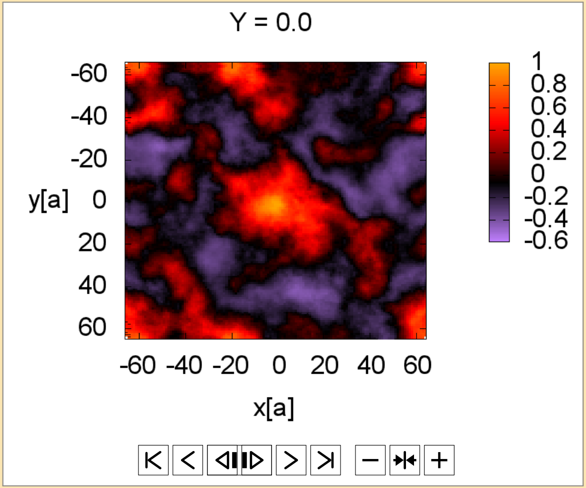
Normalization, at parton level



Correlation vs. $\Delta\varphi$.

Including the "best known" approximation for the 6-pt function **increases the ($\Delta\varphi$-independent) correlation** by a factor of $\sim$100% (p_T-dependent) compared to the approximation of Marquet.

Visualization of correlations in JIMWLK



Correlation between origin $(0,0)$ and (x,y)

$$\frac{1}{N_c} \mathrm{Re}\,\mathrm{Tr}\, U^\dagger(0,0)U(x,y)$$

$\Longrightarrow$ correlation length decreases for increasing energy.

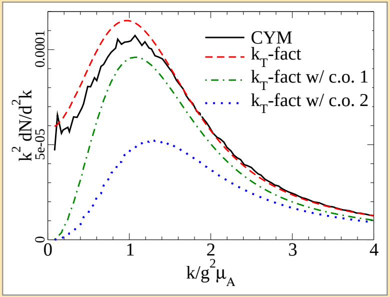
Conclusion

General features of CGC framework

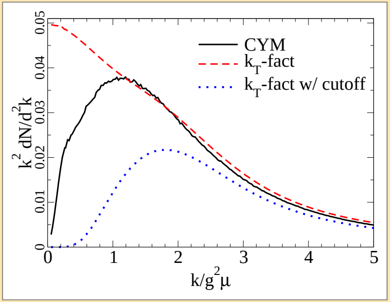
- Gluon saturation at small x $\Longrightarrow$ semihard bulk is one scale problem, Q_s
- Energy, rapidity dependence from JIMWLK/BK RGE
- Used for:
  - Single inclusive particle production
  - Correlations

CYM vs. $\mathbf{k}_T$-factorization

Blaizot, T.L., Mehtar-Tani 2010



pA: $\mathbf{k}_T$-factorization works



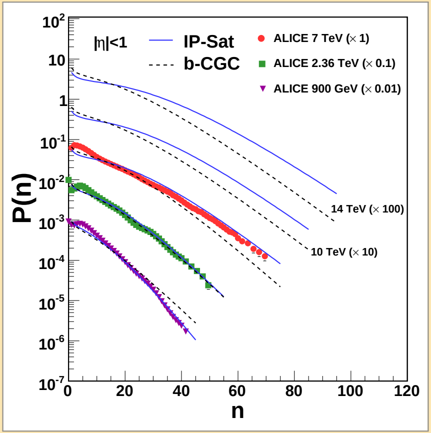
AA: no k_T-factorization

Not directly observable

Do not measure gluon spectrum with $\mathbf{p}_T \lesssim 1\text{GeV}$!
Centrality, rapidity, energy dependence from $N \sim S_\perp Q_s^2$

Suggested interpretation Levin, 2010 : Sudakov suppression factor.

Backup: existence of high multiplicity events



Multiplicity distribution in pp-collisions, Tribedy, Venugopalan, Nucl. Phys. A **850** (2011) 136