



# Towards Thermalization in Heavy Ion Collisions

Stellenbosch, South Africa, February 2012

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Initial state

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Summary

François Gelis  
IPHT, Saclay



## ① Initial state factorization

## ② Final state evolution

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## In collaboration with :

K. Dusling

(NCSU)

T. Epelbaum

(IPhT)

R. Venugopalan

(BNL)

# Introduction

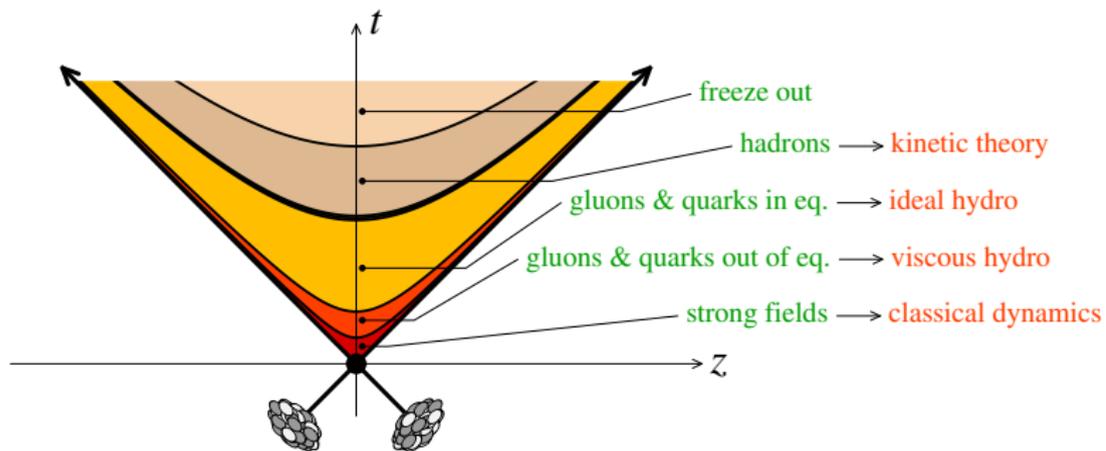
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# Stages of a collision



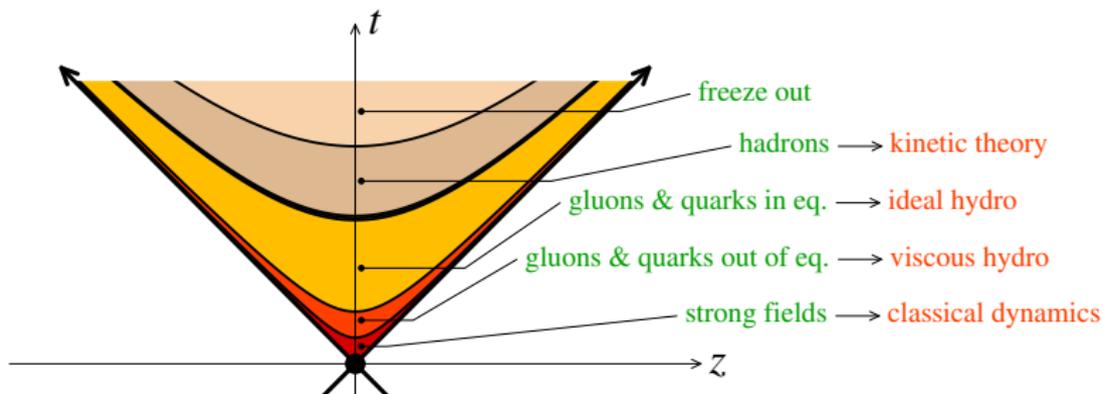
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## Stages of a collision



**This talk : evolution up to times  $\sim 1$  fm/c**

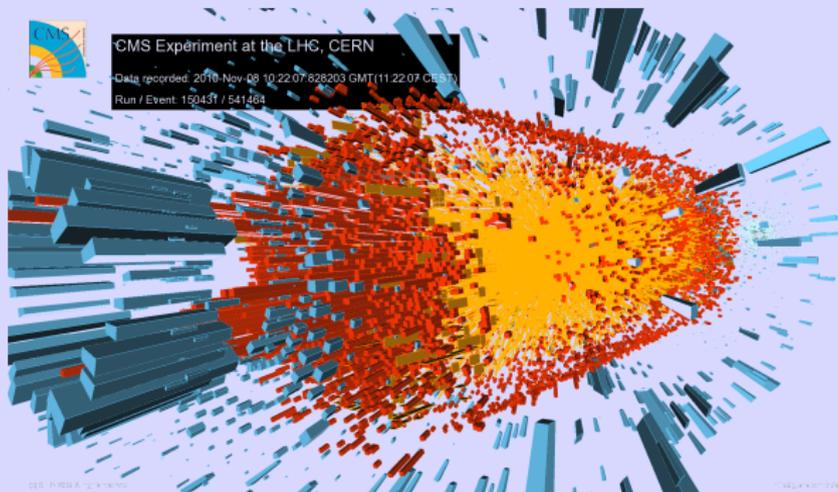
- i.** Partonic content of high energy nuclei
- ii.** Gluon production in the collision
- iii.** Evolution shortly after the collision, Thermalization

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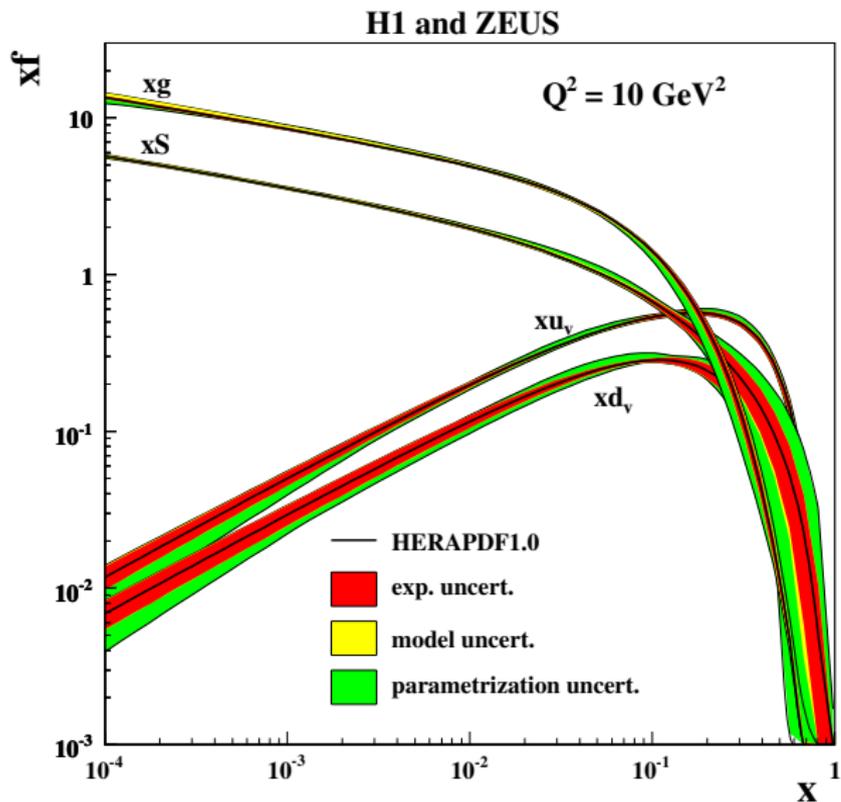
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- Low typical final state transverse momentum  $p_{\perp} \lesssim 1 \text{ GeV}$
- Incoming partons have low momentum fractions  $x \sim p_{\perp}/E$ 
  - $x \sim 10^{-2}$  at RHIC ( $E = 200 \text{ GeV}$ )
  - $x \sim 4 \cdot 10^{-4}$  at the LHC ( $E = 2.76 - 5.5 \text{ TeV}$ )

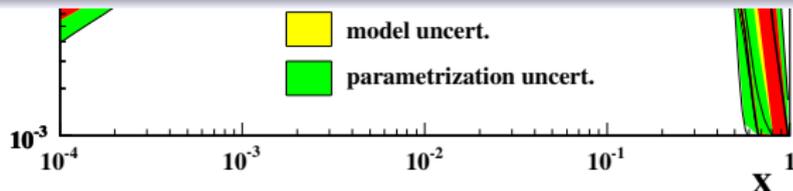
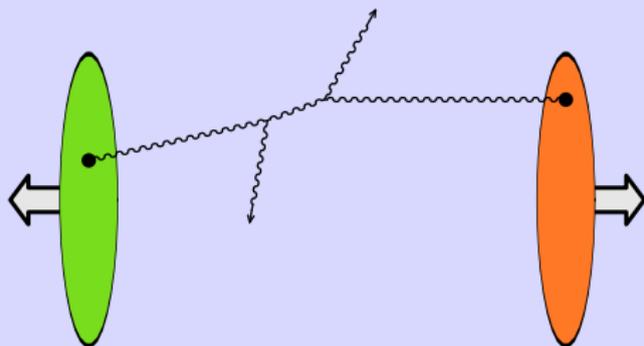
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## H1 and ZEUS



Large  $x$  : dilute regime



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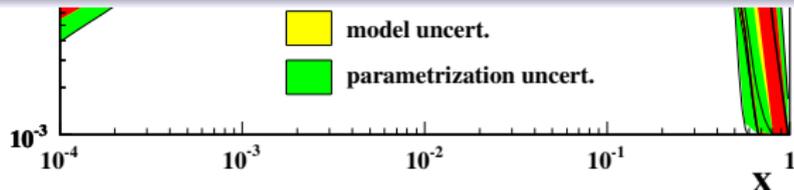
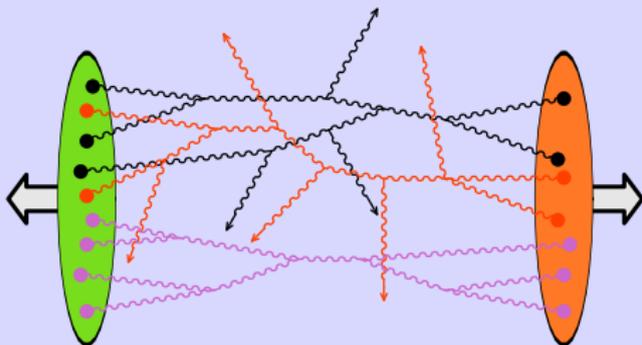
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# Nucleon parton distributions

H1 and ZEUS



Small  $x$  : dense regime, gluon saturation



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## Color Glass Condensate = effective theory of small $x$ gluons

[McLerran, Venugopalan (1994), Jalilian-Marian, Kovner, Leonidov, Weigert (1997), Iancu, Leonidov, McLerran (2001)]

- The **fast partons** ( $k^+ > \Lambda^+$ ) are frozen by time dilation
  - ▷ described as **static color sources** on the light-cone :

$$J^\mu = \delta^{\mu+} \rho(x^-, \vec{x}_\perp) \quad (0 < x^- < 1/\Lambda^+)$$

- The color sources  $\rho$  are **random**, and described by a probability distribution  $W_{\Lambda^+}[\rho]$
- **Slow partons** ( $k^+ < \Lambda^+$ ) may evolve during the collision
  - ▷ treated as standard gauge fields
  - ▷ eikonal coupling to the current  $J^\mu$  :  $J_\mu A^\mu$

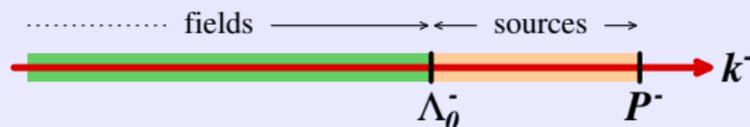
$$S = \underbrace{-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu}}_{S_{\text{YM}}} + \int \underbrace{J^\mu A_\mu}_{\text{fast partons}}$$

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- The cutoff between the sources and the fields is not physical, and should not enter in observables
- Loop corrections contain logs of the cutoff
- These logs can be cancelled by letting the distribution of the sources depend on the cutoff

$$\Lambda \frac{\partial W[\rho]}{\partial \Lambda} = \mathcal{H} \left( \rho, \frac{\delta}{\delta \rho} \right) W[\rho] \quad (\text{JIMWLK equation})$$

# Initial State Factorization

[FG, Venugopalan (2006)]

[FG, Lappi, Venugopalan (2008)]

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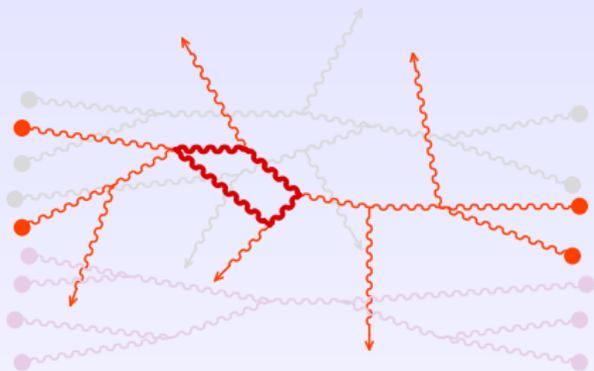
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## Power counting



**In the saturated regime:**  $J \sim g^{-1}$

$$g^{-2} g^{\# \text{ of external legs}} g^{2 \times (\# \text{ of loops})}$$

- No dependence on the number of sources  $J$ 
  - ▷ infinite number of graphs at each order

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# Inclusive gluon spectrum

## Inclusive gluon spectrum to all orders

$$\frac{dN_1}{d^3\vec{p}} \sim \int d^4x d^4y e^{ip \cdot (x-y)} \square_x \square_y \left[ A_+(x) A_-(y) + G_{+-}(x, y) \right]$$

$A_{\pm}, G_{+-}$  = Schwinger-Keldysh 1- and 2-point functions

- Structure of the expansion in  $g^2$  :

$$A_{\pm} = \frac{1}{g} \left[ \underbrace{a_0}_{\text{tree}} + \underbrace{a_1 g^2}_{\text{1-loop}} + \dots \right] \quad G_{+-} = \underbrace{b_0}_{\text{tree}} + \underbrace{b_1 g^2}_{\text{1-loop}} + \dots$$

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## Leading Order

- LO : we need only  $A_+(x)$  and  $A_-(y)$ , at tree level
- These functions obey the classical equation of motion :

$$\square \mathcal{A} + V'(\mathcal{A}) = J$$

- Boundary conditions : retarded, with  $\mathcal{A} \rightarrow 0$  at  $x_0 = -\infty$

### Inclusive spectra at LO

$$\left. \frac{dN_1}{d^3\vec{p}} \right|_{LO} \sim \int d^4x d^4y e^{ip \cdot (x-y)} \square_x \square_y \mathcal{A}(x) \mathcal{A}(y)$$

$$\left. \frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n} \right|_{LO} = \left. \frac{dN_1}{d^3\vec{p}_1} \right|_{LO} \cdots \left. \frac{dN_1}{d^3\vec{p}_n} \right|_{LO}$$

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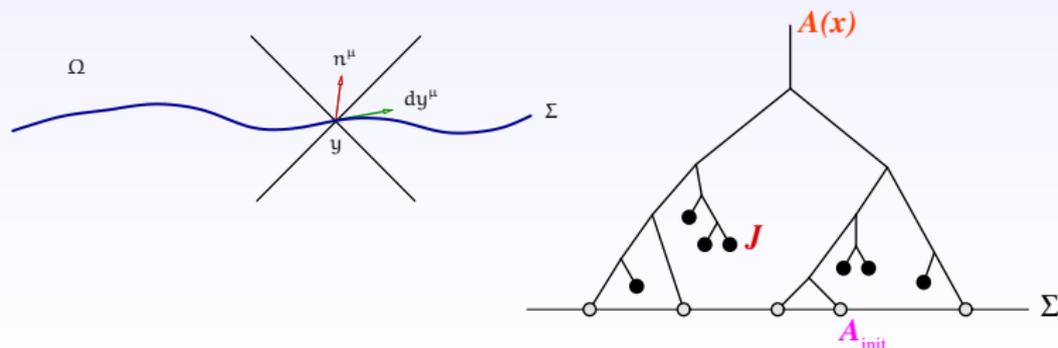


## Cauchy problem for classical fields

- In some situations, one needs to express the classical field in terms of the source  $J$  and its value on a surface  $\Sigma$

### Green's formula

$$\mathcal{A}(x) = i \int_{y \in \Omega} G_R^0(x, y) [J(y) - V'(\mathcal{A}(y))] + i \int_{y \in \Sigma} G_R^0(x, y) (\mathbf{n} \cdot \overleftrightarrow{\partial}_y) \mathcal{A}_{init}(y)$$



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## Small perturbations of a classical field

### Disturbance propagating over a classical background

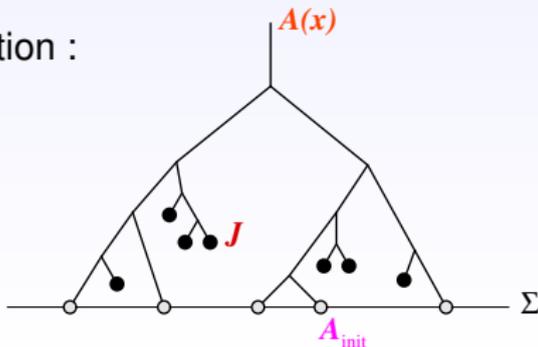
$$\left[ \square_x + V''(\mathcal{A}(x)) \right] \mathbf{a}(x) = 0 \quad , \quad \mathbf{a}(x) = \alpha(x) \text{ on } \Sigma$$

### Formal solution

$$[\alpha \mathbb{T}]_{\mathbf{y}} \equiv \alpha(\mathbf{y}) \frac{\delta}{\delta \mathcal{A}_{\text{init}}(\mathbf{y})} + (\mathbf{n} \cdot \partial \alpha(\mathbf{y})) \frac{\delta}{\delta (\mathbf{n} \cdot \partial \mathcal{A}_{\text{init}}(\mathbf{y}))}$$

$$\alpha(x) \equiv \int_{\mathbf{y} \in \Sigma} [\alpha \mathbb{T}]_{\mathbf{y}} \mathcal{A}(x)$$

- Diagrammatic interpretation :



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## Small perturbations of a classical field

### Disturbance propagating over a classical background

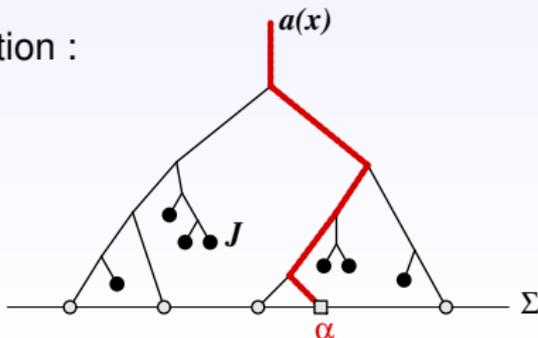
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$$\mathbf{a}(x) \equiv \int_{\mathbf{y} \in \Sigma} [\alpha \mathbb{T}]_{\mathbf{y}} \mathcal{A}(x)$$

- Diagrammatic interpretation :



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## Next to Leading Order

- What do we need at NLO?

$$A_{\pm} = \frac{1}{g} \left[ a_0 + \underline{\underline{a_1 g^2}} + \dots \right] \quad G_{+-} = \underline{\underline{b_0}} + b_1 g^2 + \dots$$

- These two subleading quantities can be expressed in terms of perturbations to the retarded classical field
- For instance, at tree level:

$$G_{+-}(x, y) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3 2k} \mathbf{a}_{\mathbf{k}}(x) \mathbf{a}_{\mathbf{k}}^*(y)$$

$$\left[ \square_x + V''(\mathcal{A}(x)) \right] \mathbf{a}_{\mathbf{k}}(x) = 0 \quad , \quad \lim_{x_0 \rightarrow -\infty} \mathbf{a}_{\mathbf{k}}(x) = e^{i\mathbf{k} \cdot \mathbf{x}}$$



## Next to Leading Order

### Master relation between LO and NLO

$$\left. \frac{dN_1}{d^3\vec{p}} \right|_{\text{NLO}} = \left[ \frac{1}{2} \int \int_{\mathbf{k}} [\mathbf{a}_{\mathbf{k}} \mathbb{T}]_{\mathbf{u}} [\mathbf{a}_{\mathbf{k}}^* \mathbb{T}]_{\mathbf{v}} + \int_{\mathbf{u} \in \Sigma} [\boldsymbol{\alpha} \mathbb{T}]_{\mathbf{u}} \right] \left. \frac{dN_1}{d^3\vec{p}} \right|_{\text{LO}}$$

- Valid for all inclusive multi-gluon spectra, and for the energy-momentum tensor
- Valid for any Cauchy surface  $\Sigma$
- Not specific to scalar theories
- In the CGC, upper cutoff on the loop momentum :  $k^\pm < \Lambda$ , to avoid double counting with the sources  $J_{1,2}$ 
  - ▷ large logarithms of the cutoff

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# Initial state logarithms

## Central result

$$\frac{1}{2} \int \int_{\mathbf{k}} [\mathbf{a}_{\mathbf{k}} \mathbb{T}]_{\mathbf{u}} [\mathbf{a}_{\mathbf{k}}^* \mathbb{T}]_{\mathbf{v}} + \int_{\mathbf{u} \in \Sigma} [\boldsymbol{\alpha} \mathbb{T}]_{\mathbf{u}} =$$

$$= \log(\Lambda^+) \mathcal{H}_1 + \log(\Lambda^-) \mathcal{H}_2 + \text{terms w/o logs}$$

$\mathcal{H}_{1,2}$  = JIMWLK Hamiltonians of the two nuclei

- No mixing between the logs of  $\Lambda^+$  and  $\Lambda^-$
- Since the LO $\leftrightarrow$ NLO relationship is the same for all inclusive observables, these logs have a universal structure

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## Factorization of the logarithms

- By integrating over  $\rho_{1,2}$ 's, one can absorb the logarithms into universal distributions  $W_{1,2}[\rho_{1,2}]$
- $\mathcal{H}$  is a self-adjoint operator :

$$\int [D\rho] W (\mathcal{H} \vartheta) = \int [D\rho] (\mathcal{H} W) \vartheta$$

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### Single inclusive gluon spectrum at Leading Log accuracy

$$\left\langle \frac{dN_1}{d^3\vec{p}} \right\rangle_{\text{Leading Log}} = \int [D\rho_1 D\rho_2] W_1[\rho_1] W_2[\rho_2] \underbrace{\frac{dN_1[\rho_{1,2}]}{d^3\vec{p}}}_{\text{fixed } \rho_{1,2}} \Big|_{LO}$$

- Logs absorbed into the evolution of  $W_{1,2}$  with the scales

$$\Lambda \frac{\partial W}{\partial \Lambda} = \mathcal{H} W \quad (\text{JIMWLK equation})$$



## Multi-gluon correlations at Leading Log

- The previous factorization can be extended to multi-particle inclusive spectra :

$$\begin{aligned}
 \left\langle \frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n} \right\rangle_{\text{Leading Log}} &= \\
 &= \int [D\rho_1 D\rho_2] W_1[\rho_1] W_2[\rho_2] \left. \frac{dN_1[\rho_{1,2}]}{d^3\vec{p}_1} \cdots \frac{dN_1[\rho_{1,2}]}{d^3\vec{p}_n} \right|_{\text{LO}}
 \end{aligned}$$

- At Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions  $W[\rho_{1,2}]$ 
  - ▷ they are a property of the pre-collision initial state
- Predicts long range ( $\Delta y \sim \alpha_s^{-1}$ ) correlations in rapidity

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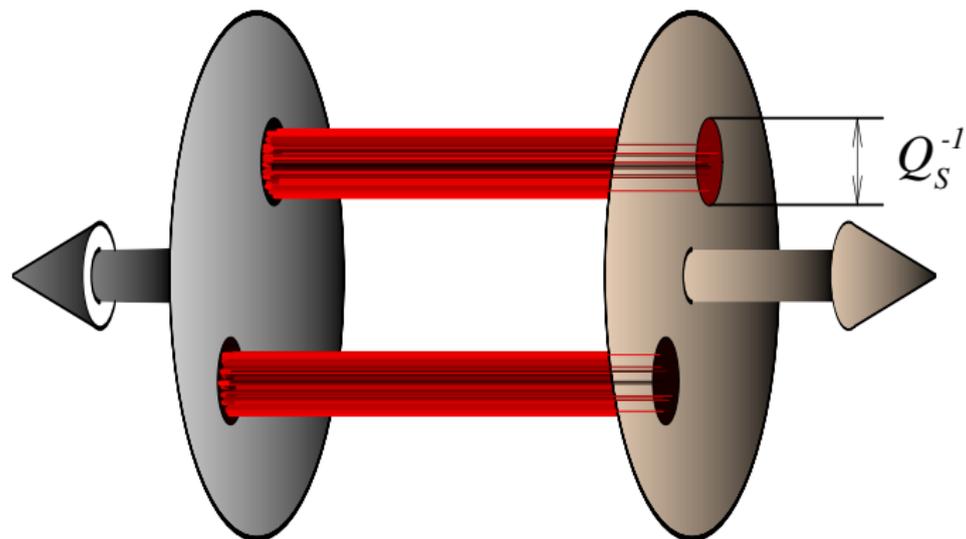
# Final state evolution

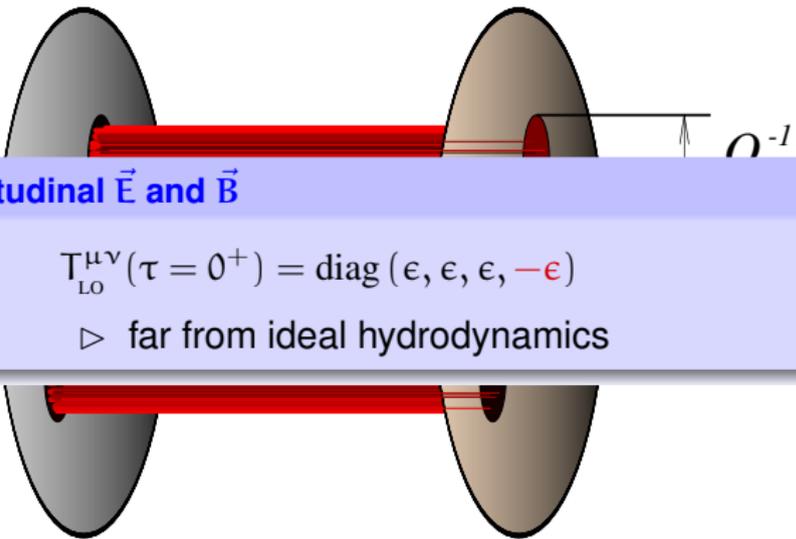
[Dusling, Epelbaum, FG, Venugopalan (2010)]

[Dusling, FG, Venugopalan (2011)]

[Epelbaum, FG (2011)]

## Energy momentum tensor at LO





$T^{\mu\nu}$  for longitudinal  $\vec{E}$  and  $\vec{B}$

$$T_{LO}^{\mu\nu}(\tau = 0^+) = \text{diag}(\epsilon, \epsilon, \epsilon, -\epsilon)$$

▷ far from ideal hydrodynamics



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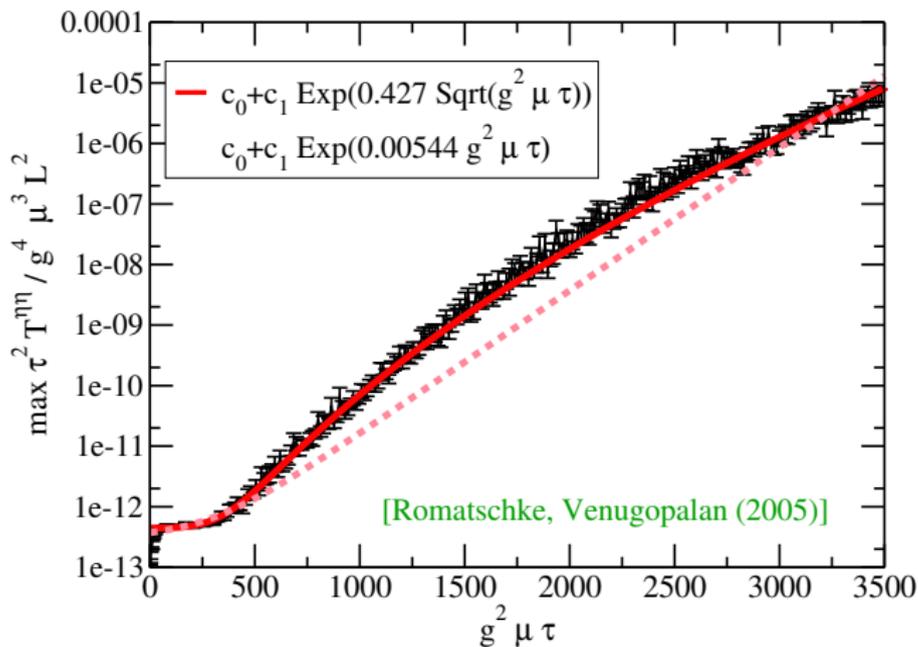
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## Weibel instabilities for small perturbations

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007),...]





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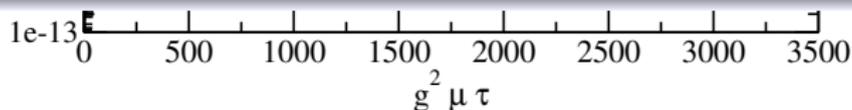
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- Some of the field fluctuations  $\alpha_k$  diverge like  $\exp \sqrt{\mu\tau}$  when  $\tau \rightarrow +\infty$
- Some components of  $T^{\mu\nu}$  have secular divergences when evaluated at fixed loop order
- When  $\alpha_k \sim \mathcal{A} \sim g^{-1}$ , the power counting breaks down and additional contributions must be resummed :

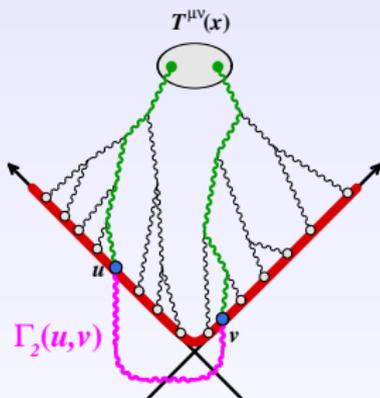
$$g e^{\sqrt{\mu\tau}} \sim 1 \quad \text{at} \quad \tau_{\max} \sim \mu^{-1} \log^2(g^{-1})$$





# Improved power counting

$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T}_{\mathbf{u}} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop :  $(ge^{\sqrt{\mu\tau}})^2$

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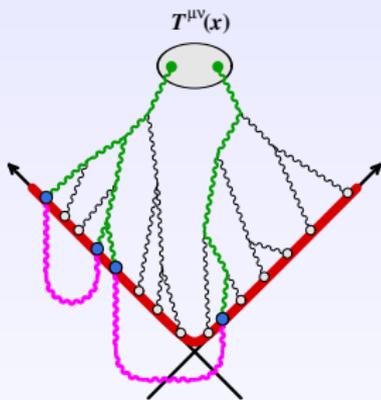
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# Improved power counting

$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T}_{\mathbf{u}} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop :  $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops :  $(ge^{\sqrt{\mu\tau}})^4$

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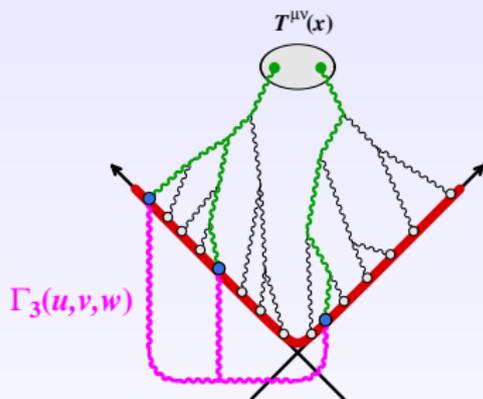
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## Improved power counting

$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T}_{\mathbf{u}} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop :  $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops :  $(ge^{\sqrt{\mu\tau}})^4$
- 2 nested loops :  $g(ge^{\sqrt{\mu\tau}})^3 \triangleright$  subleading

### Leading terms at $\tau_{\max}$

- All disjoint loops to all orders
  - $\triangleright$  exponentiation of the 1-loop result

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$$T_{\text{resummed}}^{\mu\nu} = \exp \left[ \frac{1}{2} \int_{\mathbf{u}, \mathbf{v} \in \Sigma} \underbrace{\int_{\mathbf{k}} [\mathbf{a}_{\mathbf{k}} \mathbb{T}]_{\mathbf{u}} [\mathbf{a}_{\mathbf{k}}^* \mathbb{T}]_{\mathbf{v}}}_{\mathcal{G}(\mathbf{u}, \mathbf{v})} + \int_{\mathbf{u} \in \Sigma} [\boldsymbol{\alpha} \mathbb{T}]_{\mathbf{u}} \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}}]$$

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$$\begin{aligned}
 T_{\text{resummed}}^{\mu\nu} &= \exp \left[ \frac{1}{2} \int_{\mathbf{u}, \mathbf{v} \in \Sigma} \underbrace{\int_{\mathbf{k}} [\mathbf{a}_{\mathbf{k}} \mathbb{T}]_{\mathbf{u}} [\mathbf{a}_{\mathbf{k}}^* \mathbb{T}]_{\mathbf{v}}}_{\mathcal{G}(\mathbf{u}, \mathbf{v})} + \int_{\mathbf{u} \in \Sigma} [\boldsymbol{\alpha} \mathbb{T}]_{\mathbf{u}} \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}}] \\
 &= \int [\mathcal{D}\boldsymbol{\chi}] \exp \left[ -\frac{1}{2} \int_{\mathbf{u}, \mathbf{v} \in \Sigma} \boldsymbol{\chi}(\mathbf{u}) \mathcal{G}^{-1}(\mathbf{u}, \mathbf{v}) \boldsymbol{\chi}(\mathbf{v}) \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}} + \boldsymbol{\chi} + \boldsymbol{\alpha}]
 \end{aligned}$$

- The evolution remains classical, but we must average over a Gaussian ensemble of initial conditions
- Note : the constant shift  $\boldsymbol{\alpha}$  can be absorbed into a redefinition of  $\mathcal{A}_{\text{init}}$



## More on this resummation

- The Gaussian fluctuations around the classical field  $\mathcal{A}_{\text{init}}$  promote it to a **coherent quantum state** (they add 1/2 particle to every mode)
- Dual formulation of QM in the classical phase-space :

Density	$\hat{\rho}$		$W(Q, P)$
Evolution	$\partial_t \hat{\rho} + i[\hat{H}, \hat{\rho}] = 0$	Wigner trans. $\longrightarrow$	$\partial_t W + \{\{W, H\}\} = 0$
Initial condition	$ \mathcal{A}_{\text{init}}\rangle\langle\mathcal{A}_{\text{init}} $		$\exp -\frac{1}{2} \int \chi \mathcal{G}^{-1} \chi$

Approximations :

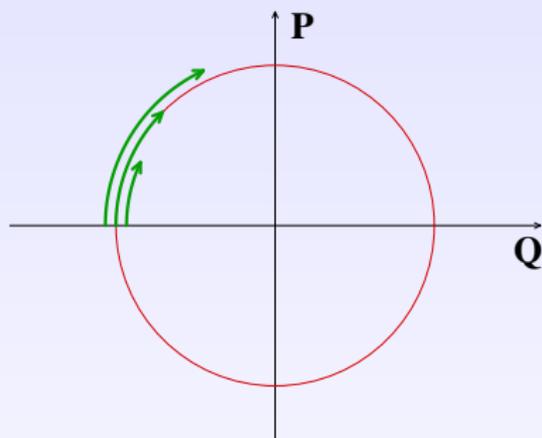
- Moyal bracket  $\{\{ \cdot, \cdot \}\}$  replaced by classical Poisson bracket
- Non-gaussianities of the initial distribution are ignored
- Independent (and anterior..) uses of this scheme :
  - Cosmology [Polarski, Starobinsky (1995), Son (1996), Khlebnikov, Tkachev (1996)]
  - Cold atoms [Davis, Morgan, Burnett (2002), Norrie, Ballagh, Gardiner (2004)]

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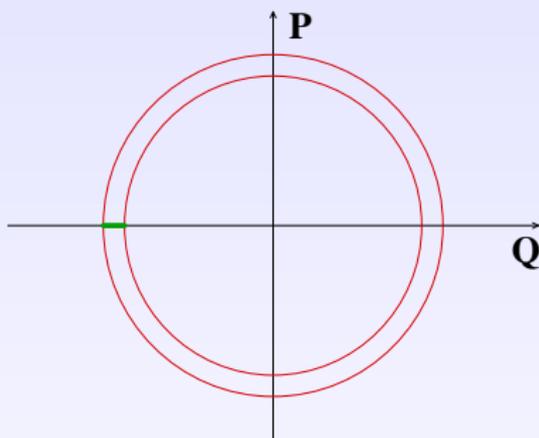
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- The oscillation frequency depends on the initial condition



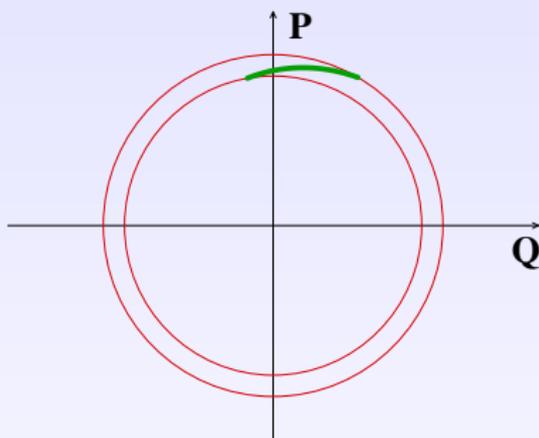
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- The oscillation frequency depends on the initial condition



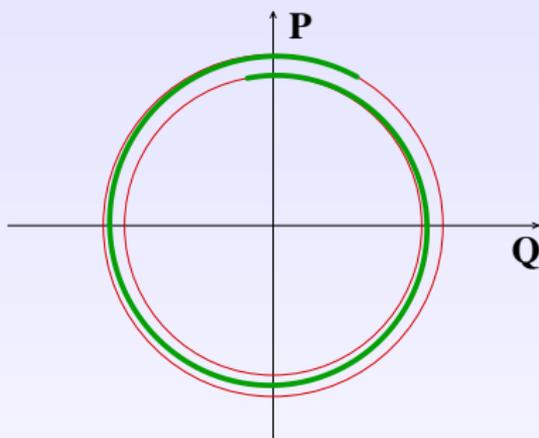
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- The oscillation frequency depends on the initial condition
- An ensemble of initial configurations spreads in time



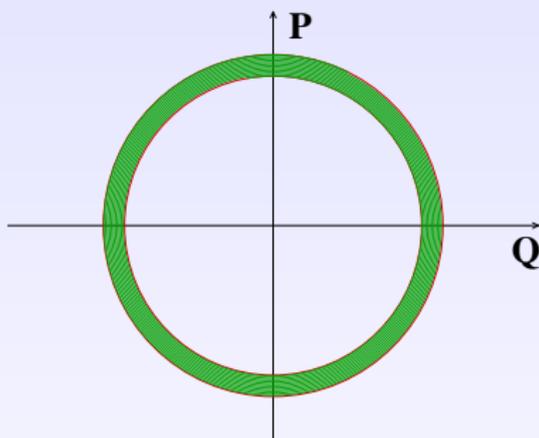
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- The oscillation frequency depends on the initial condition
- An ensemble of initial configurations spreads in time



- The oscillation frequency depends on the initial condition
- An ensemble of initial configurations spreads in time
- At large times, the ensemble fills densely all the region allowed by energy conservation



## Similar problem in a simpler toy model

### $\phi^4$ field theory coupled to a source

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha\phi)^2 - \frac{g^2}{4!}\phi^4 + J\phi$$

$$J \propto \theta(-x^0) \frac{Q^3}{g}$$

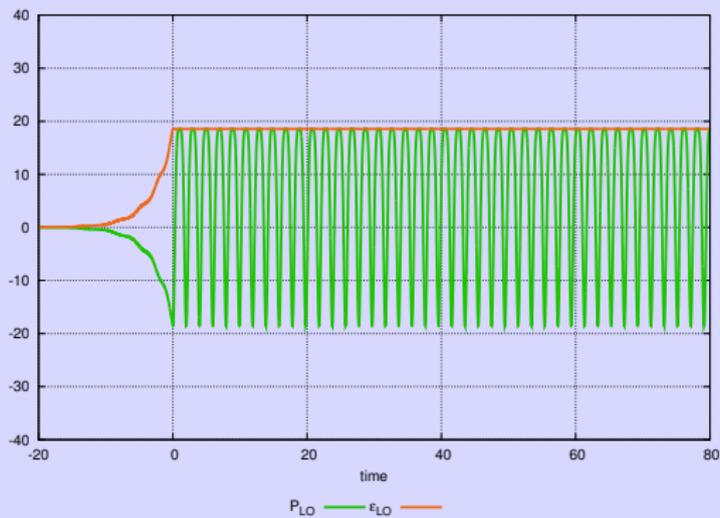
- In 3+1-dim,  $g$  is dimensionless, and the only scale in the problem is  $Q$ , provided by the external source
- The source is active only at  $x^0 < 0$ , and is turned off adiabatically when  $x^0 \rightarrow -\infty$
- This theory has unstable modes (parametric resonance)

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# Secular divergences in fixed order calculations

## Tree

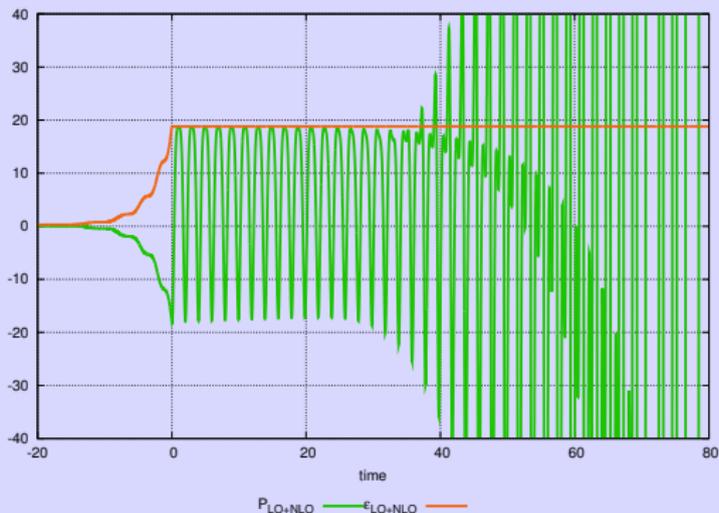


- Oscillating pressure at LO : no equation of state

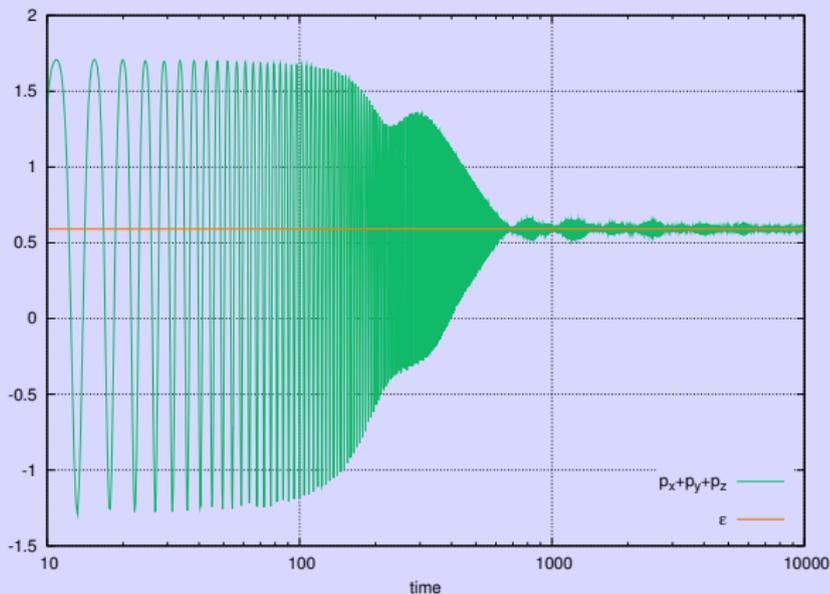
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# Secular divergences in fixed order calculations

## Tree + 1-loop



- Oscillating pressure at LO : no equation of state
- Small NLO correction to the energy density (protected by energy conservation)
- Secular divergence in the NLO correction to the pressure

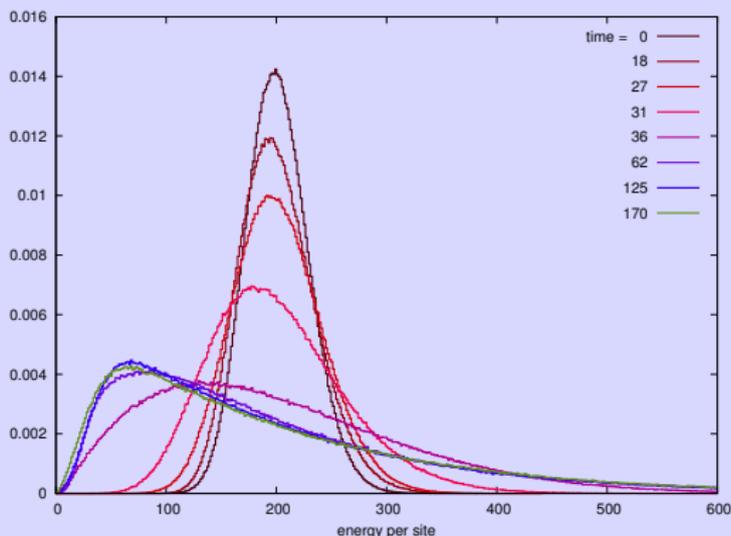
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- No secular divergence in the resummed pressure
- The pressure relaxes to the equilibrium equation of state



## Energy fluctuations in a small subvolume

Probability distribution  $P(e)$  ( $e = \text{energy on one site, } g = 0.5$ )



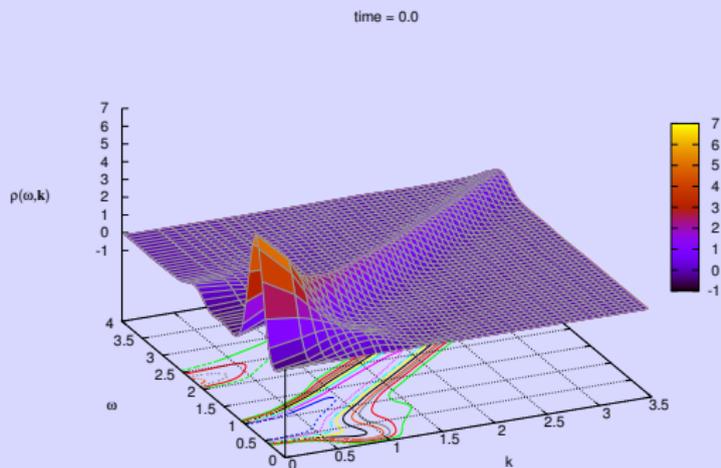
- At  $t = 0$ , narrow Gaussian fluctuations
- Very rapid change of shape
- Shape close to that expected from the canonical ensemble

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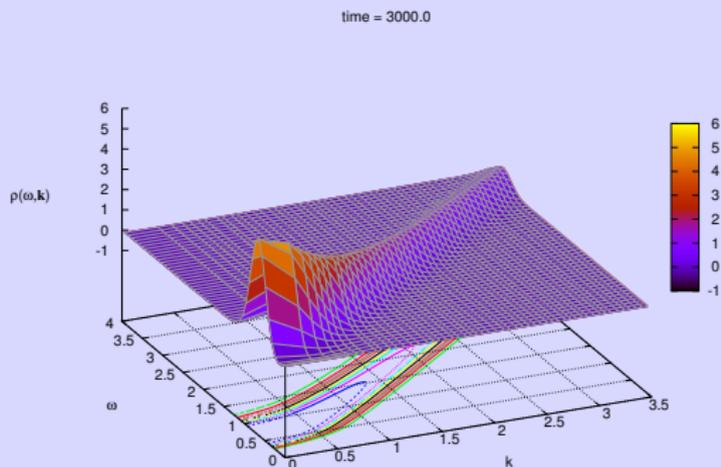
 $\tau = 0$ 

- Complicated spectral density at early times

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$\tau = 3000$



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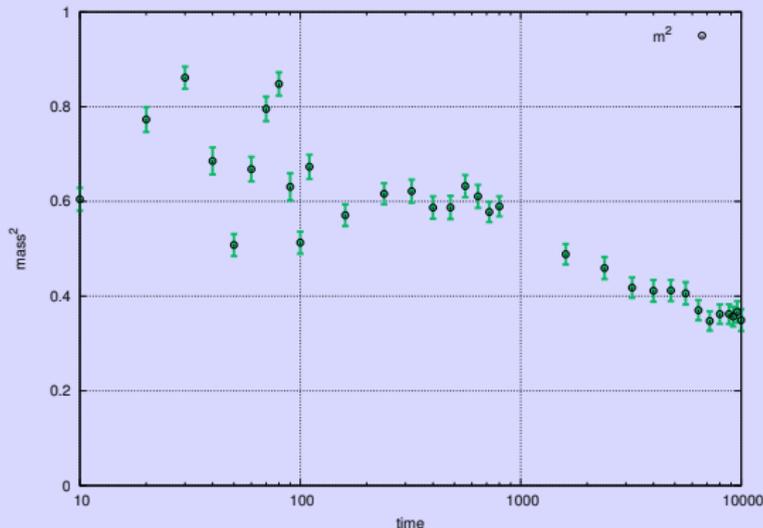
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- Complicated spectral density at early times
- Single quasiparticle peak at late times

## Medium induced mass

Fit of the spectral peak by  $\omega^2 = k^2 + m^2$



- Note : at weak coupling, the mass fitted from the spectral peak agrees with

$$m^2 = \frac{g^2}{2} \langle \phi^2 \rangle$$

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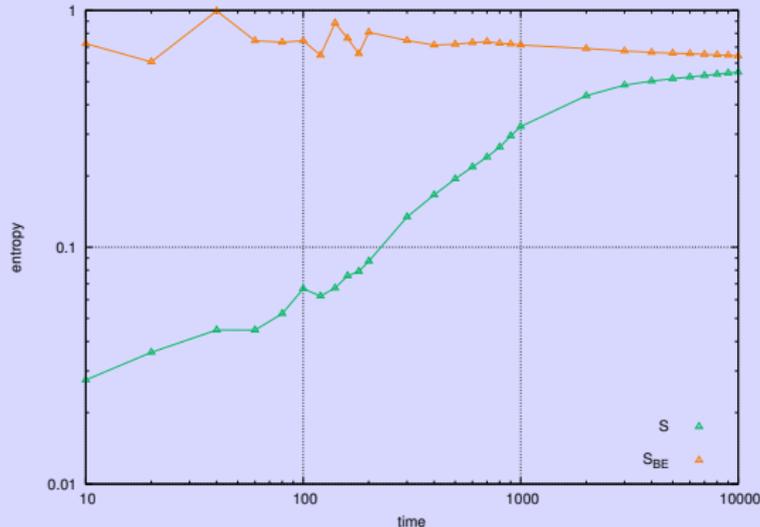
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$$\mathcal{S} \equiv \int_{\mathbf{k}} \left[ (1 + f_{\mathbf{k}}) \log(1 + f_{\mathbf{k}}) - f_{\mathbf{k}} \log(f_{\mathbf{k}}) \right]$$

## Time evolution of the entropy



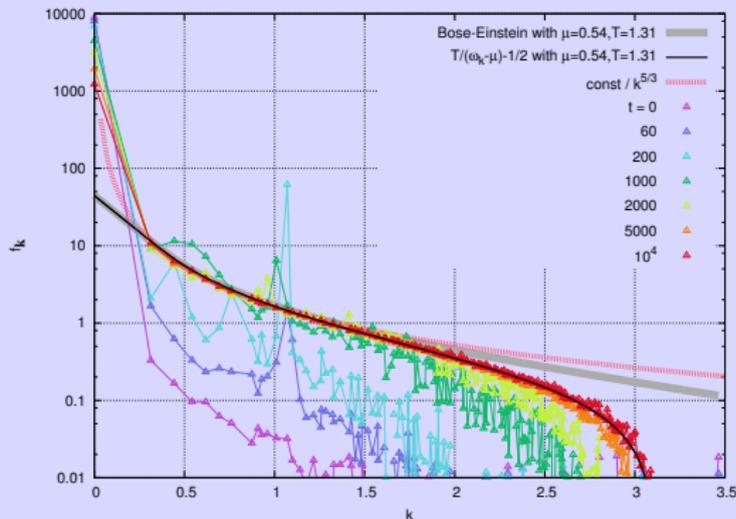
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# Time evolution of the occupation number



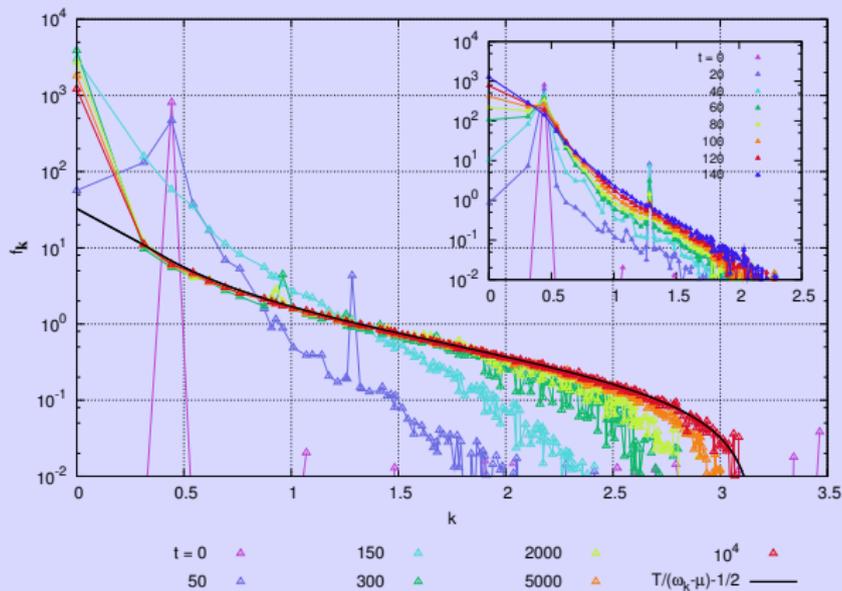
- Resonant peak at early times
- Turbulent Kolmogorov spectrum in the intermediate  $k$ -range?
- Late times : classical equilibrium with a chemical potential
- $\mu \approx m$  + excess at  $k = 0$  : Bose-Einstein condensation?

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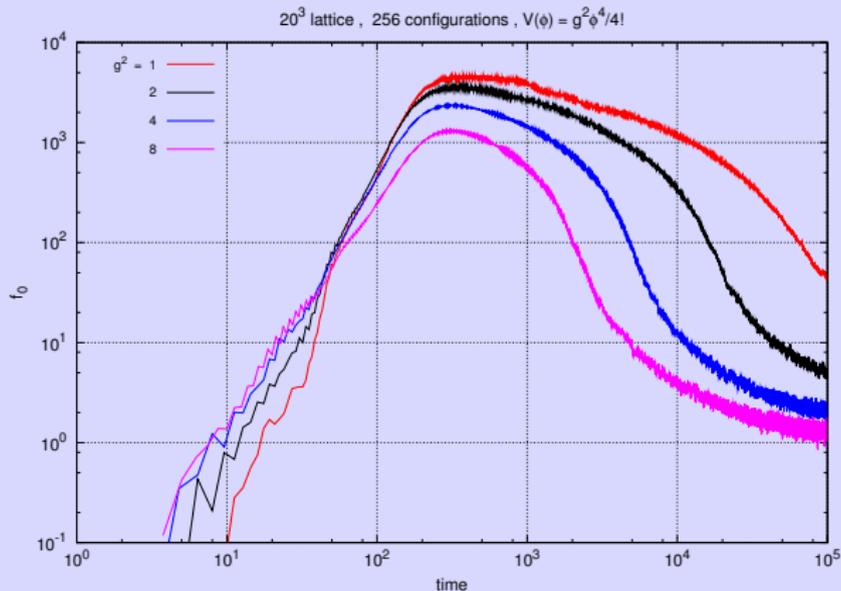
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- Start with the same energy density, but an empty zero mode
- Very quickly, the zero mode becomes highly occupied
- Same distribution as before at late times

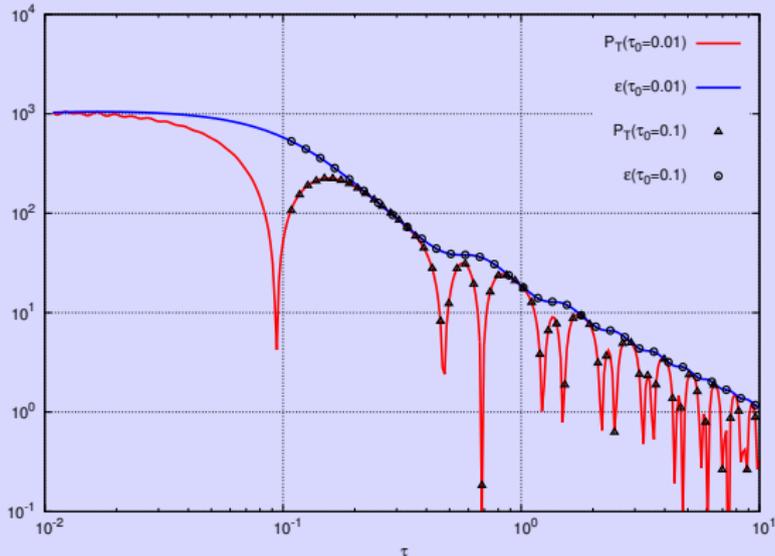
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- Formation time almost independent of the coupling
- Condensate lifetime much longer than its formation time
- Smaller amplitude and faster decay at large coupling



## Effect of longitudinal expansion

- The EoM is singular when  $\tau \rightarrow 0$  : one must start at  $\tau_0 \neq 0$
- With the proper spectrum of field fluctuations (that depends on  $\tau_0$ ) and zero point subtraction, the result does not depend on the initial  $\tau_0$



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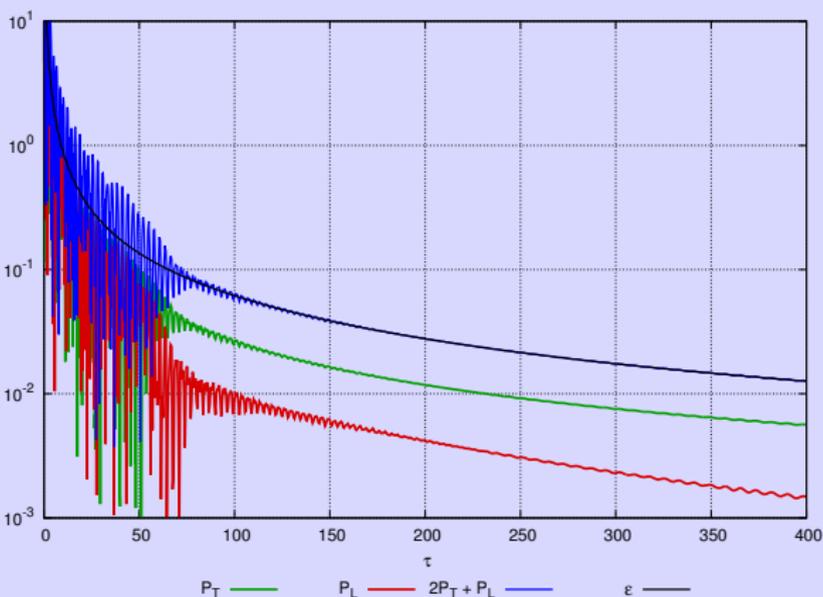
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## Effect of longitudinal expansion

- After some time, the pressures relax and we have the expected equation of state :  $\epsilon = 2P_T + P_L$
- However :  $P_T \neq P_L$



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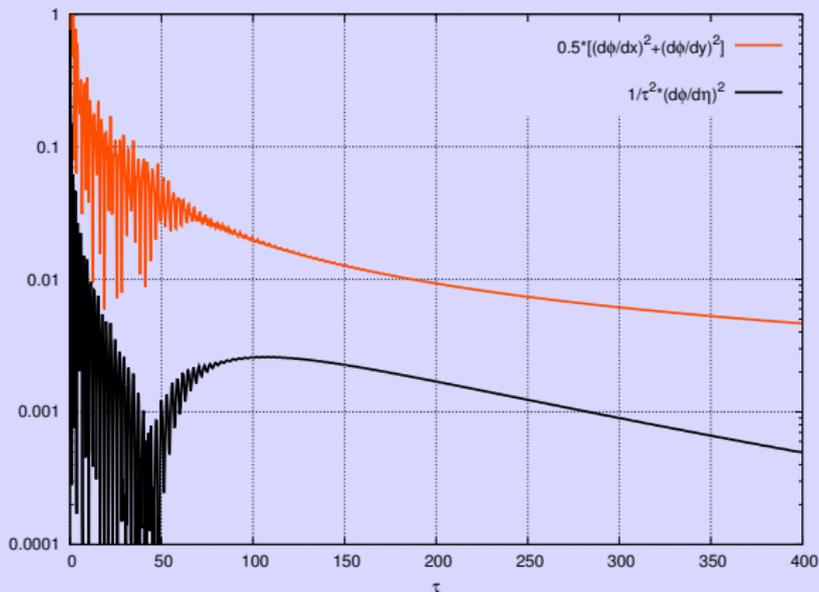


## Effect of longitudinal expansion

- $P_T = P_L$  requires

$$\frac{1}{\tau^2} \left( \frac{\partial \phi}{\partial \eta} \right)^2 = \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right]$$

But instead...



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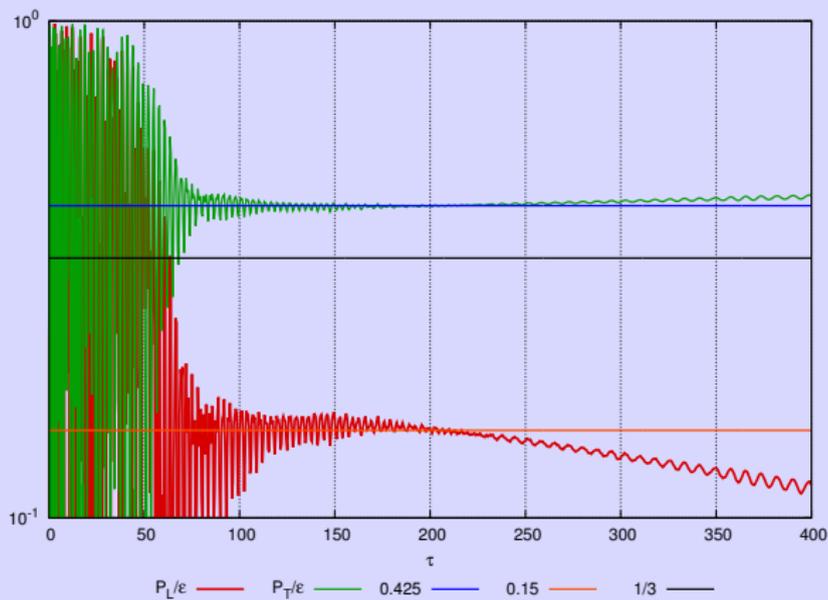
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## Effect of longitudinal expansion

- Constant anisotropy (the drop of  $P_L/\epsilon$  at  $\tau \geq 200$  is likely a lattice artifact)



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# Summary and Outlook

## Summary

- **Factorization of high energy logarithms in AA collisions**
  - limited to inclusive observables
  - controls the rapidity dependence of correlations
  - links nucleus-nucleus collisions to other reactions (pA, DIS)
- **Resummation of secular terms in the final state evolution**
  - stabilizes the NLO calculation
  - leads to the equilibrium equation of state
  - full thermalization on much longer time-scales
  - Bose-Einstein condensation for overoccupied initial state
  - $\phi^4$  theory : instabilities too weak to resist against expansion

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## Outlook

- thermalization in QCD, w/ longitudinal expansion?
- if a BEC is formed, phenomenological implications?



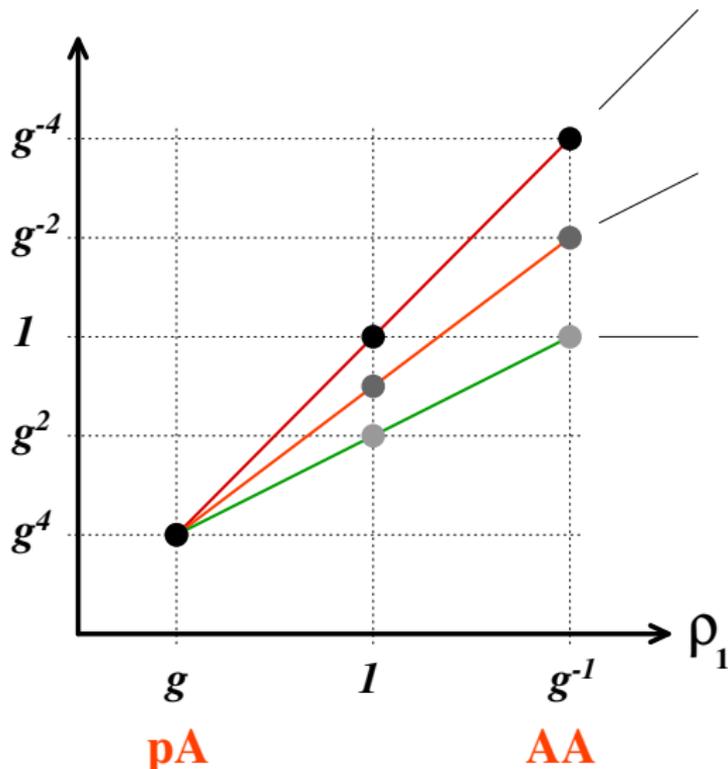
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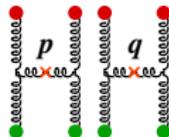
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# Extra



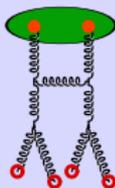
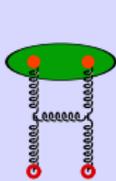
# Dense-dilute collisions



## Expected complications

- More diagrams to consider even at Leading Order
- More terms in the evolution Hamiltonian if  $\rho \sim g$ :

$$g^2 \rho^2 \left( \frac{\partial}{\partial \rho} \right)^2 \sim g^4 \rho^2 \left( \frac{\partial}{\partial \rho} \right)^4$$



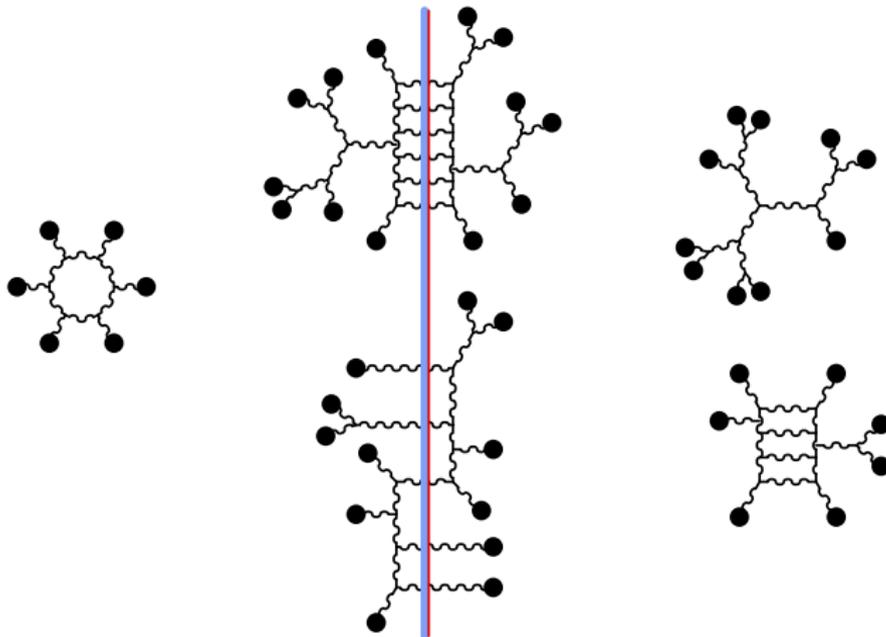
$g$   
**pA**

$1$

$g^{-1}$   
**AA**

$\sim \rho^1$

# Exclusive processes

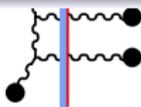


## Exclusive processes

### Example : differential probability to produce 1 particle at LO

$$\left. \frac{dP_1}{d^3\vec{p}} \right|_{LO} = F[0] \times \int d^4x d^4y e^{ip \cdot (x-y)} \square_x \square_y \mathcal{A}_+(x) \mathcal{A}_-(y) \Big|_{z=0}$$

- The vacuum-vacuum graphs do not cancel in exclusive quantities :  $F[0] \neq 1$  (in fact,  $F[0] = \exp(-c/g^2) \ll 1$ )
- $\mathcal{A}_+$  and  $\mathcal{A}_-$  are classical solutions of the Yang-Mills equations, but with **non-retarded boundary conditions**



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- Recent analytical work : [Kurkela, Moore \(2011\)](#)
- Going from scalars to gauge fields :
  - More fields per site (3 Lorentz components  $\times$  8 colors)
  - More complicated spectrum of initial conditions
  - Expansion : UV overflow on a fixed grid in  $\eta$

# BEC and dilepton production

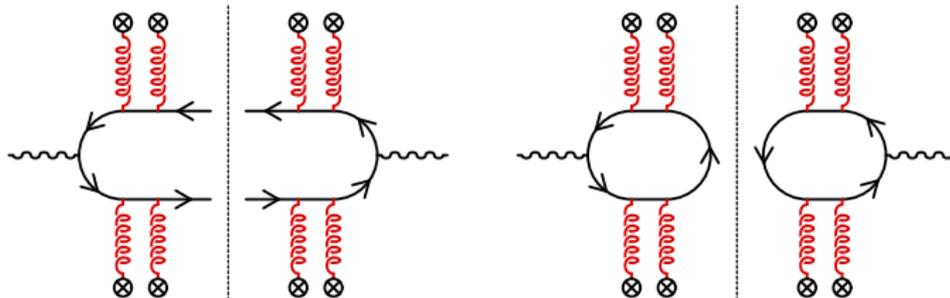


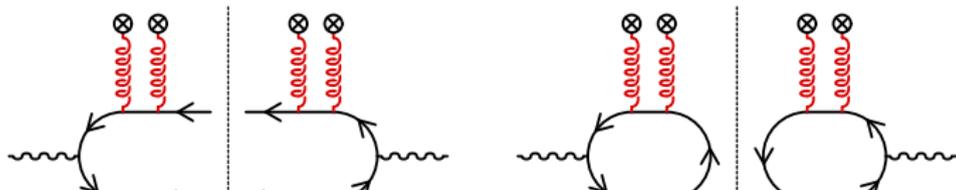
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## Two topologies for virtual photons at LO

Connected	$\omega \sim M_{\text{inv}} \sim Q_s$	$k_{\perp} \sim Q_s$
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Disconnected	$\omega \sim M_{\text{inv}} \sim Q_s$	$k_{\perp} \ll Q_s$
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▷ excess of dileptons with  $k_{\perp} \ll M_{\text{inv}}$

- Quantum Chaos : how does the chaos at the classical level manifests itself in quantum mechanics?

- **Berry's conjecture** [M.V. Berry (1977)]

High lying eigenstates of such systems have nearly random wavefunctions. The corresponding Wigner distribution is almost uniform on the energy surface

- **Srednicki's eigenstate thermalization hypothesis**  
[M. Srednicki (1994)]

For sufficiently inclusive measurements, these high lying eigenstates look thermal. If the system starts in a coherent state, decoherence is the main mechanism to thermalization