# A MODEL FOR SOFT INTERACTIONS MOTIVATED BY AdS/CFT AND QCD

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(work done with Genya Levin and Uri Maor) **Outline** 

- GLM model for high energy soft interactions incorporating multi eikonal scattering plus multi-Pomeron vertices.
- Comparison with competing models
- LHC Results Model Predictions Inclusive Distributions
- Summary

### **Introduction**

- Hard pQCD deals with high  $p_T$  partons. These are short distance interactions calculated perturbatively.
- Soft npQCD applies to low  $p_T$  partons, separated by large distances. Phenomological calculations are usually based on the Regge pole model, in which the Pomeron  $I\!\!P$  is the leading term.
- The total and elastic (but NOT diffractive) cross sections in the ISR-Tevatron range are well described by the DL model  $\alpha_{I\!\!P} \; = \; 1 \; + \; \Delta_{I\!\!P} \; + \; \alpha^{'}_{I\!\!P} t , \, \Delta_{I\!\!P} \; = \; 0.08, \; \; \; \alpha^{'}_{I\!\!P} \; = \; 0.25 \; GeV^2.$ For a super critical  $I\!\!P$  ( $\Delta_{I\!\!P} > 0$ ),  $\sigma_{el}$  grows much faster than  $\sigma_{tot}$  and will, eventually become larger! This paradox is solved by imposing a s-channel unitarity bound on  $\sigma_{el}$ .
- Enforcing unitarity is model dependent. For <sup>a</sup> scattering matrix, where the initial elastic re-scatterings insure s-channel unitarity  $2Im A_{el}(s, b) = |A_{el}(s, b)|^2 + G^{in}(s, b).$ This is just the statement that  $\sigma_{tot} = \sigma_{el} + \sigma_{in}$
- Its general solution is  $A_{el}(s, b) = i(1 e^{-\Omega(s, b)/2})$ so  $Im A_{el}(s, b) = \frac{\Omega}{2} - (\frac{\Omega^2}{8}) + ...$ which displays the multi- $I\!\!P$  corrections to the bare  $I\!\!P$   $(\frac{\Omega}{2})$ , that tames the power growth of the cross section with energy. and  $G^{in}(s, b) = (1 - e^{-\Omega(s,b)})$ .
- The opacity  $\Omega(s,b)$  is arbitrary, inducing a unitarity bound  $|A_{el}(s,b)|\ \le\ 2.$
- The total, elastic and inelastic cross sections are given by:

$$
\sigma_{tot} = 2 \int d^2b (1 - e^{-\Omega(s,b)/2})
$$
  
\n
$$
\sigma_{el} = 2 \int d^2b (1 - e^{-\Omega(s,b)/2})^2
$$
  
\n
$$
\sigma_{inel} = \sigma_{tot} - \sigma_{el} = \int d^2b (1 - e^{-\Omega(s,b)})
$$

- $\sigma_{el}$  is bounded by  $\frac{\sigma_{tot}}{2}$ .
- The Froissart-Martin bound

$$
\sigma_{tot} \ \leq \ C \ log^2(s/s_0) \ \text{where} \ C \ = \ \pi/(2m_\pi^2).
$$

The coefficient C is far too large to make this bound useful.

### Good-Walker Formalism

In our Introduction we have ignored two important items:

- Diffractive channels which are also due to  $I\!\!P$  exchange processes.
- $\bullet$  t-channel unitarity expressed through multi- $I\!\!P$  exchange.

The Good-Walker (G-W) formalism, considers the diffractively produced hadrons as <sup>a</sup> single hadronic state described by the wave function  $\Psi_D$ , which is orthonormal to the wave function  $\Psi_h$  of the incoming hadron (proton in the case of interest) i.e.  $\langle \Psi_h | \Psi_D \rangle = 0$ .

One introduces two wave functions  $\psi_1$  and  $\psi_2$  that diagonalize the 2x2 interaction matrix T

$$
A_{i,k} = \langle \psi_i \psi_k | \mathbf{T} | \psi_{i'} \psi_{k'} \rangle = A_{i,k} \, \delta_{i,i'} \, \delta_{k,k'}.
$$

In this representation the observed states are written in the form  $\psi_h = \alpha \psi_1 + \beta \psi_2$ ,

$$
\psi_D = -\beta \psi_1 + \alpha \psi_2
$$

where, 
$$
\alpha^2 + \beta^2 = 1
$$

Good-Walker Formalism-2

The s-channel Unitarity constraints for (i,k) are analogous to the single channel equation:

$$
Im\,A_{i,k}\left(s,b\right)=|A_{i,k}\left(s,b\right)|^{2}+G_{i,k}^{in}(s,b),
$$

 $G_{i,k}^{in}$  is the summed probability for all non-G-W inelastic processes, including non-G-W "high mass diffraction" induced by multi- $I\!\!P$  interactions. A simple

solution to the above equation is:

$$
A_{i,k}(s,b) = i\left(1 - \exp\left(-\frac{\Omega_{i,k}(s,b)}{2}\right)\right), G_{i,k}^{in}(s,b) = 1 - \exp(-\Omega_{i,k}(s,b)).
$$

The opacities  $\Omega_{i,k}$  are real, determined by the Born input.

Good-Walker Formalism-3

Amplitudes in two channel formalism are:

$$
a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2 \beta^2 A_{1,2} + \beta^4 A_{2,2}\},\,
$$

$$
a_{sd}(s,b) = i\alpha\beta\{-\alpha^2A_{1,1} + (\alpha^2 - \beta^2)A_{1,2} + \beta^2A_{2,2}\},\,
$$

$$
a_{dd}(s,b) = i\alpha^2 \beta^2 \{A_{1,1} - 2A_{1,2} + A_{2,2}\}.
$$

With the G-W mechanism  $\sigma_{el}$ ,  $\sigma_{sd}$  and  $\sigma_{dd}$  occur due to elastic scattering of  $\psi_1$  and  $\psi_2$ , the correct degrees of freedom.

Guiding criteria for GLM Model

- The model should be built using Pomerons and Reggeons.
- The intercept of the Pomeron should be relatively large. In AdS/CFT correspondence we expect  $\Delta_{I\!\!P} = \alpha_{I\!\!P}(0) - 1 = 1 - 2/\sqrt{\lambda} \approx 0.11 \div 0.33$ . The estimate for  $\lambda$  from the cross section for multiparticle production as well as from DIS at HERA is  $\lambda = 5 \div 9$ ;
- $\alpha'_{\mathbb{P}}(0) = 0;$
- <sup>A</sup> large Good-Walker component is expected, as in the AdS/CFT approach the main contribution to shadowing corrections comes from elastic scattering and diffractive production.
- The Pomeron self-interaction should be small (of the order of  $2/\sqrt{\lambda}$  in AdS/CFT correspondence), and much smaller than the vertex of interaction of the Pomeron with a hadron, which is of the order of  $\lambda$ ;
- The last requirement follows from the natural matching with perturbative QCD: where the only vertex that contributes is the triple Pomeron vertex.  $\lambda = 4\pi N_c \alpha_s^{YM}$



Pomeron diagrams that lead to <sup>a</sup> different source of the diffractive dissociation that cannot be described in the framework of the G-W mechanism. (a) is the simplest diagram that describes the process of diffraction in the region of large mass  $Y - Y_1 = \ln(M^2/s_0)$ . (b) and (c) are examples of more complicated diagrams in the region of large mass. The dashed line shows the cut Pomeron, which describes the production of hadrons.

### Example of enhanced and semi-enhanced diagram



Different contributions to the Pomeron Green's function a) examples of enhanced diagrams ; (occur in the renormalisation of the Pomeron propagator) b) examples of semi-enhanced diagrams (occur in the renormalisation of the  $I\!\!P$  -p vertex ) Multi-Pomeron interactions are crucial for the production of LARGE MASS DIFFRACTION

Tel Aviv approach for summing interacting Pomeron diagrams

In the spirit of LO pQCD we write a generating function

$$
Z(y, u) = \sum_{n} P_n(y) u^n,
$$

 $P_n(y)$  is the probability to find  $n\textrm{-}\mathsf{Pomerons}$  (dipoles) at rapidity  $y$ .

The solution, with boundry conditions, gives us the sum of enhanced diagrams.

For the function  $Z\left( u\right)$  the following evolution equation can be written

$$
-\frac{\partial Z(y,\,u)}{\partial\,y}\;=\;-\,\Gamma(1\to 2)\,u\,(1-u)\;\frac{\partial\,Z(y,\,u)}{\partial\,u}\;+\;\Gamma(2\to 1)\,u\,(1-u)\;\frac{\partial^2\,Z(y,\,u)}{\partial^2\,u},
$$

This is no more than the Fokker-Planck diffusion equation

 $\Gamma(1\to 2)$  describes the decay of one Pomeron (dipole) into two Pomerons (dipoles) while  $\Gamma(2\to 1)$  relates to the merging of two Pomerons (dipoles) into one Pomeron (dipole). Tel Aviv approach for summing interacting Pomeron diagrams contd.

Using the functional  $Z$ , we find the scattering amplitude, using the following formula:

$$
N(Y) \equiv \operatorname{Im} A_{el}(Y) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n Z(y, u)}{\partial^n u} |_{u=1} \gamma_n(Y = Y_0, b),
$$

 $\gamma_n(Y=Y_0,b)$  is the scattering amplitude of  $n$ -partons (dipoles) at low energy.

Using the MPSI approximation (where only large  ${I\!\!P}$  loops of rapidity size  $\mathsf{O}(Y)$  contribute) we obtain the exact Pomeron Green's function

$$
G_F\left(Y\right) \;\;=\;\; 1 \;\; - \;\exp\left(\frac{1}{T(Y)}\right)\;\frac{1}{T(Y)}\; \Gamma\left(0,\frac{1}{T(Y)}\right),
$$

 $\Gamma(0, x)$  is the incomplete gamma function and

$$
T(Y) = \gamma e^{\Delta p Y}.
$$

 $\gamma$  is the amplitude of the parton (colorless dipole) interaction with the target at arbitrary Y. MPSI approximation is valid only for  $Y\leq \frac{1}{\gamma}.$ 

### Full set of diagrams that need to be summed



A) shows the sum of enhanced diagrams in two channel approach,

B) shows the full set of the diagrams which in C) is pictured in the way that is most suitable to illustrate the MPSI approach.

The bold wave line stands for the exact Pomeron Green's function that includes all enhanced diagrams. The summed amplitude has the form:

$$
A_{i,k}\left(Y;b\right) \;\; = \;\; 1 \;\; - \;\exp\left\{-\;\frac{1}{2} \,\int d^2 b' \;\; \frac{\left(\,\tilde{g}_i\left(\vec{b}'\right)\,\,\tilde{g}_k\left(\vec{b}-\vec{b}'\right)\,T(Y)\right)}{1\; + \;T(Y)\,\left[\tilde{g}_i\left(\vec{b}'\right)+\tilde{g}_k\left(\vec{b}-\vec{b}'\right)\right]}\right\}
$$

Parameters for our model fit includes G-W

 $+$  enhanced  $+$  semi-enhanced Pomeron diagrams



- $\bullet$   $g_1(b)$  and  $g_2(b)$  describe the vertices of interaction of the Pomeron with state 1 and state 2
- $\bullet\,$  The Pomeron intercept is  $\Delta_{I\!\!P}(0)$   $1$
- $\bullet~~\gamma$  denotes the low energy amplitude of the dipole-target interaction
- $\tilde{g}_i = g_i / \sqrt{\gamma}$
- $\bullet$  For  $\tilde{g}_i(b)$ , we use the phenomenological assumption  $\tilde{g}_i(b)$   $\;=\; \tilde{g}_i\,S(b) \;=\;$  $\tilde{g}_i$  $\frac{\tilde{g}_{i}}{4\pi}$   $m^{3}_{i}$   $b$   $K_{1}$   $(m_{i}$   $b)$   $,$ where  $S(b)$  is the Fourier transform of the dipole formula for the form factor  $1/(1+q^2/m_i^2)$  $\binom{2}{i}$ <sup>2</sup>.

## Energy dependence of GLM1 cross sections



### GLM2 Results



Block and Halzen (PRL 107,212002) have "converted" <sup>a</sup> recent measurement of the Pierre Auger Observatory collaboration of  $\sigma_{inel}^{p-air}$  at W =57  $\pm$  6 TeV to  $\sigma_{inel}$  for p-p and obtain a value of  $\sigma_{inel} = 90 \pm 7$ (stat)  $\pm 1.5$ (Glauber) +9/-11(syst) mb and predict  $\sigma_{tot} = 134.8 \pm 1.5$  mb

### Comparison of results obtained in GLM, Ostapchenko, K-P and KMR models

Ostapchenko (Phys.Rev.D81,114028(2010)) has made <sup>a</sup> comprehensive calculation in the framework of Reggeon Field Theory based on the resummation of both enhanced and semi-enhanced Pomeron diagrams.

To fit the total and diffractive cross sections he assumes TWO POMERONS: "SOFT  $\textrm{POMERON}^{\prime\prime} \quad \alpha^{Soft}=1.14\,+\,0.14t$  "HARD  $\textrm{POMERON}^{\prime\prime} \quad \alpha^{Hard}=1.31\,+\,0.085t$ 

The Durham Group (Khoze, Martin and Ryskin), have <sup>a</sup> model which is similar in spirit to GLM, the main difference lies in the technique of summing the "Pomeron loop" diagrams. KMR couplings are  $g_n^n$  $\frac{n}{m} = \frac{1}{2}$  $\frac{1}{2}g_Nnm\lambda^{n+m-2}=\frac{1}{2}nmG_3{_{\rm I\!P}}\lambda^{n+m-3}$  $\lambda$  is a free parameter,  $n + m > 2$ ,  $G_{3IP} = \lambda g_N$ . Over the years they have improved their model, the latest version (EPJ C71, 1617(2011) includes  $k_t$  evolution.

Kaidalov-Poghosyan have <sup>a</sup> model which is based on Reggeon calculus, they attempt to describe data on soft diffraction taking into account all possible non-enhanced absorptive corrections to 3 Reggeon vetices and loop diagrams. However, it is a single  $I\!\!P$  model and with secondary Regge poles, they have  $\Delta_{I\!\!P}~=~0.12$  and  $\alpha^{'}_{I\!\!P}~=~0.22$ .

### Comparison of results continued



Donnachie and Landshoff in <sup>a</sup> recent preprint [arXiv:1112.2485], have attempted to fit the new TOTEM results at  $W = 7$  TeV

They find that they have to introduce an additional Hard  $I\!\!P$  with  $\alpha_{I\!\!P}^{hard} = 1.2$ 

$$
\qquad \qquad \text{and } \alpha_{I\!\!P}^{hard'} \, = \, 0.1 GeV^{-2} \qquad \qquad \text{and } \mathsf{L} \text{ predict } \sigma_{tot}^{D+L} \, = \, 113 \pm 5 \text{ mb at W = }14 \text{ TeV}
$$

With no screening mechanism included, at energy of W  $=$  57 TeV,  $\sigma_{tot}^{D+L}$  $_{tot}^{D+L}$   $\,\approx 500$  mb.



# LHC Results

 $\sigma_{inel}$  at  $W = 7$  TeV

```
ATLAS: 69.4 \pm 2.4 (exper) + 6.9 (extrap) mb
```
CMS:  $68.0 \pm 2.0$  (syst)  $\pm 2.4$  (lumi) + 4 (extrap) mb

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TOTEM: 73.5 \pm 0.6 (stat) +1.8/-1.3(syst) mb
```
ALICE:  $72.7 + 1.1 + 5.1$  mb

 $\sigma_{tot}$  at  $W = 7$  TeV

TOTEM:  $98.3 \pm 0.2$  (stat)  $\pm 2.7$ (syst) + 0.8/-0.2 (from  $\rho$ ) mb



#### MODEL RESULTS AT 7 TEV

Experimental Results for Cross Sections

# **Total Inelastic pp Cross Section**

- Full current picture on total cross section (from TOTEM) - ATLAS and CMS central values lower than TOTEM after extrap'n into region of very low  $\xi$  (extrapolation error is dominant)





A. Experimental Results for Inclusive Production

The three experimental groups ALICE, CMS and ATLAS have slight differences in the presentation of their results for for psuedo-rapidity distributions at the LHC.

$$
\sigma_{tot} = \sigma_{ND} + \sigma_{el} + \sigma_{SD} + \sigma_{DD} = \sigma_{el} + \sigma_{inel}
$$

ATLAS give results for  $\sigma_{ND}$ 

CMS display

$$
\sigma_{NSD} = \sigma_{ND} + \sigma_{DD} = \sigma_{tot} - \sigma_{el} - \sigma_{SD}
$$

 $\sigma_{inel}=\sigma_{ND}+\sigma_{SD}+\sigma_{DD}$ 

ALICE also present  $\sigma_{NSD}$  for W  ${=}0.9$  and 2.36 TeV, however, for W  ${=}$  7 TeV they impose an additional constraint requiring at least one charged particle in the interval  $\mid \eta \mid < 1$   $(inel > 0_{\mid \eta \mid < 1}).$ 

### Single inclusive cross section 1

We expand our approach to describe rapidity distributions at high energies e.g. the single inclusive cross section.

#### **Assumptions**

- $\alpha'_{\mathcal{P}}=0.$
- Only the triple Pomeron vertex is included to describe the interaction of the soft Pomerons.
- The single inclusive cross section in the framework of the Pomeron calculus can be calculated using Mueller diagrams shown.
- $a_{PP}$  denotes the emission of hadrons from the Pomeron:  $a_{PR}$  denotes the emission of hadrons from Reggeons.



### Single Inclusive cross section 2

They lead to the following expression for the single inclusive cross section

$$
\frac{1}{\sigma_{NSD}} \frac{d\sigma}{dy} = \frac{1}{\sigma_{NSD}(Y)} \left\{ a_{FP} \left( \int d^2b \left( \alpha^2 G_1(b, Y/2 - y) + \beta^2 G_2(b, Y/2 - y) \right) \right) \right\}
$$
  
\n
$$
\times \int d^2b \left( \alpha^2 G_1(b, Y/2 + y) + \beta^2 G_2(b, Y/2 + y) \right)
$$
  
\n
$$
- a_{FP} \left( \alpha^2 g_1^R + \beta^2 g_2^R \right) \left[ \alpha^2 \int d^2b \left( \alpha^2 G_1(b, Y/2 - y) + \beta^2 G_2(b, Y/2 - y) \right) e^{\Delta} e^{(Y/2+y)} \right]
$$
  
\n
$$
+ \int d^2b \left( \alpha^2 G_1(b, Y/2 + y) + \beta^2 G_2(b, Y/2 + y) \right) e^{\Delta} e^{(Y/2-y)} \right] \Big\}.
$$
  
\n
$$
G_i(b, Y) \text{ denotes the sum of 'fan' diagrams}
$$
  
\n
$$
G_i(b, Y) = (g_i(b) / \gamma) G_{enh}(y) / (1 + (G_{3FP}/\gamma) g_i(b) G_{enh}(y)),
$$
  
\nwhere the Green's function of the Pomeron, obtained by the summation of the enhanced

 $G_{enh}\left(Y\right) \;=\; 1 \,-\, \exp\left(\frac{1}{T\left(Y\right)}\right)\, \frac{1}{T\left(Y\right)}\, \Gamma\left(0,\frac{1}{T\left(Y\right)}\right) \,. \;\;\; T\left(Y\right) \;=\; \gamma e^{\Delta_{I\!\!P}\,Y}$ 

diagrams, is equal to

Single Inclusive cross section 3

- We need to introduce two new phenomenological parameters:  $a_{I\!\!P I\!\!P}$  and  $a_{I\!\!R I\!\!P}$  to describe the emission of hadrons from the Pomeron and the Reggeon.
- $\bullet~$  As well as two dimensional parameters  $Q$  and  $Q_0, \, Q$  is the average transverse momentum of produced minijets, and  $\frac{Q_0}{2}$  denotes the mass of the slowest hadron produced in the decay of the minijet.
- $\bullet$  We extract the three new parameters:  $a_{I\!\!P I\!\!P}, a_{I\!\!P I\!\!R}$  and  $Q_0/Q$  from the experimental inclusive data.
- The ratio  $Q_0/Q$  determines the shape of the inclusive spectra.
- We made two separate fits:
	- (a) fitting only the CMS data at different LHC energies and
	- (b) fitting all inclusive data for  $W \geq 546 \, GeV$ .



### Results of GLM for single inclusive cross section



The single inclusive density versus energy.

The data were taken from ALICE,CMS, and ATLAS Collaborations and from PDG. The fit to the CMS data is plotted in (a), while (b) presents the description of all inclusive spectra with  $W \geq 546 \, GeV$  .

# **Summary**

- We present a model for soft interactions having two components: (i) G-W mechanism for elastic and low mass diffractive scattering. (ii) Pomeron enhanced contributions for high mass diffractive production.
- Enhanced IP diagrams, make important contributions to both  $\sigma_{sd}$  and  $\sigma_{dd}$ .
- Diffractive processes are important, at Tevatron energies they constitute more than 20% of the  $\sigma_{tot}$ .

Consequently, single channel models that attempt to describe "soft" scattering, are neglecting an essesential feature of the process.

- Monto Carlo generators which were successful in describing data for  $W \le 1.8$  TeV need to be RETUNED to describe LHC data.
- GLM find their original fit determined from data for  $W \le 1.8$  TeV underestimates LHC data, need a new fit GLM2 (which also includes measurements at  $W = 7$  TeV) to be successful.

## BACKUPS -







Energy dependence of the inelastic  $p - p(\bar{p}p)$  cross section (left) and of midrapidity charged hadron multiplicity density (right).

### RESULTS 3 CMS 2

#### CMS Collaboration JHEP (2010)041



Average value of  $dN_{ch}/d\eta$  in the central  $\eta$  region as a function of the c.m. energies in pp and  $\bar{p}$ p collisions

# Comparison of cross sections obtained in GLM, Phythia6(MC09) and Phojet

and Experimental Data

