Glasma to plasma: classical coherence, quantum decoherence & thermalization in the little Bang

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Lecture iV, UCT, February 2012

Outline of lectures

- Lecture I: QCD and the Quark-Gluon Plasma
- Lecture II: Gluon Saturation and the Color Glass Condensate
- Lecture III: Quantum field theory in strong fields. Factorization. the Glasma and long range correlations
- Lecture IV: Quantum field theory in strong fields. Instabilities and the spectrum of initial quantum fluctuations
- Lecture V: Quantum field theory in strong fields. Decoherence, hydrodynamics, Bose-Einstein Condensation and thermalization
- Lecture VI: Future prospects: RHIC, LHC and the EIC

Talk Outline

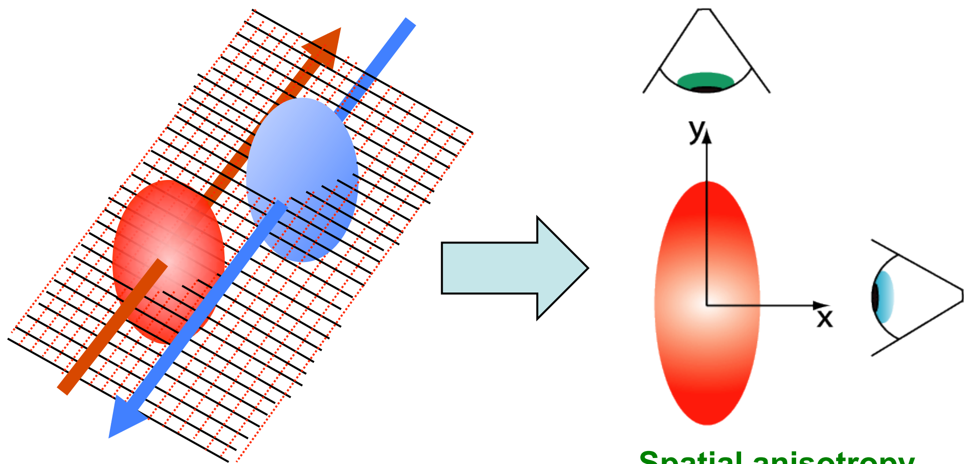
- Motivation: the unreasonable effectiveness of hydrodynamics in heavy ion collisions

<u>An ab initio weak coupling approach:</u>

- Paradigm: Classical coherence in nuclear wavefunctions
- Quantum fluctuations: Factorization, Evolution, Decoherence
- Isotropization, Bose-Einstein Condensation, Thermalization ?

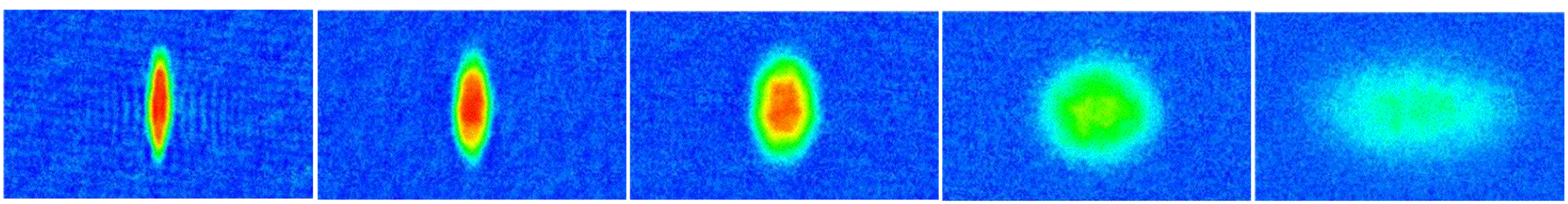
HI theory draws concretely on concepts in perturbative and non-perturbative QCD, string holography, reaction-diffusion systems, topological effects, plasma physics, thermodynamics and stat. mech, quantum chaos, Bose-Einstein condensates, pre-heating in inflationary cosmology

Strong flow = (nearly) ideal hydrodynamics



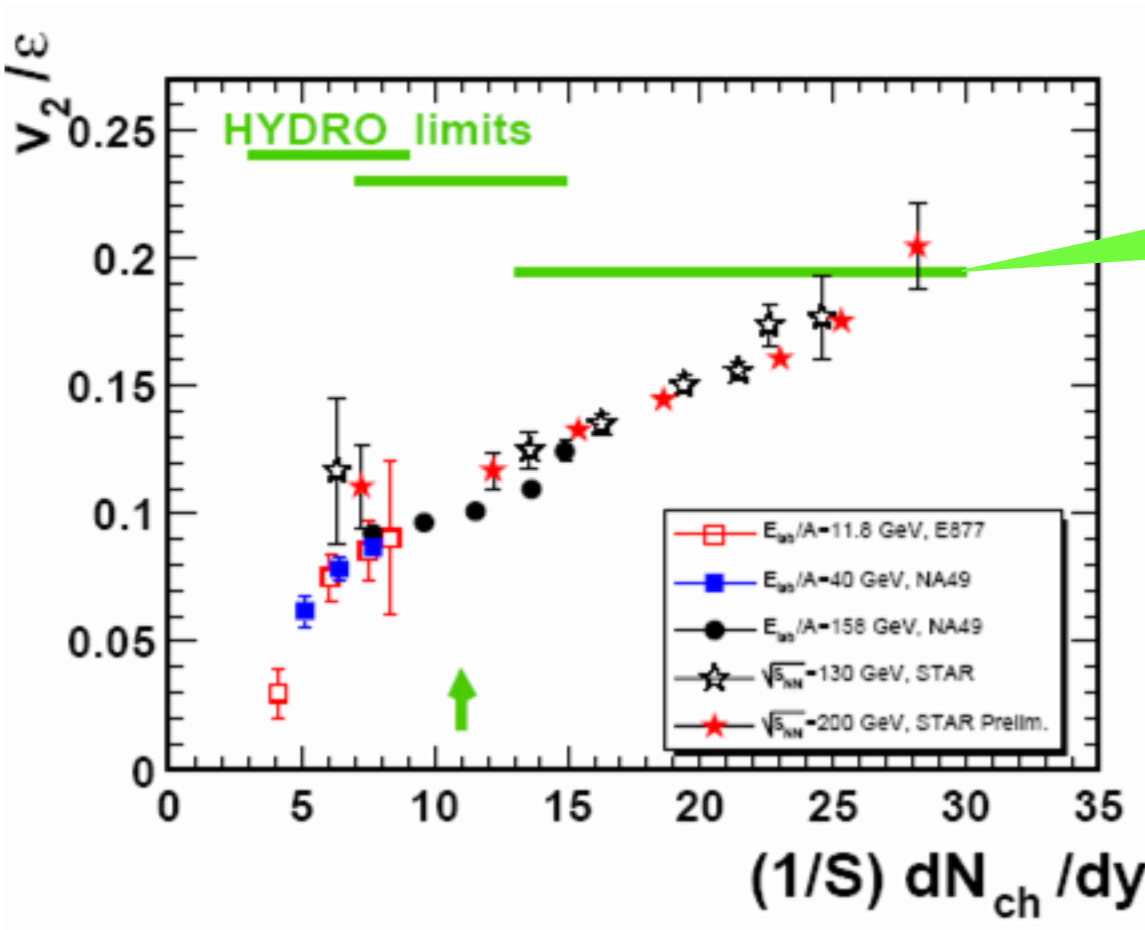
$$\frac{dN}{dy\, p_\perp dp_\perp d\phi} = \frac{1}{2\pi}\frac{dN}{dy\, p_\perp dp_\perp}\left(1 + 2\, v_2(p_\perp)\cos 2(\phi - \Psi_{\rm RP}) + \cdots\right)$$

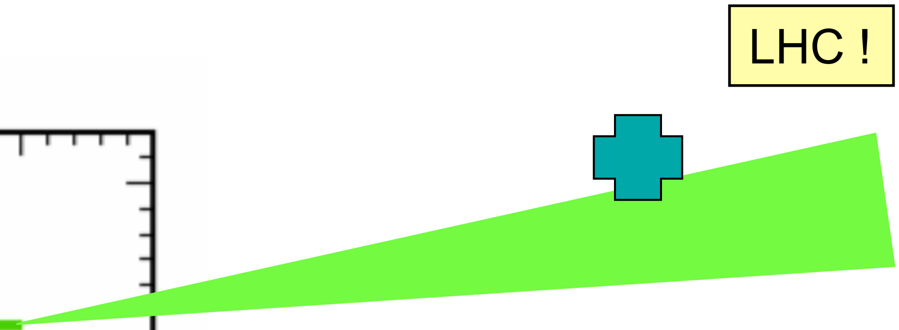
Spatial anisotropy



v_2 measures how efficiently hot matter converts spatial anisotropies to momentum anisotropy
– most efficient way is hydrodynamics

Strong flow = (nearly) ideal hydrodynamics



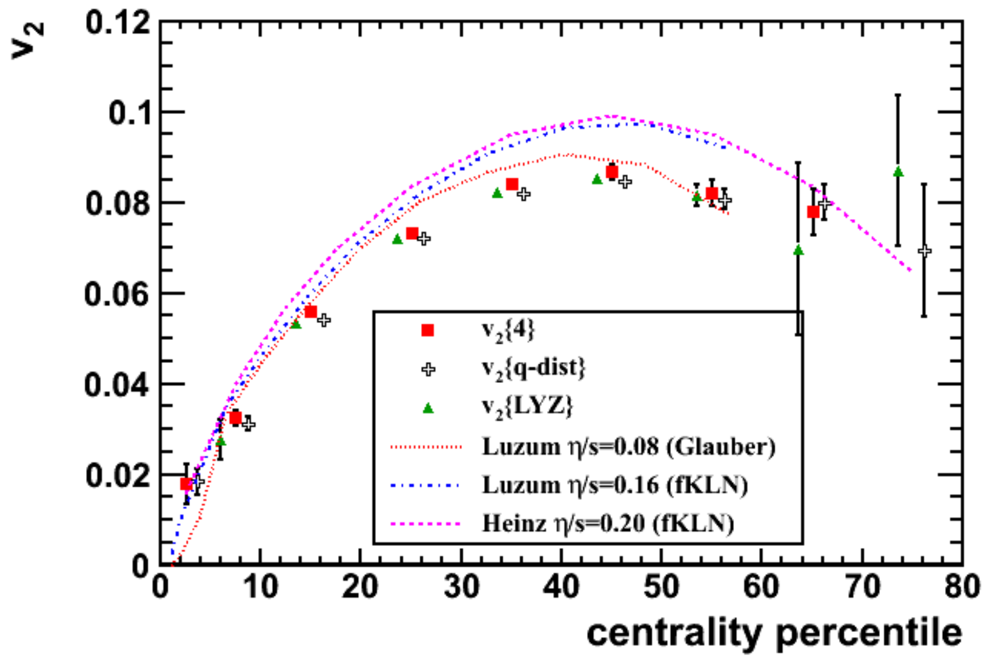


CERN Press release, November 26, 2010:

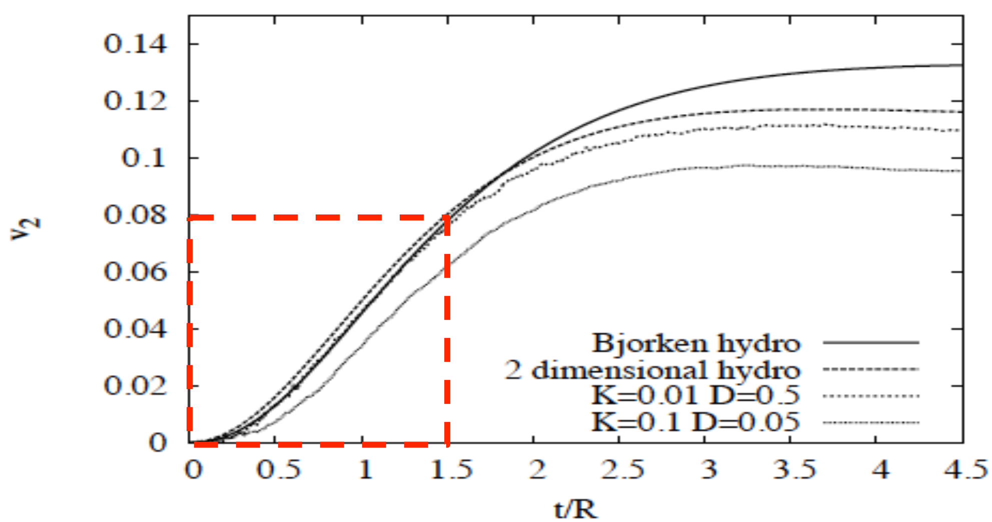
'confirms that the much hotter plasma produced at the LHC behaves as a very low viscosity liquid (a perfect fluid)..'

v_2 at RHIC and the LHC is large

Strong flow = (nearly) ideal hydrodynamics



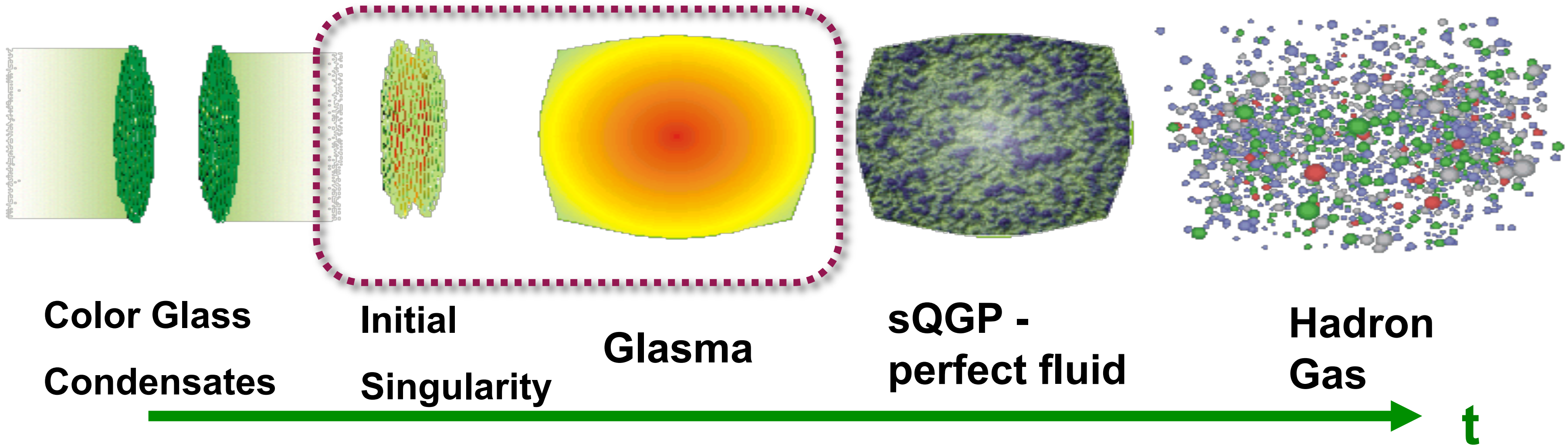
v_2 at LHC in agreement with (slightly) viscous relativistic hydrodynamics



Takes a long time ~ R/c_S to build up v_2

Flow must set in very early (≤ 1 fm)

Quantum decoherence from classical coherence



Glasma (\Glahs-maa\): ***Noun:*** **non-equilibrium matter between CGC and QGP**

Computational framework

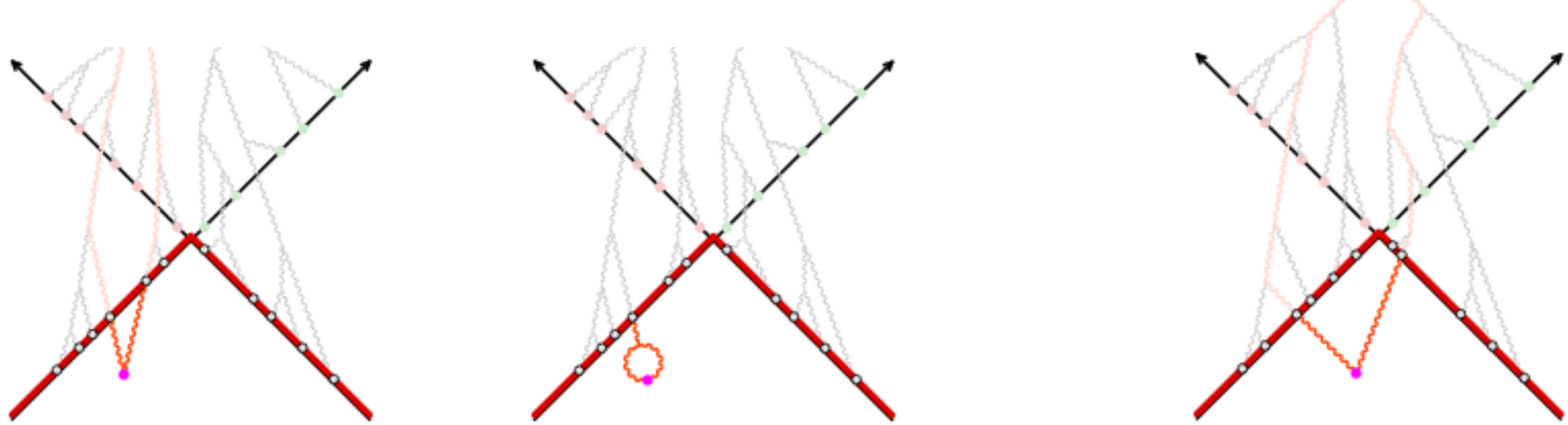
Gelis,RV NPA (2006)

Quantum field theory for strong time dependent sources (ρ ~ 1/g) ,

For eg., Schwinger mechanism for pair production in QED,
Hawking radiation on Black Hole horizon, …

Quantum fluctuations in classical backgrounds: I

Gelis,Lappi,RV: 0804.2630



Factorized into energy evolution of wavefunctions

Suppressed

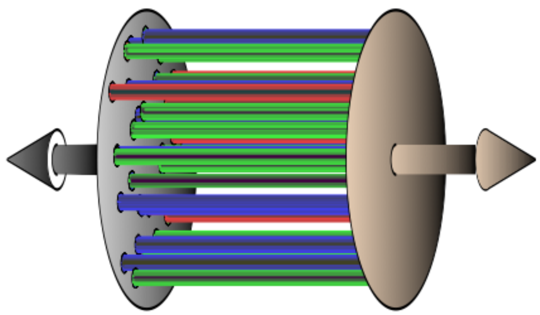
$p^\eta=0$ (small x !) modes that are coherent with the nuclei can be factorized for inclusive observables - JIMWLK factorization

$$\langle T^{\mu\nu}(\tau,\underline{\eta},x_\perp)\rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau,x_\perp)$$

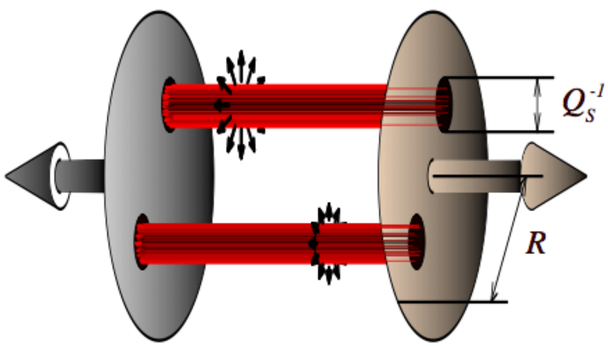
$$Y_1 = Y_{\rm beam} - \eta\,;\; Y_2 = Y_{\rm beam} + \eta$$

The W's are universal "functional density matrices"
and can be extracted from DIS or hadronic collisions

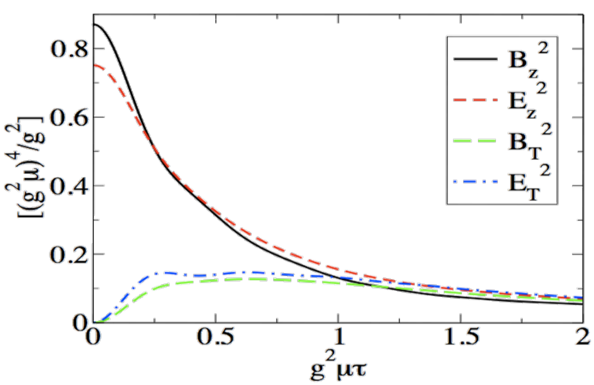
Classical features of the Glasma



Solutions of Yang-Mills equations produce (nearly) boost invariant gluon field configurations: “Glasma flux tubes”



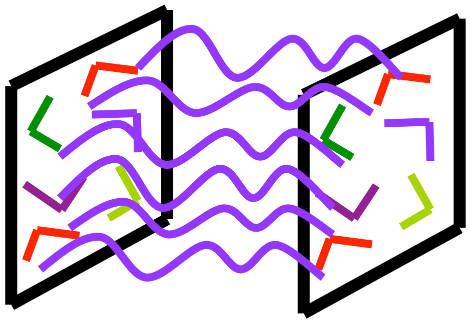
Lumpy gluon fields are color screened in transverse plane over distances ~ $1/Q_S$
- Negative Binomial multiplicity distribution.



“Glasma flux tubes” have non-trivial longitudinal color E & B fields at early times
--generate Chern-Simons topological charge

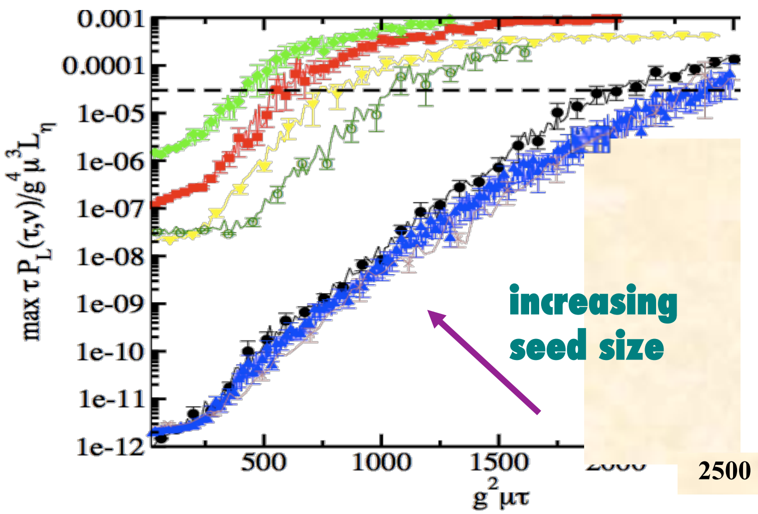
Quantum fluctuations in classical backgrounds: II

Romatschke,Venugopalan
Fukushima,Gelis,McLerran



Quant. fluct.
grow exponentially
after collision

As large as classical
field at 1/Qs !



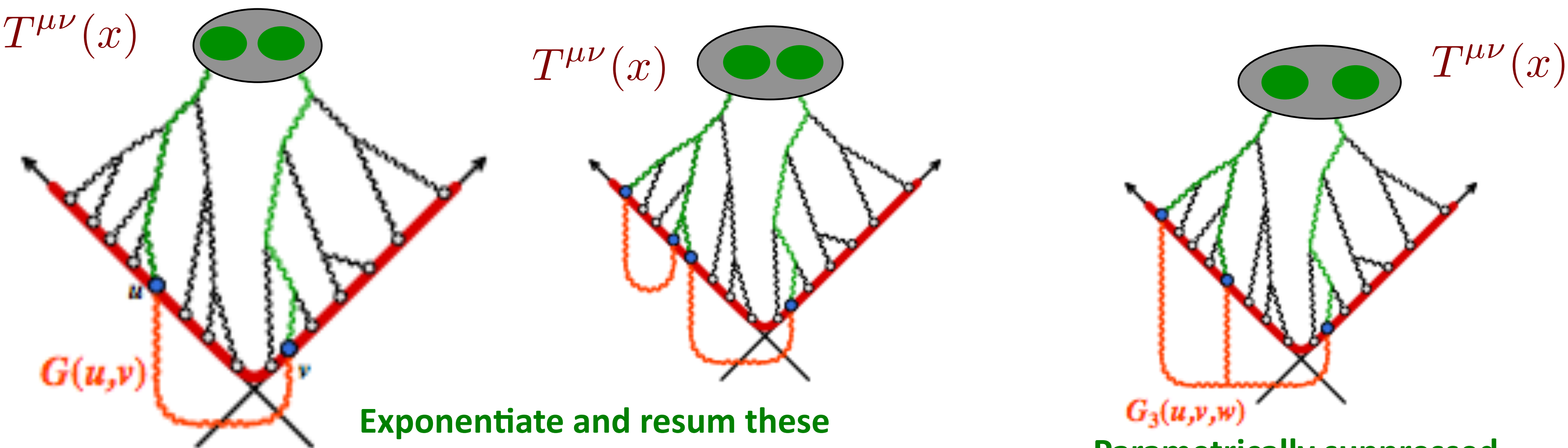
$p^\eta \neq 0$ (generated after collision) modes grow exponentially with proper time can have to be resummed to all orders

$$\left[g \exp\left(\sqrt{Q_S \tau} \right) \right]^n$$

So called "secular divergences" known in condensed matter physics

The Boltzmann equation is a specific example…

Glasma spectrum of initial quantum fluctuations



Leading quantum corrections to all orders give:

$$\langle\langle T^{\mu\nu}\rangle\rangle_{\mathrm{LLx+Linst.}} = \int [D\rho_1][D\rho_2]\, W_{\mathrm{Y_{beam}-Y}}[\rho_1]\, W_{\mathrm{Y_{beam}+Y}}[\rho_2] \times \int [da(u)]\; F_{\mathrm{init}}[a]\; T^{\mu\nu}_{\mathrm{LO}}[A_{\mathrm{cl}}(\rho_1,\rho_2)+a]$$

Gauge invariant Gaussian spectrum of quantum fluctuations computed *ab initio*

(in inflation, see Son; Khlebnikov,Tkachev; Kofman,Linde,Starobinsky)

Glasma spectrum of initial quantum fluctuations

Path integral over small fluctuations equivalent to

$$A(x_\perp,\tau,\eta) = A_{\rm cl.}(x_\perp,\tau) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\, c_{\nu k}\, e^{i\nu\eta}\, \chi_k(x_\perp)\, H_{i\nu}(\lambda_k \tau) + c.c$$

Gaussian random variables

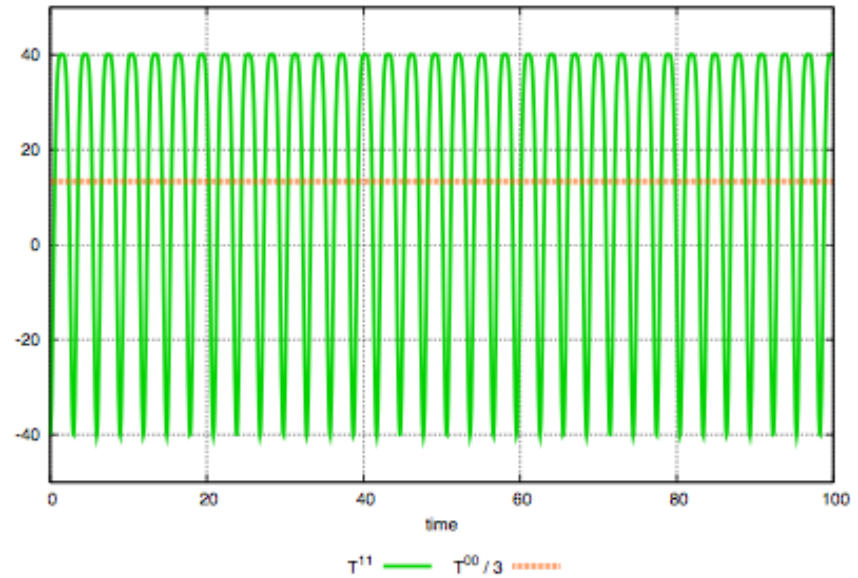
Berry conjecture: High lying quantum eigenstates of classically chaotic systems, linear superpositions of Gaussian random variables

Srednicki: Systems that satisfy Berry's conjecture exhibit "eigenstate thermalization"

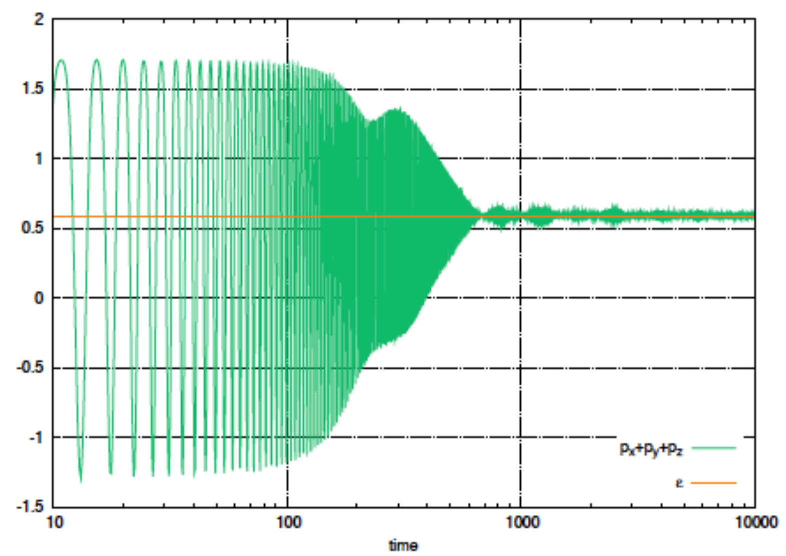
Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

scalar Φ^4 theory:



Energy density and pressure without averaging over fluctuations



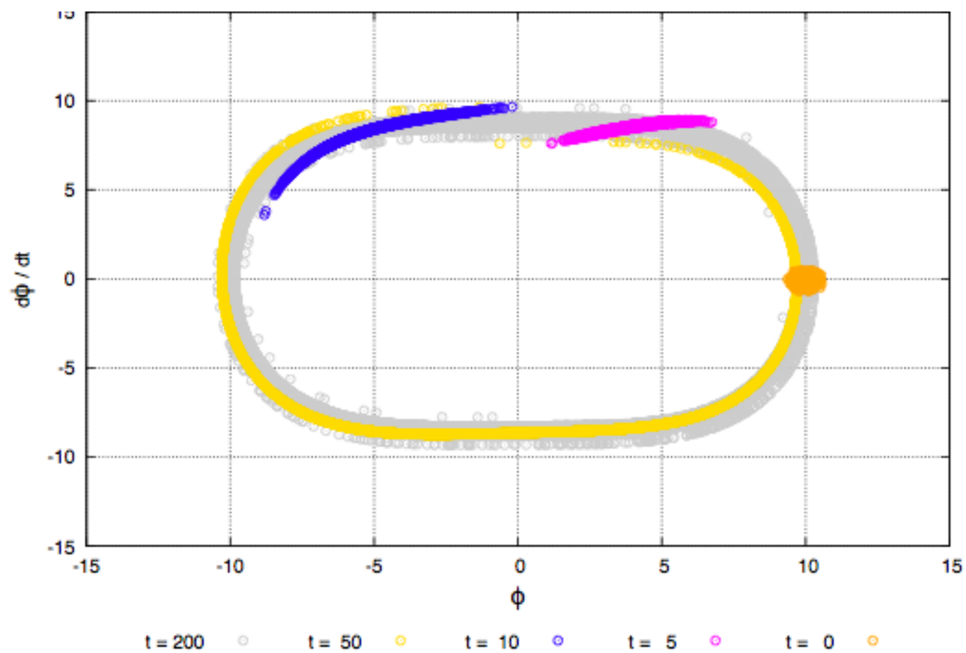
Energy density and pressure after averaging over fluctuations

→ **Converges to single valued relation "EOS"**

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

Anatomy of phase decoherence:



$\Delta\Theta = \Delta\omega\ t$

$T_{period} = 2\pi\ /\ \Delta\omega$

➔ $T_{period} \cong 18.2\ /\ g\ \Delta\Phi_{max}$

Different field amplitudes from different initializations of the classical field

$$\langle T^\mu_\mu \rangle = \int d\phi\ d\dot{\phi}\ \rho_t(\phi,\dot{\phi})\ T^\mu_\mu(\phi,\dot{\phi}) \equiv \int dE\ d\theta\ \tilde{\rho}_t(E,\theta)\ T^\mu_\mu(E,\theta)$$

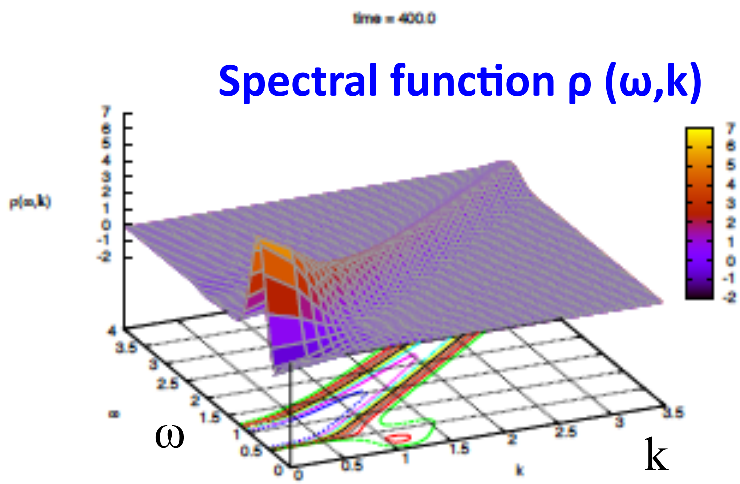
$$t \overset{\approx}{\rightarrow} \infty \quad \int dE\ \tilde{\rho}_t(E) \int d\theta\ T^\mu_\mu(E,\theta) = 0$$

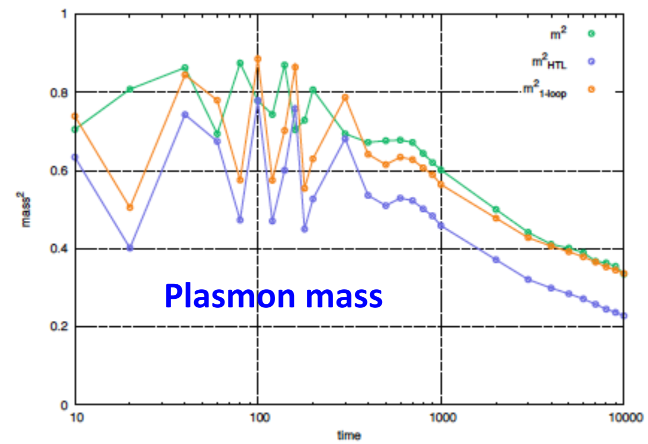
$$\int d\theta\ T^\mu_\mu(E,\theta) = \frac{2\pi}{T} \int_t^{t+T} d\tau\ T^\mu_\mu(\phi(\tau),\dot{\phi}(\tau)) = 0$$

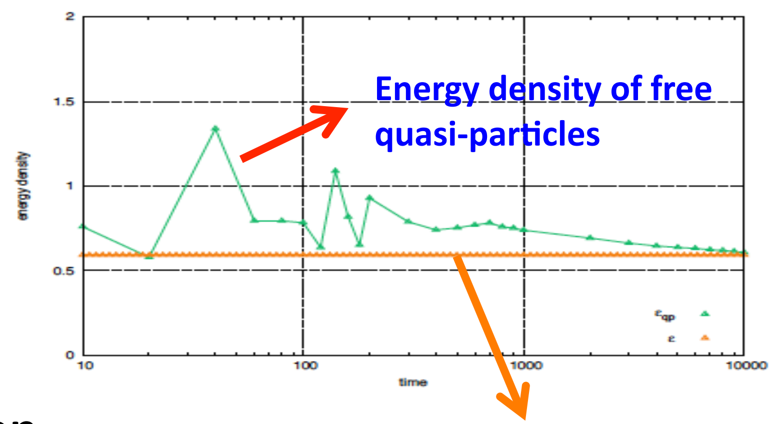
Because T_μ^μ for scalar theory is a total derivative and ϕ is periodic

Quasi-particle description?

Epelbaum,Gelis (2011)





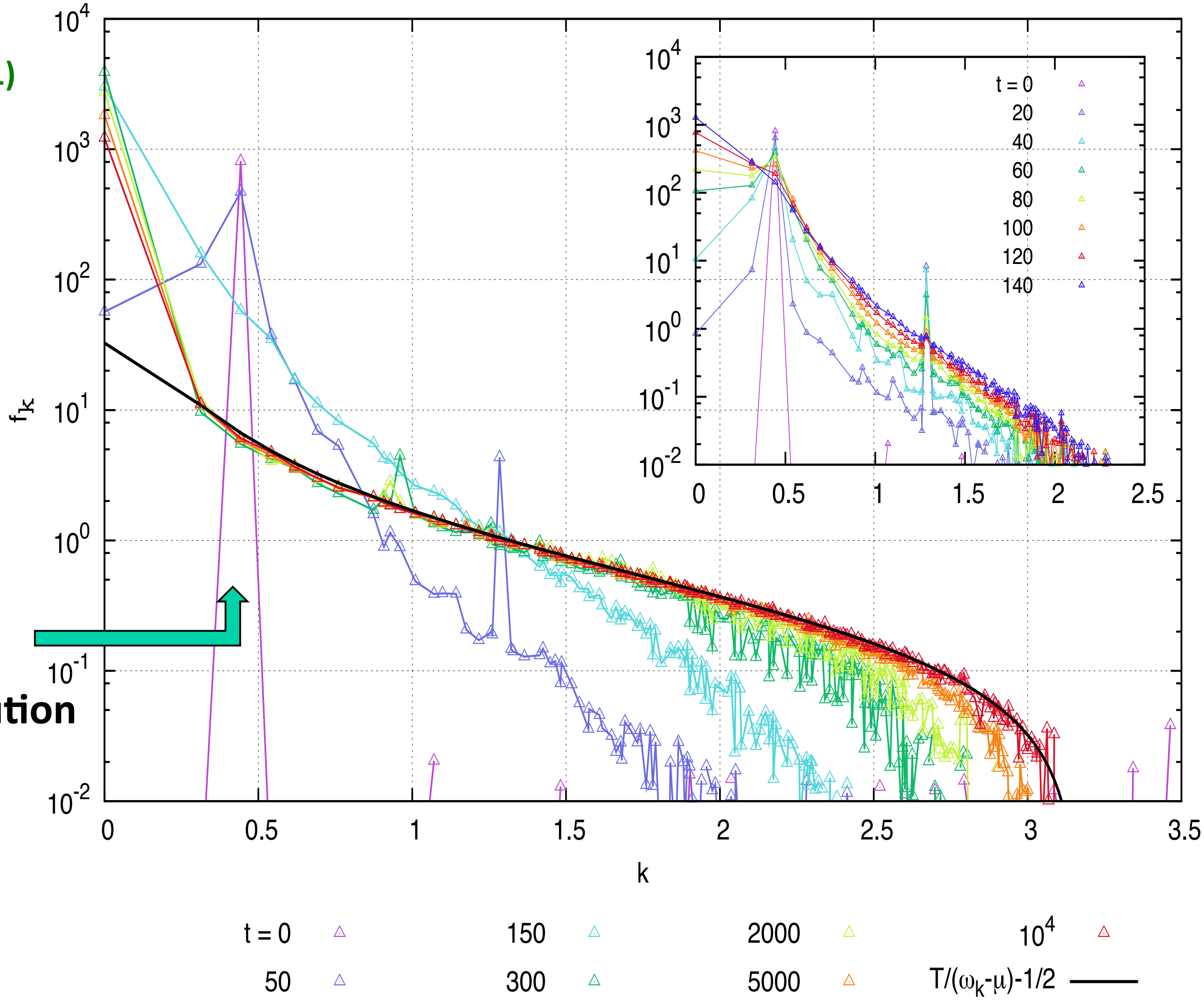


Energy density on the lattice

- ❑ **At early times, no quasi-particle description**
- ❑ **May have quasi-particle description at late times. Effective kinetic "Boltzmann" description in terms of interacting quasi-particles at late times ?**

Quasi-particle occupation number

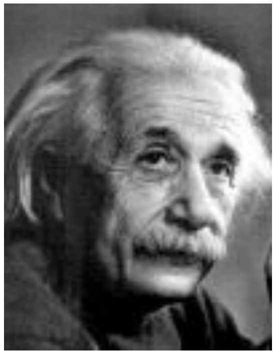
Gelis,Epelbaum (2011)

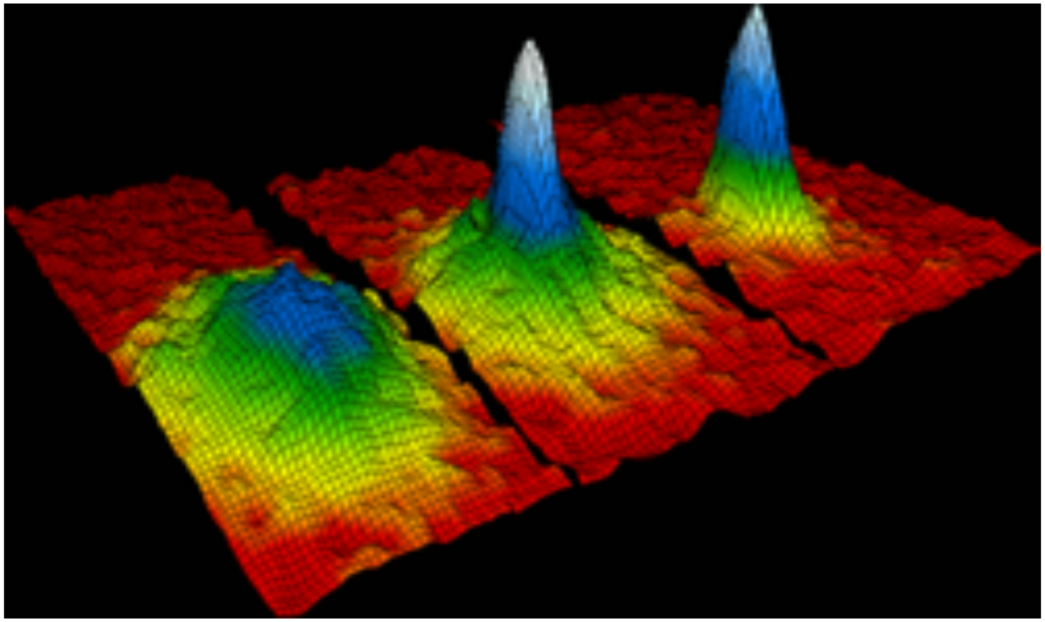


Initial mode distribution

System becomes over occupied relative to a thermal distribution…

Bose-Einstein Condensation in HI Collisions ?





Cold rubidium atoms in a magnetic trap

Gell-Mann's Totalitarian Principle of Quantum Mechanics:
Everything that is not forbidden is Compulsory

Bose-Einstein Condensation and Thermalization

Blaizot,Gelis,Liao,McLerran,RV: arXiv:1107.5295v2

Assumption: Evolution of "classical" fields in the Glasma can be matched to a quasi-particle transport description

See also, Mueller, Son (2002)

All estimates are "parametric": $\alpha_S \ll 1$

System is over-occupied: $n \approx Q_S^3/\alpha_s$; $\varepsilon = Q_S^4/\alpha_S$

$\Rightarrow n \bullet \varepsilon^{-3/4} \approx 1/\alpha_S^{1/4} >> 1$

In a thermal system, $n \bullet \varepsilon^{-3/4} = 1$

If a system is over-occupied near equilibrium and elastic scattering dominates, it can generate a Bose-Einstein condensate

Known also in context of inflation:
Khlebnikov, Tkachev (1996)
Berges et al. (2011)

Bose-Einstein Condensation and Thermalization

$$n_{\rm eq} = \int_{\mathbf{p}} f_{\rm eq}(\mathbf{p}) \;;\; \varepsilon_{\rm eq} = \int_{\mathbf{p}} \omega_{\mathbf{p}}\, f_{\rm eq}(\mathbf{p})$$

$$f_{\rm eq}(\mathbf{p}) = \frac{1}{e^{\beta(\omega_p - \mu)} - 1}$$

In a many-body system, gluons develop a mass
$\omega_{p=0} = m \approx \alpha_S^{1/2}\, T$

If over-occupation persists for μ = m, system develops a condensate

$$f_{\rm eq}(\mathbf{p}) = n_c \delta^3(\mathbf{p}) + \frac{1}{e^{\beta(\omega_p - m)} - 1}$$

$$n_c = \frac{Q_s^3}{\alpha_S}\left(1 - \alpha_S^{1/4}\right)$$

As α_S ➔ 0, most particles go into the condensate

$$\varepsilon_c = m\, n_c \approx \alpha_S^{1/4}\, T^4 << T^4$$

It however carries a small fraction of the energy density…

Transport in the Glasma

“Landau” equation for small angle 2 → 2 scattering:

$$\frac{df}{dt}|_{\rm coll} \sim \frac{\Lambda_S^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[\frac{df}{dp} + \frac{\alpha_S}{\Lambda_S} f(p)(1+f(p)) \right] \right\}$$

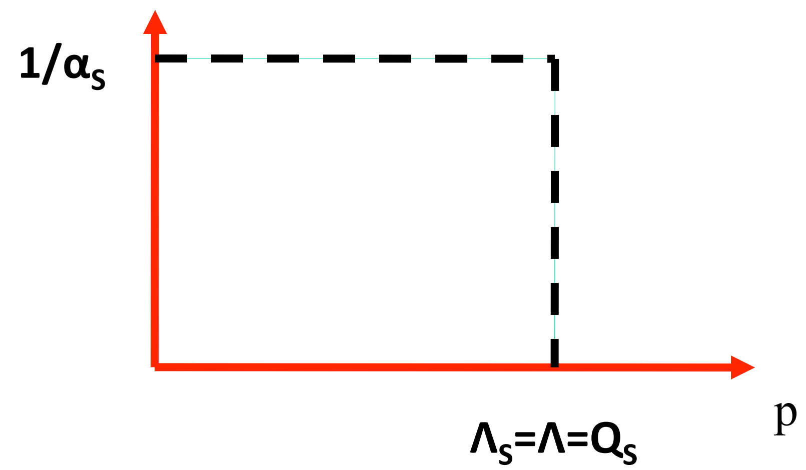
This is satisfied by a distribution where

$$f \sim \frac{1}{\alpha_S} \; ; p < \Lambda_S \qquad \sim \frac{1}{\alpha_S} \frac{\Lambda_S}{p} \; ; \Lambda_S < p < \Lambda \qquad \sim 0 \; ; \Lambda < p$$

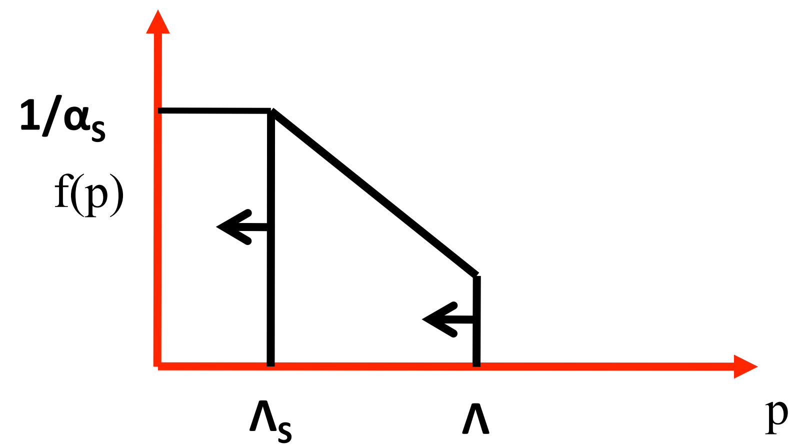
Λ_S and Λ are dynamical time dependent scales determined by the transport equation

Transport in the Glasma

At $\tau \sim 1/Q_S$



At $\tau > 1/Q_S$



When $\Lambda_S = \alpha_S \Lambda$, the system thermalizes;
one gets the ordering of scales: $\Lambda = T$, $m = \Lambda \Lambda_S = \alpha^{1/2} T$, $\Lambda_S = \alpha_S T$

Thermalization: from Glasma to Plasma

Fixed box: Energy conservation gives $\Lambda^3 \Lambda_S$ = constant

From moments of transport eqn., , $\tau_{coll} = \Lambda / \Lambda_S^2 \sim t$

From these two conditions, $\Lambda_S \sim Q_s \left(\frac{t_0}{t}\right)^{3/7}$ $\Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{1/7}$

Thermalization time: $t_{\mathrm{therm.}} \sim \frac{1}{Q_S}\left(\frac{1}{\alpha_S}\right)^{7/4}$

Also, Kurkela, Moore (2011))

Entropy density s = Λ^3 increases and saturates at t_{therm} as T^3

We showed that system is strongly interacting with itself due to coherence of fields

Thermalization: from Glasma to Plasma

Expanding box : **matter is now strongly self interacting for fixed momentum anisotropy**

$$\varepsilon_g(t) \sim \varepsilon(t_0) \left(\frac{t_0}{t}\right)^{1+\delta}$$

$0 < \delta \leq 1/3$

$$\Lambda_S \sim Q_S \left(\frac{t_0}{t}\right)^{(4+\delta)/7} \qquad \Lambda \sim Q_S \left(\frac{t_0}{t}\right)^{(1+2\delta)/7}$$

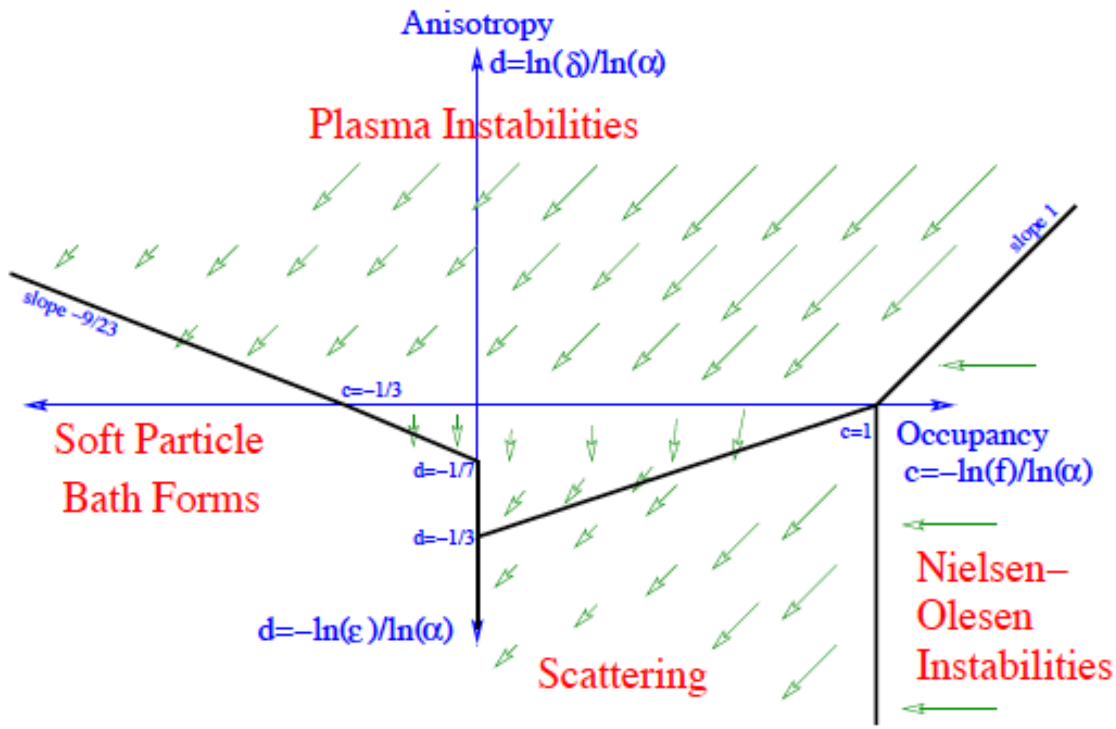
Thermalization time t_{therm} $= \frac{1}{Q_S}\left(\frac{\tau_0}{\tau}\right)^{7/(3-\delta)}$

For δ= -1, recover fixed box results…

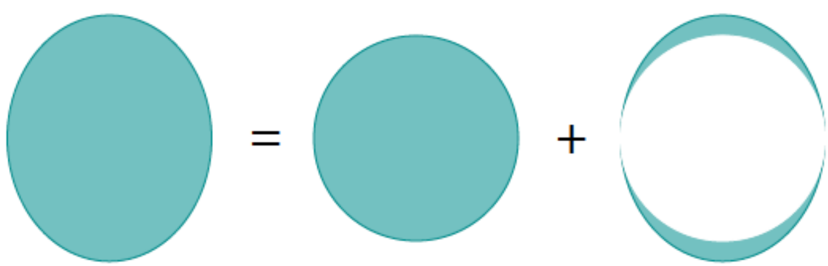
A condensate can still form in the expanding case for δ > 1/5

What about plasma instabilities ?

Another mechanism for isotropization, hydrodynamics, thermalization



Kurkela, Moore(2011)



Likely weak anisotropy relevant:
Needs careful study to gauge impact on scaling solutions

Summary

- Presented *ab initio* picture of collective features of multi-particle production and thermalization in heavy ion collisions

- Thermalization is a subtle business even in weak coupling

- Hydrodynamics is unreasonably effective because it requires rapid decoherence of classical fields and strong self-interactions, not thermalization

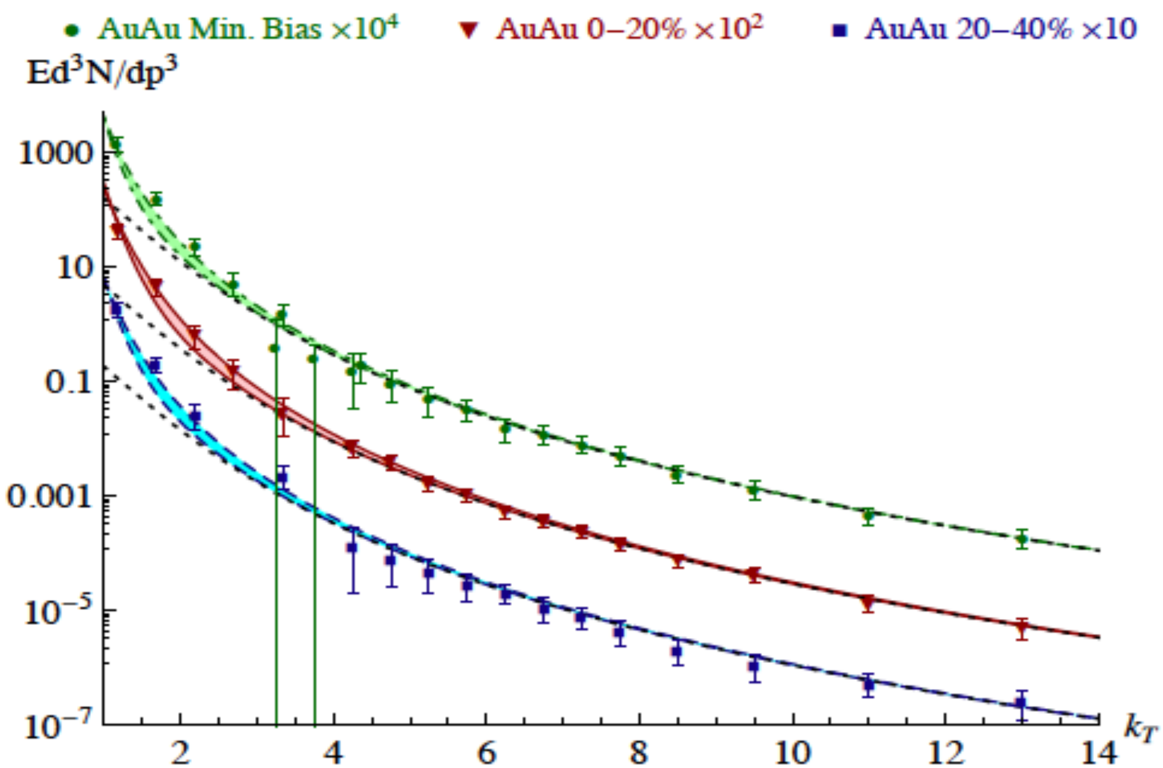
- Exciting possibility of a transient Bose-Einstein Condensate interesting phenomenological consequences

THE END

Photon & di-lepton emission in HI collisions

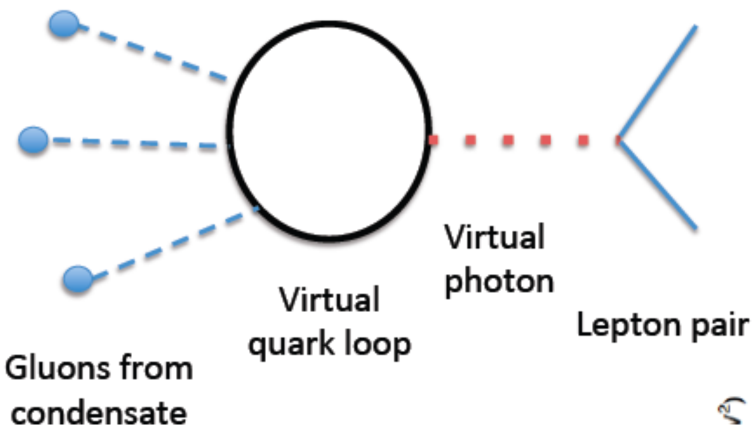
- **Photons: excess at transverse momenta 1-3 GeV; strong N_{part}^2 dependence**

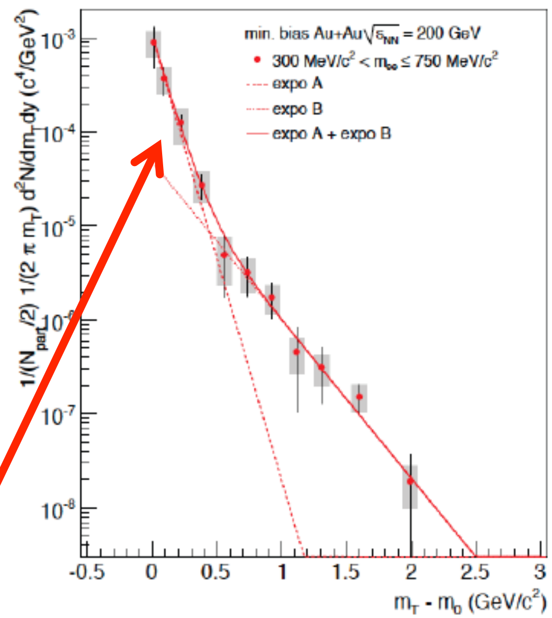
McLerran et al: distributions sensitive to anisotropy δ



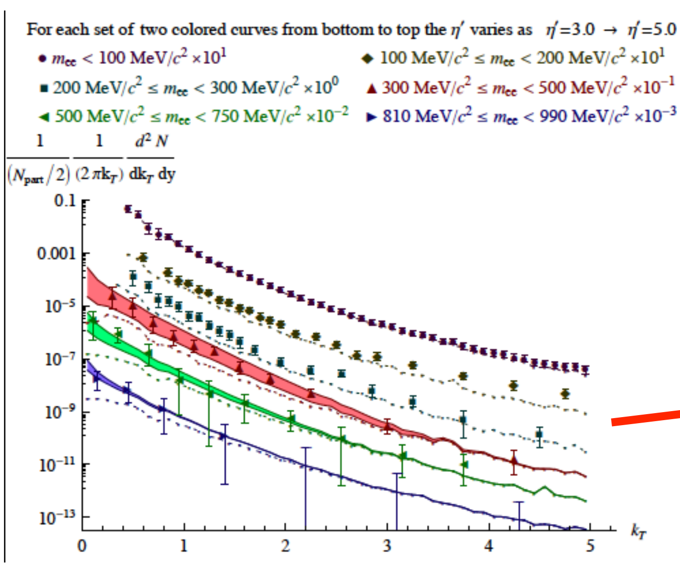
Photon & di-lepton emission in HI collisions

- **Di-leptons: excess in region pT < M; M ≤0.5**



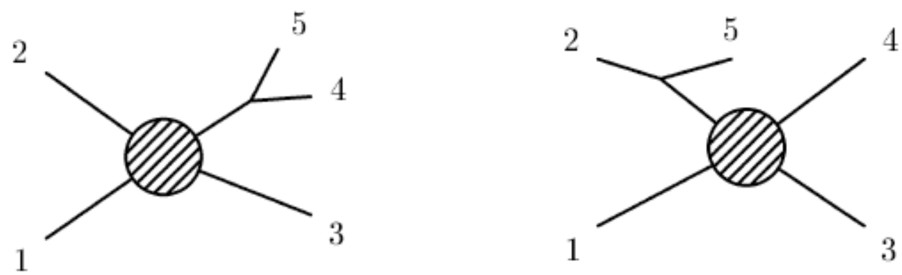


Fit T ~ 100 MeV; inconsistent with thermal emission or hard scattering



Min. bias PHENIX data – predict $N_{part}^{3/2}$ dependence

Role of inelastic processes ?



Wong (2004)
Mueller,Shoshi,Wong (2006)

Power counting for n ➔ m processes contributions to the collision integral

Vertices contribute α_S^{n+m-2}

Factor of $(\Lambda_S/\alpha_S)^{n+m-2}$ from distribution functions

Screened infrared singularity: $(1/\Lambda\ \Lambda_S)^{n+m-4}$

Remaining phase space integrals Λ^{n+m-5}

Net result is $\tau_{inelas} \sim \Lambda / \Lambda_S^2 = \tau_{elas}$

At most parametrically of the same order as elastic scattering.
So a transient Bose-Einstein condensate can form.

Numerical simulations will be decisive

Dusling,Epelbaum,Gelis,RV, in progress
Blaizot, Liao, McLerran