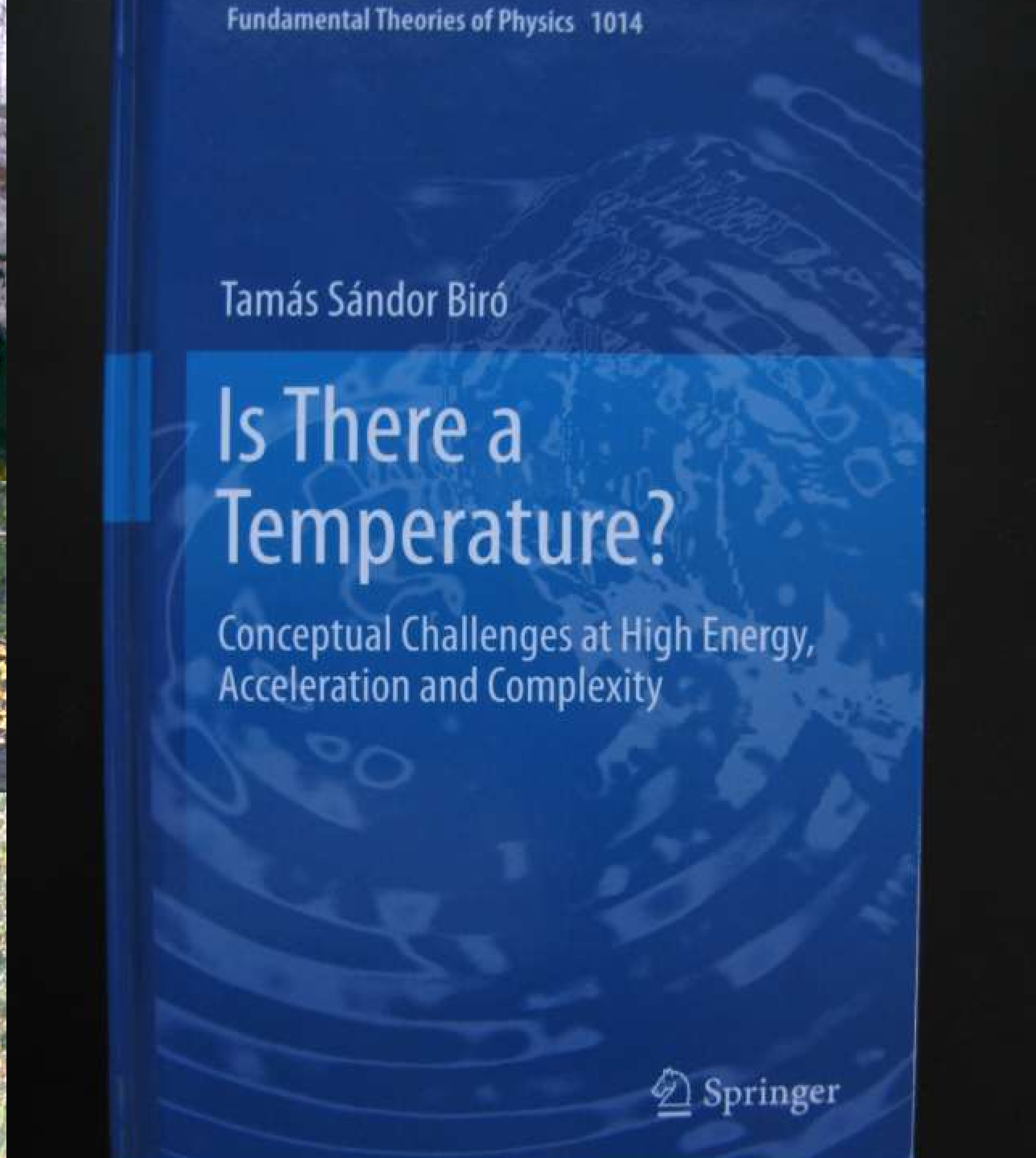

30º





Fundamental Theories of Physics 1014
Tamás Sándor Biró
Is There a Temperature?
Conceptual Challenges at High Energy, Acceleration and Complexity
Springer

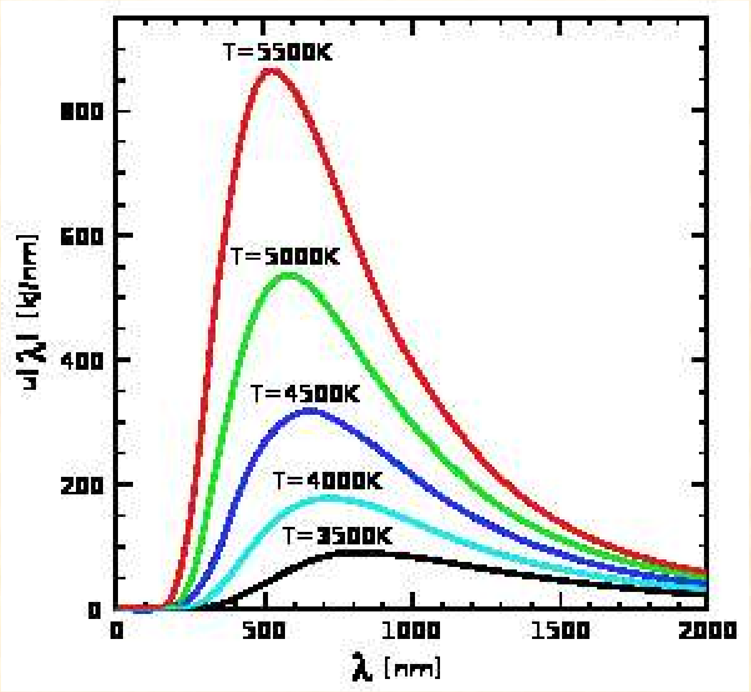
30°


Measuring the temperature

- Thermometer (direct contact)
  - dilatation
  - air pressure
  - mechanical or electric stress

- Chemistry (mixture)
  - color
  - mass ratios
  - hadronic composition

- Spectra (telemetrics)
  - astronomy (photons)
  - pT spectra of light and heavy particles
  - multiplicity fluctuations



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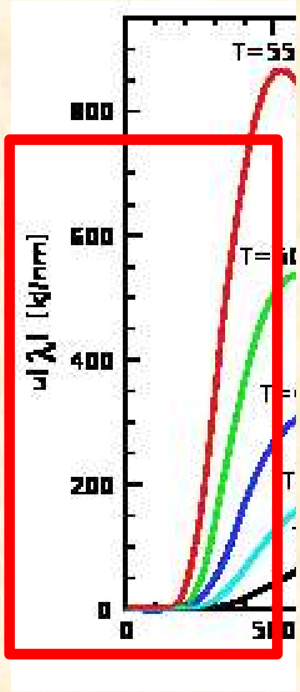
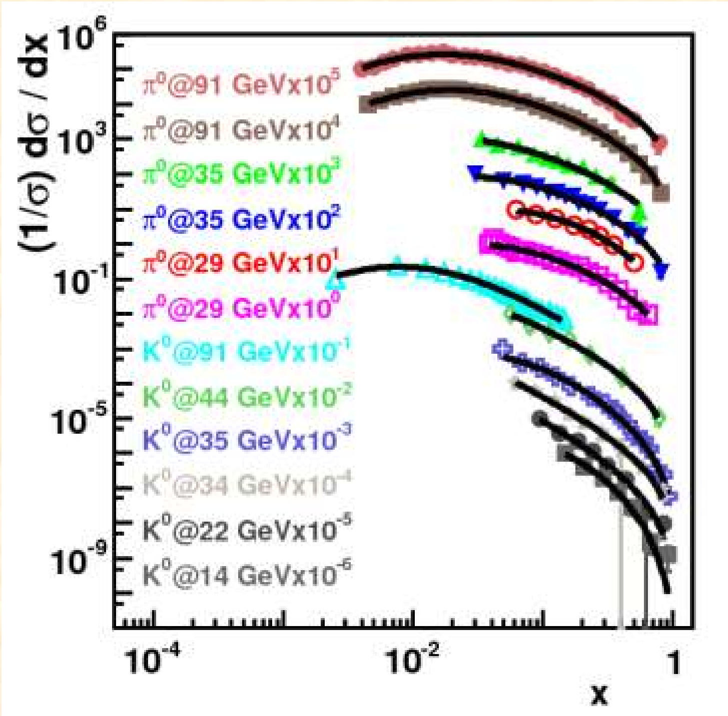


Interpreting the temperature

- Thermodynamics (universality of equilibrium)
  - zeroth theorem (Biro+Van PRE 2011)
  - derivative of entropy
  - Boyle-Mariotte type equation of state

- Kinetic theory (equipartition)
  - energy / degree of freedom
  - fluctuation - dissipation
  - other average values

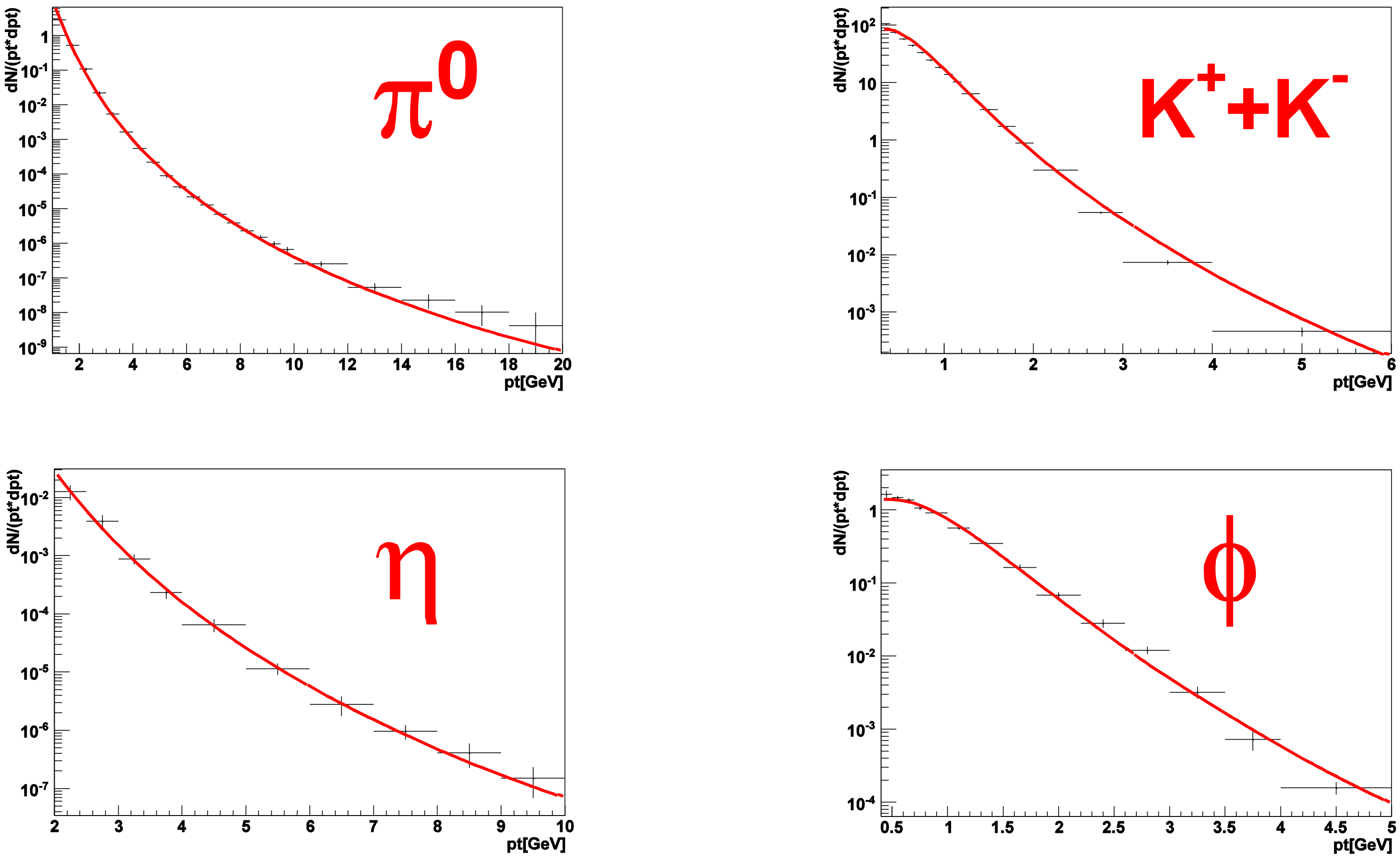
- Spectral statistics (abstract)
  - logarithmic slope parameter
  - observed energy scale
  - dispersion / power -law effects



Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

with Károly Ürmössy

RHIC data



JPG: SQM 2008, Beijing

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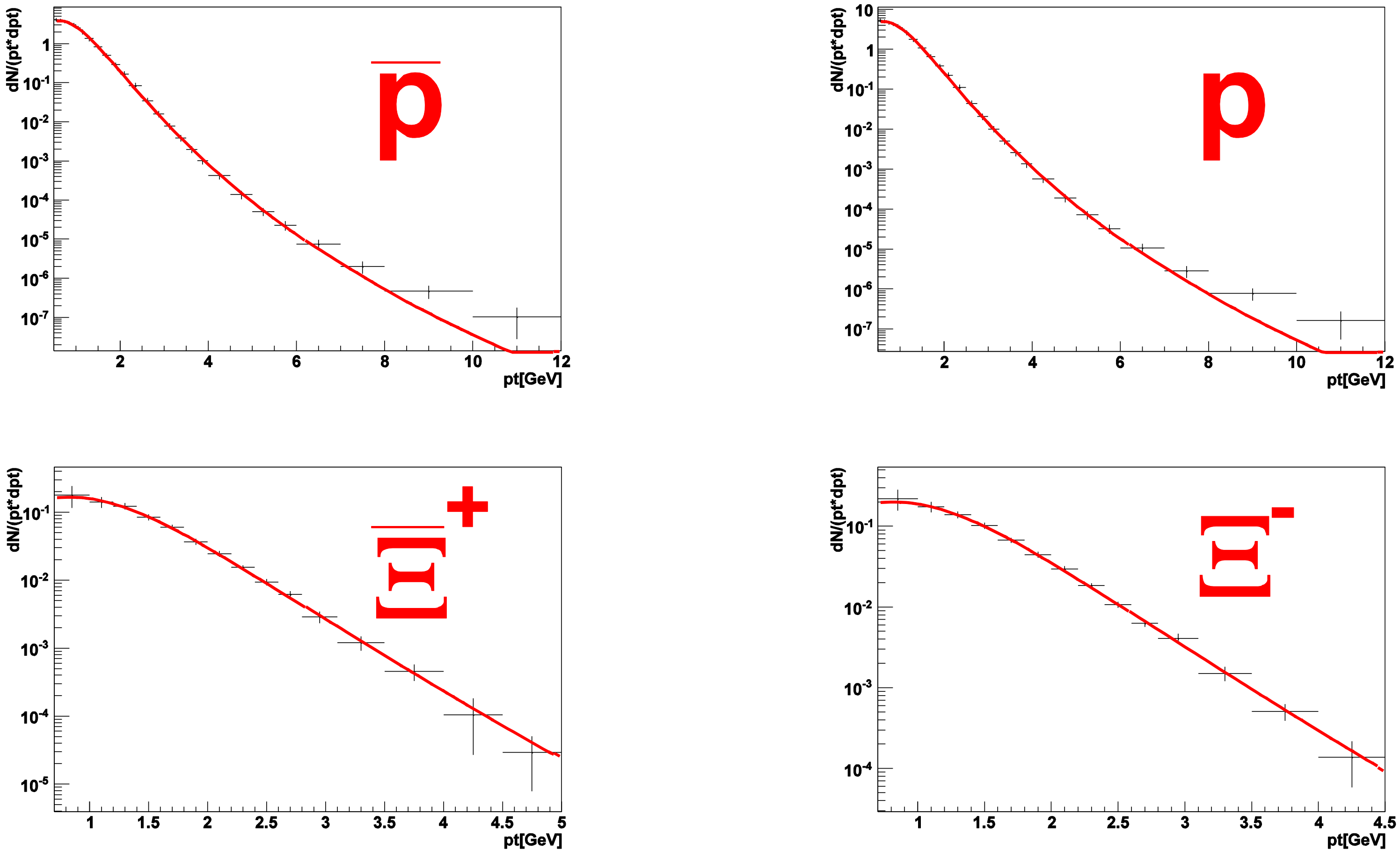


JPG: SQM 2008, Beijing

arXiV: 1111.4817 -> PLB: Unruh gamma radiation at RHIC ?
with Miklós Gyulassy and Zsolt Schram

Mimicking thermal sources
by Unruh radiation

T.S.Biró[1], M.Gyulassy[2] and Z.Schram[3]
[1] MTA KFKI RMKI → MTA Wigner Research Centre RMI
[2]University of Columbia, [3]University of Debrecen

Constant acceleration* is alike temperature…
Soft bremsstrahlung: high k_T exp, low k_T 1 / square;
long acceleration: Bjorken
short acceleration: Landau hydro

*** What about non-constant, non time-symmetric acceleration?**

Exploring QCD Frontiers: from RHIC and LHC to EIC, Jan. 30. – Feb. 03. 2012, Stellenbosch, Republic of South-Africa

Why do statistics work ?

- **Independent** observation over 100M events
- **Universal** laws for large numbers
- **Steady noise** in environment: ‚reservoir'
- **Phase space** dominance
- **By chance** the dynamics mimics thermal behavior

Canonical distribution with Rényi entropy

$$\frac{1}{1-q}\ln\sum p_i^q - \alpha\sum p_i - \beta\sum p_i E_i = \max$$

$$\frac{1}{1-q}\frac{q\,p_i^{q-1}}{\sum p_i^q} = \alpha + \beta E_i$$

This cut power-law distribution is an **excellent** fit to particle spectra in high-energy experiments!

$$p_i = \frac{1}{e^{\hat{L}(S)}}\left(1 + (1-q)\frac{(E_i - \langle E \rangle)}{qT}\right)^{\frac{1}{q-1}}$$

Fit and physics with Rényi entropy

$$p_i^{eq} = \frac{1}{Z}\left(1 + \frac{\hat{\beta} E_i}{c}\right)^{-c} \rightarrow \frac{1}{Z} e^{-\hat{\beta} E_i}$$

$$q = 1 - 1/c$$

$$T = \hat{T} + \frac{1}{c-1}\left(\hat{T} + \langle E \rangle\right)$$

The cut power-law distribution is an excellent fit, but it gives smaller values for the parameter hat(T) at the same T than the Boltzmann form!

NBD = Euler ○ Poisson

Power Law = Euler ○ Gibbs

$$P_{n,k} = \int_0^\infty \frac{(x\bar{f})^n}{n!} e^{-\bar{f}x} \cdot \frac{x^k}{k!} e^{-x}\, dx$$

$$w_i^{eq} = \int_0^\infty \frac{1}{Z} e^{-\frac{\beta E_i}{k+1+\alpha}x} \cdot \frac{x^k}{k!} e^{-x}\, dx$$

$$q = \frac{k}{k+1}$$

Superstatistics

Our view

Arxiv: 1111.4817

Phys.Lett. B, 2012

Unruh gamma radiation at RHIC?

Tamás S. Biró
Theory Division, MTA KFKI Res. Inst. for Particle and Nuclear Physics, Budapest, Hungary

Miklós Gyulassy
Dept. of Physics, Columbia University, NY, USA

Zsolt Schram
Dept. of Theoretical Physics, University of Debrecen, Hungary

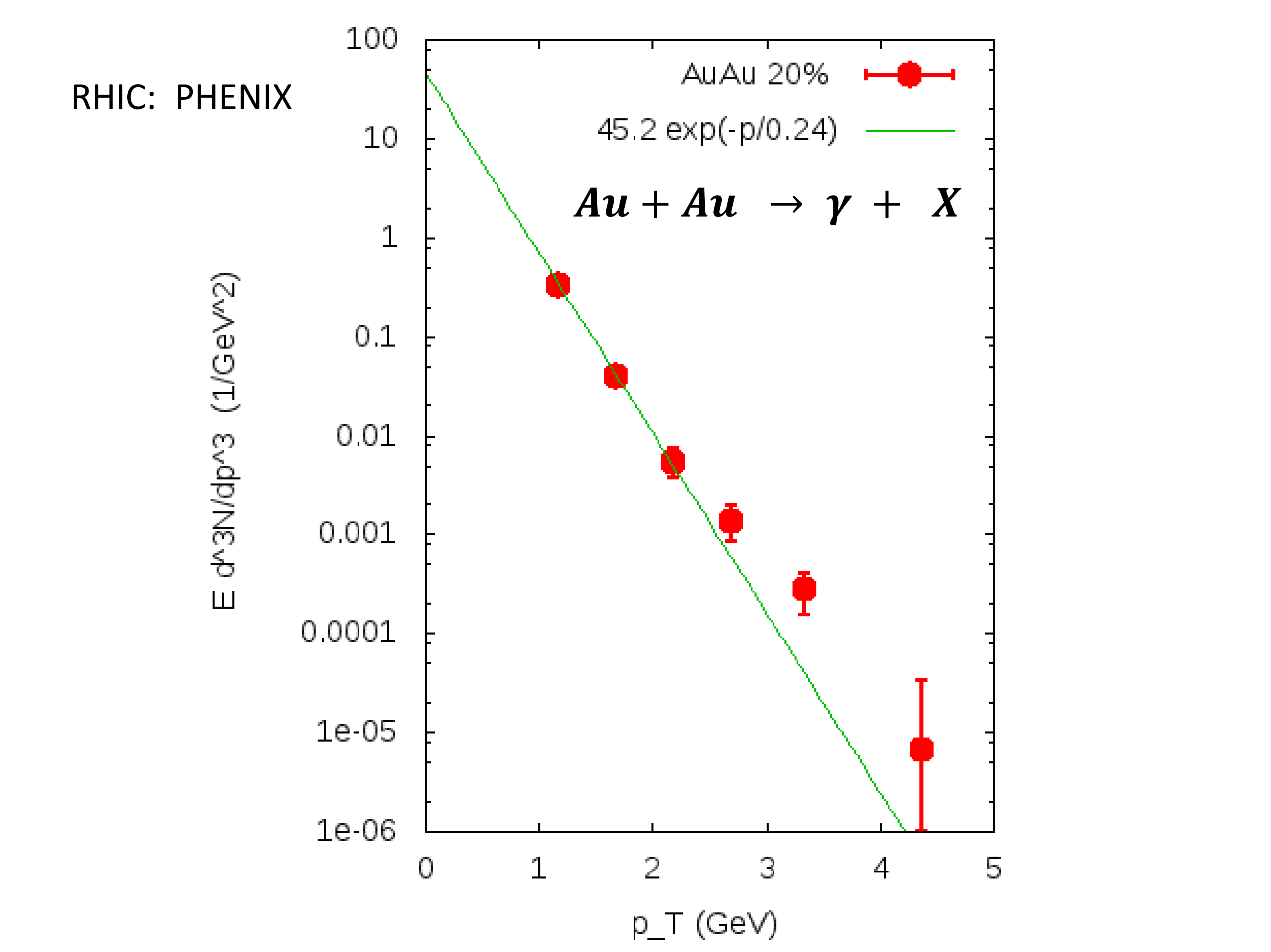
(Dated: October 23, 2011)

Varying the proposition that acceleration itself would simulate a thermal environment, we investigate the semiclassical coherent photon radiation as a possible telemetric thermometer of accelerated charges. Based on the classical Jackson formula we obtain the equivalent photon intensity spectrum stemming from a constantly accelerated charge and demonstrate its resemblances to a thermal distribution for high transverse momenta. The transverse slope temperature *differs* from the famous Unruh formula: it is larger by a factor of π. We compare the resulting direct photon spectrum with experimental data for AuAu collisions at RHIC and speculate about further, analytically solvable acceleration histories.

PACS numbers: 24.10.Pa, 25.75.Ag, 25.20.Lj
Keywords: Thermal models, Unruh temperature, bremsstrahlung, photon spectra

THERMAL LOOKING CLASSICAL RADIATION Satz and Kharzeev [19–22]. A calculation demonstrating

Experimental motivation: apparently thermal photons



Theoretical motivation

- Deceleration due to stopping
- Schwinger formula + Newton + Unruh = Boltzmann

$$E_p \frac{dN}{d^3p} \propto e^{-2\pi m_T^2/q\boldsymbol{E}}, \quad q\boldsymbol{E} = m_T a, \quad T = \frac{a}{2\pi}$$

$$E_p \frac{dN}{d^3p} \propto e^{-m_T/T}$$

Satz, Kharzeev, …

Why Photons (gammas) ?

- **Zero mass**: flow – Doppler, easy kinematics
- **Color neutral**: escapes strong interaction
- **Couples to charge**: Z / A sensitive
- **Classical** field theory also predicts spectra

Soft bremsstrahlung

- Jackson formula for the amplitude:

$\vec{u}$

$$\vec{A} = K \int e^{i\phi} \, \frac{d}{dt} \boxed{\left(\frac{\vec{n} \times (\vec{n} \times \vec{\beta})}{1 - \vec{n} \cdot \vec{\beta}} \right)} \mathrm{dt}$$

With $$K^2 = \frac{e^2}{8\pi c^2}, \quad \vec{\beta} = \frac{\vec{v}}{c} = \frac{1}{c} \frac{d\vec{r}}{dt}$$

and the retarded phase $$\phi = \omega \left(t - \frac{\vec{n} \cdot \vec{r}}{c} \right) = k \cdot x$$

Soft bremsstrahlung

- Covariant notation:

$$k = (\omega, \omega\vec{n}) = k_\perp(\cosh\eta, \sinh\eta, \cos\psi, \sin\psi\ )$$

$$u = (\gamma, \gamma\vec{v}) = (\cosh\xi\ , \sinh\xi\ , 0,0)$$

$$\aleph\ = \int e^{i\varphi}\ \frac{d}{d\tau}\left(\frac{\epsilon\cdot u}{k\cdot u}\right)\ d\tau$$

IR div, coherent effects

Feynman graphs

The Unruh effect cannot be calculated by any finite number of Feynman graphs!

Kinematics, source trajectory

- Rapidity: $$\beta = \frac{v}{c} = \tanh(\xi + \xi_0)$$ $$\xi = \frac{g}{c}\,\tau$$

Trajectory:

$$t = t_0 + \frac{c}{g}\left(\sinh(\xi + \xi_0) - \sinh \xi_0\right)$$

$$z = z_0 + \frac{c^2}{g}\left(\cosh(\xi + \xi_0) - \cosh \xi_0\right)$$

Let us denote $\xi + \xi_0$ by ξ in the followings!

Kinematics, photon rapidity

- Angle and rapidity:

$$\cos\theta = \tanh\eta$$

$$\sin\theta = \frac{1}{\cosh\eta}$$

$$\cot\theta = \sinh\eta$$

$$\boldsymbol{\eta = \ln\cot\frac{\theta}{2}}$$

Kinematics, photon rapidity

- Doppler factor:

$$\mathrm{k} \cdot \mathrm{u} = \omega\gamma(1 - \beta\cos\theta) = \omega\,\frac{\cosh(\xi-\eta)}{\cosh\eta} = \frac{d\phi}{d\tau}$$

Phase:

$$\phi = \frac{\omega c}{g}\frac{\sinh(\xi-\eta)}{\cosh\eta} = \ell k_{\perp}\sinh(\xi-\eta)$$

Magnitude of projected velocity:

$$u = \frac{\sinh\xi}{\cosh(\xi-\eta)}, \qquad \frac{du}{d\xi} = \frac{\cosh\eta}{\cosh^2(\xi-\eta)}$$

Intensity, photon number

Amplitude as an integral over rapidities on the trajectory:

$$\vec{A} = K\vec{e}\ \int_{\xi_1}^{\xi_2} e^{i\ell k_\perp \sinh(\xi-\eta)}\ \frac{\cosh\eta}{\cosh^2(\xi-\eta)}\, d\xi$$

Here $\ell = \dfrac{c^2}{g}$ is a characteristic length.

Intensity, photon number

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from –c to +c):

$$\vec{A} = 2K\vec{e}\,\ell k_\perp \cosh\eta \;\; K_1(\ell k_\perp)$$

With K1 Bessel function!

$$\boxed{\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi}\,\ell^2\, K_1^2(\ell k_\perp)}$$

Flat in rapidity !

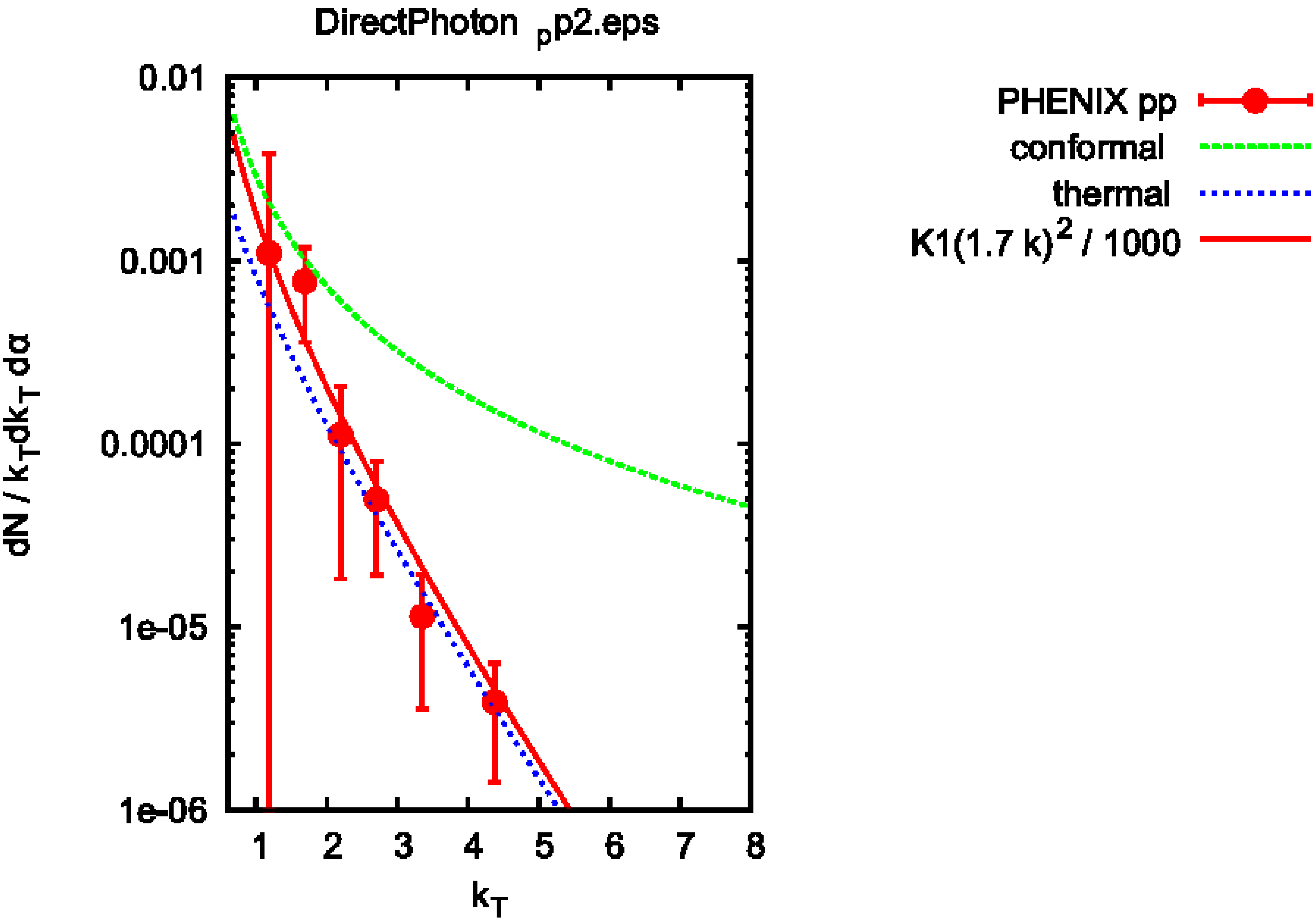
Photon spectrum, limits

Amplitude as an integral over infinite rapidities on the trajectory (velocity goes from –c to +c):

$$\frac{dN}{k_\perp dk_\perp d\eta d\psi} = \frac{4\alpha_{EM}}{\pi} \; \frac{1}{k_\perp^2} \qquad \text{for } \ell k_\perp \to 0$$

$$\frac{dN}{k_\perp dk_\perp d\eta \, d\psi} = 2\alpha_{EM} \; \frac{\ell}{k_\perp} \, e^{-2\ell k_\perp} \quad \text{for } \ell k_\perp \to \infty$$

Photon spectrum from pp backgroound, PHENIX experiment



Apparent temperature

- High - $k_\perp$ infinite proper time acceleration:

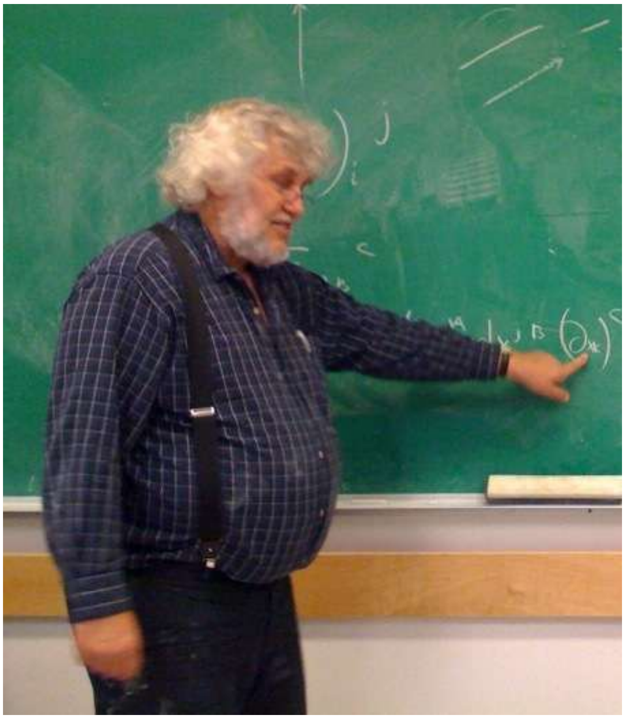
$$k_B\, T = \frac{\hbar c}{2\ell} = \frac{\hbar g}{2c} = \pi\, k_B\, T^{Unruh}$$

Connection to Unruh:

$$\frac{du}{d\tau} \rightarrow e^{-i\nu\tau}$$

proper time Fourier analysis of a monochromatic wave

Unruh temperature



Unruh

- **Entirely classical effect**
- **Special Relativity suffices**

$$I(\nu) \propto \left| \int e^{i\left[\int \omega \sqrt{\frac{1-V(\tau)/c}{1+V(\tau)/c}}\, d\tau - \nu\tau\right]} d\tau \right|^2$$

$$I(\nu) \propto \left| \int_0^\infty e^{ic\omega z/g} z^{-i\nu c/g-1} dz \right|^2 \boxed{\propto \frac{1}{e^{2\pi c\nu/g}-1}}$$

Constant ‚g’ acceleration in a comoving system: $dv/d\tau = -g(1-v^2)$

Unruh temperature

Planck-interpretation:

$$\frac{2\pi\,\mathrm{c}}{\mathrm{g}}\nu = \frac{\hbar\nu}{k_B T}$$

The temperature in Planck units:

$$\boxed{T = \frac{g}{2\pi}}$$

The temperature more commonly:

$$k_B T = \frac{\hbar}{c}\frac{g}{2\pi} = M_P g \cdot \frac{L_P}{2\pi}$$

Unruh temperature

Small for Newtonian gravity

$$g = \frac{GM}{R^2}$$

$$k_B T = \frac{Mc^2}{2\pi} \cdot \frac{L_P^2}{R^2}$$

On Earth' surface ist is 10^(-19) eV, while at room temperature about 10^(-3) eV.

Unruh temperature

Not small in heavy ion collisions

$$g = \frac{c^2}{2L} = \frac{mc^3}{\hbar}$$

$$k_B T = \frac{mc^2}{2\pi}$$

Braking from +c to -c in a Compton wavelength:

kT ~ 150 MeV if mc^2 ~ 940 MeV (proton)

Connection to Unruh

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{\alpha_{EM}}{2\pi k_\perp^2 \cosh^2\eta} \left| \int_{-\infty}^{+\infty} e^{i\phi(\tau)} \frac{du}{d\tau} d\tau \right|^2$$

Fourier component for the retarded phase:

$$f_k = \int_{-\infty}^{+\infty} e^{i\phi(\tau)} e^{i\nu\tau} d\tau = \frac{\ell}{c} \int_{-\infty}^{+\infty} e^{i\ell k_\perp \sinh\xi} e^{ik\xi} d\xi$$

Connection to Unruh

Fourier component for the projected acceleration:

$$a_k = \int_{-\infty}^{+\infty} \frac{du}{d\tau} e^{i\nu\tau}\, d\tau = \cosh\eta \int_{-\infty}^{+\infty} \frac{1}{\cosh^2\xi}\, e^{ik\xi} d\xi$$

Photon spectrum in the incoherent approximation:

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} \approx \frac{\alpha_{EM}}{2\pi k_\perp^2\, \cosh^2\eta} \int_{-\infty}^{+\infty} |f_k|^2 |a_k|^2\, \frac{c}{\ell} \frac{dk}{2\pi}$$

Connection to Unruh

Fourier component for the retarded phase at constant acceleration:

$$f_k = \frac{\ell}{c}\int_{-\infty}^{+\infty} e^{i\ell k_\perp \sinh\xi}\, e^{ik\xi} d\xi = \frac{2\ell}{c}\, K_{ik}(\ell k_\perp)\, e^{-\pi k/2}$$

KMS relation and Planck distribution:

$$f_{-k} = e^{k\pi}\, f_k^*, \qquad |f_{-k}|^2 = e^{2\pi k}\, |f_k|^2$$

$$-\mathrm{n}(-\nu) = e^{2\pi\ell\nu/c}\, n(\nu) = 1 + n(\nu)$$

$$n(\nu) = \frac{1}{e^{2\pi\ell\nu/c} - 1}$$

Connection to Unruh

KMS relation and Planck distribution:

$$2\pi k = \frac{2\pi c}{g}\,\nu = \frac{\hbar}{k_B T_U}\,\nu\ ;$$

$$T_U = \frac{\hbar}{2\pi k_B c}\,g$$

Connection to Unruh

Note:

$$a_k = \cosh\eta \, \frac{k\pi}{\sinh k\pi/2}$$

It is peaked around k = 0, but relatively wide!   (an unparticle…)

Transverse flow interpretation

Mathematica knows: (I derived it using Feynman variables)

$$\int_0^\pi \frac{d\theta}{\sin\theta}\; K_2\left(\frac{z}{\sin\theta}\right) = K_1^2\left(\frac{z}{2}\right) \qquad \frac{1}{\sin\theta} = \cosh\eta$$

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}\,\hbar c}{\pi(2\pi k_B T_U)^2} \int_{-\infty}^{+\infty} K_2\left(\frac{\hbar c\, k_\perp}{\pi k_B T_U}\cosh(\zeta-\eta)\right) d\zeta$$

$$\boxed{\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi\, g} \int_{-\infty}^{+\infty} K_2\left(\frac{k\cdot u}{\pi T_U}\right) d\tau}$$

Alike Jüttner distributions integrated over the flow rapidity...

Finite time (rapidity) effects

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4K^2}{\hbar\, {k_\perp}^2} \left| \int_{w_1}^{w_2} \frac{e^{i\ell k_\perp w}}{(1+w^2)^{3/2}} \right|^2$$

with $w = \sinh(\xi - \eta)$

Short-time deceleration → Non-uniform rapidity distribution; → Landau hydrodynamics

Long-time deceleration → uniform rapidity distribution; → Bjorken hydrodynamics

Short time constant acceleration

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi} \frac{1}{k_\perp^2} \frac{(w_2 - w_1)^2}{(1+w_0^2)^3}$$

$$\frac{dN}{k_\perp dk_\perp d\eta\, d\psi} = \frac{4\alpha_{EM}}{\pi} \frac{4}{\omega^2} \frac{1}{\cosh^2 \eta}$$

Non-uniform rapidity distribution;

→ Landau hydrodynamics

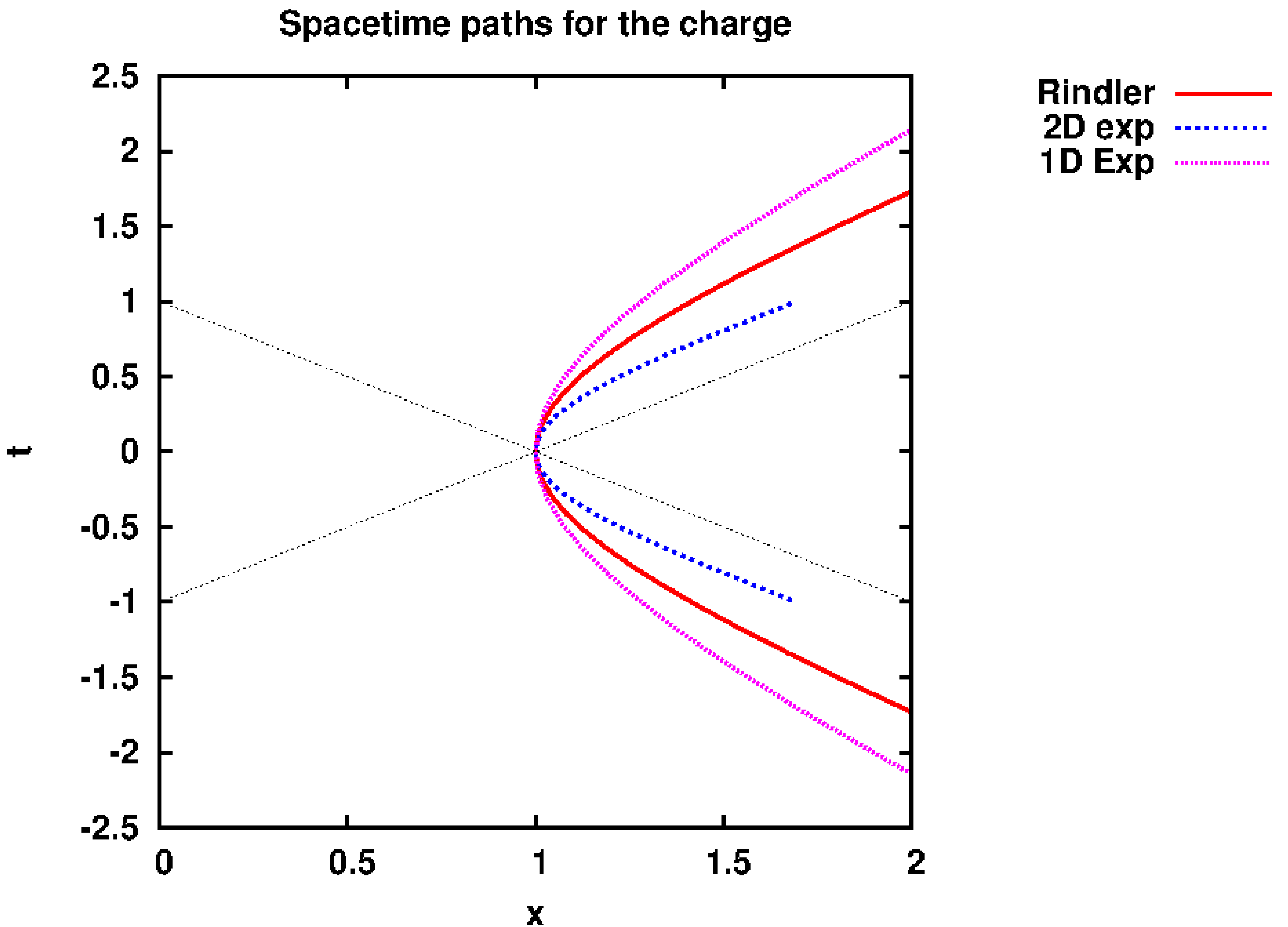
Analytic results

x(t), v(t), g(t), τ(t), $\mathcal{A}$, dN/kdkdη limit

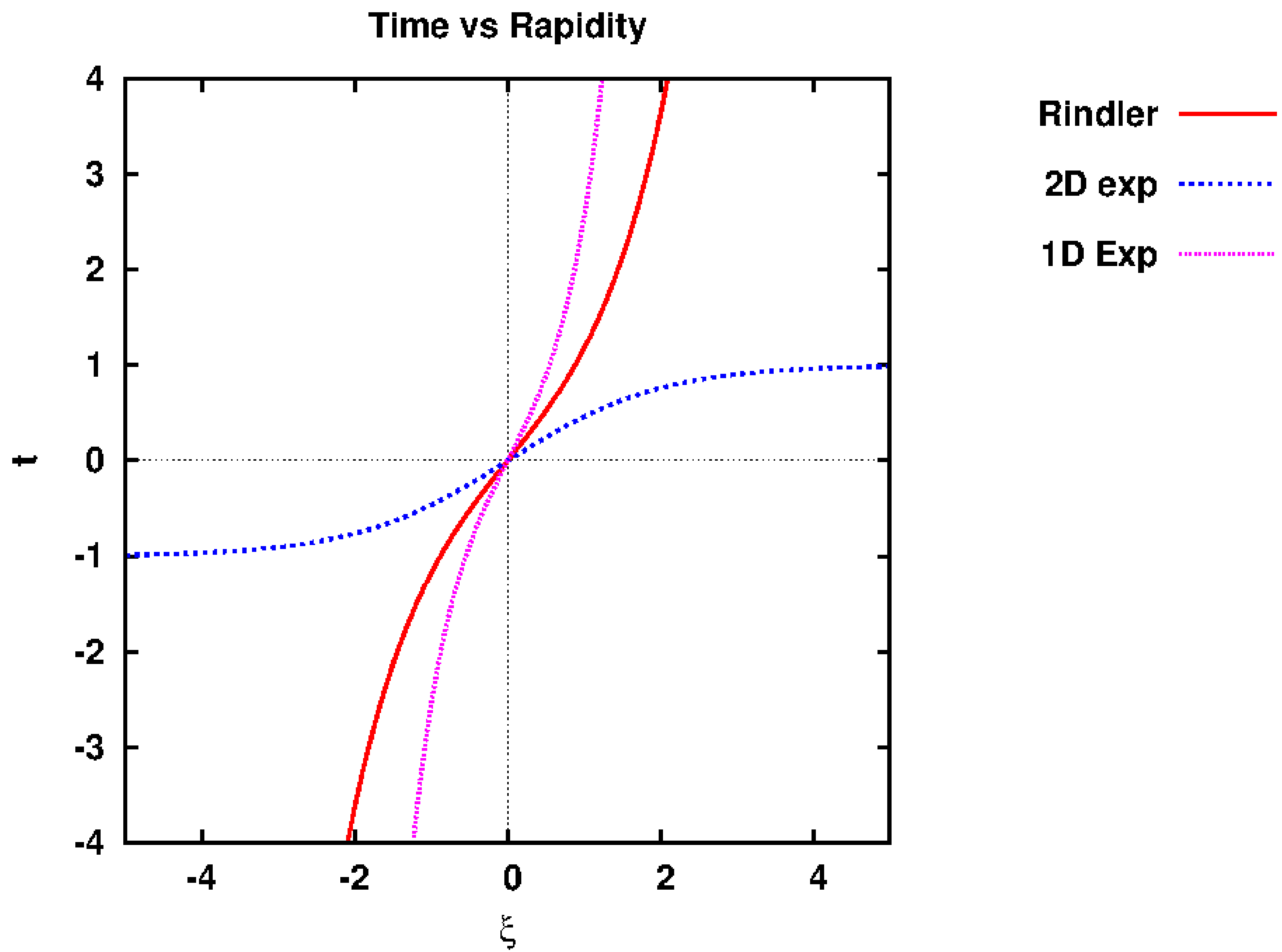
$$\sqrt{1+t^2},\quad \frac{t}{\sqrt{1+t^2}},\quad 1,\quad \mathrm{Arc\,sh}\,t\,,\quad bK_1(b),\quad \frac{\ell}{k}e^{-2\ell k}$$

$$1+\ln(1+t^2),\quad \frac{2t}{1+t^2},\quad \frac{2(1+t^2)}{(1-t^2)^2},\quad 2\,\mathrm{atn}\,t\ -t, be^{-b}, \ell^2 e^{-2\ell k}$$

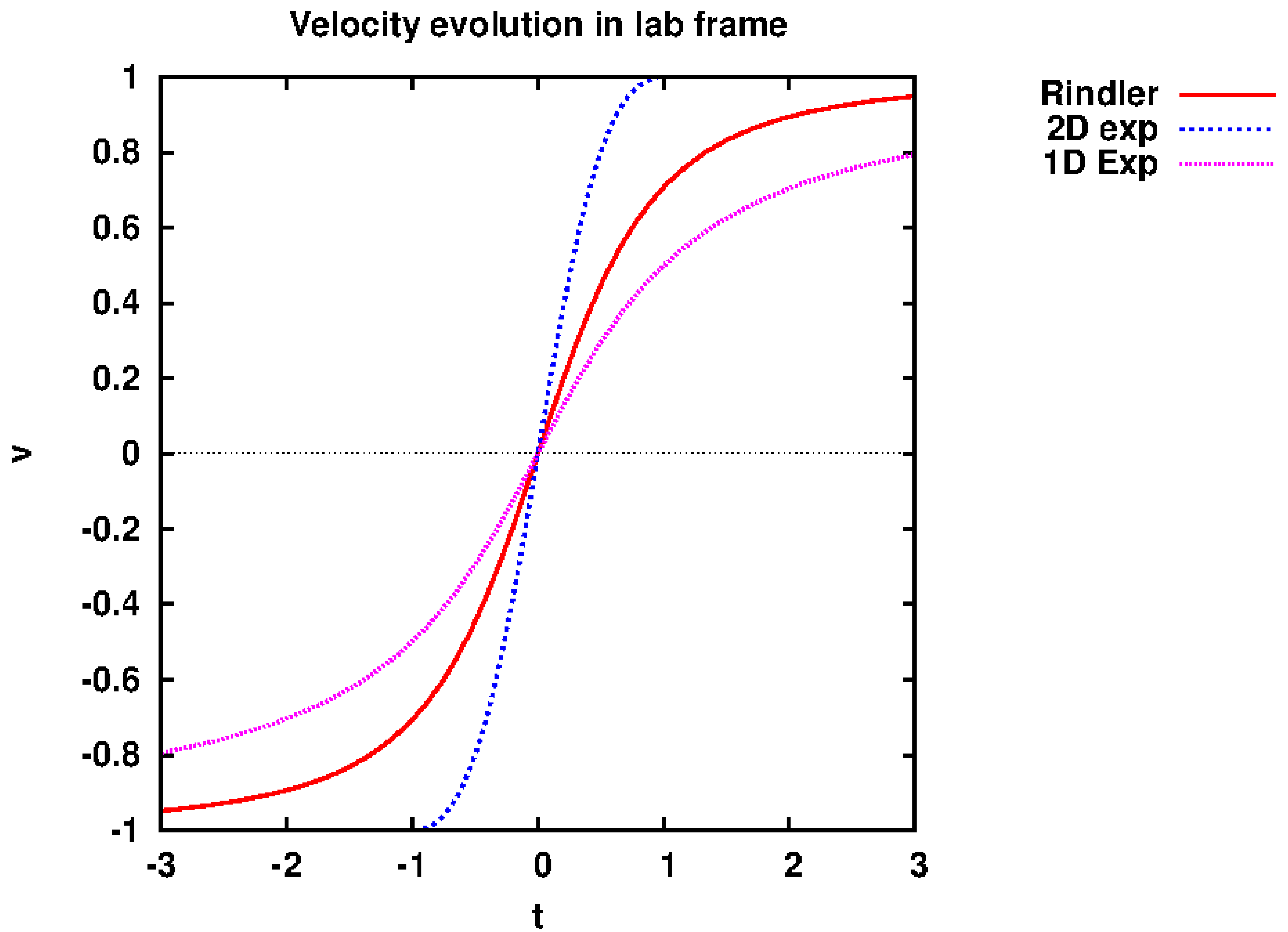
$$1+\frac{2t}{\pi}\mathrm{atn}\left(\frac{t}{\pi}\right)-\ln\left(1+\frac{t^2}{\pi^2}\right), \frac{2}{\pi}\mathrm{atn}\left(\frac{t}{\pi}\right), \frac{2\gamma^3}{\pi^2+t^2}, \frac{\pi^2}{2}\int\frac{\sqrt{1-v^2}\,dv}{\cos^2\left(\frac{\pi v}{2}\right)},\quad e^{-b},\quad \frac{1}{k^2}e^{-2\ell k}$$



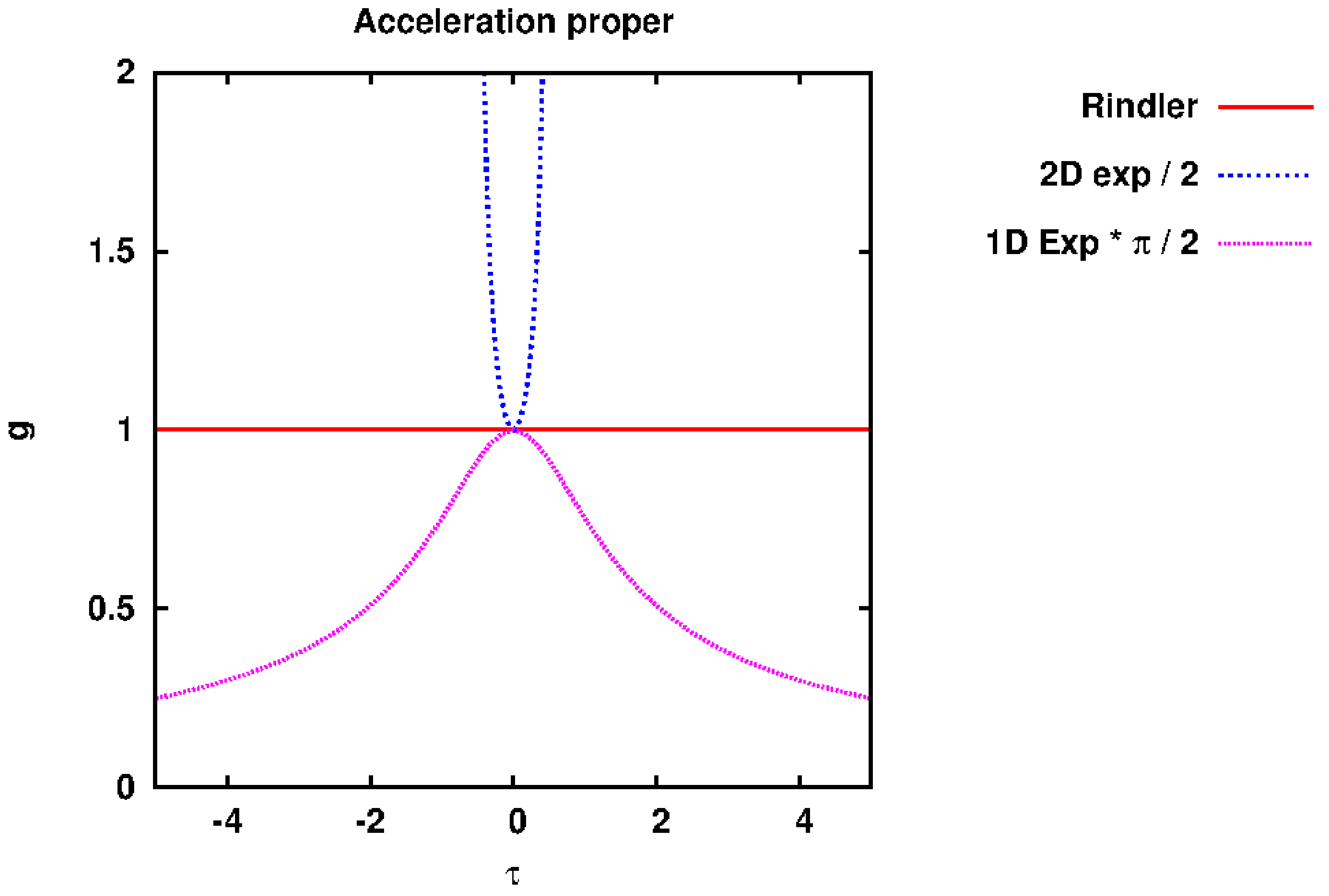
Spacetime paths for the charge
Rindler
2D exp
1D Exp
t
x
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2



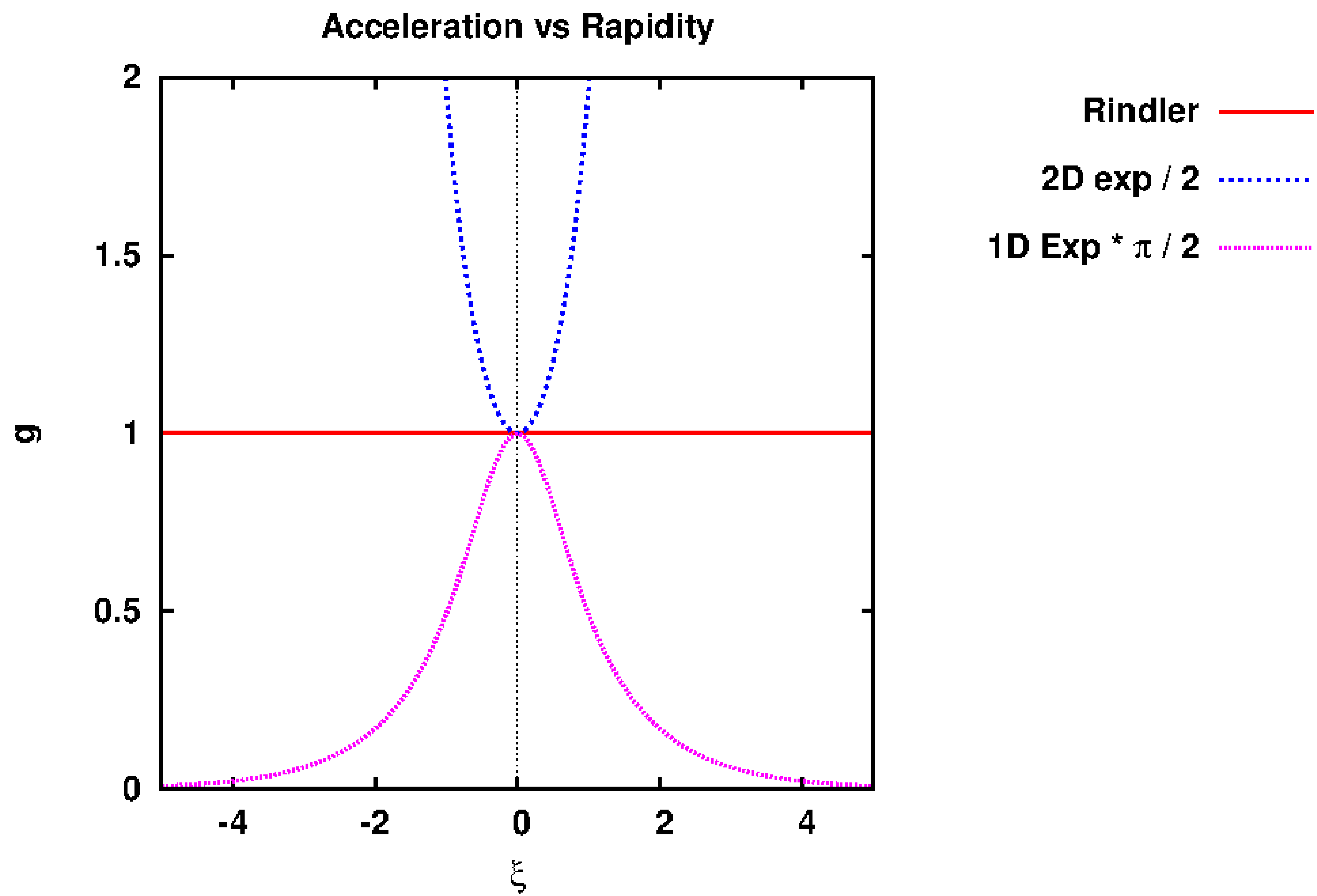
Time vs Rapidity
t
ξ
4
3
2
1
0
-1
-2
-3
-4
-4
-2
0
2
4
Rindler
2D exp
1D Exp

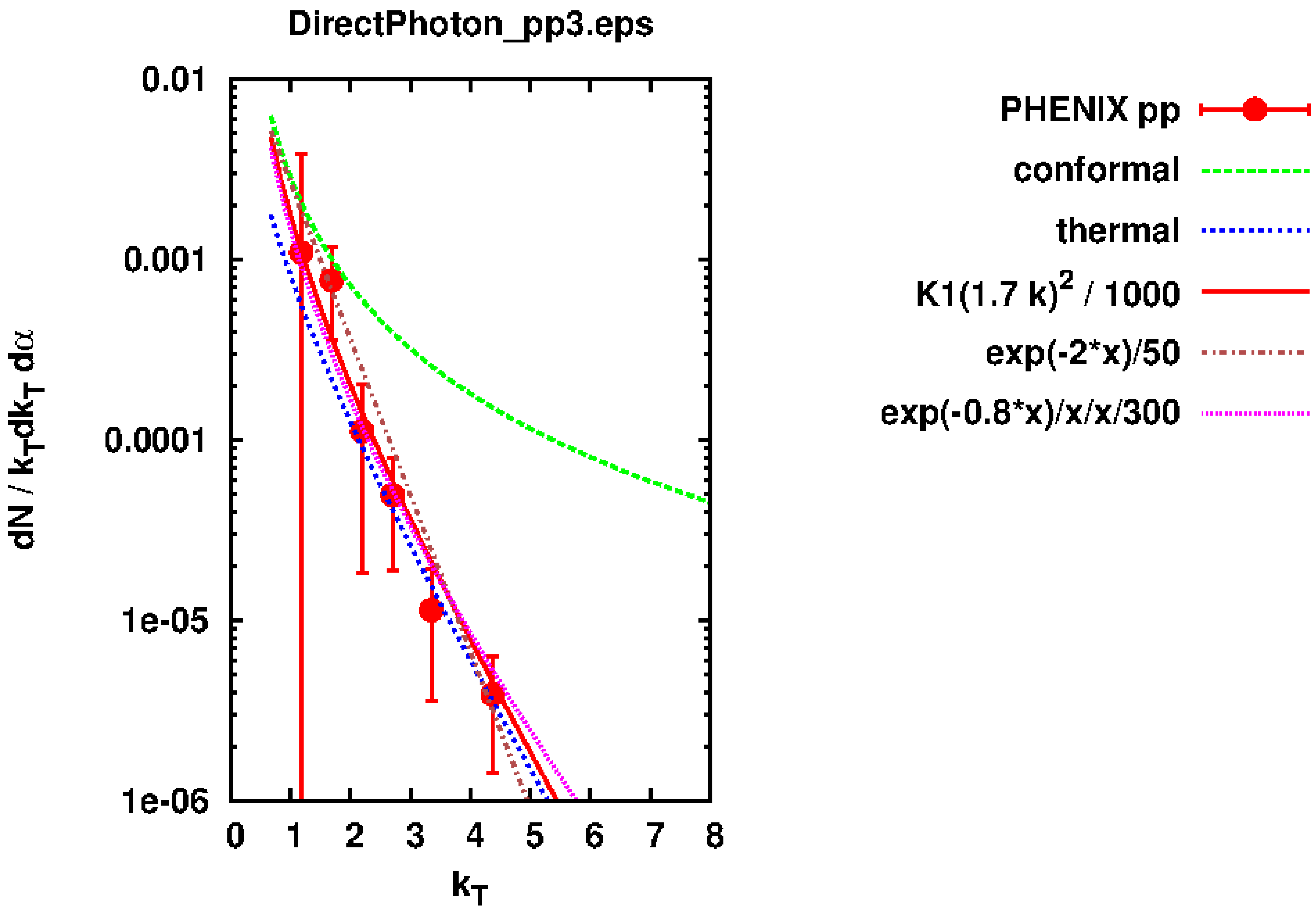


Velocity evolution in lab frame
Rindler
2D exp
1D Exp
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
v
-3
-2
-1
0
1
2
3
t



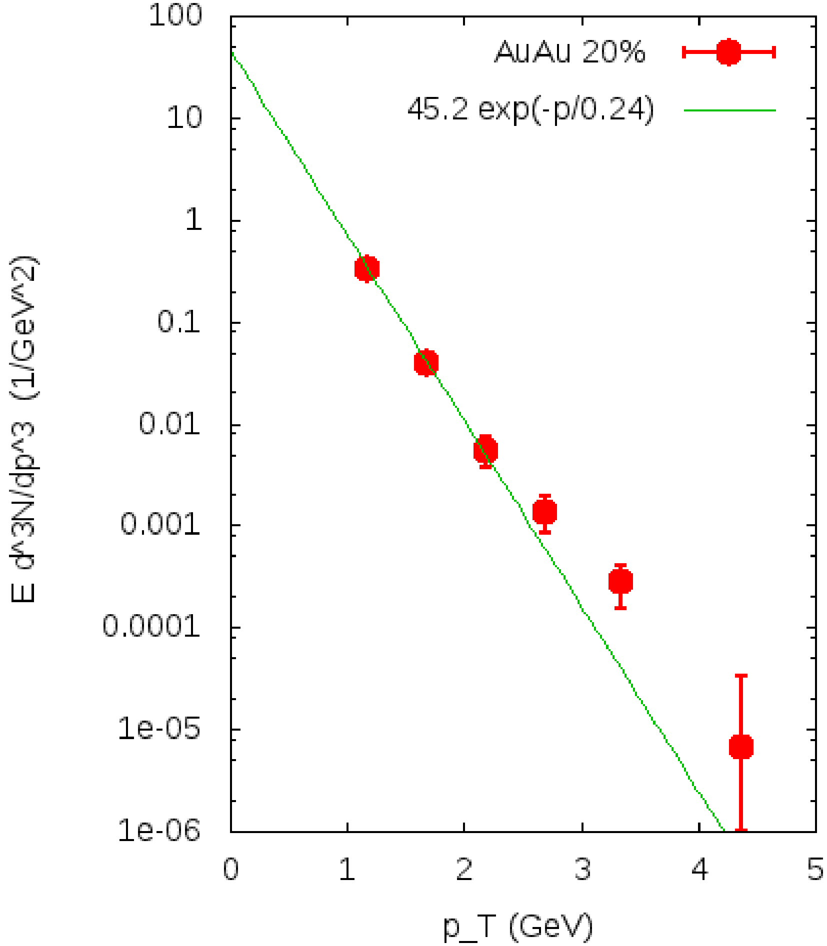
Acceleration proper
Rindler
2D exp / 2
1D Exp * π / 2
g
τ
2
1.5
1
0.5
0
-4
-2
0
2
4


Acceleration vs Rapidity
Rindler
2D exp / 2
1D Exp * π / 2
g
ξ
2
1.5
1
0.5
0
-4
-2
0
2
4

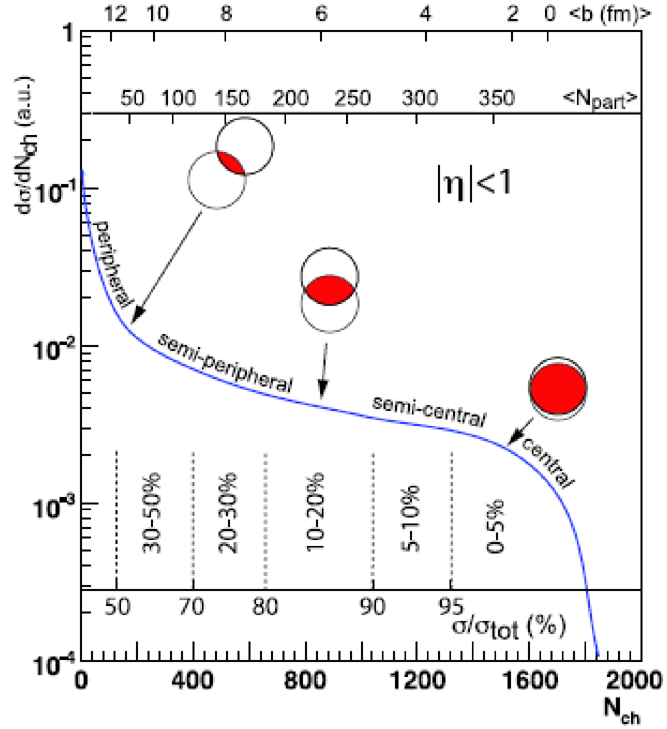


DirectPhoton_pp3.eps
0.01
0.001
0.0001
1e-05
1e-06
0
1
2
3
4
5
6
7
8
k_T
$dN / k_T dk_T d\alpha$
PHENIX pp
conformal
thermal
$K1(1.7\ k)^2$ / 1000
exp(-2*x)/50
exp(-0.8*x)/x/x/300

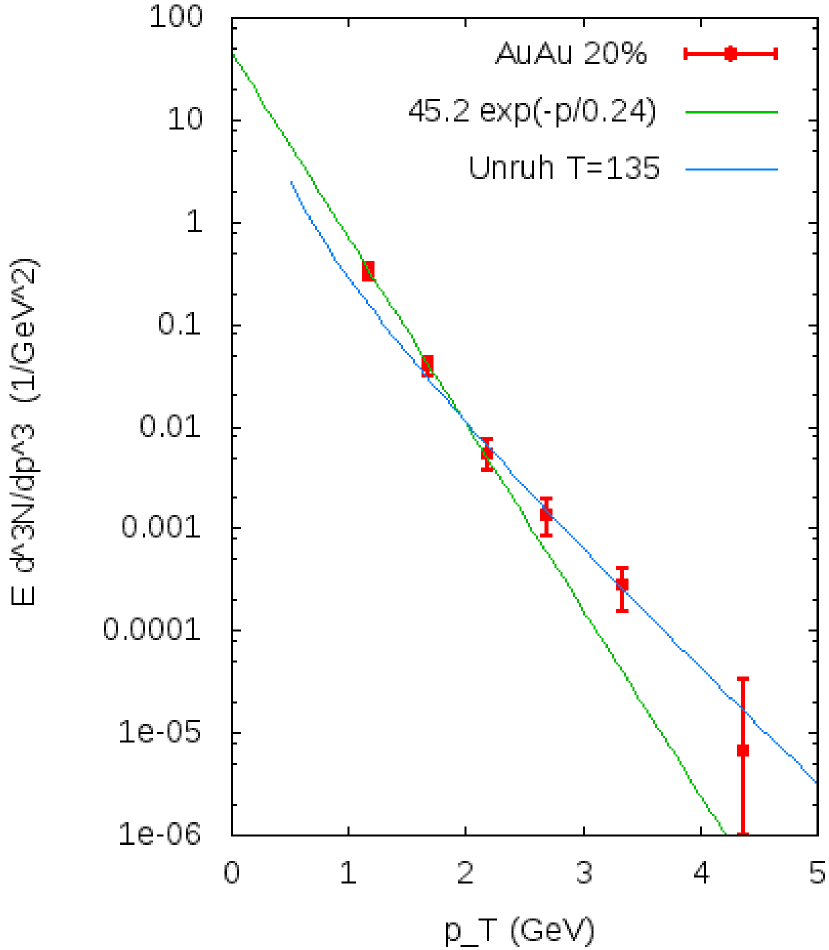
$$Au + Au \rightarrow \gamma + X$$



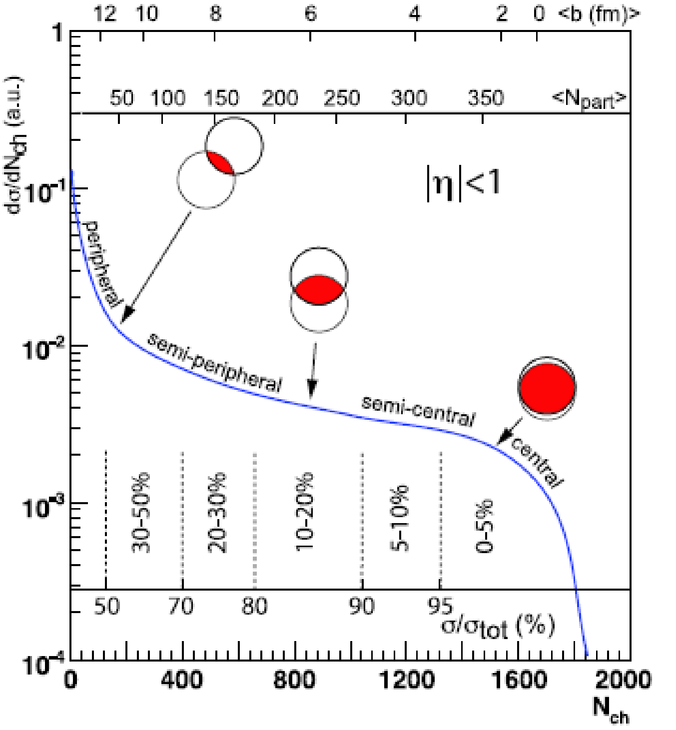
Glauber model



$$Au + Au \rightarrow \gamma + X$$

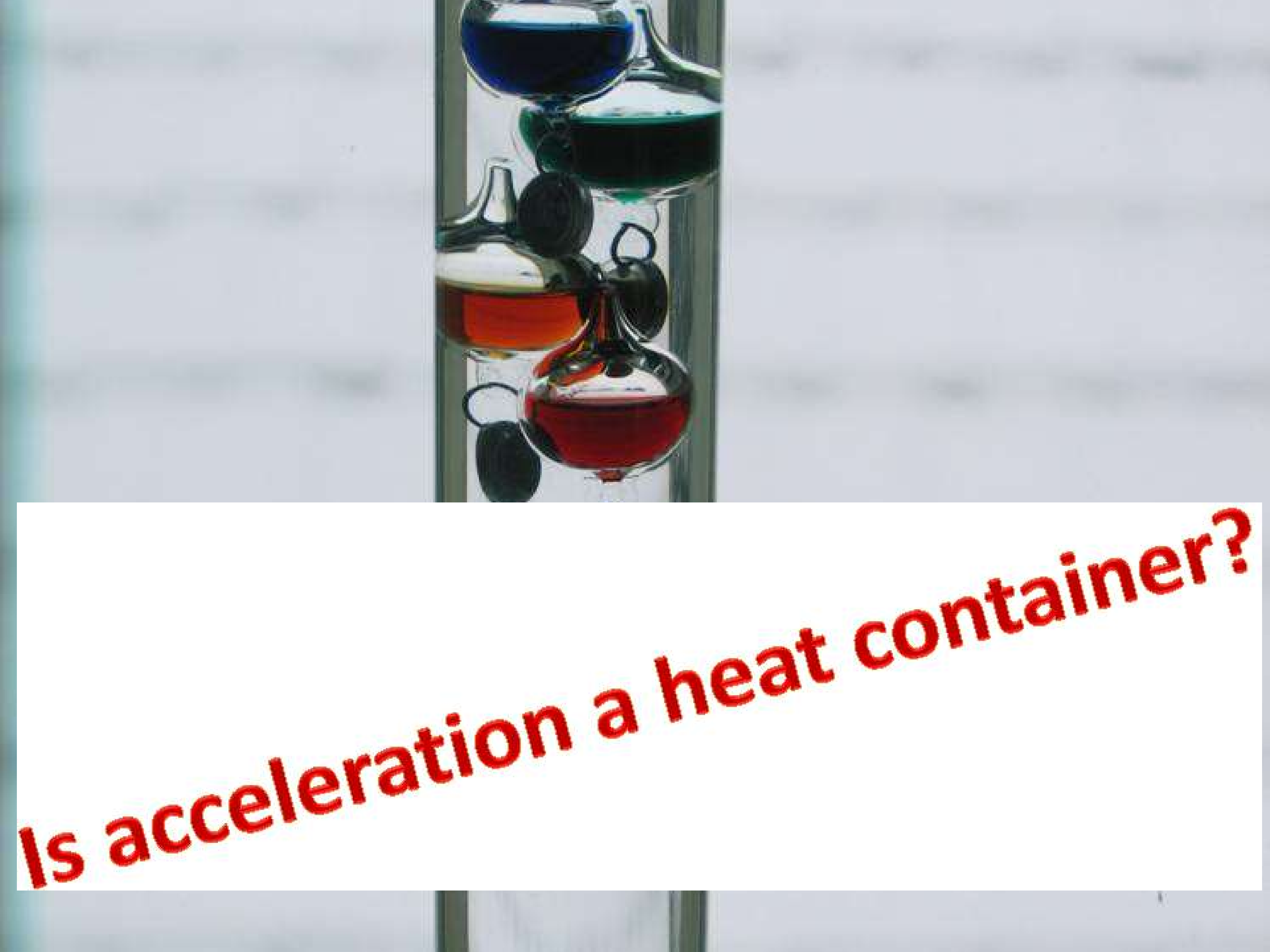


Glauber model



Summary

- Semiclassical radiation from constant accelerating point charge occurs **rapidity-flat** and **thermal**
- The thermal tail develops at high enough k_perp
- At low k_perp the **conformal** NLO result emerges
- Finite time/rapidity acceleration leads to **peaked** rapidity distribution, alike **Landau** - hydro
- Exponential fits to surplus over NLO pQCD results reveal a ’’**pi-times Unruh-**’’ temperature


Is acceleration a heat container?