Nonlocal effective theories

Jakovác Antal

BME Institute of Physics, Budapest, Hungary

Outlines

Outlines

Outlines

Energy regimes of QCD

QCD $\Rightarrow$ interacting theory of quark and gluon fields

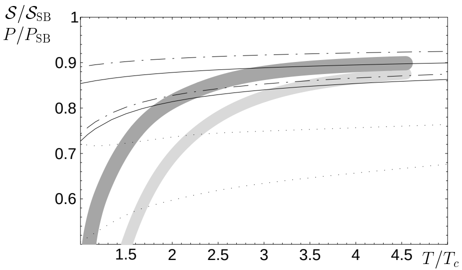
Is it a theory of quarks and gluons?

Energy regimes of QCD

QCD ⇒ interacting theory of quark and gluon fields

Is it a theory of quarks and gluons?

- at high energy: quark, gluons
  lower energies new concepts
  (pomeron, CGC; HTL,
  screened PT, 2PI. . .)



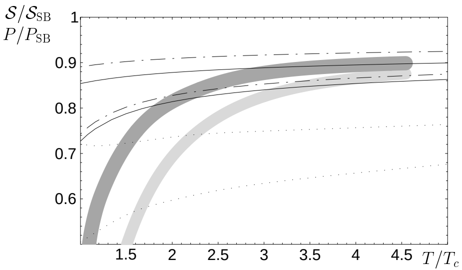
(J. P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. Lett. **83**, 2906 (1999) [arXiv:hep-ph/9906340])

Energy regimes of QCD

QCD $\Rightarrow$ interacting theory of quark and gluon fields

Is it a theory of quarks and gluons?

- at high energy: quark, gluons lower energies new concepts (pomeron, CGC; HTL, screened PT, 2PI...)
- near T_c: liquid – description?



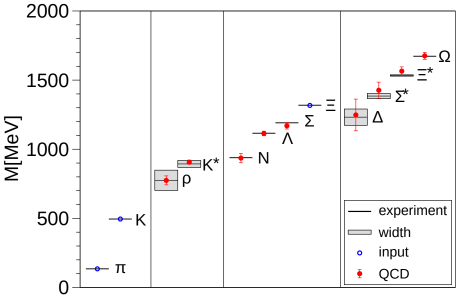
(J. P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. Lett. **83**, 2906 (1999) [arXiv:hep-ph/9906340])

Energy regimes of QCD

QCD $\Rightarrow$ interacting theory of quark and gluon fields

Is it a theory of quarks and gluons?

- at high energy: quark, gluons
  lower energies new concepts (pomeron, CGC; HTL, screened PT, 2PI...)
- near T_c: liquid – description?
- at low energy
  QCD $\equiv$ hadrons



(S. Durr *et al.*, Science **322**, 1224 (2008) [arXiv:0906.3599 [hep-lat]])

Energy regimes of QCD

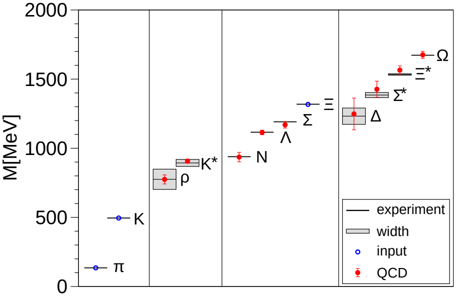
QCD ⇒ interacting theory of quark and gluon fields

Is it a theory of quarks and gluons?

- at high energy: quark, gluons
  lower energies new concepts (pomeron, CGC; HTL, screened PT, 2PI. . .)
- near T_c: liquid – description?
- at low energy
  QCD ≡ hadrons

⇓

description (physical picture) depends on the energy range



(S. Durr *et al.*, Science **322**, 1224 (2008) [arXiv:0906.3599 [hep-lat]])

Physical picture

- Perturbation theory: basic model & small interactions
- basic model must depend on the energy range (or, in general, on the environment)
- "physical picture" $\sim$ basic model

$$\Downarrow$$

To understand QCD we need effective models

Outlines

Symmetries

Symmetries of the effective model $\equiv$ symmetries of the fundamental model

eg. symmetries of (u,d,s) QCD:
$U(3) \times U(3) \rightarrow U_B(1) \times U_A(1) \times SU_V(3) \times SU_A(3)$
$\Rightarrow$ sigma models
(nonlinear σ-model, chiral PT, linear σ-models, chiral σ-models, $O(N)$ models etc.)

Spectrum

basic excitation spectrum must be close to the real spectrum

- otherwise: to correct the spectrum we need strong interactions is needed
  it seems nonperturbative, but we just use the false excitations.
- parameters of the basic model must be fitted to experiments

characterization of the spectrum

spectrum: energy levels belonging to certain Q numbers (eg. momentum).

measure the spectrum: A operator with fixed quantum numbers:

$$\varrho_A(x) = \langle 0|[A(x), A(0)]|0\rangle \quad \Rightarrow \quad \varrho_A(k)_{k_0>0} = \sum_n \alpha_{n,\mathbf{k}} \delta(k_0 - E_{n,\mathbf{k}})$$

spectral function wrt. A.

- projects out energy levels with the given Q numbers
- $\alpha_{n,\mathbf{k}} = 2\pi |\langle 0|A|n,\mathbf{k}\rangle|^2$
- normalization is not too important $\Rightarrow$ reflects the measurement of the spectrum
- for $V \to \infty$ discrete levels + continuum (if $m = 0$ excitations $\Rightarrow$ only continuum!)

Outlines

Outlines

The basic model

- at large energy: QCD weakly interacting
- elementary excitations: free quarks and gluons
  $\Rightarrow$ energy and momentum eigenstates with $E_{\mathbf{k}}^2 = \mathbf{k}^2 + m^2$ dispersion relation

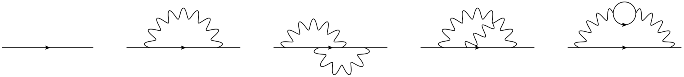
Basic model

Non-interacting, free particles (infinite lifetime)

Perturbation theory

Weak interaction: expand expectation values with respect of the coupling constant $\Rightarrow$ perturbation theory (PT)

- direct PT: Feynman-diagrams

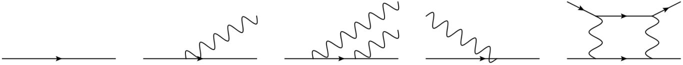


- divergences (UV and IR) $\Rightarrow$ renormalization, resummation (self-energy, RG, OPE, thermal masses, dimensional reduction, screened PT, HTL, 2PI, etc.)

Spectrum

Result of PT: states with the same quantum numbers mix

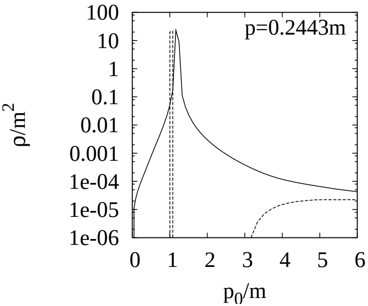
e.g. one-particle states mix with multi-particle states



multi-particle states have no "mass-shell"

(2-particle state with $\mathbf{k} = 0$ net momentum $E = E_{\mathbf{p}} + E_{\mathbf{k}-\mathbf{p}}$ possible $\forall \mathbf{p}$).

Typical spectrum



Φ^4 model, 2-loop renormalized 2PI resummation

$(T = 0, m, \lambda = 10)$

(AJ, PRD76 (2007) 125004 [hep-ph/0612268])

- $T = 0$: mass-shell shifts, multiparticle thresholds
- $T > 0$: mass-shell shifts and acquires width, $\varrho > 0$ everywhere

Quasiparticles

- supports quasiparticle approximation:
  like fundamental particles, but with modified mass and finite lifetime
- BUT no consistent PT can be built on this basic model:
  finite lifetime $\Rightarrow$ decaying particle $\Rightarrow$ violates E-conservation, unitarity
- we must keep all the energy levels $\Rightarrow$ 2PI approximation

2PI approximation

(2PI approximation: 2-particle irreducible)

Idea of 2PI

use the "exact" excitation spectrum for the quasiparticles

Consistent resummed PT: all energy levels are taken into account!
technically for scalar field theory we start from the form:

$$\mathcal{L} = \frac{1}{2}\Phi G^{-1}\Phi + \mathcal{L}_{int}$$

$\Rightarrow$ G comes from self-consistent propagator equation (2PI)

(J. M. Cornwall,R. Jackiw and E. Toumbolis, Phys. Rev. D10, 2428 (1974).)
(J. Berges and J. Cox, Phys. Lett. B 517 (2001) 369)

$$G^{-1}(p) = G_0^{-1}(p) - \Sigma[G](p)$$

and in the self-energy calculation we use the G propagator.

2PI approximation

(2PI approximation: 2-particle irreducible)

Idea of 2PI

use the "exact" excitation spectrum for the quasiparticles

Consistent resummed PT: all energy levels are taken into account!
technically for scalar field theory we start from the form:

$$\mathcal{L} = \frac{1}{2}\Phi G^{-1}\Phi + \mathcal{L}_{int}$$

$\Rightarrow$ G comes from self-consistent propagator equation (2PI)

(J. M. Cornwall,R. Jackiw and E. Toumbolis, Phys. Rev. D10, 2428 (1974).)
(J. Berges and J. Cox, Phys. Lett. B 517 (2001) 369)

$$G^{-1}(p) = G_0^{-1}(p) - \Sigma[G](p)$$

and in the self-energy calculation we use the G propagator.

basic model has a G^{-1} non-local kernel!

Consistency

- renormalizability ✓

  (H. van Hees, J. Knoll, PRD66 (2002) 025028)
  (A. Jakovac, Zs. Szep PRD71 (2005) 105001 [hep-ph/0405226])
  (A. Patkos, Zs. Szep, Nucl.Phys. A811 (2008) 329, [arXiv:0806.2554])

- unitarity: no missing state ✓
- global symmetries ✓
- local symmetries (gauge) ✗

  (U. Reinosa, J. Serreau, Ann.Phys. 325 (2010) 969, [arXiv:0906.2881])

- deep IR physics ✗

  (AJ., P. Mati, arXiv:1112.3476 [hep-ph])

Lessons

1. For representation of finite width we need non-local theory
2. 2PI framework treats non-local theories consistently

Outlines

QCD at low energy/temperature

- Strongly interacting, nonperturbative from the point of view of the quark-gluon picture
- observation: "weakly" interacting bound states (hadrons)

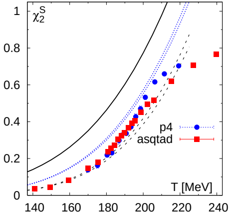
Basic model

non-interacting hadrons

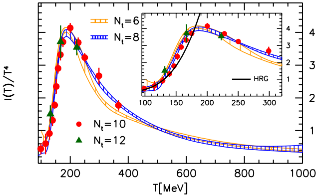
Taking into account all hadrons as stable particles
$\Rightarrow$ hadron resonance gas model (HRG)

Basic model

HRG (hadron resonance gas) – masses from the experiments



(P. Huovinen and P. Petreczky, Nucl. Phys. A **837**) (26 (2010) [arXiv:0912.2541 [hep-ph]].)



(Sz. Borsanyi, G. Endrodi, Z. Fodor, A.J., S. D. Katz) (S. Krieg, C. Ratti, K.K. Szabo, JHEP 1011 (2010) 077)

basic model works reasonably well for thermodynamics!

$\Rightarrow$ How shall we represent a realistic spectrum of bound states?

Problems

- Spectrum and symmetries: HRG introduces a lot of new conserved quantities! (the partcile numbers for different hadrons)
  $\Rightarrow$ changes the symmetries of the basic model

  does it matter?

- very short lifetime and “overlapping” hadronic states

  how shall we treat them?

  (J. Knoll, Yu.B. Ivanov and D.N. Voskresensky, Ann. of Phys. 293 (2001) 126)

Example

Example: 1 component free scalar model at high temperatures

$\mathcal{L} = \frac{1}{2}\Phi\mathcal{K}\Phi$ where $\mathcal{K} = -\partial^2 - m^2$

spectral function $\varrho(k) = 2\pi\mathrm{sgn}(k_0)\delta(k^2 - m^2)$

energy density: $\varepsilon = \frac{\pi^2}{30}T^4$ at high T

Take a 2-component representation!

$\mathcal{L} = \frac{1}{2}(\Phi_1\ \Phi_2)\begin{pmatrix}\mathcal{K} & 0\\ 0 & \mathcal{K}\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix}$

spectral function of $\Phi = \frac{\Phi_1+\Phi_2}{\sqrt{2}}$ is the same!

energy density: $\varepsilon = 2\times\frac{\pi^2}{30}T^4$ ✗: wrong with factor of 2

Take a non-independent 2-component representation!

$\mathcal{L} = \frac{1}{2}(\Phi_1\ \Phi_2)\begin{pmatrix}\mathcal{K} & \mathcal{K}\\ \mathcal{K} & \mathcal{K}\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \frac{1}{2}(\Phi_1+\Phi_2)\mathcal{K}(\Phi_1+\Phi_2)$

spectral function of $\Phi = \frac{\Phi_1+\Phi_2}{\sqrt{2}}$ is the same!

energy density: $\varepsilon = \frac{\pi^2}{30}T^4$ ✓

Lesson

Fields with the same quantum numbers may represent non-independent degrees of freedom!

Problematic also in QM, H gas

- if all excited states was independent, the H gas would be always ionized

  (Landau, Lifsitz; Peierls: Surprises in theoretical physics)

- ad hoc solution: highly excited states are too large, omit them

Overlapping peaks

- Scattering theory (Beth-Uhlenbeck formula): resonances give contribution to the free energy $\Rightarrow$ degrees of freedom

  (Landau, Lifsitz V.)

- well-separated peaks are independent

  (R.F Dashen, R. Rajaraman, PRD10 (1974), 694.)

- non-well separated peaks contribute to the S-matrix with complex amplitudes

  $\Rightarrow$ analytic: means relations between the amplitudes

  (M. Svec, PRD64 (2001) 096003 [hep-ph/0009275])

Lesson

Overlapping peaks of a spectral function represent non-independent degrees of freedom!

Outlines

1. Introduction
   - Effective models
   - Guiding principles to build effective theories
2. Effective models at different energy/temperature scales
   - The perturbative regime
   - The low energy/temperature regime
   - Treating resonances in field theory
3. Applications
   - Thermodynamics
   - QCD near the critical temperature
   - Black body radiation of a strongly interacting system
4. Conclusion

Nonlocal Lagrangian

Strategy

Represent a spectral function at fixed Q numbers with a single field.

(AJ. arXiv:1102.5629)

Similar to the 2PI resummation

$$\mathcal{L} = \frac{1}{2}\Phi(x)\mathcal{K}(i\partial)\Phi(x)$$

- relation of $\mathcal{K}$ kernel and ϱ spectral function:

  $$G_R(k_0,\mathbf{k}) = \mathcal{K}^{-1}(k_0+i\varepsilon,\mathbf{k}), \qquad \varrho = -2\,\mathrm{Im}\,G_R$$

  $$G_R(k_0,\mathbf{k}) = \int\frac{d\omega}{2\pi}\frac{\varrho(\omega,\mathbf{k})}{k_0-\omega+i\varepsilon}, \qquad \mathcal{K} = \mathrm{Re}\,G_R^{-1}$$

  spectrum completely determines phyisics!

Consistency

$$\mathcal{L} = \frac{1}{2}\Phi(x)\mathcal{K}(i\partial)\Phi(x)$$

- unitarity fulfilled if $\varrho(\omega > 0) \geq 0$ ✓
  physically: we take into account all possible states
  (J. Polonyi, A. Siwek, Phys. Rev. D81, 085040 (2010).)
- causality: x space-like vector
  $\langle[A(x), B(0)]\rangle = 0 \quad \Leftrightarrow \quad \varrho(x) = 0$
  Now ϱ is an input $\quad\Rightarrow\quad$ causality ✓
- energy and momentum conservation: consequence of the space and time translation symmetry ✓
- Lorentz-invariance: if kernel is Lorentz-invariant ✓

(similar to 2PI resummation case)

Consistency

We constructed a consistent (unitary, causal, E-conserving, Lorentz-invariant) non-local effective theory with correct symmetries!

$\Rightarrow$ This theory should be used to represent the (finite width) bound states.

Outlines

Outlines

Energy density

Construction

- time translation symmetry $\Rightarrow$ energy density (Noether-thm)
- finite temperature averaging (KMS relation)
- renormalization

$$\varepsilon = T_{00} = \int \frac{d^4p}{(2\pi)^4}\,\Theta(p_0)\left(p_0\frac{\partial\mathcal{K}}{\partial p_0} - \mathcal{K}\right) n(p_0)\varrho(p)$$

Consequences

- pressure, entropy, etc come from standard thermodynamics
- nonlinear functional of ϱ! (because $\varrho \Rightarrow \mathcal{K}$)
- rescaling invariant $\varrho \to Z\varrho$ yields the same energy density
  $\Rightarrow$ only the energy levels count, not the way we measure them!

Number of degrees of freedom

More instructive characterization of the system:
Number of the bound states?

- not evident in case of a general spectrum!
- consistency (for independent particles, or for one Breit-Wigner form)
- consistent with usual physical picture (Williams-Weizsacker)

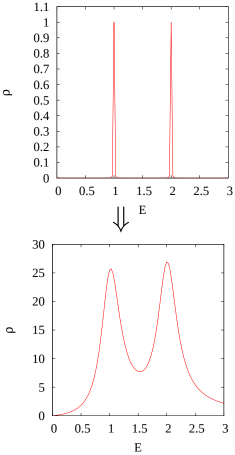
$$N_{dof} = \int_0^\infty \frac{dp_0}{2\pi}\frac{1}{p_0}\left(p_0\frac{\partial\mathcal{K}}{\partial p_0} - \mathcal{K}\right)\varrho(p).$$

Consequences

- for $\varrho(\omega) = \sum_{i=1}^{n} Z_i\delta(\omega - E_i) \quad \Rightarrow \quad N_{dof} = n$!
  independent of the normalization
- number of DoF is a dynamical quantity.

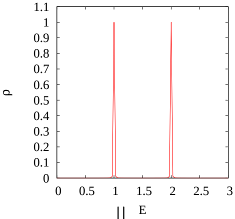
On the independence of the bound states

Change the width and compute the number of degrees of freedom!

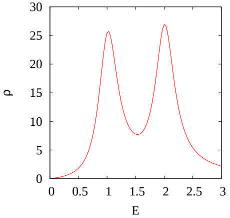


On the independence of the bound states

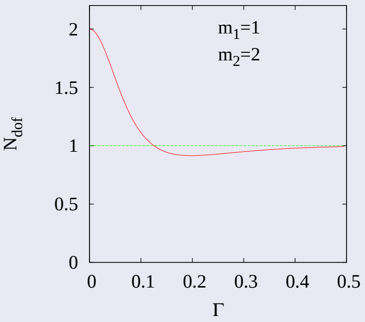
Change the width and compute the number of degrees of freedom!



⇓

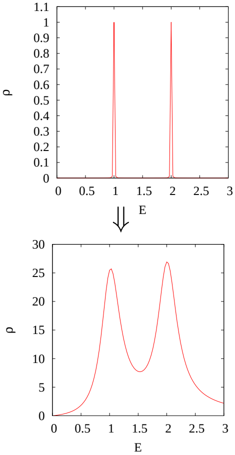


number of DoF

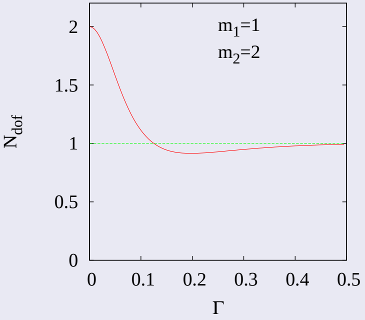


On the independence of the bound states

Change the width and compute the number of degrees of freedom!

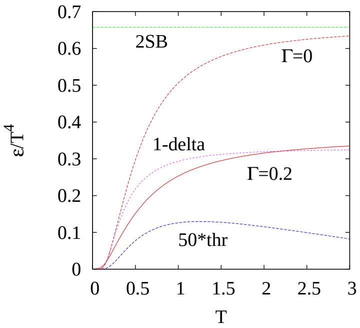


number of DoF



independence: separation is larger then width

Independece of bound states from thermodynamics



$m_1 = 1, \ m_2 = 2$

- $\Gamma = 0$: 2 Dirac-delta ($\Gamma_1 = \Gamma_2 = 0$)
- $\Gamma = 0.2$: finite width peaks
  $\Gamma_1 = \Gamma_2 = 0.2$: if we had only one particle!
  $\Rightarrow$ reduction of the number of degrees of freedom is observable in thermodynamics, too

Lowest curve: multiparticle threshold $\varrho(p) = \sqrt{1 - \frac{m^2}{p^2}}$

- negligible contribution to thermodynamics!
- overlapping Breit-Wigners $\Rightarrow$ destructive interference

Outlines

experimental evidence: liquid-like matter ("almost perfect liquid")

- more precisely: $\frac{\eta}{s} \sim \eta\ell^3 \sim \frac{1}{4\pi}$ small (on ℓ internal scale)

  $\Rightarrow$ very far from an ideal gas

- kinetic theory: $\frac{\eta}{s} \sim E\tau$ small

  $\Rightarrow$ very short lifetime excitations are needed (!?)
  (cf. jet suppression)

$\Rightarrow$ nonperturbative regime both from hadronic and quark-gluon side

how to treat it?

I. method: exactly solvable model

$\mathcal{N} = 4$ SYM theory with large N_c and $\lambda = g^2 N_c$

- CFT $\Rightarrow$ AdS/CFT duality $\Rightarrow$ 5D AdS gravitation $\Rightarrow$ computable
- indeed liquid: $\eta/s = 1/4\pi$ if $\lambda \to \infty$

  (P. Kovtun, D.T. Son, A.O. Starinets JHEP 0310, (2003) 064.)

  (A. Buchel, R.C. Myers, M.F. Paulos, A. Sinha, Phys.Lett.B669:364-370,2008.)

BUT: $\mathcal{N} = 4$ SYM $\neq$ QCD (symmetries, particle content)

- similar when we apply Φ^3 model instead QCD
- Hope: some universality is in the background, and so the details are not important

Universality?

How specific is QCD?

Generic fluid:

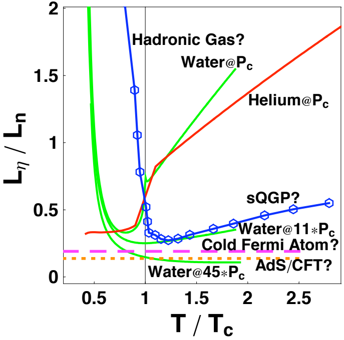
- fluidity measure $\frac{\eta}{s} \to \frac{L_\eta}{L_n}$
- smallest to supercritical fluids

Lesson:

- QCD not extraordinary
- behaviour near T_C

$\Downarrow$

supercritical fluids?



(J. Liao, V. Koch, Phys. Rev. C81, 014902 (2010))

II. method: nonlocal model

can we build a quadratic model which describes liquid?

$$\mathcal{L} = \frac{1}{2}\Phi(x)\mathcal{K}(i\partial)\Phi(x) \qquad \mathcal{K} \Leftrightarrow \varrho$$

- in the spectrum must be no sharp peaks $\Rightarrow$ they would lead to large free mean path, gas-like behaviour
- excitations are not particle-like "non-particles", "non-shell particles", "unparticles"

  (N.P. Landsman, Annals Phys. 186 (1988) 141)

  (H. Georgi, Phys. Rev. Lett. **98**, 221601 (2007). [hep-ph/0703260].)

Viscosity for broad spectral functions

One can calculate η/s for a generic spectral function

(AJ., PRD81 (2010) 045020 [arXiv:0911.3248])

generic structure:

$$\frac{\eta}{s} \sim \frac{\int f_1 \varrho^2}{\int f_2 \varrho + \ln \int f_3 \varrho} \xrightarrow{rescaling} \frac{\langle \varrho^2 \rangle}{\langle \varrho \rangle, \ln \langle \varrho \rangle}.$$

sum rule: $\int \varrho = 1$

- large peak in ϱ $\Rightarrow$ even larger peak in ϱ^2 $\Rightarrow$ η/s large
- shallow ϱ $\Rightarrow$ ϱ^2 even shallower $\Rightarrow$ η/s small

robust result: broad spectral function describes liquid!

is it the universality in the background. . . ?

- lower bound $\frac{\eta}{s} \geq \frac{s}{NLT^4}$

  N number of species, L “interaction length”

  no universal lower bound!

Outlines

Particle yields

In the plasma: distribution function $e^{-\beta E}$ $\Rightarrow$ what is the outgoing particle current?

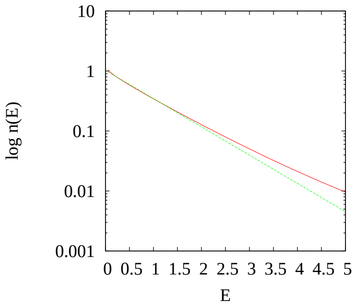
- quasiparticles in the plasma $\neq$ vacuum particles
- dressing ("hadronization"): assume some conserved quantity: energy and momentum! (works also with other assumptions)
- observed energy spectrum:

$$\omega_p n_{obs}(\omega_p) = \int_0^\infty \frac{dp_0}{2\pi} \left(p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K} \right) \varrho(p_0, p) n(p_0)$$

- if ϱ peaked near ω_p $\Rightarrow$ at small energies the peak region dominates
- at large energies peak suppressed by $n(p_0)$ exponentially $\Rightarrow$ small p_0 regime dominates $\Rightarrow$ off-shell effects

Példa

in case of a Breit-Wigner spectrum ($\Gamma = 0.1E$)



Prediction

- exponential behaviour at small energies
- power-law at large energies

(details depend on the form of the spectral function at small energies)

. . . work in progress. . .

Outlines

Conclusions

- description of quasiparticles is consistent only with taking into account the complete spectrum
  - gives nonlocal theory
  - unitary, causal, E-conserving
  - number of exciations is dynamical question
    $\Rightarrow$ independence of excitations, change in the number of excitations is possible to describe
- applications
  - quasiparticles in PT: 2PI method
  - description of bound states
  - description of liquids, transport coefficients
  - black body radiation, off-shell effects: lower-law at large energies