Observables in Anisotropic Plasma in AdS/CFT

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Outline

AdS/CFT correspondence

- The AdS/CFT correspondence, in the original and best understood form, is a duality between the $\mathcal{N} = 4$ supersymmetric Yang-Mills and type IIB superstring theory on $AdS_5 \times S^5$.
- In this correspondence there exist a map between gauge invariant operators in field theory and states in string theory.
- Example:The Wilson loop, is a physical gauge invariant object and can measure the interaction potential between the external quarks and acts as an order of confinement.
- The Wilson loop operator in the fundamental representation is dual to a string worldsheet extending in the $AdS_5 \times S^5$ with boundary the actual loop placed on the AdS boundary. *(Maldacena; Rey, Yee)*

$$< W[C] >= e^{-S_{string[C]}}$$

- Lot of effort to find more realistic gauge/gravity dualities. For example:
- Less Supersymmetry.
- Broken conformal symmetry, confinement.
- Finite temperature.
- Inclusion of dynamical quarks.
- Inclusion of Anisotropy(for our purposes)

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Motivation

- The rapid expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Properties of the supergravity solutions, that are dual to the anisotropic plasmas.
- There exist several results for observables in weakly coupled anisotropic plasmas. Do their predictions carry on in the strongly coupled limit?
- The main question we answer accurately here is: How the inclusion of anisotropy modifies the results on several observables in our dual QGP compared to the isotropic theory?

The anisotropic background

The metric in string frame *(Mateos, Trancanelli, 2011)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\, d\Omega_{S^5}^2 \, .$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy. The anisotropic parameter is α with units of inverse length. In sufficiently high temperatures, $T \gg \alpha$, and imposed boundary conditions the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

The isotropic limit $\alpha \to 0$ reproduce the well know result of the isotropic D3-brane solution (dual to $\mathcal{N} = 4$ finite sYM solution).

The temperature is given by

$$T = -\frac{\partial_u \mathcal{F}\sqrt{\mathcal{B}}}{4\pi}\Big|_{u=u_h} = \frac{1}{\pi u_h} + \alpha^2 \frac{5\log 2 - 2}{48\pi} u_h$$

The metric can be expressed in α, T parameters through

$$u_h = \frac{1}{\pi T} + \alpha^2 \frac{5log2 - 2}{48\pi^3 T^3} \ .$$

The energy and pressures can be found from the expectation value of the stress tensor, where the elements $\langle T_{11}\rangle = \langle T_{22}\rangle = P_{x_1x_2}$ denote the pressure along the x_1 and x_2 directions and $\langle T_{33}\rangle = P_{x_3}$ is the pressure along the anisotropic direction. The analytic expression read

$$\begin{aligned} P_{x_1x_2} &= \frac{\pi^2 N_c^2 T^4}{8} + \alpha^2 \frac{N_c^2 T^2}{32}. \\ P_{x_3} &= \frac{\pi^2 N_c^2 T^4}{8} - \alpha^2 \frac{N_c^2 T^2}{32}. \end{aligned}$$

$$P_{x_3} < P_{x_1x_2}$$

Static potential

- The static potential measures the interaction between two heavy quarks. The $SU(N_c)$ pure gauge theory at zero temperature has a static potential of the form

$$E = \text{const.} + \sigma L + \frac{c_1^{\text{gauge}}}{L} + \cdots$$

- The constant is non-physical and one can get rid of it by considering the static force.
- The string tension σ for small N_c, can be computed in lattice Monte Carlo simulations as the slope of the static potential, as a function of the static quark separation L.

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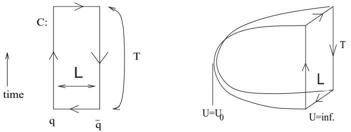
- The Coulomb like term (Lüscher term), is of special importance because its coefficient is dimensionless. Towards the IR, it stops running roughly around a particular scale, the Sommer scale and takes a constant value predicted and successfully confirmed in a Monte Carlo simulation to be

$$c_1^{\mathrm{gauge}} = -(d-2)\pi/24$$

  in d-dimensions. (Lüscher, Symanzik, Weisz)
  This result can be also reproduced using the gauge/gravity duality. (Kinat, Schreiber, Sonnenschein, Weiss, 1999; Aharony, Karzbrun, Field,2009...)
- Screening of the static force when backreacted flavor branes are taken into account in AdS/CFT has been observed. (Giataganas, Irges, 2011)

Static Potential in AdS/CFT

The static potential can be measured by introducing two infinitely heavy probe quarks on the boundary of the space. This corresponds to a Wilson loop of the following shape:



(pic taken from 0712.0689)

The normalized expectation value of the Wilson loop which involves the minimal surface of the particular world-sheet minus the infinite quark mass is

$$W[C] \sim e^{-(S - mass_Q)} \sim e^{-V_{Q\bar{Q}} T}$$

Static Potential in the anisotropic background

- We consider a string world-sheet (τ, σ) of the following form.

$$x_0 = \tau, \qquad x_p = \sigma, \qquad u = u(\sigma) \ .$$

The x_p is the direction where the pair is aligned:
$x_p = x_2 =: x_\perp$ pair along transverse direction,
$x_p = x_3 =: x_\parallel$ pair along parallel direction to anisotropy.

The solution to Nambu-Goto action is a catenary shape w-s with u_0 being the turning point.
We can work in full generality by renaming for example the anisotropic metric as

$$ds^2 = g_{00} dx_0^2 + \sum g_{ii} dx_i^2 + g_{pp} dx_p^2 + g_{uu} du^2 + \text{internal space}$$

To find the static potential we need to derive from the eoms of the NG action the length L of the Wilson loop. Then express the minimal surface ($\sim$static potential) in terms of L. The process is not always doable analytically. In general the length of the two endpoints of the string on the boundary is given by

$$L = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu} c_0^2}{(g_{00} g_{pp} + c_0^2) g_{pp}}} \;.$$

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$2\pi\alpha' V \;\; = \;\; c_0 L + 2\left[\int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}} \left(\sqrt{1 + \frac{c_0^2}{g_{pp} g_{00}}} - 1\right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}}\right] \;.$$

(Sonnenschein..)

This does not always implies that exist a term linear in L, since eventually c_0 (the Hamiltonian) is a function of L.

Therefore we can always at least numerically find the $V(L)$ expression for any background. In the anisotropic case we get:

• $V_{\parallel} < V_{\perp} < V_{iso}$ when the comparison is done with LT keeping α, T fixed.

• $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2}$. Increase of anisotropy, leads to decrease of the static potential.

• The critical length of the string beyond the quarks are not bounded is decreased in presence of anisotropy as $L_{c\parallel} < L_{c\perp} < L_{c\ iso}$.

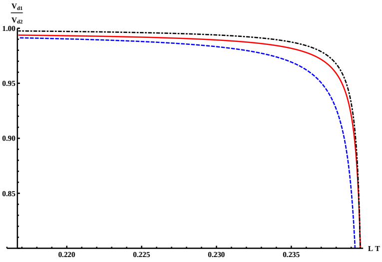


Figure: $V_{\parallel}/V_{\perp}$, $V_{\parallel}/V_{iso}$, $V_{\perp}/V_{iso}$ vs LT and $T = 3$, $\alpha = 0.35T$.

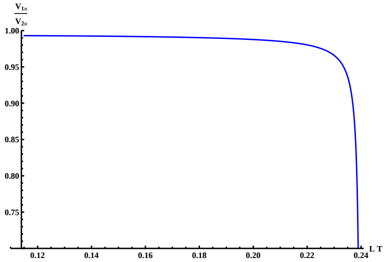


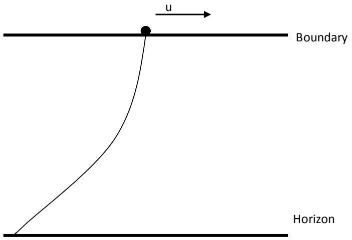
Figure: $V_{\parallel_1}/V_{\parallel_2}$. $\alpha_1 = 0.01T, \alpha_2 = 0.5T$ and $T = 3$.

Drag Force

In AdS/CFT the drag force of a single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. (Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser, 2006)

In radial gauge the trailing string motion along the $x_p := x_{\parallel,\perp}$ directions described by:

$$x_0 = \tau, \qquad u = \sigma, \qquad x_p = v\tau + \xi(u)$$



By solving the Nambu-Goto equations and after some algebra the drag force can be found for any background to be

$$F_d = -\Pi_u^1 = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0}$$

where here u_0 is given by

$$\left(g_{uu}(g_{00} + g_{pp}v^2)\right)\ |_{u=u_0} = 0\ .$$

We substitute the metric elements of our background and we find the analytic expressions which have the form

$$F_{aniso} = F_{iso} + \alpha^2 f(v)$$

They lead to

- $F_{\parallel} > F_{iso}$
- $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.
- $$\frac{F_{\parallel}}{F_{\perp}} = 1 + \alpha^2 \frac{(2 - v^2)\,\text{Log}\left[1 + \sqrt{1 - v^2}\right]}{8\pi^2 T^2 (1 - v^2)} .$$

For any velocity: $F_{drag,\parallel} > F_{drag,\perp}$

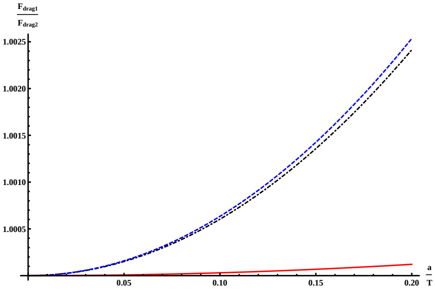


Figure: $F_{drag,\parallel}/F_{drag,\perp}$, $F_{drag,\parallel}/F_{drag,iso}$, $F_{drag,\perp}/F_{drag,iso}$, vs α/T, $v \simeq 0.98$ and $T = 1$.

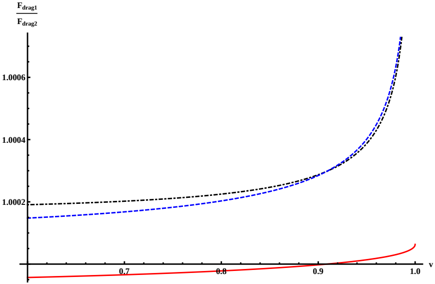


Figure: ... vs v, $\alpha = 0.1$ and $T = 1$.

Diffusion time

The drag coefficient is defined as

$$\frac{dp}{dt} = -n_D p, \text{ with } p = \frac{M_q v}{\sqrt{1-v^2}} \ .$$

Therefore the diffusion time τ_D is given by:

$$\tau_{D,\parallel,\perp} = \frac{1}{n_{D,\parallel,\perp}} = -\frac{1}{F_{drag,\parallel,\perp}} \frac{M_q v}{\sqrt{1-v^2}} \ ,$$

Relations between the diffusion times in different directions are inverse to the drag force ones.
For example for $v > v_c$, $\tau_{D,\parallel} < \tau_{D,\perp} < \tau_{D,iso}$ and

$$\frac{\tau_{D,\parallel}}{\tau_{D,\perp}} = 1 - \alpha^2 \frac{\left(2-v^2\right) \text{Log}\left[1+\sqrt{1-v^2}\right]}{8\pi^2 T^2 \left(1-v^2\right)}$$

The jet Quenching

In the gravity dual description the jet quenching can be calculated from the minimal surface of a world-sheet which ends on an orthogonal Wilson loop lying along the light-like lines. Two parallel lines of the Wilson loop, with length say L_- related to the partons moving at relativistic velocities are taken to be much more larger that the other two sides with length $L_\perp$ related to the transverse momentum of the radiated gluons.

$$\langle W(\mathcal{C})\rangle = \exp^{-\frac{1}{4\sqrt{2}}\hat{q}L_\perp^2 L_-}$$

(Liu,Rajagopal,Wiedermann,2006)

- We manage again to do the calculation in a generic background.

To calculate the corresponding Wilson loop we go to the light-cone coordinates as $\sqrt{2}x^{\pm} = x_0 \pm x_p$ where $i, p, k = 1, 2, 3$ A generic metric becomes

$$ds^2 = g_{--}(dx_+^2 + dx_-^2) + g_{+-}(dx_+ dx_-) + g_{ii(i\neq p)} dx_i^2 + g_{uu} du^2$$

$$g_{--} = \frac{1}{2}(g_{00} + g_{pp}), \qquad g_{+-} = g_{00} - g_{pp}$$

The ansatz that describes the string configuration and solves the eom is

$$x_- = \tau, \quad x_k = \sigma, \quad u = u(\sigma)$$

$$x_+, \; x_{p\neq k} \quad \text{are constant} \; ,$$

which represents a Wilson loop extending along the x_k direction and lying at a constant $x_+, x_{i\neq k}$. The index k here denotes a chosen direction.

$\hat{q}$	x_p	x_k	Energetic parton along	Momentum broadening along
$\hat{q}_{\perp(\parallel)}$	$x_\perp$	$x_\parallel$	$x_\perp$	$x_\parallel$
$\hat{q}_{\parallel(\perp)}$	$x_\parallel$	$x_\perp$	$x_\parallel$	$x_\perp$
$\hat{q}_{pl}$	$x_{\perp,1}$	$x_{\perp,2}$	$x_{\perp,1}$	$x_{\perp,2}$

After calculating the on-shell action, canceling the divergences and applying the approximations we obtain

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi\alpha'}\left(\int_0^{u_h}\frac{1}{g_{kk}}\sqrt{\frac{g_{uu}}{g_{--}}}\right)^{-1}.$$

Applying the results to our background we obtain:

- $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{pl} > \hat{q}_{iso}$.

$\hat{q}$(q motion parallel to anisotropy, broadening along transverse)> $\hat{q}$(q motion transverse to anisotropy, broadening along parallel)> $\hat{q}$(q motion transverse to anisotropy, broadening along transverse)

- $\frac{\hat{q}_{\parallel(\perp)}}{\hat{q}_{iso}} \simeq 1 + 0.122\,\frac{\alpha}{T}$

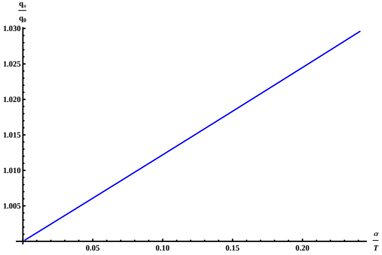


Figure: $\hat{q}_{\parallel(\perp)}/\hat{q}_{iso}$ vs α/T. $T = 5$.

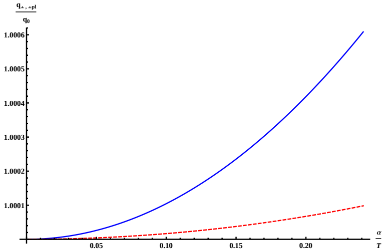


Figure: $\hat{q}_{\perp}$, $\hat{q}_{pl}$ vs α/T. $T = 5$.

Anisotropic momentum distribution function

The anisotropic distribution function that can be written as

$$f_{aniso} = c_{norm}(\xi) f_{iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

where (Romatschke, Strickland, 2003)

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

and **n** the unit vector along the anisotropic direction. To relate ξ and α define

$$\Delta := \frac{P_T}{P_L} - 1 = \frac{P_{x_1 x_2}}{P_{x_3}} - 1 \ .$$

Using the anisotropic distribution function: (Martinez, Strickland, 2009)

$$\Delta = \frac{1}{2}(\xi - 3) + \xi\left((1+\xi)\frac{\arctan\sqrt{\xi}}{\sqrt{\xi}} - 1\right)^{-1}$$

Using the supergravity model

$$\Delta = \frac{\alpha^2}{2\pi^2 T^2} \ .$$

Then

$$T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \backsimeq \frac{5\alpha^2}{8\pi^2 T^2} ,$$

Supposing we trust the estimation of the anisotropic parameter $\xi \simeq 1$. Using any comparison normalization scheme(direct or fixed energy density scheme)

$$\xi_{aSYM} \gtrsim \xi .$$

Therefore, in that case we can not make a more 'quantitative' comparison using our model in the particular limit $T \gg \alpha$. We can do it only if the values of $\xi \ll 1$.
But we have found the qualitative behavior on how the observables behave in the strong coupling in presence of anisotropy.

Conclusions-Partial List of Results

We have calculated several observables using a IIB supergravity solution in the dual anisotropic finite temperature $\mathcal{N} = 4$ sYM plasma.

- The static potential:
  •$V_{\parallel} < V_{\perp} < V_{iso}$.
  • $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2}$.
  • In weak coupling has been observed increase of the static potential but the models have many differences, that can affect the potential significantly. (Dumitru, Guo, Strickland, 2007).
- The drag Force:
  • $F_{\parallel} > F_{iso}$ and $F_{\parallel} > F_{\perp}$.
  • $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.
- The jet quenching:
  • $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{pl} > \hat{q}_{iso}$.
  • In weak coupling has been observed enhancement of the jet quenching in agreement with our results. (Dumitru, Nara, Schenke, Strickland; Baier, Mehtar-Tani, 2008,..).