Photon impact factor for BFKL pomeron at next-to-leading order

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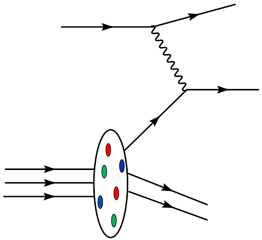
Outline

- Light-cone OPE versus OPE in color dipoles.
- High-energy scattering and Wilson lines formalism.
- Factorization in rapidity.
- NLO Photon Impact Factor.
- Brief review of the LO and NLO BK equation.
- Triple Pomeron vertex through Wilson line formalism: planar (leading N_c) and non-planar (next to-leading N_c) contribution.
- Factorization for Inclusive Hadron Production in pA collisions.
- Conclusions and outlook.

Incoherent-vs-Coherent

Incoherent Interactions



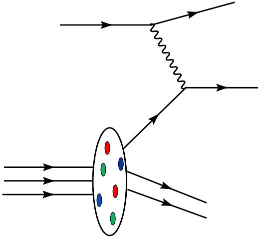
Bjorken Limit

$$Q^2 \to \infty, \; s \to \infty$$

$$x_{\rm B} = \frac{Q^2}{s} \;\; \text{fixed}$$

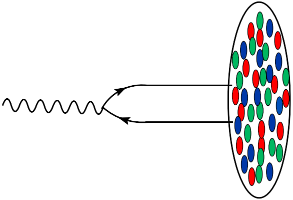
$$\text{resum} \;\; \alpha_s \ln \frac{Q^2}{\Lambda_{\rm QCD}}$$

Incoherent Interactions



vs.

Coherent Interactions



Bjorken Limit

$$Q^2 \to \infty, \; s \to \infty$$

$$x_B = \frac{Q^2}{s} \quad \text{fixed}$$

resum $\alpha_s \ln \frac{Q^2}{\Lambda_{QCD}}$

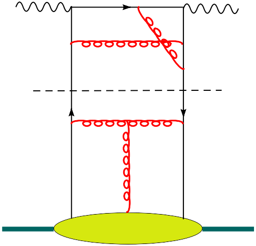
Regge Limit

$$Q^2 \;\; \text{fixed}, \; s \to \infty$$

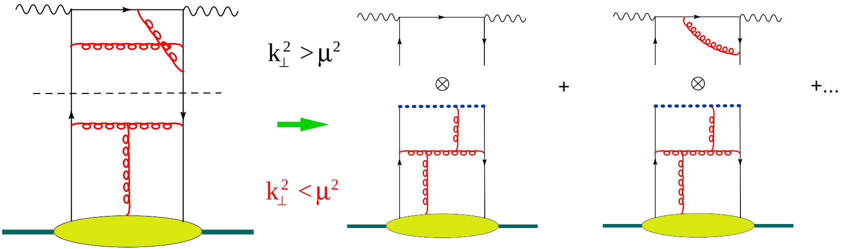
$$x_B = \frac{Q^2}{s} \to 0$$

resum $\alpha_s \ln \frac{1}{x_B}$

Light-cone expansion and DGLAP evolution in the NLO



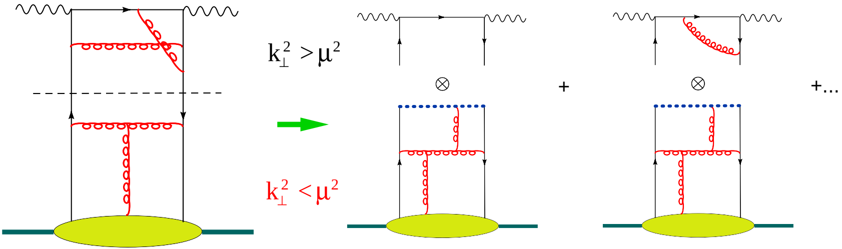
Light-cone expansion and DGLAP evolution in the NLO



μ^2 - factorization scale (normalization point)

$k_\perp^2 \; > \; \mu^2$ - coefficient functions
$k_\perp^2 \; < \; \mu^2$ - matrix elements of light-ray operators (normalized at μ^2)

Light-cone expansion and DGLAP evolution in the NLO

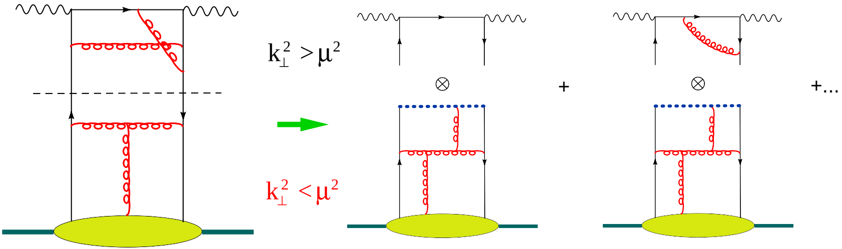


μ^2 - factorization scale (normalization point)

$k_\perp^2 > \mu^2$ - coefficient functions
$k_\perp^2 < \mu^2$ - matrix elements of light-ray operators (normalized at μ^2)
OPE in light-ray operators $(x-y)^2 \to 0$

$$T\{j_\mu(x)j_\nu(y)\} = \frac{(x-y)_\xi}{2\pi^2(x-y)^4}\Big[1 + \frac{\alpha_s}{\pi}(\ln(x-y)^2\mu^2 + C)\Big]\bar\psi(x)\gamma_\mu\gamma^\xi\gamma_\nu[x,y]\psi(y)$$

$[x,y] \equiv Pe^{ig\int_0^1 du\ (x-y)^\mu A_\mu(ux+(1-u)y)}$ - gauge link



μ^2 - factorization scale (normalization point)

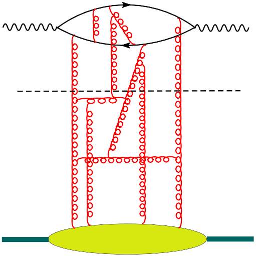
$k_\perp^2 > \mu^2$ - coefficient functions

$k_\perp^2 < \mu^2$ - matrix elements of light-ray operators (normalized at μ^2)

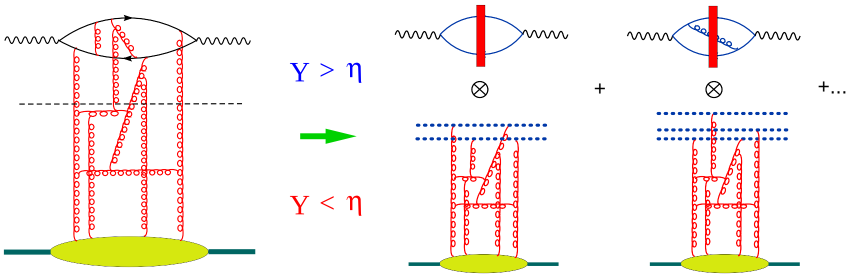
Renorm-group equation for light-ray operators $\Rightarrow$ DGLAP evolution of parton densities $(x-y)^2 = 0$

$$\mu^2 \frac{d}{d\mu^2} \bar{\psi}(x)[x,y]\psi(y) = K_{\rm LO}\bar{\psi}(x)[x,y]\psi(y) + \alpha_s K_{\rm NLO}\bar{\psi}(x)[x,y]\psi(y)$$

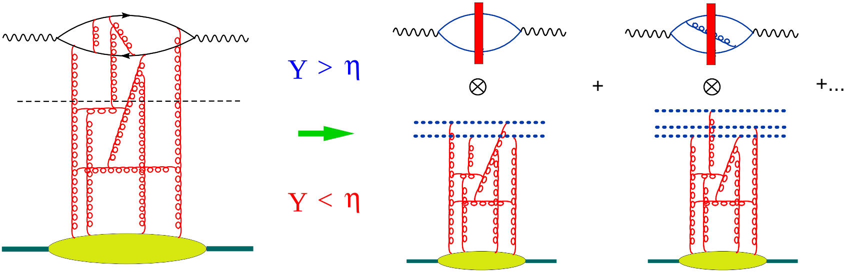
High-energy expansion in color dipoles in the NLO



High-energy expansion in color dipoles in the NLO



High-energy expansion in color dipoles in the NLO

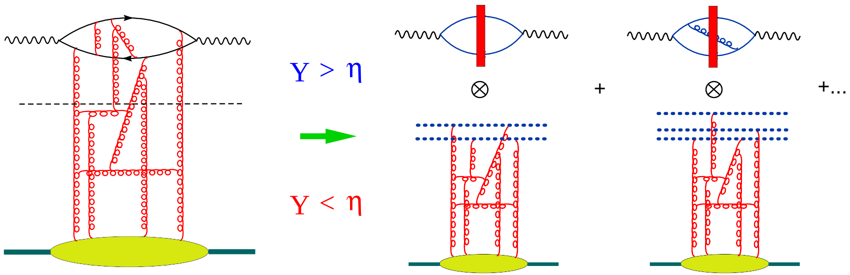


η - rapidity factorization scale

Rapidity Y > η - coefficient function (“impact factor”)
Rapidity Y < η - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$U^\eta_x = \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} dx^+ A^\eta_+(x_+, x_\perp)\Big]$$

$$A^\eta_\mu(x) = \int \frac{d^4k}{(2\pi)^4}\theta(e^\eta - |\alpha_k|)e^{-ik\cdot x}A_\mu(k)$$



The high-energy operator expansion is

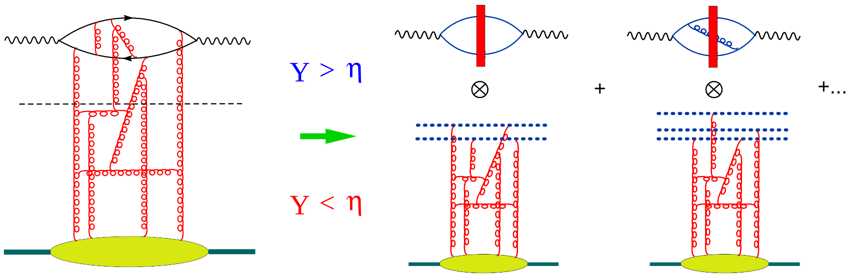
$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$+\int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

In the leading order the impact factor is Möbius invariant

In the NLO one should also expect conf. invariance since $I^{\rm NLO}_{\mu\nu}$ is given by tree diagrams

High-energy expansion in color dipoles in the NLO



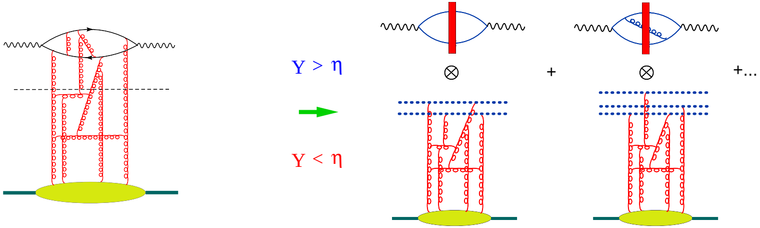
η - rapidity factorization scale

Evolution equation for color dipoles

$$\frac{d}{d\eta}\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\} = \frac{\alpha_s}{2\pi^2}\int d^2z \frac{(x-y)^2}{(x-z)^2(y-z)^2}[\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\}\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\} - N_c\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\}] + \alpha_s K_{\mathrm{NLO}}\mathrm{tr}\{U_x^\eta U_y^{\dagger\eta}\} + O(\alpha_s^2)$$

K_{NLO}=? (Linear part of K_{NLO} = $K_{\mathrm{NLO\ BFKL}}$)

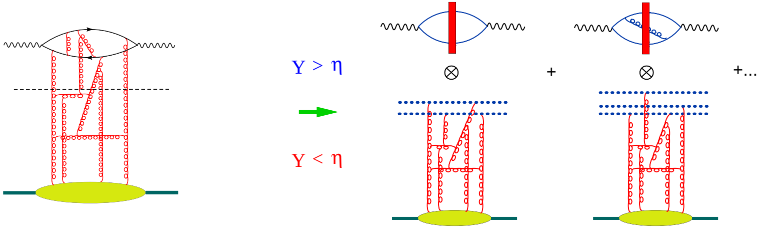
Expansion of $F_2(x)$ in color dipoles in the next-to-leading order



$$F_2(x_B) \simeq \int d^2z_1 d^2z_2\ I^{LO}(z_1,z_2)\langle \mathrm{tr}\{U^{\eta}_{z_1} U^{\dagger\,\eta}_{z_2}\}\rangle \qquad \eta = \ln\frac{1}{x_B}$$

$$+\ \frac{\alpha_s}{\pi}\int d^2z_1 d^2z_2 d^2z_3\ I^{NLO}(z_1,z_2,z_3)\langle \mathrm{tr}\{U^{\eta}_{z_1} U^{\dagger\eta}_{z_3}\}\mathrm{tr}\{U_{z_3} U^{\dagger\eta}_{z_2}\}\rangle$$

Expansion of $F_2(x)$ in color dipoles in the next-to-leading order



$$F_2(x_B) \simeq \int d^2z_1 d^2z_2 \; I^{LO}(z_1,z_2)\langle \mathrm{tr}\{U^\eta_{z_1} U^{\dagger\,\eta}_{z_2}\}\rangle \qquad \eta = \ln\frac{1}{x_B}$$

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plan

- Calculate the NLO photon impact factor.
- Calculate the NLO evolution of color dipole.
- Convolute the solution with the initial conditions for the evolution and get the amplitude.

Propagation in the shock wave: Wilson line (Spectator frame)

Each path is weighted with the gauge factor $Pe^{ig\int dx_\mu A^\mu}$. Since the external field exists only within the infinitely thin wall, quarks and gluons do not have time to deviate in the transverse direction $\Rightarrow$ we can replace the gauge factor along the actual path with the one along the straight-line path.

$$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$$

$$[x, y] = Pe^{ig\int_0^1 du (x-y)^\mu A_\mu (ux+(1-u)y)} \qquad p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

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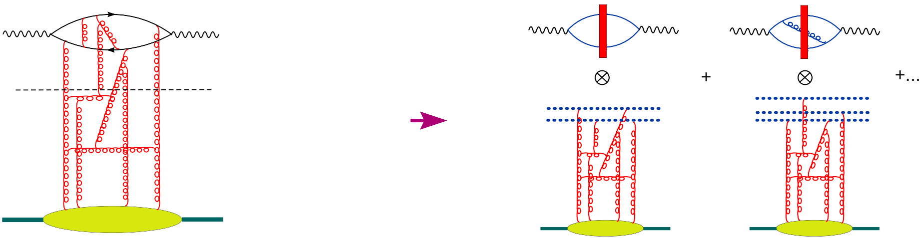
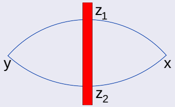


LO and NLO Impact Factor

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$+\int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

LO Impact Factor diagram: $I^{\rm LO}$



LO and NLO Impact Factor

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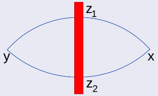
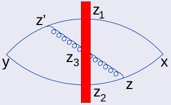
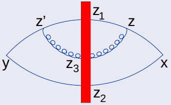


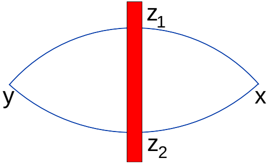
NLO Impact Factor diagrams: $I^{\rm NLO}$





LO Impact Factor

Conformal invariance: $(x^+, x_\perp)^2 = -x_\perp^2 \Rightarrow$ after the inversion $x_\perp \to x_\perp/x_\perp^2$ and $x^+ \to x^+/x_\perp^2$



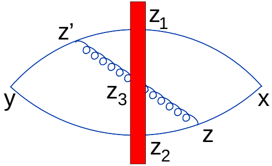
Conformal vectors:

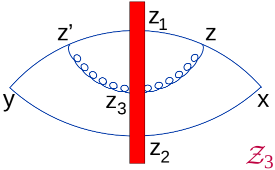
$$\kappa = \frac{\sqrt{s}}{2x_*}\Big(\frac{p_1}{s} - x^2 p_2 + x_\perp\Big) - \frac{\sqrt{s}}{2y_*}\Big(\frac{p_1}{s} - y^2 p_2 + y_\perp\Big)$$

$$\zeta_1 = \Big(\frac{p_1}{s} + z_{1\perp}^2 p_2 + z_{1\perp}\Big), \qquad \zeta_2 = \Big(\frac{p_1}{s} + z_{2\perp}^2 p_2 + z_{2\perp}\Big)$$

Here $x^2 = -x_\perp^2$, $x_* \equiv x_\mu p_2^\mu$ (similarly for y); $\mathcal{R} = \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$

$$I_{\mu\nu}^{\mathrm{LO}}(z_1, z_2) = \frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\frac{\partial^2}{\partial x^\mu \partial y^\nu}\Big[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\Big]$$

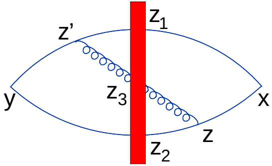


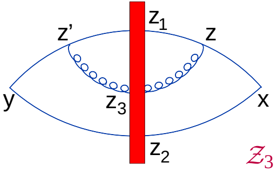


$$\mathcal{Z}_3 \equiv \frac{(x-z_3)_\perp^2}{x^+} - \frac{(y-z_3)_\perp^2}{y^+}$$

$$I_{\mu\nu}^{\rm NLO}(x,y;z_1,z_2,z_3;\eta) \;=\; -\, I_{\mu\nu}^{\rm LO} \times \frac{\alpha_s}{2\pi}\frac{z_{13}^2}{z_{12}^2 z_{23}^2}\ln\frac{\sigma s}{4}\mathcal{Z}_3 \;+\; \text{conf.}$$

The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff $\eta = \ln\sigma$ is not invariant.





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The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff $\eta = \ln\sigma$ is not invariant.

However, if we define a composite operator (a - analog of μ^{-2} for usual OPE)

$$[{\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]^{\rm conf} \ = \ {\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\} + \frac{\alpha_s}{4\pi}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\frac{1}{N_c}{\rm tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\} - {\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]\ln\frac{az_{12}^2}{z_{13}^2 z_{23}^2} \ + \ O(\alpha_s^2)$$

the impact factor becomes conformal in the NLO.

$$[\mathrm{Tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}_{a,\eta}$$
$$= \mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\} + \frac{\alpha_s}{2\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{Tr}\{T^n\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_3}T^n\hat U^\sigma_{z_3}\hat U^{\dagger\sigma}_{z_2}\} - N_c\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]\ln\frac{4az_{12}^2}{sz_{13}^2z_{23}^2} + O(\alpha_s^2)$$

choose a rapidity-dependent constant $a \to ae^{-2\eta} \Rightarrow [\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]^{\rm conf}_a$ does not depend on $\eta = \ln\sigma$ and all the rapidity dependence is encoded into a-dependence:

$$[\mathrm{Tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}_{a}$$
$$= \mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\} + \frac{\alpha_s}{2\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{Tr}\{T^n\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_3}T^n\hat U^\sigma_{z_3}\hat U^{\dagger\sigma}_{z_2}\} - N_c\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]\ln\frac{4az_{12}^2}{\sigma^2sz_{13}^2z_{23}^2} + O(\alpha_s^2)$$

Using the leading-order evolution equation

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\} = \sigma\frac{d}{d\sigma}\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\} = \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{Tr}\{T^n\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_3}T^n\hat U^\sigma_{z_3}\hat U^{\dagger\sigma}_{z_2}\}-N_c\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]$$

$\Rightarrow \frac{d}{d\eta}[\mathrm{Tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}_a = 0$ (with $O(\alpha_s^2)$ accuracy).

$$2a\frac{d}{da}[\mathrm{Tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}_a = \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{Tr}\{T^n\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_3}T^n\hat U^\sigma_{z_3}\hat U^{\dagger\sigma}_{z_2}\} - N_c\mathrm{Tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]$$

Operator expansion in conformal dipoles

Analogy:

When the UV cutoff does not respect the symmetry of a local operator, the composite local renormalized operator must be corrected by finite counter-terms order by order in perturbation theory.

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm tr}[\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf}$$
$$+\int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[\frac{1}{N_c}{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

$$I^{\rm NLO}_{\mu\nu} = -I^{\rm LO}_{\mu\nu}\frac{\alpha_s N_c}{4\pi}\int dz_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\ln\frac{z_{12}^2 e^{2\eta} a s^2}{z_{13}^2 z_{23}^2}\mathcal{Z}_3^2 + {\rm conf.}$$

The new NLO impact factor is conformally invariant.

In conformal $\mathcal{N}=4$ SYM theory one can construct the composite conformal dipole operator order by order in perturbation theory.

NLO structure functions for DIS off a large nucleus

$$(x-y)^4 T\{\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)\hat{\bar{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\Big\{I^{\mu\nu}_{\rm LO}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big][\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}$$

$$+\int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)}-2C\Big)I^{\mu\nu}_{\rm LO}+\ I_2^{\mu\nu}\Big]$$

$$\times[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\}-N_c\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}\Big\}$$

where

$$(I_2)_{\mu\nu}(z_1,z_2,z_3) = \frac{\alpha_s}{16\pi^8}\frac{\mathcal{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)}+\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}$$

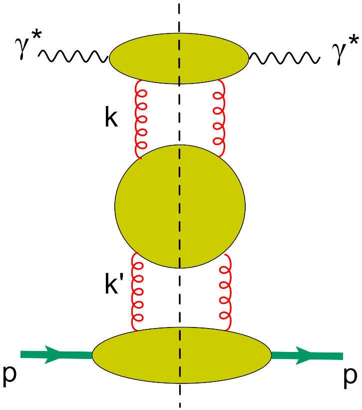
$$+\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)}-\frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\Big]+\frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)}-\frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\Big]$$

$$+(\zeta_1\leftrightarrow\zeta_2)\Big\}$$

NLO Photon Impact Factor for BFKL pomeron

Fourier transformation for the impact factors in the forward case corresponding to deep inelastic scattering at $x_B = \frac{Q^2}{2p\cdot q}$

$$\int d^4x\, e^{iqx} \int d^4z\, \delta(z_\bullet) T\{\hat{j}_\mu(x+z)\hat{j}_\nu(z)\}$$



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compare with

k_T-Factorization

$$A(s,0) \;=\; s\int \frac{d^2k}{k^2}\frac{d^2k'}{k'^2}F_A(k)F_B(k') \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i}\, f_+(\omega)\Big(\frac{s}{kk'}\Big)^{\omega} G_\omega(k,k')$$

$F_A(k)$ spectator's impact factor
$F_B(k')$ and target's impact factor
$f_+(\omega) = \frac{1-e^{-i\pi\omega}}{\sin \pi\omega}$ is the signature factor

$$\omega G_\omega(k,k') = \delta^2(k-k') + \int d^2p K(k,p)G_\omega(p,k')$$

$G_\omega(k,k')$ partial wave of the forward reggeized gluon scattering amplitude satisfying the BFKL equation

NLO Photon Impact Factor for BFKL pomeron

I. Balitsky and G. A. C. 2010

$$\Delta \equiv (x-y), \qquad x_* = x^+\sqrt{s/2}, \qquad y_* = y^+\sqrt{s/2}, \qquad R \equiv \frac{\Delta^2 z_{12\perp}^2}{x_* y_* \mathcal{Z}_1 \mathcal{Z}_2}$$

$$I^{NLO}_{\mu\nu}(x,y) = \frac{\alpha_s}{4\pi^7 \Delta^4} \frac{\partial\kappa^\alpha}{\partial x^\mu} \frac{\partial\kappa^\beta}{\partial y^\nu} \int \frac{dz_1 dz_2}{z_{12}^4}\, \mathcal{U}(z_1,z_2) R^2 \Big\{ -\frac{2}{\kappa^2}\Big(g^{\alpha\beta} - 2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\Big)$$

$$+\frac{\zeta_1^\alpha \zeta_2^\beta + \zeta_1 \leftrightarrow \zeta_2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \Big[4\mathrm{Li}_2(1-R) - \frac{2\pi^2}{3} + \frac{2\ln R}{1-R} + \frac{\ln R}{R} - 4\ln R + \frac{1}{2R} - 2 - 4C - \frac{2C}{R}$$

$$+2\big(\ln\frac{1}{R} + \frac{1}{R} - 2\big)\big(\ln\frac{1}{R} + 2C\big)\Big] + \Big(\frac{\zeta_1^\alpha \zeta_1^\beta}{(\kappa\cdot\zeta_1)^2} + \zeta_1 \leftrightarrow \zeta_2\Big)\Big[\frac{\ln R}{R} - \frac{2C}{R} + 2\frac{\ln R}{1-R} - \frac{1}{2R}\Big]$$

$$+\Big[-2\frac{\ln R}{1-R} - \frac{\ln R}{R} + \ln R - \frac{3}{2R} + \frac{5}{2} + 2C + \frac{2C}{R}\Big]\Big[\frac{\zeta_1^\alpha \kappa^\beta + \zeta_1^\beta \kappa^\alpha}{(\kappa\cdot\zeta_1)\kappa^2} + \zeta_1 \leftrightarrow \zeta_2\Big]$$

$$+\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[\frac{2\pi^2}{3} - 4\mathrm{Li}_2(1-R) - 2\big(\ln\frac{1}{R} + \frac{1}{R} + \frac{1}{2R^2} - 3\big)\big(\ln\frac{1}{R} + 2C\big)$$

$$+6\ln R - \frac{2}{R} + 2 + \frac{3}{2R^2}\Big]\Big\}$$

Photon Impact Factor at NLO

Conformal vectors

$$\kappa^\mu = \frac{\sqrt{s}}{2x_*}\left(\frac{p_1^\mu}{s} - x^2 p_2^\mu + x_\perp^\mu\right) - \frac{\sqrt{s}}{2y_*}\left(\frac{p_1^\mu}{s} - y^2 p_2^\mu + y_\perp^\mu\right)$$

$$\zeta_1^\mu = \left(\frac{p_1^\mu}{s} + z_{1\perp}^2 p_2^\mu + z_{1\perp}^\mu\right), \qquad \zeta_2^\mu = \left(\frac{p_1^\mu}{s} + z_{2\perp}^2 p_2^\mu + z_{2\perp}^\mu\right)$$

DIS photon impact factor is a linear combination of the following tensor basis

$$\mathcal{I}_1^{\mu\nu} = g^{\mu\nu} \qquad \mathcal{I}_2^{\mu\nu} = \frac{\kappa^\mu\kappa^\nu}{\kappa^2}$$

$$\mathcal{I}_3^{\mu\nu} = \frac{\kappa^\mu\zeta_1^\nu + \kappa^\nu\zeta_1^\mu}{\kappa\cdot\zeta_1} + \frac{\kappa^\mu\zeta_2^\nu + \kappa^\nu\zeta_2^\mu}{\kappa\cdot\zeta_2}$$

$$\mathcal{I}_4^{\mu\nu} = \frac{\kappa^2\zeta_1^\mu\zeta_1^\nu}{(\kappa\cdot\zeta_1)^2} + \frac{\kappa^2\zeta_2^\mu\zeta_2^\nu}{(\kappa\cdot\zeta_2)^2} \qquad \mathcal{I}_5^{\mu\nu} = \frac{\zeta_1^\mu\zeta_2^\nu + \zeta_2^\mu\zeta_1^\nu}{\zeta_1\cdot\zeta_2}$$

Cornalba, Costa, Penedones (2010)

Projection of the LO impact factor on the eigenfunctions

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^2} I_{LO}^{\mu\nu}(z_1, z_2)\Big(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\Big)^\gamma = \frac{1}{\Delta^2 x_* y_*} B(1-\gamma)\Gamma(\gamma+2)\Gamma(3-\gamma)$$
$$\times\Big\{\frac{\gamma(1-\gamma)D_1^{\mu\nu}}{12(1+\gamma)(2-\gamma)} + \frac{D_2^{\mu\nu}}{2(1+\gamma)(2-\gamma)} - \frac{D_3^{\mu\nu}}{8(1+\gamma)(2-\gamma)}$$
$$-\frac{\gamma(1-\gamma)D_4}{16(1+2\gamma)(3-2\gamma)(1+\gamma)(2-\gamma)} - \frac{D_1^{\mu\nu}+D_2^{\mu\nu}}{8}\Big\}_{\mu\nu}\Big(\frac{\kappa^2}{(\kappa\cdot\zeta_0)^2}\Big)^\gamma$$

where

$$(D_1+D_2)^{\mu\nu} = -2\Delta^2 x_* y_* \kappa^{-2}\partial_x^\mu \partial_y^\nu \kappa^2$$
$$D_2^{\mu\nu} = -\Delta^2 x_* y_* \partial_x^\mu(\ln\kappa^2)\partial_y^\nu \ln\kappa^2$$
$$D_3^{\mu\nu} = 4\gamma\Delta^2 x_* y_*\big[(\partial_x^\mu \ln\kappa^2)\partial_\nu^y \ln(\kappa\cdot\zeta_0) + (\partial_y^\nu \ln\kappa^2)\partial_\mu^x \ln(\kappa\cdot\zeta_0) - (\partial_x^\mu \ln\kappa^2)\partial_y^\nu \ln\kappa^2\big]$$
$$D_4^{\mu\nu} = 4\gamma(1+2\gamma)\Delta^2 x_* y_*\big[-\frac{1}{3}\partial_x^\mu\partial_y^\nu \ln\kappa^2 - \partial_x^\mu(\ln\kappa^2)\partial_y^\nu \ln\kappa^2$$
$$+(\partial_x^\mu \ln\kappa^2)\partial_\nu^y \ln(\kappa\cdot\zeta_0) + (\partial_y^\nu \ln\kappa^2)\partial_\mu^x \ln(\kappa\cdot\zeta_0) - 2\partial_\mu^x \ln(\kappa\cdot\zeta_0)\partial_\nu^y \ln(\kappa\cdot\zeta_0)\big]$$

Mellin representation of the NLO photon impact factor

Conformal spin 0: NLO impact factor for the unpolarized forward structure functions

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^4} I^{\mu\nu}_{NLO}(z_1,z_2)\Big(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\Big)^\gamma = \alpha_s \frac{B(1-\gamma)\Gamma(3-\gamma)\Gamma(2+\gamma)}{(2-\gamma)(1+\gamma)} \times$$

$$\Bigg\{ \frac{D_1^{\mu\nu}}{3\sin^2(\gamma\pi)}\Bigg[(1-\cos(2\gamma\pi))\left(\chi - 1 - \gamma(1-\gamma)\left(C\chi - \frac{1}{2}\right)\right) - \gamma(1-\gamma)\frac{\pi^2}{3}(5+\cos(2\gamma\pi))\Bigg]$$

$$+D_2^{\mu\nu}\left[-\frac{3}{\gamma(1-\gamma)} + 2\chi\left(\frac{1}{\gamma(1-\gamma)} - 2C + 1\right) + \frac{4}{3}\pi^2(1 - \frac{3}{\sin^2(\gamma\pi)})\right]$$

$$+D_3^{\mu\nu}\left[C\chi - \frac{1}{2} - \frac{1}{\gamma(1-\gamma)} - \frac{\chi}{4}\left(1 + \frac{2}{\gamma(1-\gamma)}\right) - \frac{\pi^2}{3}(1 - \frac{3}{\sin^2(\gamma\pi)})\right]$$

$$+\frac{D_4^{\mu\nu}}{4[3+4\gamma(1-\gamma)]}\left[\frac{15}{\gamma(1-\gamma)} + 10 + \gamma(1-\gamma) - \chi - 2\gamma(1-\gamma)\left(C\chi - \frac{\pi^2}{3} + \frac{\pi^2}{\sin^2(\gamma\pi)}\right)\right]$$

$$+\frac{D_1^{\mu\nu} + D_2^{\mu\nu}}{[2+\gamma(1-\gamma)]^{-1}}\left[-\frac{1}{2} - \frac{\pi^2}{3} + \frac{\pi^2}{\sin^2\pi\gamma} + \frac{4\gamma(1-\gamma)+3}{2\gamma(1-\gamma)(1+\gamma)(2-\gamma)}\right.$$

$$\left.+C\chi(\gamma) - \frac{1+2\gamma(1-\gamma)}{\gamma(1-\gamma)(1+\gamma)(2-\gamma)}\chi(\gamma)\right]\Bigg\}$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

C is the Euler constant.

Mellin representation of the NLO photon impact factor

Conformal spin 2: NLO impact factor for the polarized forward structure functions

$$\frac{N_c}{\pi^6(x-y)^4}\int\frac{d^2z_1d^2z_2}{z_{12}^4}\left[I^{\mu\nu}_{LO}(z_1,z_2)+I^{\mu\nu}_{NLO}(z_1,z_2)\right]\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^\gamma\left(\frac{\tilde{z}_{12}}{\tilde{z}_{10}\tilde{z}_{20}}\right)\left(\frac{\bar{z}_{12}}{\bar{z}_{10}\bar{z}_{20}}\right)^{-1}$$

$$= -\frac{N_c}{\pi^4\Delta^4}\frac{B(2-\gamma)}{2\Delta^2}\Gamma(3-\gamma)\Gamma(\gamma+2)\left(\frac{\Delta^2}{x_*y_*\mathcal{Z}_0^2}\right)^\gamma$$

$$\times\left[g^{\mu1}-ig^{\mu2}+2x^\mu\frac{\bar{x}y_*-\bar{y}x_*}{x_*y_*\mathcal{Z}_0}+p_2^\mu\frac{x^2\bar{y}-y^2\bar{x}}{x_*y_*\mathcal{Z}_0}\right]\left[g^{\nu1}-ig^{\nu2}+2y^\nu\frac{\bar{x}y_*-\bar{y}x_*}{x_*y_*\mathcal{Z}_0}+p_2^\nu\frac{x^2\bar{y}-y^2\bar{x}}{x_*y_*\mathcal{Z}_0}\right]$$

$$\times\left[1+\frac{\alpha_sN_c}{4\pi}\left\{\frac{4\pi^2}{\sin^2\pi\gamma}-\frac{4\pi^2}{3}+4C\chi(2,\gamma)-\frac{4}{\gamma^2}-\frac{4}{\bar{\gamma}^2}-2-6\frac{1+\chi(2,\gamma)}{2+\bar{\gamma}\gamma}\right\}\right]$$

$\chi(2,\gamma)=\chi(\gamma)-\frac{1}{\gamma(1-\gamma)}$ $\quad\bar{x}=x^1-ix^2$

"energy scale" $a_0 = -\kappa^{-2}$ for color dipoles depends on x and y
$\hat{\mathcal{U}}_{a_0}$ in terms of $\hat{\mathcal{U}}_{a_m}$ with a_m independent of coordinates x and y. We choose $a_m = 1/x_B$.

$$\hat{\mathcal{U}}_{a_0}(\nu, z_0) \;=\; \hat{\mathcal{U}}_{a_m}(\nu, z_0)\big(a_0 x_B\big)^{\frac{\omega(\nu)}{2}}$$

$$\hat{\mathcal{U}}^{(2)}_{a_0}(\nu, z_0) \;=\; \hat{\mathcal{U}}^{(2)}_{a_m}(\nu, z_0)\big(a_0 x_B\big)^{\frac{\omega(2,\nu)}{2}}$$

$$\frac{1}{N_c}\int d^4x d^4y\;\delta(y_\bullet)\;e^{iq\cdot(x-y)}\;T\{\bar{\psi}\gamma_\mu\psi(x)\bar{\psi}\gamma_\nu\psi(y)\}$$

$$= \int\frac{d\nu}{\pi^3}\int d^2z_0\;\Big\{\frac{\Gamma\big(\bar{\gamma}+\frac{\omega(\nu)}{2}\big)\Gamma^2\big(2-\gamma+\frac{\omega(\nu)}{2}\big)}{\Gamma(4-2\gamma+\omega(\nu))\Gamma\big(2+\gamma+\frac{\omega(\nu)}{2}\big)}$$

$$\times\,\frac{2\gamma-1}{2\gamma+1}\Gamma(2-\gamma)\Big[(\gamma\bar{\gamma}+2)P_1^{\mu\nu}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\Phi_1(\nu)\Big)$$

$$+(3\gamma\bar{\gamma}+2)P_2^{\mu\nu}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\Phi_2(\nu)\Big)\Big]\mathcal{U}_{a_m}(z_0,\nu)$$

Photon Impact Factor for BFKL pomeron in momentum space

k_T-factorization form

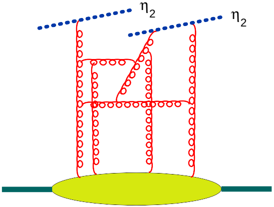
$$
\begin{aligned}
&I^{\mu\nu}(q,k_\perp)\\
&= \frac{N_c}{32}\int\frac{d\nu}{\pi\nu}\frac{\sinh\pi\nu}{(1+\nu^2)\cosh^2\pi\nu}\left(\frac{k_\perp^2}{Q^2}\right)^{\frac{1}{2}-i\nu}\left\{\left[\left(\frac{9}{4}+\nu^2\right)\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_1(\nu)\right)P_1^{\mu\nu}\right.\right.\\
&\left.+\left(\frac{11}{4}+3\nu^2\right)\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_2(\nu)\right)P_2^{\mu\nu}\right]\\
&\left.+\frac{\frac{1}{4}+\nu^2}{2k_\perp^2}\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_3(\nu)\right)\left[\tilde{P}^{\mu\nu}\bar{k}^2+\bar{P}^{\mu\nu}\tilde{k}^2\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
P_1^{\mu\nu} &= g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\\
P_2^{\mu\nu} &= \frac{1}{q^2}\left(q^\mu-\frac{p_2^\mu q^2}{q\cdot p_2}\right)\left(q^\nu-\frac{p_2^\nu q^2}{q\cdot p_2}\right)\\
\bar{P}^{\mu\nu} &= \left(g^{\mu1}-ig^{\mu2}-p_2^\mu\frac{\bar{q}}{q\cdot p_2}\right)\left(g^{\nu1}-ig^{\nu2}-p_2^\nu\frac{\bar{q}}{q\cdot p_2}\right)\\
\tilde{P}^{\mu\nu} &= \left(g^{\mu1}+ig^{\mu2}-p_2^\mu\frac{\tilde{q}}{q\cdot p_2}\right)\left(g^{\nu1}+ig^{\nu2}-p_2^\nu\frac{\tilde{q}}{q\cdot p_2}\right)
\end{aligned}
$$

k_T-factorization form

$$
\begin{aligned}
\mathcal{F}_{1(2)}(\nu) &= \Phi_{1(2)}(\nu) + \chi_\gamma \Psi(\nu), \quad \mathcal{F}_3(\nu) = F_6(\nu) + \Big(\chi_\gamma - \frac{1}{\bar\gamma\gamma}\Big)\Psi(\nu), \\
\Psi(\nu) &\equiv \psi(\bar\gamma) + 2\psi(2-\gamma) - 2\psi(4-2\gamma) - \psi(2+\gamma), \\
F_6(\gamma) &= F(\gamma) - \frac{2C}{\bar\gamma\gamma} - 1 - \frac{2}{\gamma^2} - \frac{2}{\bar\gamma^2} - 3\frac{1+\chi_\gamma - \frac{1}{\gamma\bar\gamma}}{2+\bar\gamma\gamma}, \\
\Phi_1(\nu) &= F(\gamma) + \frac{3\chi_\gamma}{2+\bar\gamma\gamma} + 1 + \frac{25}{18(2-\gamma)} + \frac{1}{2\bar\gamma} - \frac{1}{2\gamma} - \frac{7}{18(1+\gamma)} + \frac{10}{3(1+\gamma)^2} \\
\Phi_2(\nu) &= F(\gamma) + \frac{3\chi_\gamma}{2+\bar\gamma\gamma} + 1 + \frac{1}{2\bar\gamma\gamma} - \frac{7}{2(2+3\bar\gamma\gamma)} + \frac{\chi_\gamma}{1+\gamma} + \frac{\chi_\gamma(1+3\gamma)}{2+3\bar\gamma\gamma}, \\
F(\gamma) &= \frac{2\pi^2}{3} - \frac{2\pi^2}{\sin^2\pi\gamma} - 2C\chi_\gamma + \frac{\chi_\gamma - 2}{\bar\gamma\gamma}
\end{aligned}
$$

Matrix elements of Wilson lines: $\langle \mathrm{Tr}\{U(x)U^\dagger(y)\}\rangle_A$ are divergent



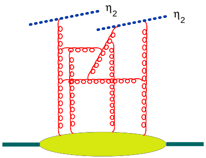
For light-like Wilson lines loop integrals are divergent in the longitudinal direction

$$\int_0^\infty \frac{d\alpha}{\alpha} = \int_{-\infty}^{\infty} d\eta = \infty$$

$$F(x_B) \simeq \int d^2z_1 d^2z_2 \, I^{LO}(z_1,z_2)\langle \mathrm{tr}\{U^\eta_{z_1} U^{\dagger\,\eta}_{z_2}\}\rangle \qquad \eta = \ln\frac{1}{x_B}$$
$$+ \frac{\alpha_s}{\pi}\int d^2z_1 d^2z_2 d^2z_3 \, I^{NLO}(z_1,z_2,z_3)\langle \mathrm{tr}\{U^\eta_{z_1} U^{\dagger\eta}_{z_3}\}\mathrm{tr}\{U_{z_3} U^{\dagger\eta}_{z_2}\}\rangle$$

Regularization of the rapidity divergence

Matrix elements of Wilson lines: $\langle \mathrm{Tr}\{U(x)U^\dagger(y)\rangle_A\}$ are divergent



For light-like Wilson lines loop integrals are divergent in the longitudinal direction

$$\int_0^\infty \frac{d\alpha}{\alpha} = \int_{-\infty}^{\infty} d\eta = \infty$$

Regularization by: slope

$$U^\eta(x_\perp) = \mathrm{Pexp}\Big\{ig\int_{-\infty}^{\infty} du\; n_\mu\, A^\mu(un + x_\perp)\Big\} \qquad n^\mu = p_1^\mu + e^{-2\eta} p_2^\mu$$

Regularization by: Rigid cut-off (used in NLO)

$$U_x^\eta = \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} du\; p_1^\mu A_\mu^\eta(up_1 + x_\perp)\Big]$$

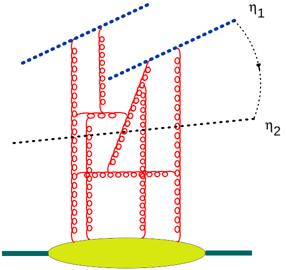
$$A_\mu^\eta(x) = \int \frac{d^4k}{(2\pi)^4}\theta(e^\eta - |\alpha_k|)e^{-ik\cdot x}A_\mu(k)$$

$k^\mu = \alpha_k\, p_1^\mu + \beta_k\, p_2^\mu + k_\perp^\mu$

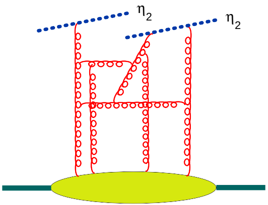
$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} \Rightarrow \frac{d}{d\eta}\langle\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle$$

To get the evolution equation, consider the dipole with the rapidies up to η_1 and integrate over the gluons with rapidity $\eta_1 > \eta > \eta_2$. This integral gives the kernel of the evolution equation (multiplied by the dipole(s) with rapidity up to η_2).

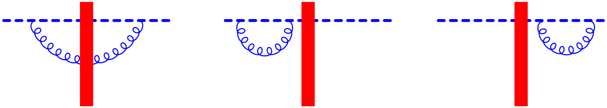
In the frame $||$ to η_1 the gluons with $\eta < \eta_1$ are seen as pancake.



$$\alpha_s(\eta_1 - \eta_2)K_{\mathrm{evol}} \otimes$$



Particles with different rapidity perceive each other as Wilson lines.



$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta \int \frac{d^2 z_\perp}{(x-z)_\perp^2}\left[\langle \mathrm{tr}\{U_x^{\eta_2} U_z^{\eta_2 \dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle \frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

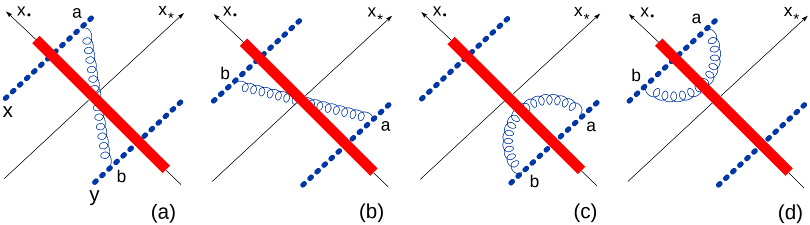
$\Delta = \eta_1 - \eta_2$

$$\{U_x^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\eta_1} U_y^{\eta_1}\}_{ij}, \quad \{U_x^{\eta_1} U_y^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\dagger\eta_1} U_y^{\dagger\eta_1}\}_{ij}$$

Leading order: BK equation

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \dots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$

Non-linear evolution equation: BK equation

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \quad \Rightarrow (U_x U_y^\dagger)^{\eta_1} \to (U_x U_y^\dagger)^{\eta_2} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

Non-linear evolution equation: BK equation

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \quad \Rightarrow (U_x U_y^\dagger)^{\eta_1} \to (U_x U_y^\dagger)^{\eta_2} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

BK equation: **Ian Balitsky (1996), Yu. Kovchegov (1999)**

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\,(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y) - \hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

Alternative approach: JIMWLK (1997-2000)

Non-linear evolution equation: BK equation

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \quad \Rightarrow (U_x U_y^\dagger)^{\eta_1} \to (U_x U_y^\dagger)^{\eta_2} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

BK equation: **Ian Balitsky (1996), Yu. Kovchegov (1999)**

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\,(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y) - \hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

Alternative approach: JIMWLK (1997-2000)

LLA for DIS in pQCD $\Rightarrow$ BFKL (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$)

Non linear evolution equation: BK equation

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \quad \Rightarrow (U_x U_y^\dagger)^{\eta_1} \rightarrow (U_x U_y^\dagger)^{\eta_2} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

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BK equation: **Ian Balitsky (1996), Yu. Kovchegov (1999)**

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\,(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y) - \hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

Alternative approach: JIMWLK (1997-2000)

LLA for DIS in pQCD $\Rightarrow$ BFKL (LLA: $\alpha_s \ll 1,\ \alpha_s\eta \sim 1$)

LLA for DIS in sQCD $\Rightarrow$ BK eqn (LLA: $\alpha_s \ll 1,\ \alpha_s\eta \sim 1,\ \alpha_s^2 A^{1/3} \sim 1$)

(s for semi-classical)

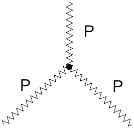
The triple Pomeron vertex: Fan Diagrams

The Balitsky equation becomes the BK equation when

$$\langle \mathrm{tr}\{1 - U_x U_z^\dagger\}\mathrm{tr}\{1 - U_z U_y^\dagger\}\rangle$$
$$= \frac{N_c^2}{2(N_c^2-1)}\Big\{2\langle\mathcal{U}_{xz}\rangle\langle\mathcal{U}_{zy}\rangle + \frac{1}{N_c^2}\Big[2\langle\mathcal{U}_{xy}\rangle\big(\langle\mathcal{U}_{xy}\rangle - \langle\mathcal{U}_{xz}\rangle - \langle\mathcal{U}_{yz}\rangle\big) + \langle\mathcal{U}_{zy}\rangle\langle\mathcal{U}_{zy}\rangle + \langle\mathcal{U}_{xz}\rangle\langle\mathcal{U}_{xz}\rangle - \langle\mathcal{U}_{xy}\rangle\langle\mathcal{U}_{xy}\rangle\Big]\Big\}$$

We extract the non planar (next-to-leading in N_c) contribution from $\langle\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\rangle$ for diffractive processes and for "fan" diagrams.

G.A.C, L.Szymanowski and S.Wallon 2010



We get

$$\int d^2\rho_a d^2\rho_b\, 16\, h_\alpha(h_\alpha-1)\bar{h}_\alpha(\bar{h}_\alpha-1)E_{h_\alpha\bar{h}_\alpha}(\rho_{a\alpha},\rho_{b\alpha})\Bigg[\int d^2\rho_c\,\frac{1}{|\rho_{ab}|^2|\rho_{ac}|^2|\rho_{bc}|^2}E_{h_\beta\bar{h}_\beta}(\rho_{a\beta},\rho_{c\beta})E_{h_\gamma\bar{h}_\gamma}(\rho_{b\gamma},\rho_{c\gamma})$$
$$-\frac{2\pi}{N_c^2}\frac{1}{|\rho_{ab}|^4}\mathrm{Re}\{\psi(1)+\psi(h_\alpha)-\psi(h_\beta)-\psi(h_\gamma)\}E_{h_\beta\bar{h}_\beta}(\rho_{a\beta},\rho_{b\beta})E_{h_\gamma\bar{h}_\gamma}(\rho_{b\gamma},\rho_{c\gamma})\Bigg]$$

which agrees with Bartels and Wusthoff (1995)

Conformal invariance of the BK equation

Formally, a light-like Wilson line

$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] = \text{Pexp}\left\{ig\int_{-\infty}^{\infty} dx^+ \, A_+(x^+, x_\perp)\right\}$$

is invariant under inversion (with respect to the point with $x^- = 0$).

Indeed,
$(x^+, x_\perp)^2 = -x_\perp^2 \Rightarrow$ after the inversion $x_\perp \to x_\perp/x_\perp^2$ and $x^+ \to x^+/x_\perp^2 \Rightarrow$

$$[\infty p_1 + x_\perp, -\infty p_1 + x_\perp] \to \text{Pexp}\left\{ig\int_{-\infty}^{\infty} d\frac{x^+}{x_\perp^2} A_+\left(\frac{x^+}{x_\perp^2}, \frac{x_\perp}{x_\perp^2}\right)\right\} = [\infty p_1 + \frac{x_\perp}{x_\perp^2}, -\infty p_1 + \frac{x_\perp}{x_\perp^2}]$$

$\Rightarrow$The dipole kernel is invariant under the inversion $V(x_\perp) = U(x_\perp/x_\perp^2)$

$$\frac{d}{d\eta}\text{Tr}\{V_x V_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int \frac{d^2z}{\not{z}^4} \frac{(x-y)^2 \not{z}^4}{(x-z)^2(z-y)^2}[\text{Tr}\{V_x V_z^\dagger\}\text{Tr}\{V_z V_y^\dagger\} - N_c\text{Tr}\{V_x V_y^\dagger\}]$$

Conformal invariance of the BK equation

SL(2,C) for Wilson lines

$$\hat{S}_- \equiv \frac{i}{2}(K^1 + iK^2), \quad \hat{S}_0 \equiv \frac{i}{2}(D + iM^{12}), \quad \hat{S}_+ \equiv \frac{i}{2}(P^1 - iP^2)$$

$$[\hat{S}_0, \hat{S}_\pm] = \pm\hat{S}_\pm, \quad \frac{1}{2}[\hat{S}_+, \hat{S}_-] = \hat{S}_0,$$

$$[\hat{S}_-, \hat{U}(z,\bar{z})] = z^2\partial_z\hat{U}(z,\bar{z}), \quad [\hat{S}_0, \hat{U}(z,\bar{z})] = z\partial_z\hat{U}(z,\bar{z}), \quad [\hat{S}_+, \hat{U}(z,\bar{z})] = -\partial_z\hat{U}(z,\bar{z})$$

$z \equiv z^1 + iz^2, \bar{z} \equiv z^1 + iz^2, \quad U(z_\perp) = U(z,\bar{z})$

Conformal invariance of the evolution kernel

$$\frac{d}{d\eta}[\hat{S}_-, \mathrm{Tr}\{U_x U_y^\dagger\}] = \frac{\alpha_s N_c}{2\pi^2}\int dz\, K(x,y,z)[\hat{S}_-, \mathrm{Tr}\{U_x U_y^\dagger\}\mathrm{Tr}\{U_x U_y^\dagger\}]$$

$$\Rightarrow \left[x^2\frac{\partial}{\partial x} + y^2\frac{\partial}{\partial y} + z^2\frac{\partial}{\partial z}\right]K(x,y,z) = 0$$

In the leading order - OK. In the NLO - ?

Non-linear evolution equation at NLO

$$\frac{d}{d\eta}Tr\{U_xU_y^\dagger\} =$$
$$\int \frac{d^2z}{2\pi^2}\left(\alpha_s\frac{(x-y)^2}{(x-z)^2(z-y)^2} + \alpha_s^2 K_{NLO}(x,y,z)\right)[Tr\{U_xU_z^\dagger\}Tr\{U_zU_y^\dagger\} - N_c Tr\{U_xU_y^\dagger\}] +$$
$$\alpha_s^2\int d^2z d^2z'\left(K_4(x,y,z,z')\{U_x, U_{z'}^\dagger, U_z, U_y^\dagger\} + K_6(x,y,z,z')\{U_x, U_{z'}^\dagger, U_{z'}, U_z, U_z^\dagger, U_y^\dagger\}\right)$$

K_{NLO} is the next-to-leading order correction to the dipole kernel and K4 and K6 are the coefficients in front of the (tree) four- and six-Wilson line operators with arbitrary white arrangements of color indices.

Definition of the NLO kernel

In general

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} = \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + O(\alpha_s^3)$$

$$\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} = \frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} - \alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + O(\alpha_s^3)$$

We calculate the “matrix element” of the r.h.s. in the shock-wave background

$$\langle\alpha_s^2 K_{\mathrm{NLO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle = \frac{d}{d\eta}\langle\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle - \langle\alpha_s K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle + O(\alpha_s^3)$$

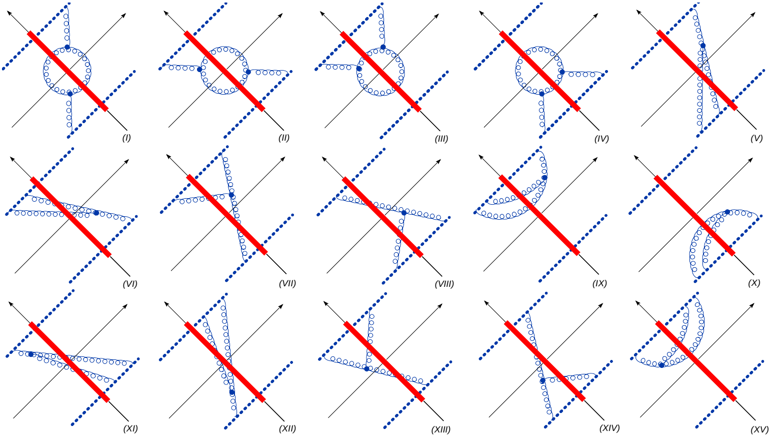
Subtraction of the (LO) contribution (with the rigid rapidity cutoff)

$\Rightarrow$ $\left[\frac{1}{v}\right]_+$ prescription in the integrals over Feynman parameter v

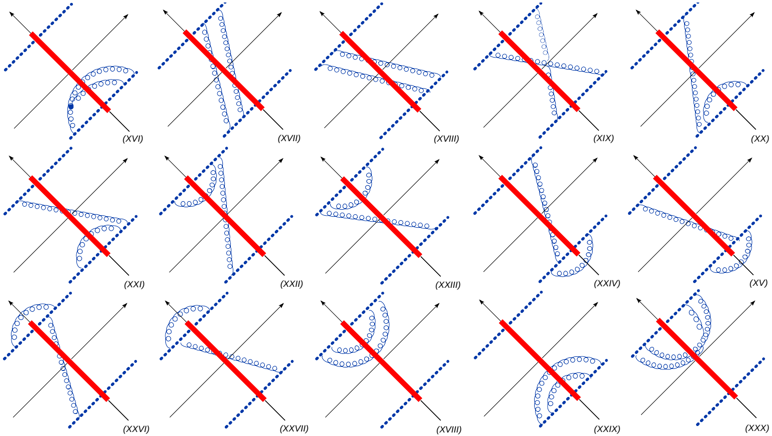
Typical integral

$$\int_0^1 dv\, \frac{1}{(k-p)_\perp^2 v + p_\perp^2(1-v)}\left[\frac{1}{v}\right]_+ = \frac{1}{p_\perp^2}\ln\frac{(k-p)_\perp^2}{p_\perp^2}$$

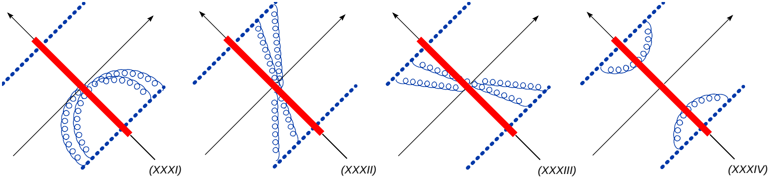
Diagrams with 2 gluons interaction



Diagrams with 2 gluons interaction

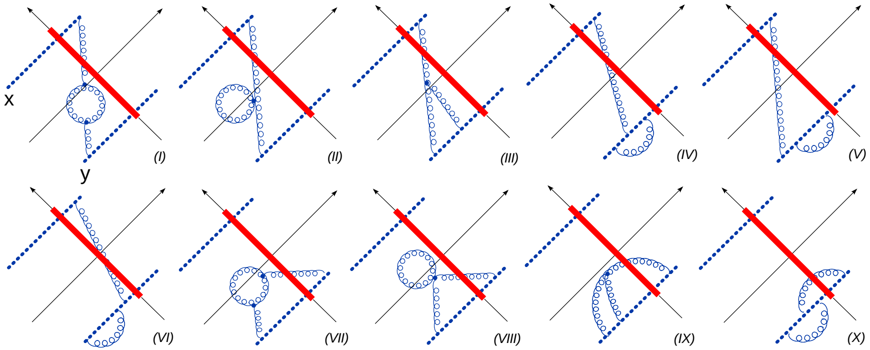


Diagrams with 2 gluons interaction

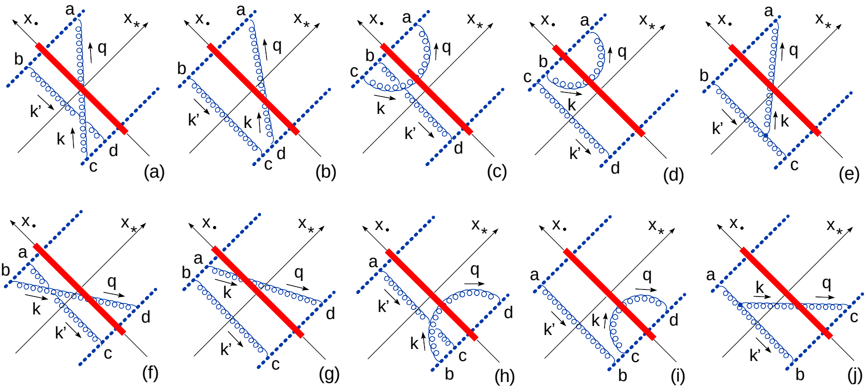


Diagrams of the NLO gluon contribution

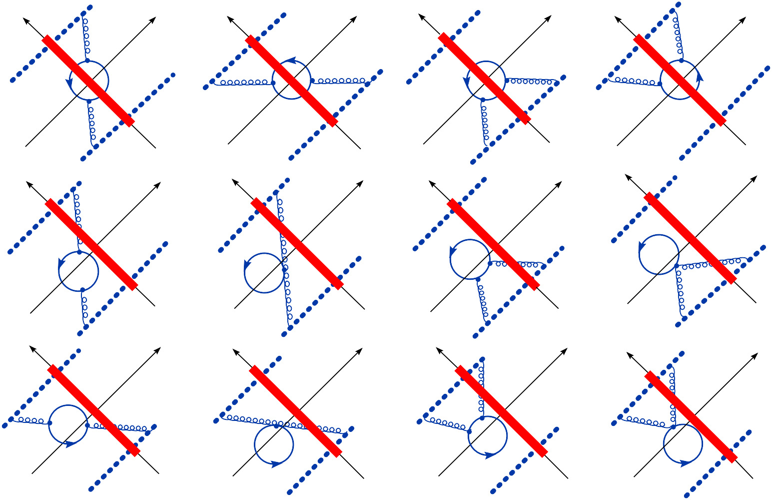
"Running coupling" diagrams



1 → 2 dipole transition diagrams



$\mathcal{N} = 4$ SYM diagrams (scalar and gluino loops)



Gluon contribution to the NLO kernel (I. Balitsky and G.A.C, 2007)

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big(\big[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}\big] \\
& \times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\Big(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3}\Big)\Big] \\
& -\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2} - \frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& +\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{\big[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\} \\
& -(z'\to z)\big]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big] \\
& +\big[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\} - (z'\to z)\big] \\
& \times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big)
\end{aligned}
$$

Our result Agrees with NLO BFKL

(Comparing the eigenvalue of the forward kernel)

It respects unitarity

Gluon contribution to the NLO kernel (I. Balitsky and G.A.C, 2007)

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_xU^\dagger_y\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_y\}-N_c\mathrm{Tr}\{U_xU^\dagger_y\}]\\
&\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\big(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3}\big)\Big]\\
&-\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}-\frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}\\
&+\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_{z'}\}\{U_{z'}U^\dagger_y\}-\mathrm{Tr}\{U_xU^\dagger_zU_{z'}U^\dagger_yU_zU^\dagger_{z'}\}\\
&-(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big]\\
&+[\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_{z'}\}\{U_{z'}U^\dagger_y\}-\mathrm{Tr}\{U_xU^\dagger_{z'}U_zU^\dagger_yU_{z'}U^\dagger_z\}-(z'\to z)]\\
&\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big)
\end{aligned}
$$

NLO kernel = Running coupling terms + Non-conformal term + Conformal term

Evolution equation for color dipoles in $\mathcal{N}=4$

(I. Balitsky and G.A.C. 2009)

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$$
$$= \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big\{1-\frac{\alpha_sN_c}{4\pi}\Big[\frac{\pi^2}{3}+2\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}\Big]\Big\}$$
$$\times\,[\mathrm{Tr}\{T^a\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^a\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\}-N_c\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]$$
$$-\,\frac{\alpha_s^2}{4\pi^4}\int\frac{d^2z_3d^2z_4}{z_{34}^4}\,\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{23}^2z_{14}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}$$
$$\times\,\mathrm{Tr}\{[T^a,T^b]\hat U^\eta_{z_1}T^{a'}T^{b'}\hat U^{\dagger\eta}_{z_2}+T^bT^a\hat U^\eta_{z_1}[T^{b'},T^{a'}]\hat U^{\dagger\eta}_{z_2}\}(\hat U^\eta_{z_3})^{aa'}(\hat U^\eta_{z_4}-\hat U^\eta_{z_3})^{bb'}$$

NLO kernel = Non-conformal term + Conformal term.

Non-conformal term is due to the non-invariant cutoff $\alpha<\sigma=e^{2\eta}$ in the rapidity of Wilson lines.

Evolution equation for color dipoles in $\mathcal{N}=4$

(I. Balitsky and G.A.C. 2009)

$$
\begin{aligned}
&\frac{d}{d\eta}\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\\
&= \frac{\alpha_s}{\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big\{1-\frac{\alpha_sN_c}{4\pi}\Big[\frac{\pi^2}{3}+2\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}\Big]\Big\}\\
&\times\,[\mathrm{Tr}\{T^a\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^a\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\}-N_c\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]\\
&-\frac{\alpha_s^2}{4\pi^4}\int\frac{d^2z_3d^2z_4}{z_{34}^4}\,\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{23}^2z_{14}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}\\
&\times\,\mathrm{Tr}\{[T^a,T^b]\hat U^\eta_{z_1}T^{a'}T^{b'}\hat U^{\dagger\eta}_{z_2}+T^bT^a\hat U^\eta_{z_1}[T^{b'},T^{a'}]\hat U^{\dagger\eta}_{z_2}\}(\hat U^\eta_{z_3})^{aa'}(\hat U^\eta_{z_4}-\hat U^\eta_{z_3})^{bb'}
\end{aligned}
$$

NLO kernel = Non-conformal term + Conformal term.

Non-conformal term is due to the non-invariant cutoff $\alpha<\sigma=e^{2\eta}$ in the rapidity of Wilson lines.

For the conformal composite dipole the result is Möbius invariant

$$
\begin{aligned}
\big[\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]^{\rm conf} \;=\; & \mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\} \\
+ \frac{\alpha_s}{4\pi}\int d^2z_3\; & \frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[\frac{1}{N_c}\mathrm{tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\} - \mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\Big]\ln\frac{az_{12}^2}{z_{13}^2z_{23}^2} \;+\; O(\alpha_s^2)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{d}{d\eta}\big[\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]^{\rm conf} \\
&= \frac{\alpha_s}{\pi^2}\int d^2z_3\;\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1-\frac{\alpha_sN_c}{4\pi}\frac{\pi^2}{3}\Big]\big[\mathrm{Tr}\{T^a\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^a\hat U_{z_3}\hat U^{\dagger\eta}_{z_2}\} - N_c\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}\big]^{\rm conf} \\
&\quad - \frac{\alpha_s^2}{4\pi^4}\int d^2z_3d^2z_4\frac{z_{12}^2}{z_{13}^2z_{24}^2z_{34}^2}\Big\{2\ln\frac{z_{12}^2z_{34}^2}{z_{14}^2z_{23}^2} + \Big[1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{14}^2z_{23}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}\Big\} \\
&\quad\times \mathrm{Tr}\{[T^a,T^b]\hat U^\eta_{z_1}T^{a'}T^{b'}\hat U^{\dagger\eta}_{z_2} + T^bT^a\hat U^\eta_{z_1}[T^{b'},T^{a'}]\hat U^{\dagger\eta}_{z_2}\}[(\hat U^\eta_{z_3})^{aa'}(\hat U^\eta_{z_4})^{bb'} - (z_4\to z_3)]
\end{aligned}
$$

Now Möbius invariant!

NLO evolution of composite “conformal” dipoles in QCD

$$\frac{d}{d\eta}[\mathrm{tr}\{\hat U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{conf}} = \frac{\alpha_s}{2\pi^2}\int d^2z_3\,\Big([\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\}-N_c\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\mathrm{conf}}$$

$$\times\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\Big(b\ln z_{12}^2\mu^2+b\frac{z_{13}^2-z_{23}^2}{z_{13}^2z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2}+\frac{67}{9}-\frac{\pi^2}{3}\Big)\Big]$$

$$+\frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\Big\{\Big[-2+\frac{z_{14}{}^2z_{23}^2+z_{24}{}^2z_{13}^2-4z_{12}^2z_{34}^2}{2(z_{14}{}^2z_{23}^2-z_{24}{}^2z_{13}^2)}\ln\frac{z_{14}{}^2z_{23}^2}{z_{24}{}^2z_{13}^2}\Big]$$

$$\times[\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\{\hat U_{z_4}\hat U^\dagger_{z_2}\}-\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\hat U_{z_4}\hat U^\dagger_{z_2}\hat U_{z_3}\hat U^\dagger_{z_4}\}-(z_4\to z_3)]$$

$$+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}{}^2}\Big[2\ln\frac{z_{12}^2z_{34}^2}{z_{14}^2z_{23}^2}+\Big(1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{14}^2z_{23}^2}\Big)\ln\frac{z_{13}^2z_{24}{}^2}{z_{14}{}^2z_{23}^2}\Big]$$

$$\times[\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\mathrm{tr}\{\hat U_{z_4}\hat U^\dagger_{z_2}\}-\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_4}\hat U_{z_3}\hat U^\dagger_{z_2}\hat U_{z_4}\hat U^\dagger_{z_3}\}-(z_4\to z_3)]\Big\}$$

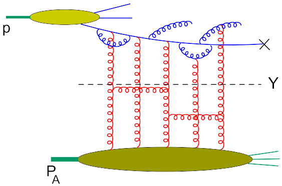
$b = \frac{11}{3}N_c - \frac{2}{3}n_f$ I. Balitsky and G.A.C

$K_{\mathrm{NLO\ BK}}$ = Running coupling part + Conformal "non-analytic" (in j) part
$+$ Conformal analytic ($\mathcal{N}=4$) part

Linearized $K_{\mathrm{NLO\ BK}}$ reproduces the known result for the forward NLO BFKL kernel Fadin and Lipatov (1998).

One-loop Factorization for Inclusive Hadron Production in pA

G.A.C, B-W. Xiao, F. Yuan (2011)

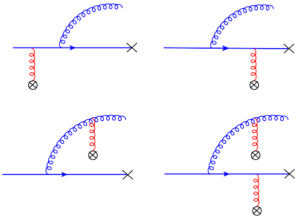


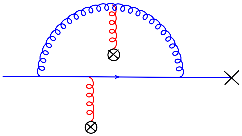
$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\,\xi\, x f_a(x,\mu) D_{h/c}(z,\mu) \int [dx_\perp]\, S^Y_{a,c}([x_\perp])\, \mathcal{H}_{a\to c}(\alpha_s, \xi, [x_\perp]\mu)$$

- $s = (p + P_A)^2 \longrightarrow \infty$
- $f_a(x)$ and $D_{h/c}(z)$: are the collinear parton distribution from the incoming nucleon and fragmentation function for the final state hadron.
- $S^Y_{a,c}([x_\perp])$: response from the nucleus. Y is the gluon rapidity in the nucleus.
- $\mathcal{H}_{a\to c}$ hard factor: describes the partonic scattering amplitude of parton a into a parton c in the dense medium.
- x is the momentum fraction of the nucleon carried by the parton a; z the momentum fraction of parton c carried by the final state hadron h.
- $\xi = \frac{\tau}{xz}, \quad \tau = p_\perp \frac{e^y}{\sqrt{s}}$.
- y and $p_\perp$: rapidity and transverse momentum for the final state hadron.

One-loop Factorization for Inclusive Hadron Production in pA

Quark channel contribution: $qA \longrightarrow q + X$ at one loop order





$$-\frac{\alpha_s N_c}{2\pi^2}\int_0^1 \frac{d\xi'}{1-\xi'}\int \frac{d^2 b_\perp}{(2\pi)^2} e^{-ik_\perp\cdot r_\perp}\frac{(x-y)_\perp^2}{(x-b)_\perp^2 (y-b)_\perp^2}\left[S^{(2)}(x_\perp,y_\perp) - S^{(4)}(x_\perp,b_\perp,y_\perp)\right]$$

$$+\frac{\alpha_s C_F}{2\pi}\int_{\tau/z}^1 d\xi \left(-\frac{1}{\epsilon}\right)\left[\mathcal{P}_{qq}(\xi)e^{-ik_\perp\cdot r_\perp} + \mathcal{P}_{qq}(\xi)\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right]\frac{1}{(2\pi)^2}S^{(2)}(x_\perp,y_\perp)$$

$$S^{(2)}(x_\perp,y_\perp) = \frac{1}{N_c}\langle U(x_\perp)U^\dagger(y_\perp)\rangle_Y$$

$$S^{(4)}(x_\perp,b_\perp,y_\perp) = \frac{1}{N_c^2}\langle \mathrm{Tr}[U(x_\perp)U^\dagger(b_\perp)]\mathrm{Tr}[U(b_\perp)U^\dagger(y_\perp)]\rangle_Y$$

Quark channel contribution: $qA \longrightarrow q + X$ at one loop order

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\,\xi\, x\,[q(x,\mu)]^{1-\text{loop}}[D_{h/q}(z,\mu)]^{1-\text{loop}}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\times$$
$$\left\{[S^{(2)}(x_\perp,y_\perp)]^{1-\text{loop}}\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]+\int\frac{d^2b_\perp}{(2\pi)^2}S^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\}$$

- The collinear divergence is absorbed into the renormalization of the quark distribution and fragmentation functions
  - $[q(x,\mu)]^{1-\text{loop}}$ and $[D_{h/q}(z,\mu)]^{1-\text{loop}}$: DGLAP evolution equation
- The rapidity divergence can be absorbed into the renormalization of the dipole gluon distribution
  - $[S^{(2)}(x_\perp,y_\perp)]^{1-\text{loop}}$: Balitsky-BK evolution equation

Hard coefficients at one loop for quark channel: Infra-red and Ultra-violet finite.

$$\mathcal{H}_{2qq}^{(1)} = C_F \mathcal{P}_{qq}(\xi) \ln \frac{c_0^2}{r_\perp^2 \mu^2} \left(e^{-ik_\perp \cdot r_\perp} + \frac{1}{\xi^2} e^{-i\frac{k_\perp}{\xi} \cdot r_\perp} \right) - 3C_F \delta(1-\xi) e^{-ik_\perp \cdot r_\perp} \ln \frac{c_0^2}{r_\perp^2 k_\perp^2}$$
$$- \left(2C_F - N_c\right) e^{-ik_\perp \cdot r_\perp} \left[\frac{1+\xi^2}{(1-\xi)_+} \widetilde{I}_{21} - \left(\frac{\left(1+\xi^2\right) \ln (1-\xi)^2}{1-\xi} \right)_+ \right]$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp \cdot r_\perp} \left\{ e^{-i\frac{1-\xi}{\xi} k_\perp \cdot (x_\perp - b_\perp)} \frac{1+\xi^2}{(1-\xi)_+} \frac{1}{\xi} \frac{x_\perp - b_\perp}{(x_\perp - b_\perp)^2} \cdot \frac{y_\perp - b_\perp}{(y_\perp - b_\perp)^2} \right.$$
$$\left. -\delta(1-\xi) \int_0^1 d\xi' \frac{1+\xi'^2}{(1-\xi')_+} \left[\frac{e^{-i(1-\xi')k_\perp \cdot (y_\perp - b_\perp)}}{(b_\perp - y_\perp)^2} - \delta^{(2)}(b_\perp - y_\perp) \int d^2 r'_\perp \frac{e^{ik_\perp \cdot r'_\perp}}{r'^2_\perp} \right] \right\}$$

where $c_0 = 2e^{-\gamma_E}$ with γ_E the Euler constant, and

$$\widetilde{I}_{21} = \int \frac{d^2 b_\perp}{\pi} \left\{ e^{-i(1-\xi)k_\perp \cdot b_\perp} \left[\frac{b_\perp \cdot (\xi b_\perp - r_\perp)}{b_\perp^2 \left(\xi b_\perp - r_\perp\right)^2} - \frac{1}{b_\perp^2} \right] + e^{-ik_\perp \cdot b_\perp} \frac{1}{b_\perp^2} \right\}$$

Conclusions

- High-energy operator expansion in color dipoles works at the NLO level.
- The analytic NLO photon impact factor off large nucleus in coordinate space.
- Analytic NLO Impact Factor for BFKL in momentum space has benn calculated.
- The planar (leading N_c) and non-planar (next-to-leading N_c) contribution to the triple Pomeron vertex has been derived through the Wilson line formalism.
- The NLO BK kernel in QCD and $\mathcal{N} = 4$ SYM agrees with NLO BFKL eigenvalues.
- The NLO BK kernel in QCD is a sum of the running-coupling part and conformal part.
- Factorization for Inclusive Hadron Production in pA holds at one loop level: the collinear divergence reproduces DGLAP eq. for parton distribution and fragmentation functions, while the rapidity diverence reproduces the Balitsky-BK evolution equation.

Outlook

- Composite conformal dipole from conformal Ward identity.
- B-JIMWLK evolution equation at NLO.
- Application of NLO B-JIMWLK to gluonic TMD.