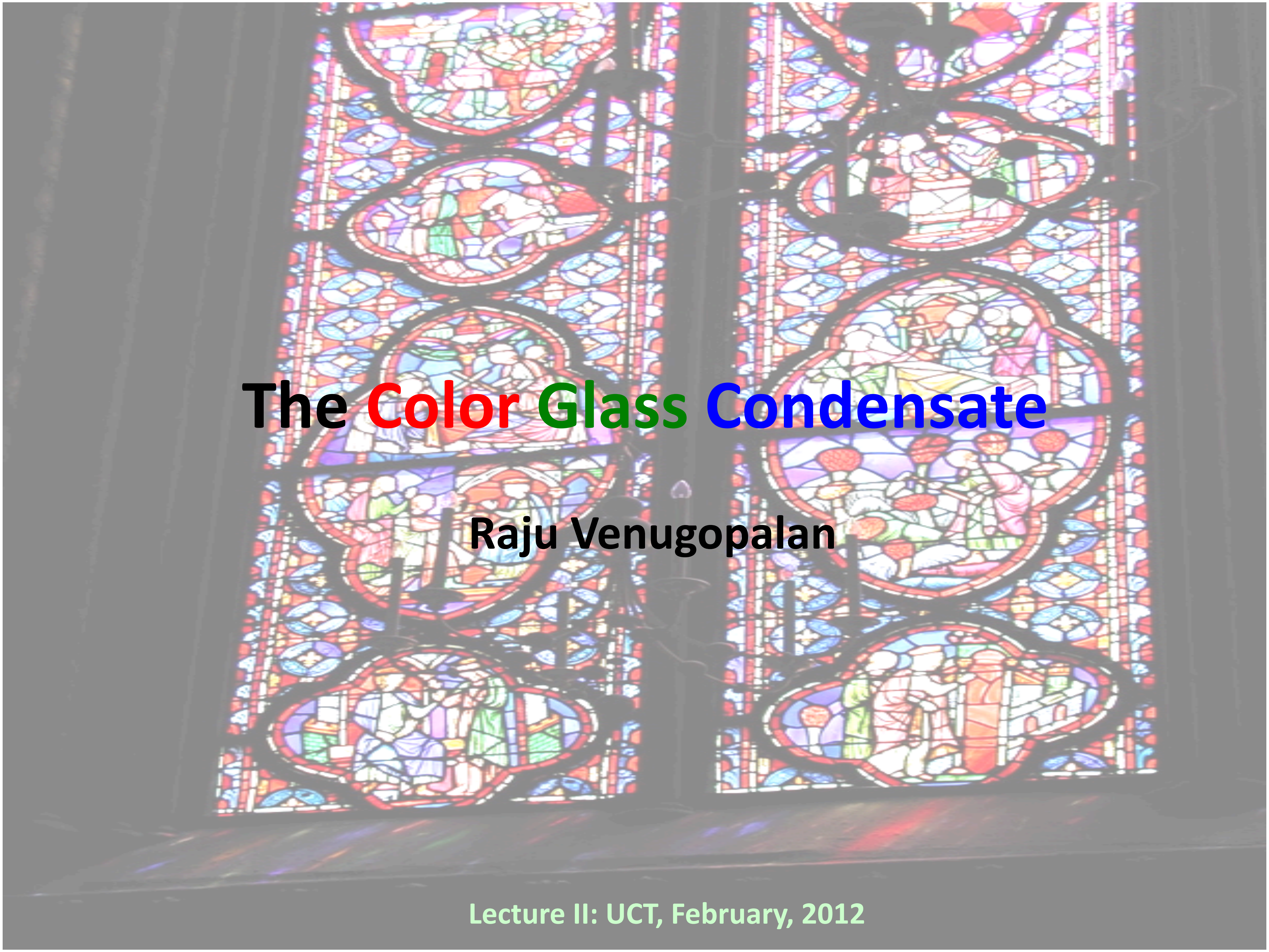


The Color Glass Condensate

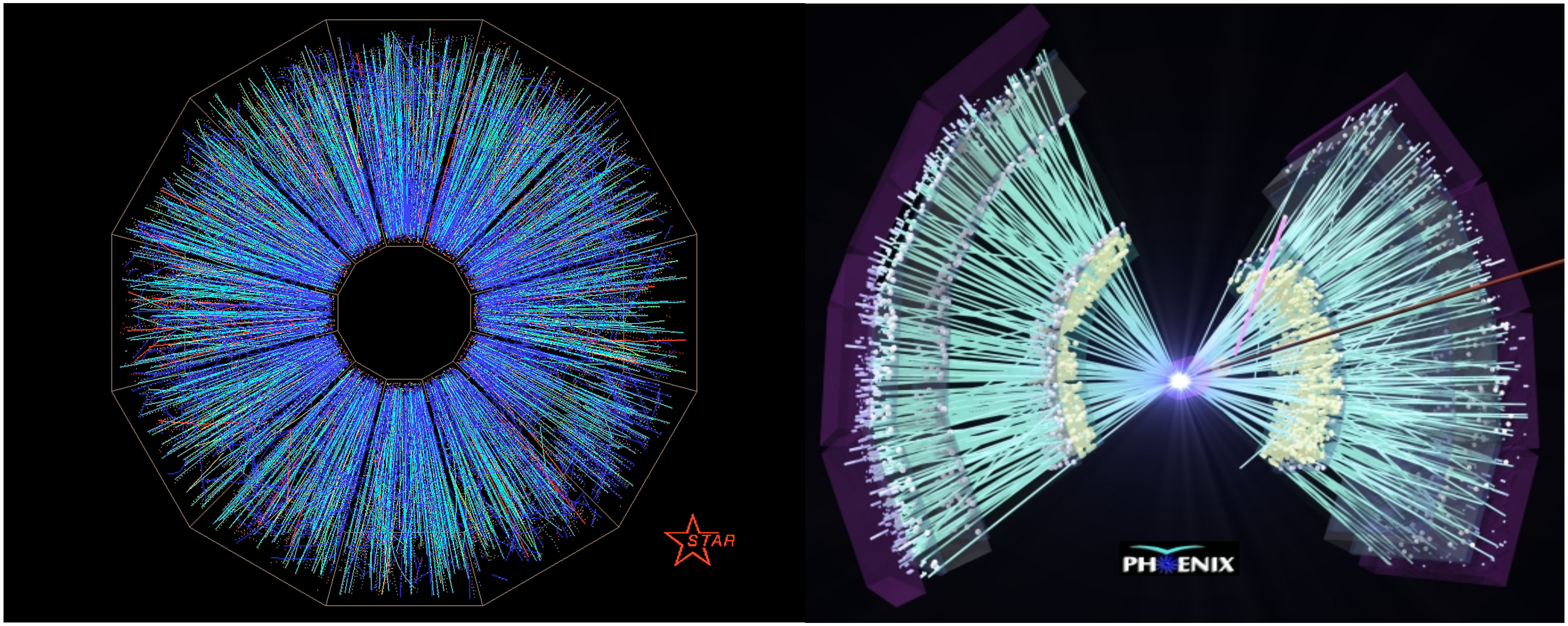
Raju Venugopalan

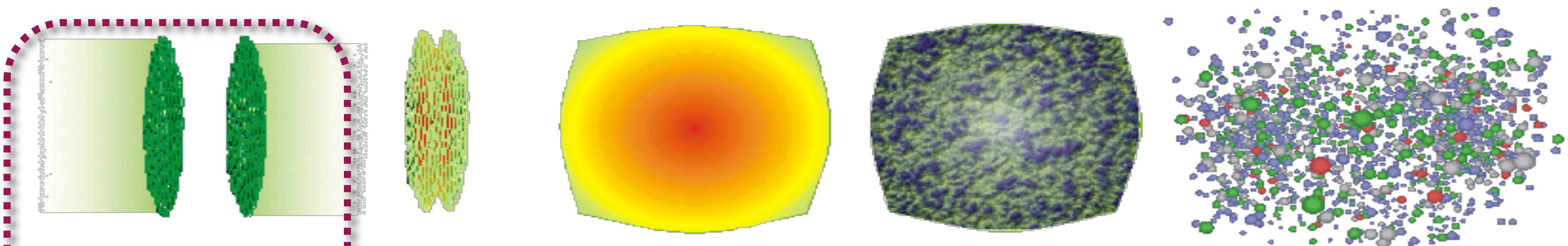
Lecture II: UCT, February, 2012

Outline of lectures

- Lecture I: QCD and the Quark-Gluon Plasma
- Lecture II: Gluon Saturation and the Color Glass Condensate
- Lecture III: Quantum field theory in strong fields. Factorization and the Glasma
- Lecture IV: Quantum field theory in strong fields. Instabilities and the spectrum of initial quantum fluctuations
- Lecture V: Quantum field theory in strong fields. Decoherence, hydrodynamics, Bose-Einstein Condensation and thermalization
- Lecture VI: Future prospects: RHIC, LHC and the EIC

What does a heavy ion collision look like ?





Color Glass Condensates

Initial Singularity

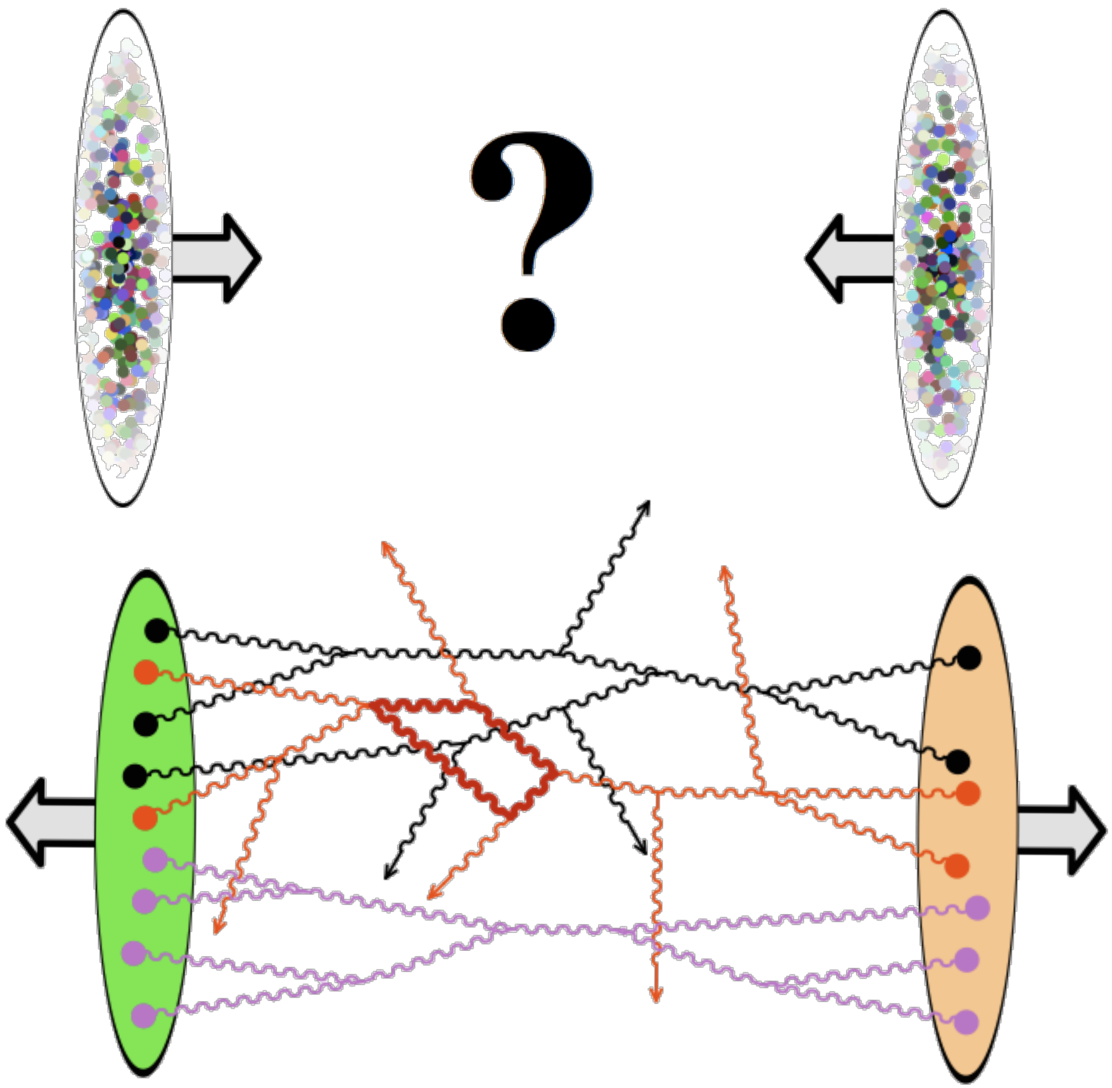
Glasma

sQGP - perfect fluid

Hadron Gas

t

The big role of wee gluons



The big role of wee glue

D. Nucleus-Nucleus Collisions at Fantastic Energies

Before leaving this subject it is fun to consider the collision of two nuclei at energies sufficiently high so that in addition to the fragmentation regions, a central plateau region can develop. Let us consider a central collision of a relatively small nucleus, say carbon, with a big one, say lead. Let us look at this collision in a center-of-mass frame for which the rapidities of both of the nucleus projectiles exceeds the critical rapidity. In such a frame they both possess the fur coat of wee-parton vacuum fluctuations. In such a central collision we see that the collision initially occurs between the fur of wee partons in each of the projectiles. Therefore the number of independent collisions will be of order of the area of overlap of the two projectiles; namely the cross-sectional area of the smaller nucleus.

(Nucleus-Nucleus Collisions at Fantastic Energies)

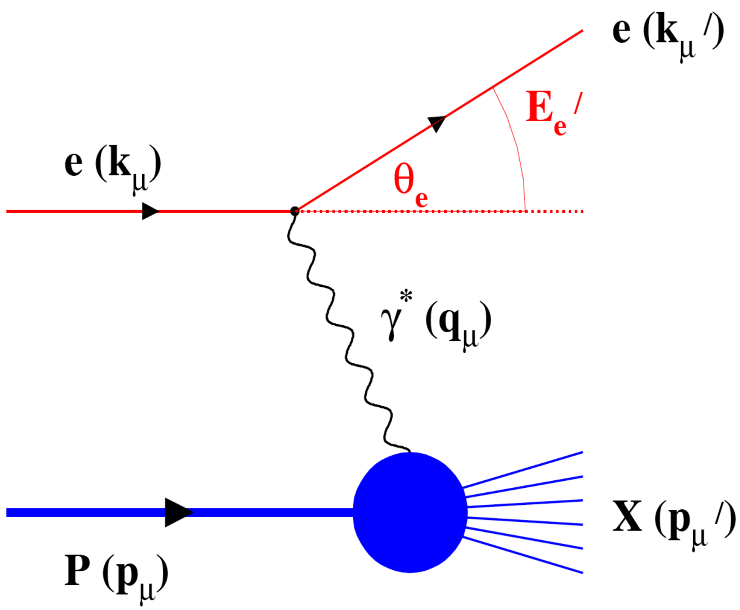
At LHC, ~14 units in rapidity!

Bj, DESY lectures (1975)

The big role of wee glue

- ❑ What is the role of wee partons ? ✔
- ❑ How do the wee partons interact and produce glue ? ✔
- ❑ Can it be understood *ab initio* in QCD ? ✔

The DIS Paradigm



$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

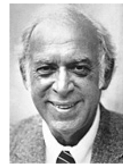
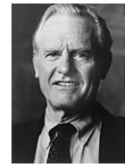
Measure of momentum fraction of struck quark

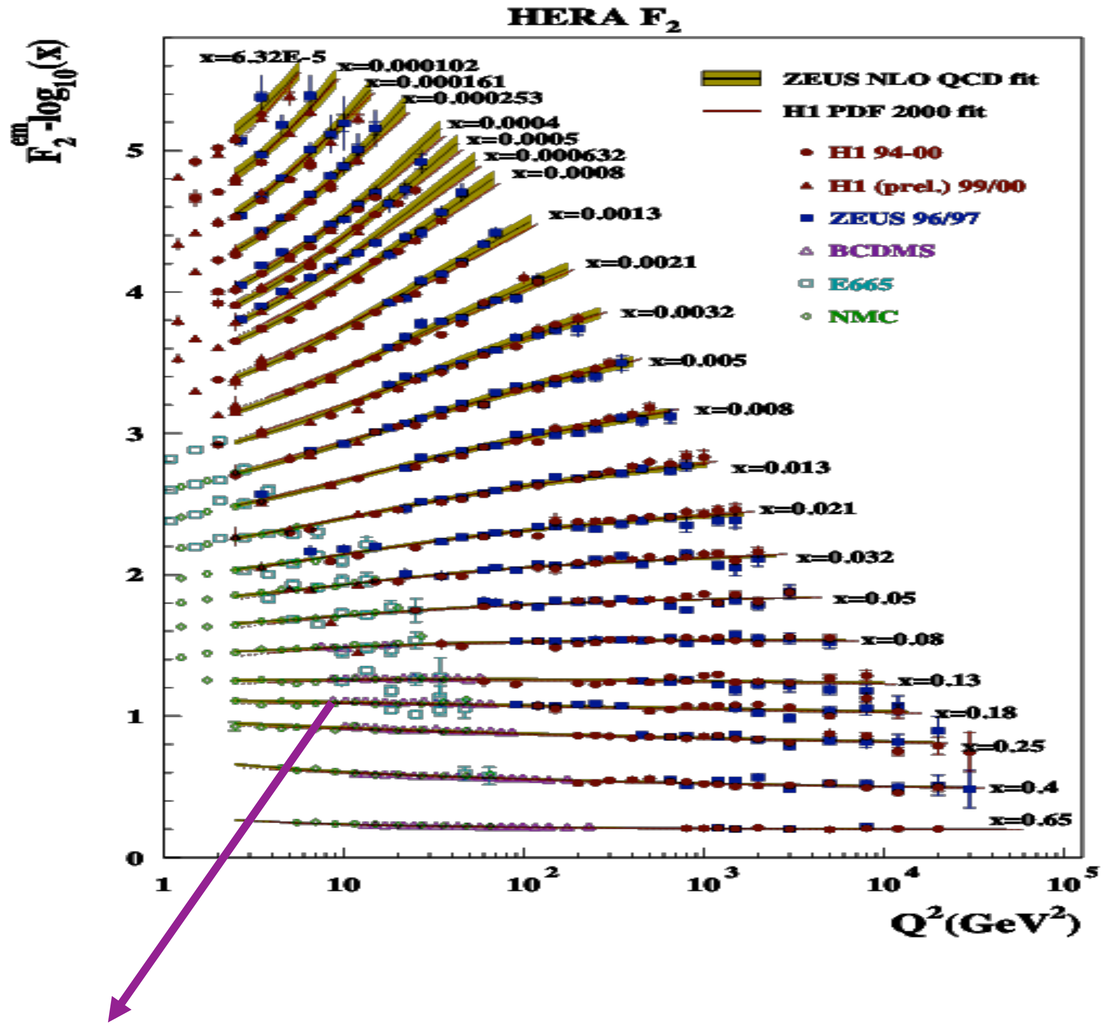
$$\frac{d^2\sigma^{eh\to eX}}{dxdQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

quark+anti-quark mom. dists.

gluon mom. dists

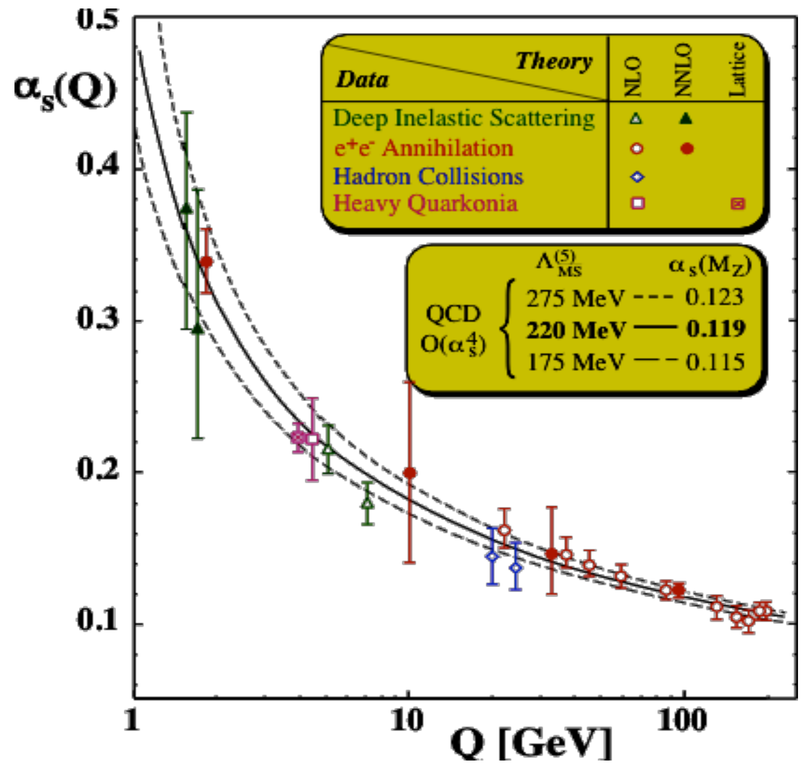
Nobel to Friedman, Kendall, Taylor

Bj-scaling: apparent scale invariance of structure functions

Puzzle resolved in QCD…



Gross, Wilczek, Politzer

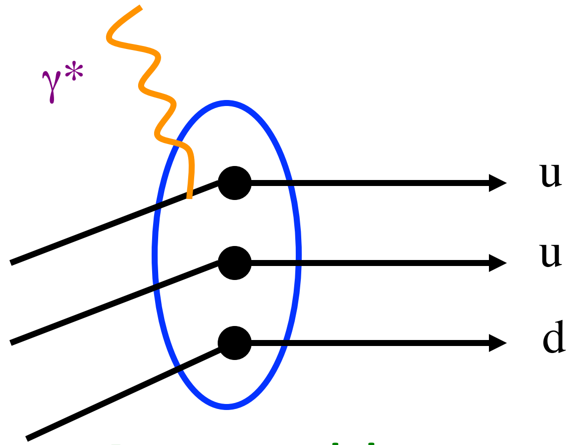


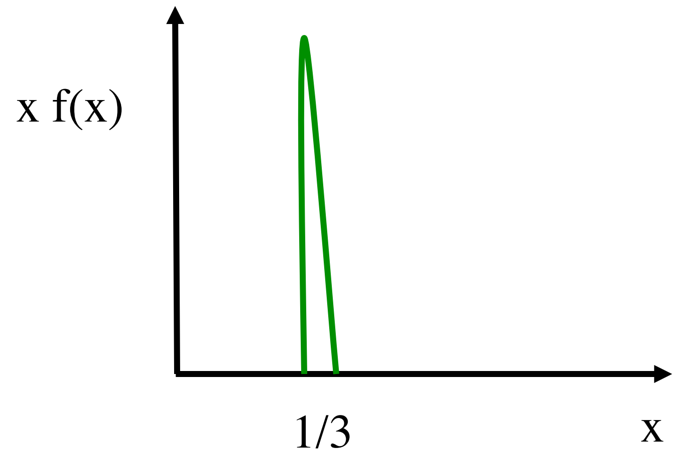

QCD ≠ Parton Model

Logarithmic scaling violations

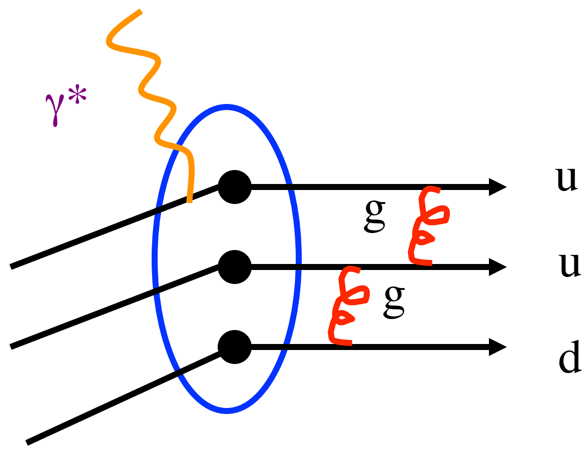
$$F_2(x,Q^2) = \sum_{\substack{q=u,c,t\\ d,s,b}} e_q^2 \left(x\, q(x,Q^2) + x\, \bar{q}(x,Q^2) \right)$$

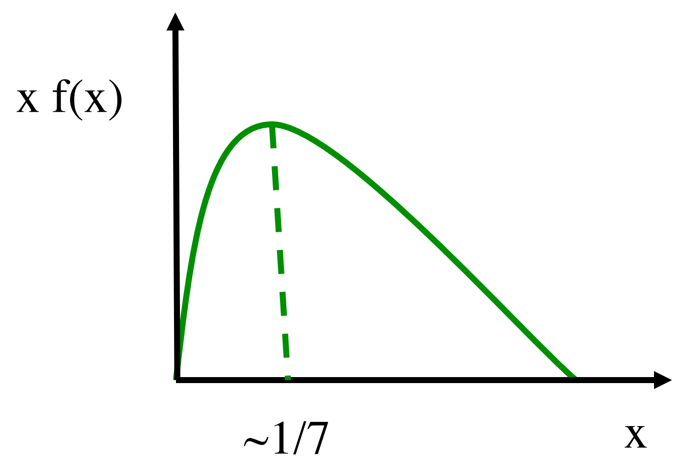
The proton at high energies



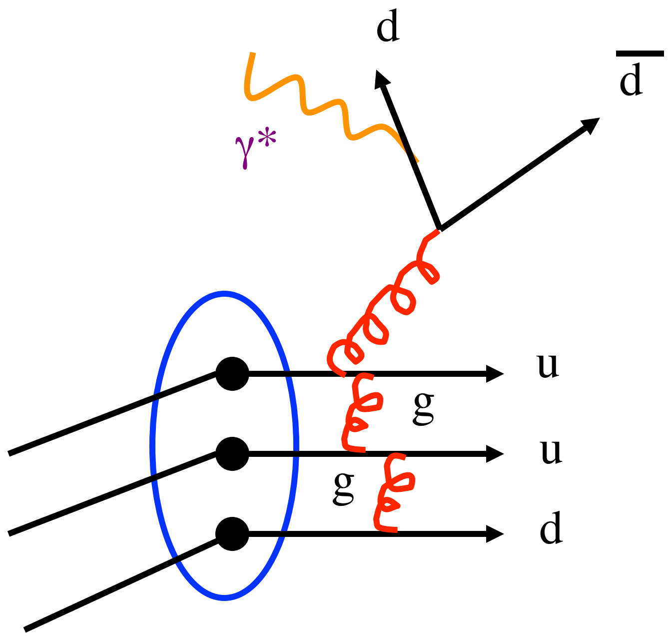


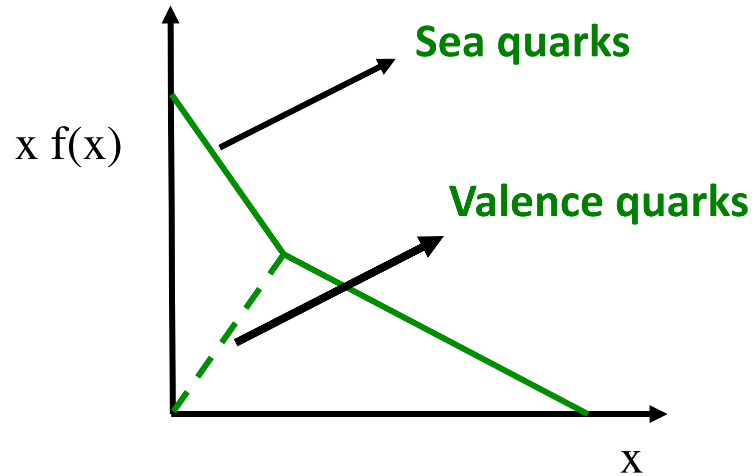
Parton model





QCD -log corrections

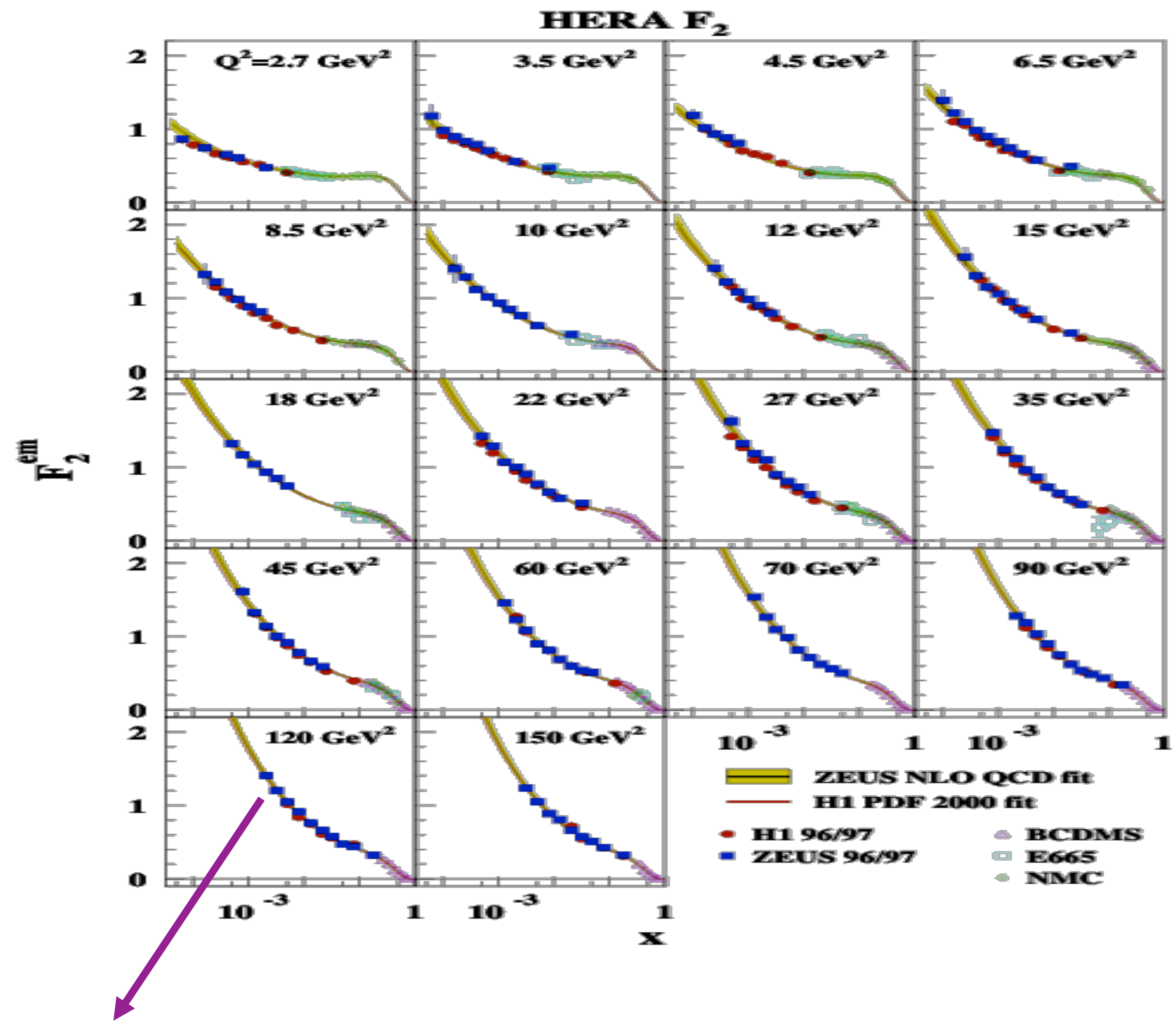




“x-QCD”- small x evolution

$$\int_0^1 \frac{dx}{x}\left(xq(x) - x\bar{q}(x)\right) = 3$$ → # of valence quarks

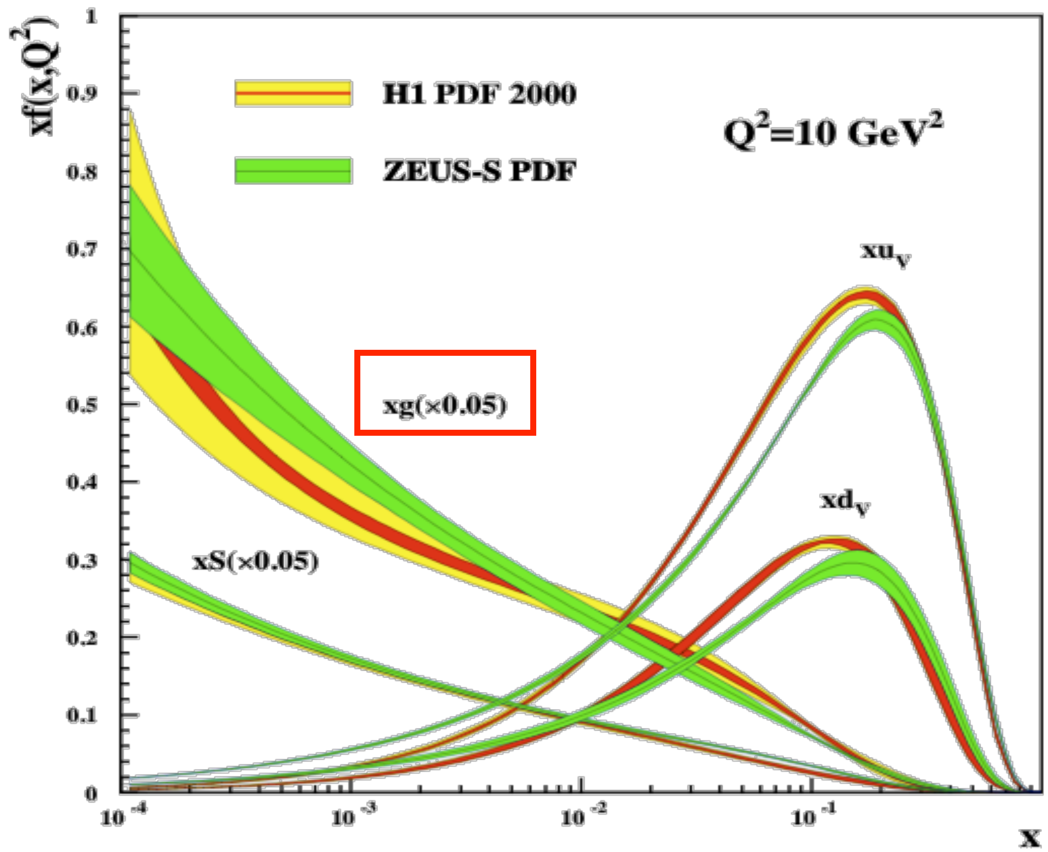
$$\int_0^1 \frac{dx}{x}\left(xq(x) + x\bar{q}(x)\right) \to \infty$$ → # of quarks



Structure functions grow rapidly at small x

Where is the glue ?

partons / unit rapidity



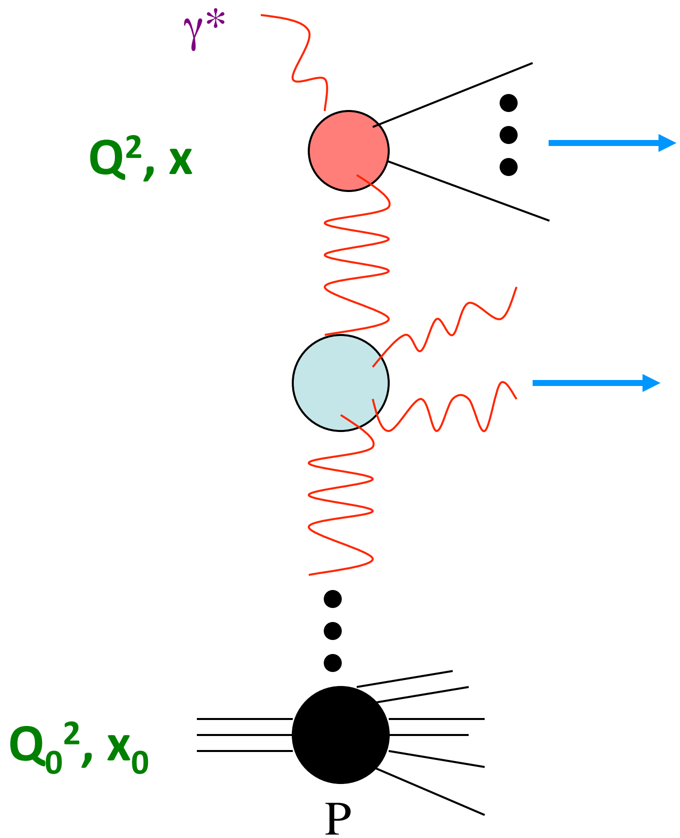
For x < 0.01, proton dominated by glue-grows rapidly
What happens when glue density is large ?

The Bjorken Limit

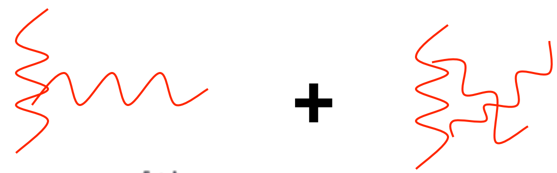
$$Q^2 \to \infty\,;\; s \to \infty\,;\; x_{\rm Bj} \approx \frac{Q^2}{s} = \text{fixed}$$

- Operator product expansion (OPE), factorization theorems, machinery of precision physics in QCD

Structure of higher order perturbative contributions in QCD



$$C_2(x,\alpha_S) = \delta(1-x) + \alpha_S C_2^{(1)}(x) + \alpha_S^2 C_2^{(2)} + \cdots$$



$\alpha_S\, P^{(1)}$ + $\alpha_S^2\, P^{(2)}$ + ...

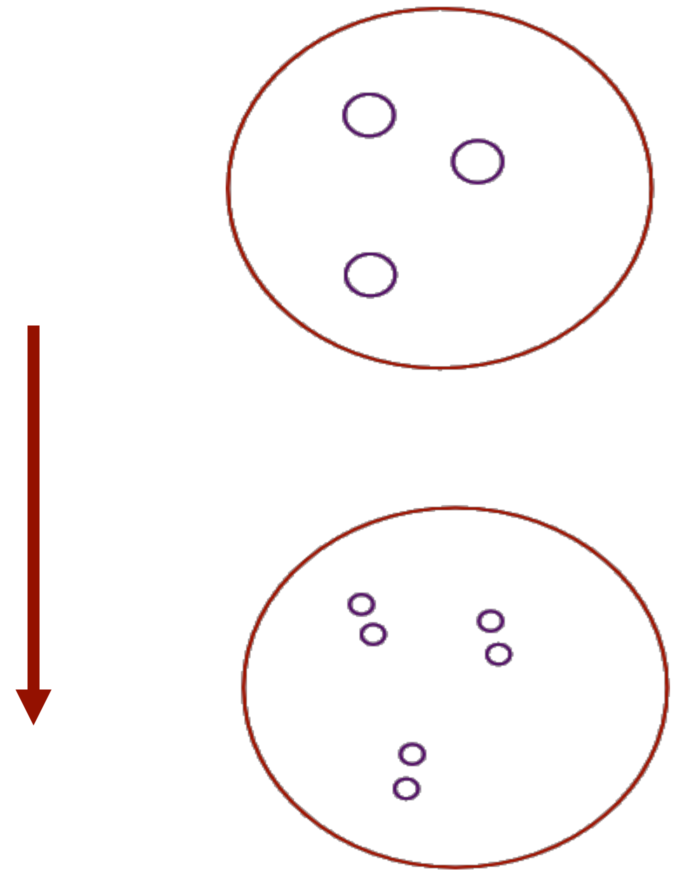
+ higher twist (power suppressed) contributions…

- Coefficient functions C - computed to NNLO for many processes
- Splitting functions P - computed to 3-loops

Resolving the hadron…

Ren.Group-DGLAP evolution
(sums large logs in Q^2)

Increasing Q^2



Phase space density (# partons / area / Q^2) decreases
- the proton becomes more dilute…

BEYOND pQCD IN THE Bj LIMIT

- Works great for inclusive, high Q^2 processes
- Higher twists important when $Q^2 \approx Q_S^2(x)$
- Problematic for diffractive/exclusive processes
- Formalism not designed to treat shadowing, multiple scattering, diffraction, energy loss, impact parameter dependence, thermalization…

The Regge-Gribov Limit

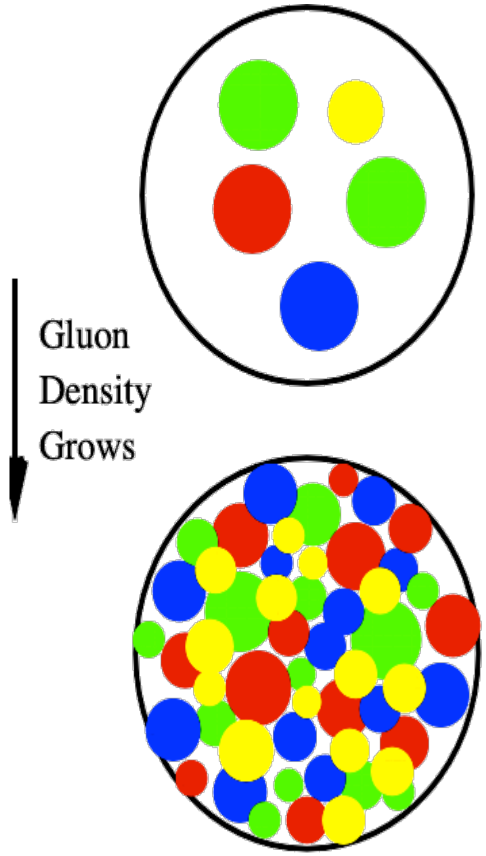
$$x_{\mathrm{Bj}} \to 0 ;\; s \to \infty ;\; Q^2 (>> \Lambda^2_{\mathrm{QCD}}) = \mathrm{fixed}$$

- **Physics of strong fields in QCD, multi-particle production,**

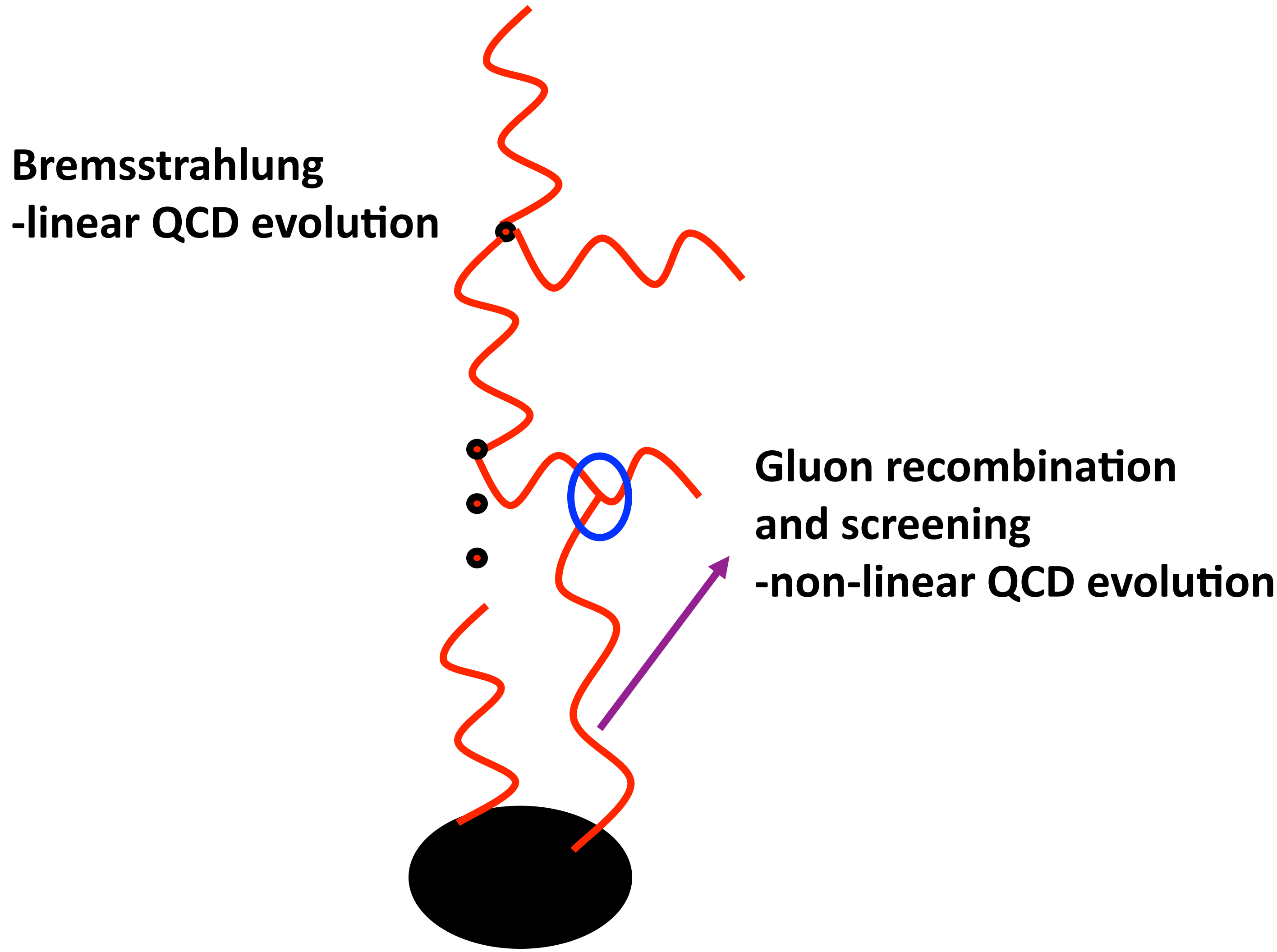
Novel universal properties of QCD ?

Resolving the hadron…

Ren.Group-BFKL evolution (sums large logs in x)



Gluon density saturates at phase space density f = 1 /α_S
- strongest (chromo-) E&M fields in nature…



Proton becomes a dense many body system at high energies

Parton Saturation

Gribov,Levin,Ryskin
Mueller,Qiu

- **Competition between attractive bremsstrahlung and repulsive recombination and screening effects**

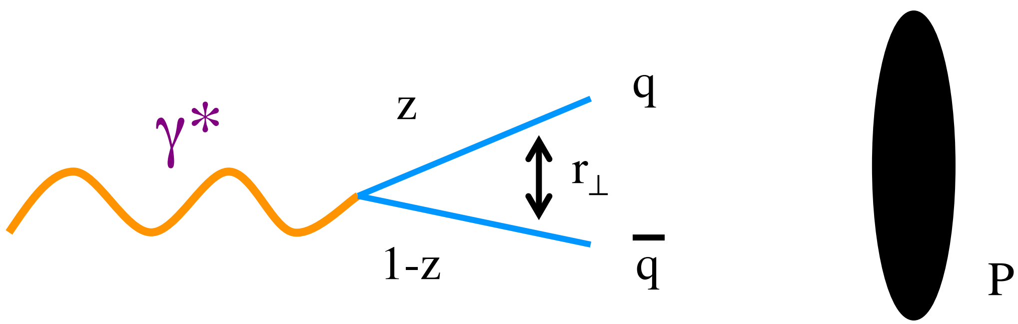
Maximum phase space density (f = $1/\alpha_S$) =>

$$\frac{1}{2(N_c^2-1)}\,\frac{x\,G(x,Q^2)}{\pi R^2\,Q^2} = \frac{1}{\alpha_S(Q^2)}$$

This relation is saturated for

$$Q = Q_s(x) >> \Lambda_{\rm QCD} \approx 0.2\ \text{GeV}$$

Parton Saturation:Golec-Biernat --Wusthoff dipole model



$$\sigma_{\mathrm{T,L}}^{\gamma^*,P} = \int d^2 r_\perp \int dz\, |\psi_{\mathrm{T,L}}(r_\perp, z, Q^2)|^2\, \sigma_{q,\bar{q},P}(r_\perp, x)$$

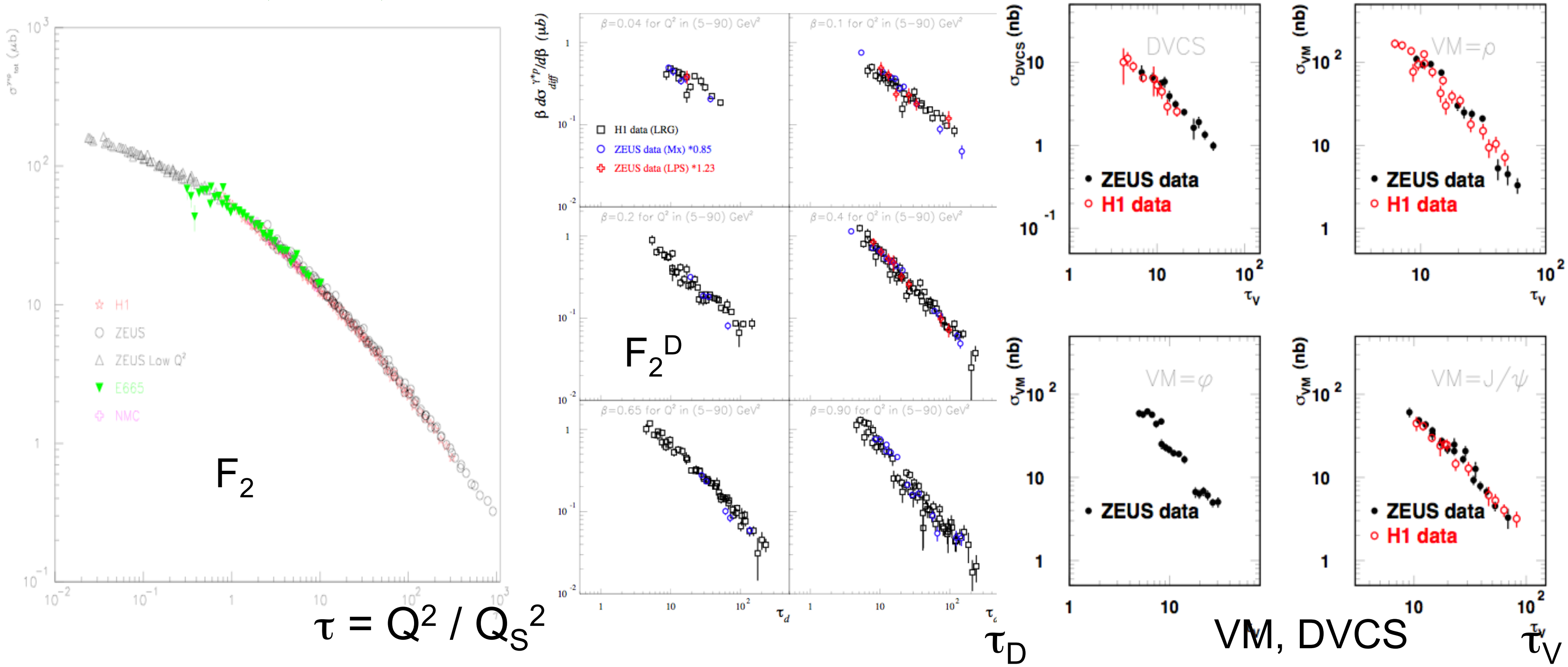
$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0\, \left[1 - \exp\left(-r_\perp^2\, Q_s^2(x)\right)\right]$$

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$$

Parameters: Q_0 = 1 GeV; λ = 0.3; x_0 = 3* 10^{-4} ; σ_0 = 23 mb

Evidence from HERA for geometrical scaling

Golec-Biernat, Stasto,Kwiecinski



Marquet, Schoeffel hep-ph/0606079

- Scaling seen for F_2^D and VM,DVCS for same Q_S as F_2

Gelis et al., hep-ph/0610435

Nuclear Physics B307 (1988) 34–60
North-Holland, Amsterdam

VIRTUAL PAIR CREATION IN A STRONG BREMSSTRAHLUNG FIELD: A QED model for parton saturation

A H MUELLER[1]

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Received 10 March 1988

Virtual pair creation in a strong, virtual, bremsstrahlung field is considered in QED as a model for parton saturation In a weak field the virtual pair density increases quadratically in the external field, however, at large values of the field the number density becomes independent of the strength of that field A similar effect is found in scalar electrodynamics

1. Introduction

At small values of the Bjorken-x-variable parton (quark and gluon) number densities are expected to grow rapidly [1] However, when, say, the gluon distribution in a hadron, $xG(x, Q^2)$, reaches a value as large as $Q^2 r^2/\alpha$, with r the radius of the hadron, these gluons are so densely packed that one expects scattering and annihilation of partons to become important, thus limiting the ultimate number density to be of the size indicated above [1, 3]

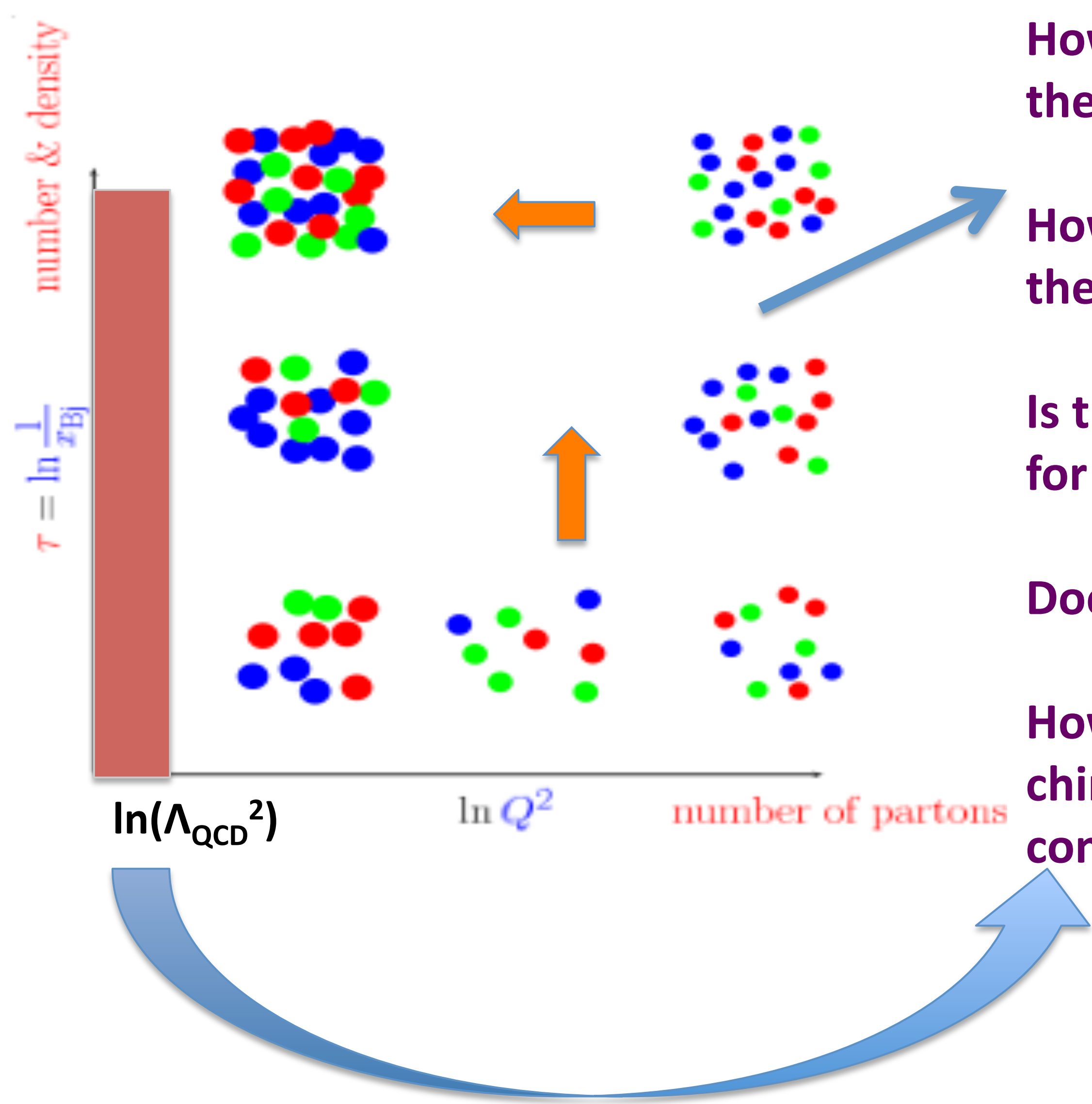
This high density quark-and-gluon system is a most fascinating regime of QCD On the one hand, if $Q^2 \geq 1$ GeV^2 the coupling, $\alpha(Q^2)$, is small and the usual non-perturbative condensates are unimportant while, on the other hand, the system is strongly interacting because of the high parton densities. That is, this regime of weak coupling but large numbers of partons is a new regime of QCD Such a high-density parton system occurs in a number of different high-energy processes (i) In deeply inelastic scattering one can directly measure such high-density systems at small x using the virtual photon as a probe [1, 3]. (ii) In the very early stages of a heavy ion collision such a system is produced over a large transverse area [4]. (iii) Two-jet correlations in high-energy reactions can trigger on local hot spots [5], high parton density regions which are smaller than the radius of a normal hadron

So far, it has not been possible to theoretically study this high density, non-equilibrium, regime of QCD directly. Lowest order gluon recombinations have been

[1] Work supported in part by the Department of Energy and NSF Grant PHY82-17853, supplemented by NASA

High energy QCD as a many body system

Many-body dynamics of universal gluonic matter



How does this happen ? What are the right degrees of freedom ?

How do correlation functions of these evolve ?

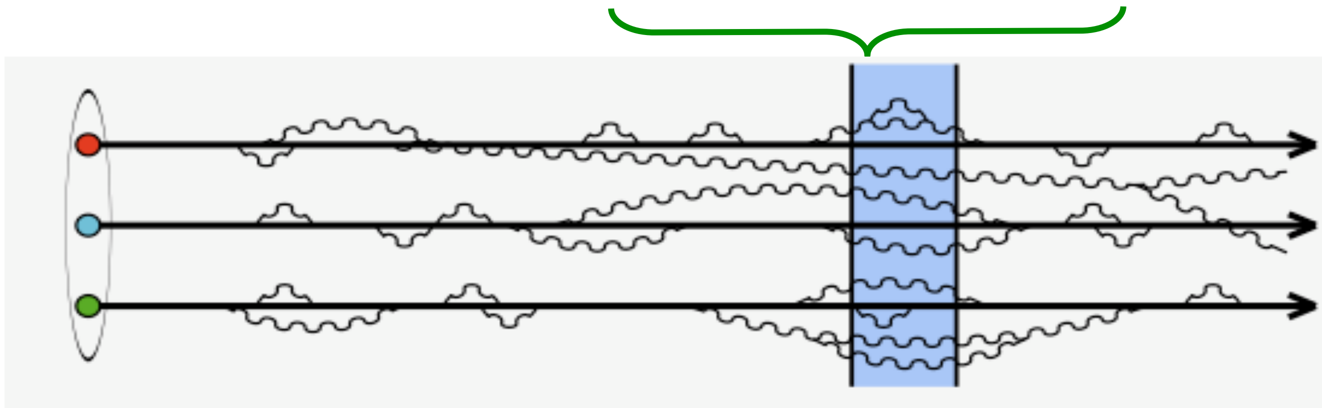
Is there a universal fixed point for the RG evolution of d.o.f

Does the coupling run with Q_s^2 ?

How does saturation transition to chiral symmetry breaking and confinement

The nuclear wavefunction at high energies

$$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\,gg\ldots g\rangle$$



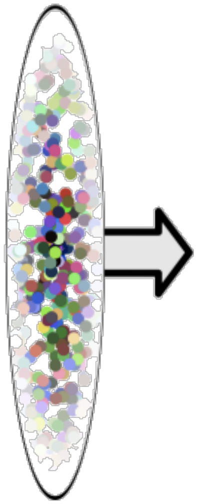
- At high energies, interaction time scales of fluctuations are dilated well beyond typical hadronic time scales

- Lots of short lived (gluon) fluctuations now seen by probe -- proton/nucleus -- dense many body system of (primarily) gluons

- Fluctuations with lifetimes much longer than interaction time for the probe function as static color sources for more short lived fluctuations

Nuclear wave function at high energies is a Color Glass Condensate

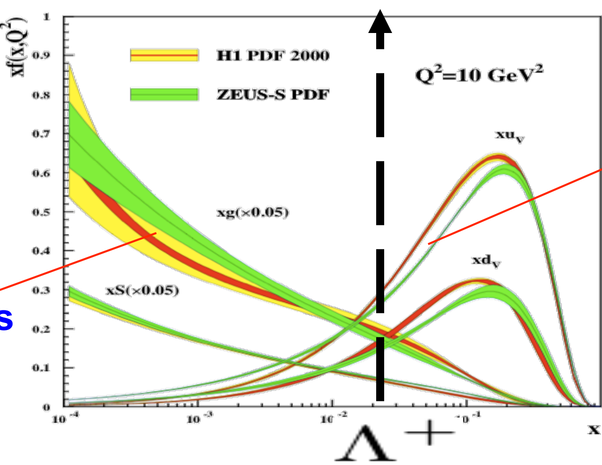
The nuclear wavefunction at high energies



$$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\,\mathbf{gg\ldots gg}\rangle$$

Higher Fock components dominate multiparticle production- construct Effective Field Theory

Dynamical Wee modes



Valence modes- are static sources for wee modes

Born--Oppenheimer LC separation natural for EFT.

RG equations describe evolution of wavefunction with energy

Effective Field Theory on Light Front

Susskind
Bardacki-Halpern

Poincare group on LF ⟷ Galilean sub-group of 2D Quantum Mechanics

isomorphism

Eg., LF dispersion relation

$$P^- = \frac{P_\perp^2}{2P^+}$$

Energy → P^-; Momentum → $P_\perp^2$; Mass → $2P^+$

Large x (P^+) modes: static LF (color) sources ρ^a

Small x ($k^+ \ll P^+$) modes: dynamical fields A_μ^a

McLerran, RV

CGC: Coarse grained many body EFT on LF

$$< P|\mathcal{O}|P > \longrightarrow \int [d\rho^a][dA^{\mu,a}]\, W_{\Lambda^+}[\rho]\, e^{iS_{\Lambda^+}[\rho,A]}\, \mathcal{O}[\rho, A]$$

$W_{\Lambda^+}[\rho]$ non-pert. gauge invariant "density matrix" defined at initial scale Λ_0^+

RG equations describe evolution of W with x

JIMWLK, BK

The Color Glass Condensate

McLerran, RV
Jalilian-Marian,Kovner,Weigert
Iancu, Leonidov,McLerran

In the saturation regime:

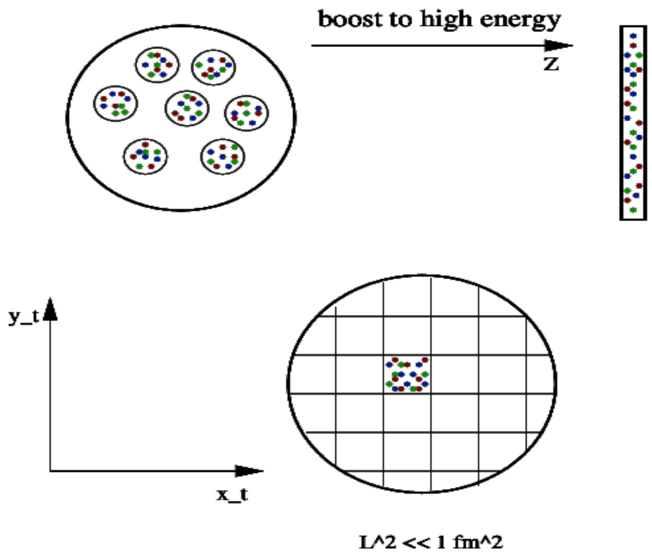
Strongest fields in nature!

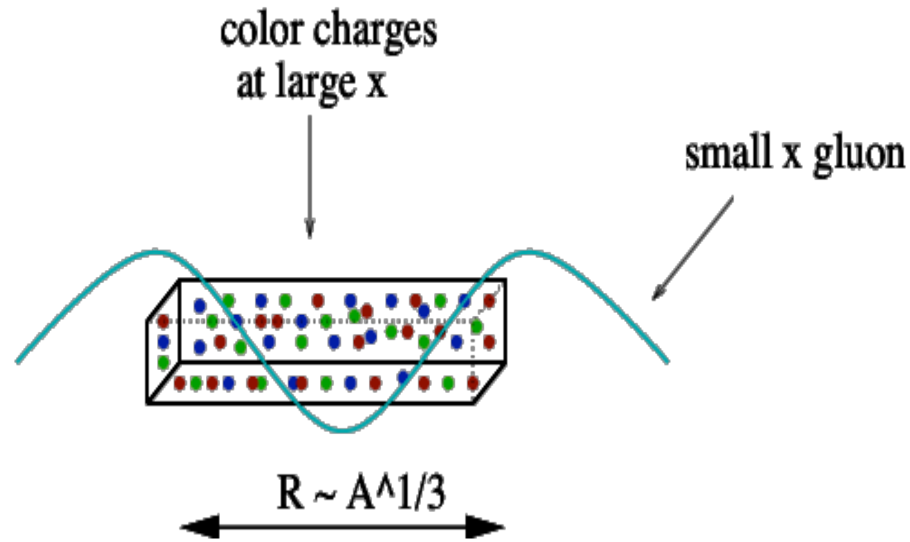
$$E^2 \sim B^2 \sim \frac{1}{\alpha_S}$$

CGC: *Classical* effective theory of QCD describing dynamical gluon fields + static color sources in non-linear regime

- **Novel renormalization group equations (JIMWLK/BK) describe how the QCD dynamics changes with energy**
- **A universal saturation scale Q_S arises naturally in the theory**

What do sources look like in the IMF ?





$$\lambda_{\rm wee} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\rm val.} \equiv \frac{Rm_p}{P^+} => x << A^{-1/3}$$

Wee partons "see" a large density of color sources at small transverse resolutions

Classical field of a large nucleus

$$< AA >_\rho = \int [d\rho] A_{\rm cl.}(\rho) A_{\rm cl.}(\rho) W_{\Lambda^+}[\rho]$$

For a large nucleus, A >>1,

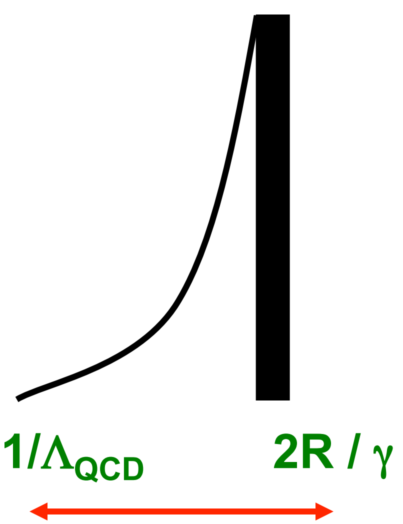
"Pomeron" excitations

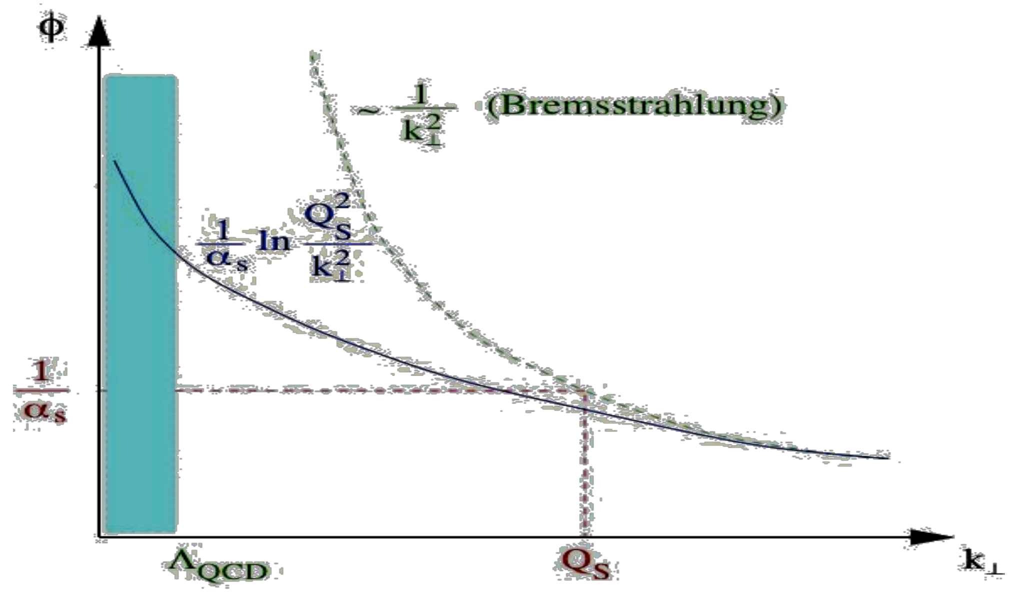
"Odderon" excitations

$$W_{\Lambda^+} = \exp\left(-\int d^2x_\perp \left[\frac{\rho^a\rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

McLerran,RV
Kovchegov
Jeon, RV

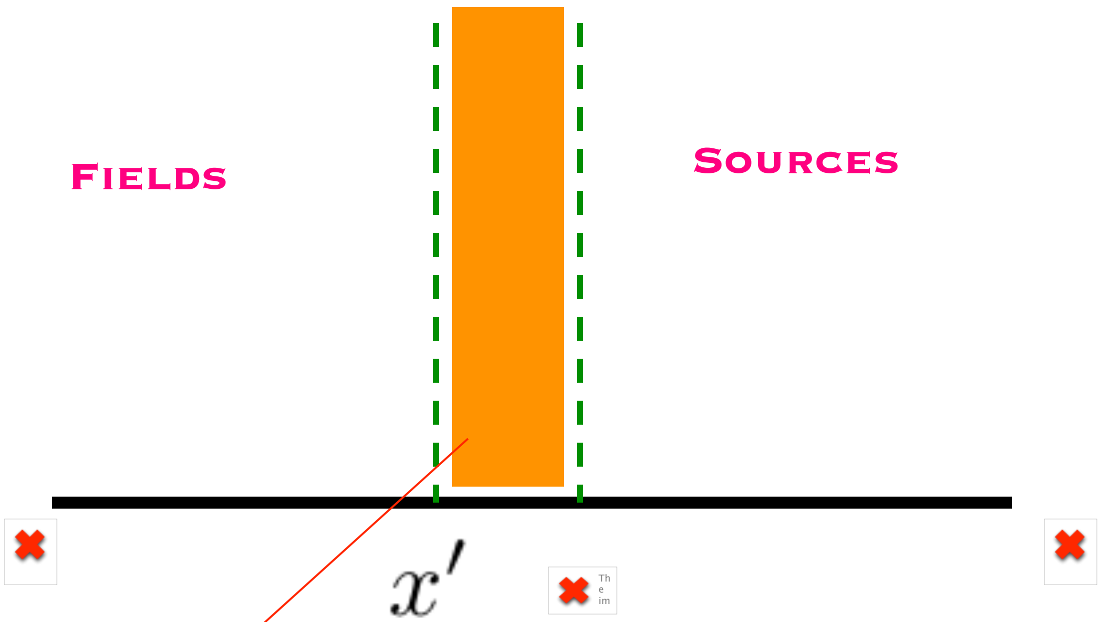
A_{cl} from → $(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu+}\,\delta(x^-)\,\rho^a(x_\perp)$





Wee parton dist. : $\frac{1}{\Lambda_{QCD}} e^{-\lambda \Delta Y/2}$ → determined from RG

Quantum evolution of classical theory: Wilson RG



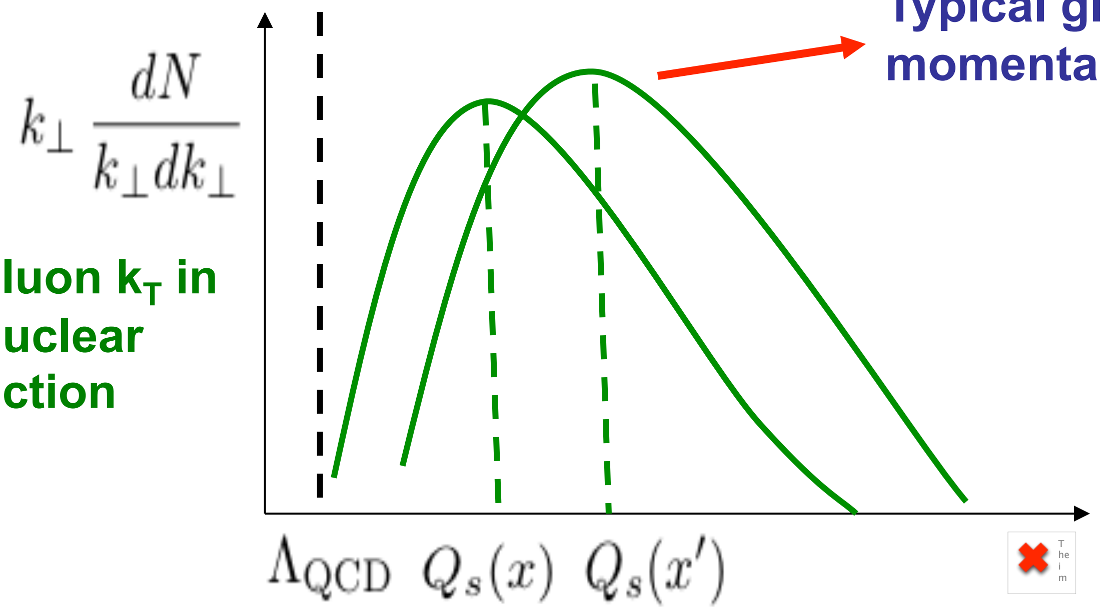
Integrate out
Small fluctuations => Increase color charge of sources

Wilsonian RG equations describe evolution of all N-point correlation functions with energy

JIMWLK Jalilian-marian, Iancu, McLerran,Weigert, Leonidov,Kovner

Saturation scale grows with energy

Typical gluon momenta are large



Typical gluon k_T in hadron/nuclear wave function

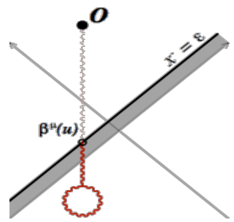
Bulk of high energy cross-sections:

a) obey dynamics of novel non-linear QCD regime

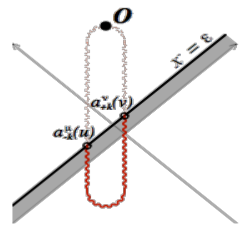
b) Can be computed systematically in weak coupling

JIMWLK RG evolution for a single nucleus:

$$\mathcal{O}_{\rm NLO} = \Bigg($$



$$+$$



$$\chi(x_\perp, y_\perp)$$

$$\Bigg)\, \mathcal{O}_{\rm LO}$$

$$= \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}\, \mathcal{O}_{\rm LO}$$

(keeping leading log divergences)

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}]\, [\mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO}]$$

$$= \int [d\tilde{\rho}] \left\{ \left[1 + \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H} \right] W_{\Lambda^+} \right\} \mathcal{O}_{\rm LO}$$

LHS independent of Λ^+ **=>**

$$\boxed{\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H}\, W[\tilde{\rho}]}$$

JIMWLK eqn.

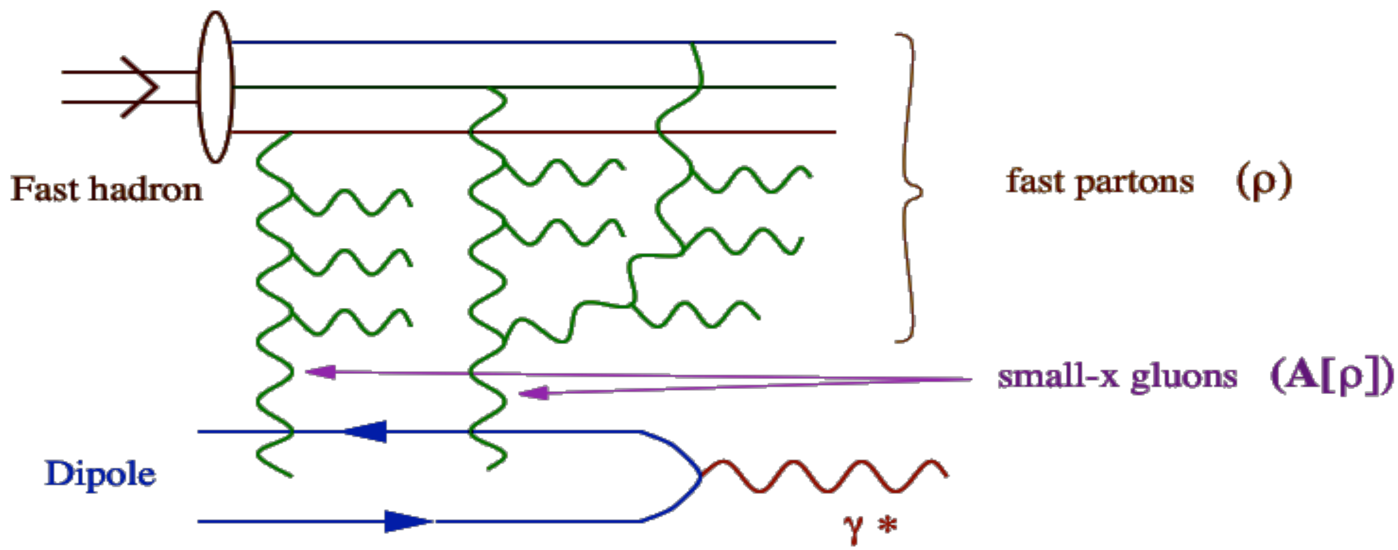
Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner

CGC Effective Theory: B-JIMWLK hierarchy of correlators

$$\frac{\partial}{\partial Y} \langle O[\alpha] \rangle_Y = \langle \frac{1}{2}\int_{x,y} \frac{\delta}{\delta\alpha_Y^a(x)} \chi^{ab}_{x,y} \frac{\delta}{\delta\alpha_Y^b(y)} O[\alpha] \rangle_Y$$

"time"

"diffusion coefficient"



At high energies, the d.o.f that describe the frozen many-body gluon configurations are novel objects: dipoles, quadrupoles, …

Universal – appear in a number of processes in p+A and e+A; how do these evolve with energy ?

Solving the B-JIMWLK hierarchy

- JIMWLK includes all multiple scattering and leading log evolution in x
- Expectation values of Wilson line correlators at small x satisfy a Fokker-Planck eqn. in functional space

Weigert (2000)

- This translates into a hierarchy of equations for n-point Wilson line correlators
- As is generally the case, Fokker-Planck equations can be re-expressed as Langevin equations – in this case for Wilson lines

Blaizot,Iancu,Weigert
Rummukainen,Weigert

First numerical solutions exist: I will report on recent developments

B-JIMWLK hierarchy: Langevin realization

Numerical evaluation of Wilson line correlators on 2+1-D lattices:

$$\left\langle \mathcal{O}[U] \right\rangle_Y = \int D[U]\, W_Y[U]\, \mathcal{O}[U] \longrightarrow \frac{1}{N} \sum_{U \epsilon W} \mathcal{O}[U]$$

Langevin eqn:

Gaussian random variable

$$\partial_Y [V_x]_{ij} = [V_x i t^a]_{ij} \left[\int d^2y\, [\mathcal{E}^{ab}_{xy}]_k\, [\xi^b_y]_k + \sigma^a_x \right]$$

$$\mathcal{E}^{ab}_{xy} = \left(\frac{\alpha_S}{\pi^2}\right)^{1/2} \frac{(x-y)_k}{(x-y)^2} \left[1 - U^\dagger_x U_y\right]^{ab}$$

"square root" of JIMWLK kernel

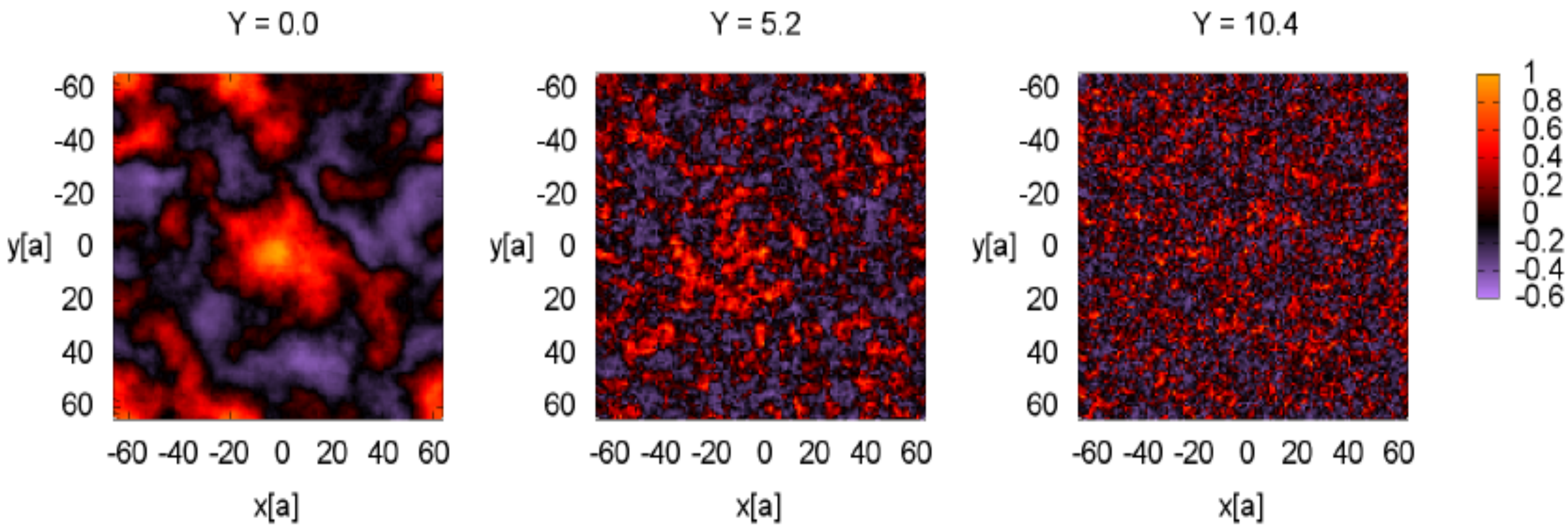
$$\sigma^a_x = -i \left(\frac{\alpha_S}{2\pi^2} \int d^2z \frac{1}{(x-z)^2} \mathrm{Tr}(T^a\, U^\dagger_x U_z) \right)$$

"drag"

- Initial conditions for V's from the MV model
- Daughter dipole prescription for running coupling

Functional Langevin solutions of JIMWLK hierarchy

Rummukainen,Weigert (2003)

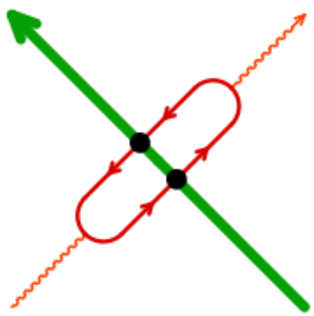


Dumitru,Jalilian-Marian,Lappi,Schenke,RV: arXiv:1108.1764

Multi-parton correlations in nuclear wavefunctions:

Event-by-event rapidity, number and impact parameter fluctuations

Inclusive DIS: dipole evolution

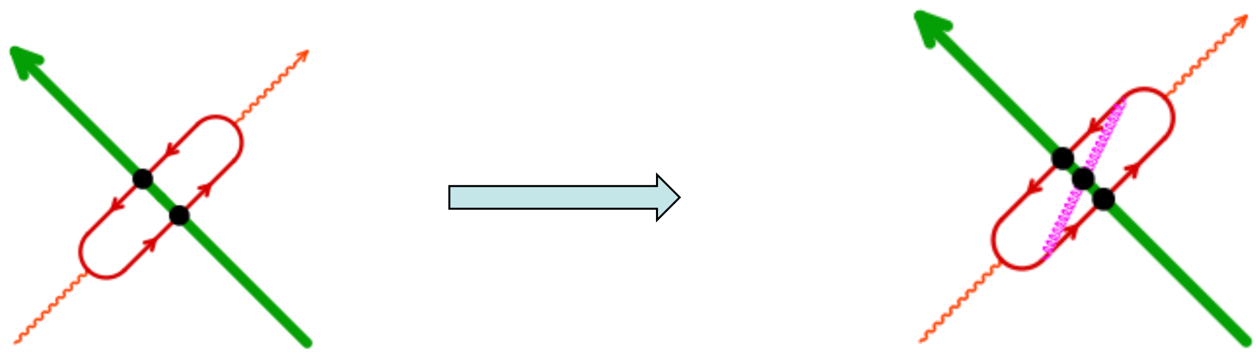


$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_\perp \, |\psi(z, r_\perp)|^2 \sigma_{\text{dipole}}(x, r_\perp)$$

$$\sigma_{\text{dipole}}(x, r_\perp) = 2 \int d^2 b \int [D\rho] \, W_{\Lambda^+}[\rho] \, T(b + \frac{r_\perp}{2}, b - \frac{r_\perp}{2})$$

$$1 - \frac{1}{N_c} \text{Tr} \left(V \left(b + \frac{r_\perp}{2} \right) V^\dagger \left(b - \frac{r_\perp}{2} \right) \right)$$

Inclusive DIS: dipole evolution



B-JIMWLK eqn. for dipole correlator

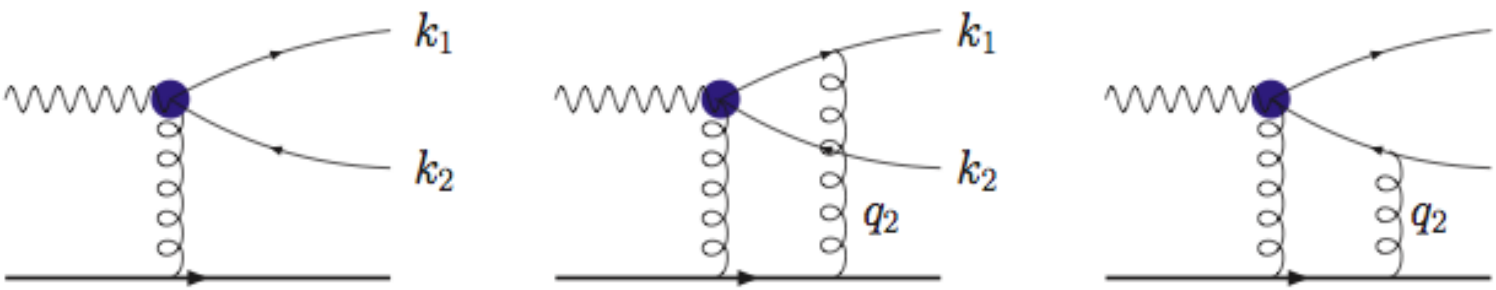
$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Dipole factorization:

$$\langle \mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger)\rangle_Y \langle \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \qquad N_c \rightarrow \infty$$

Resulting closed form eqn. is the Balitsky-Kovchegov eqn.
Widely used in phenomenological applications

Semi-inclusive DIS: quadrupole evolution



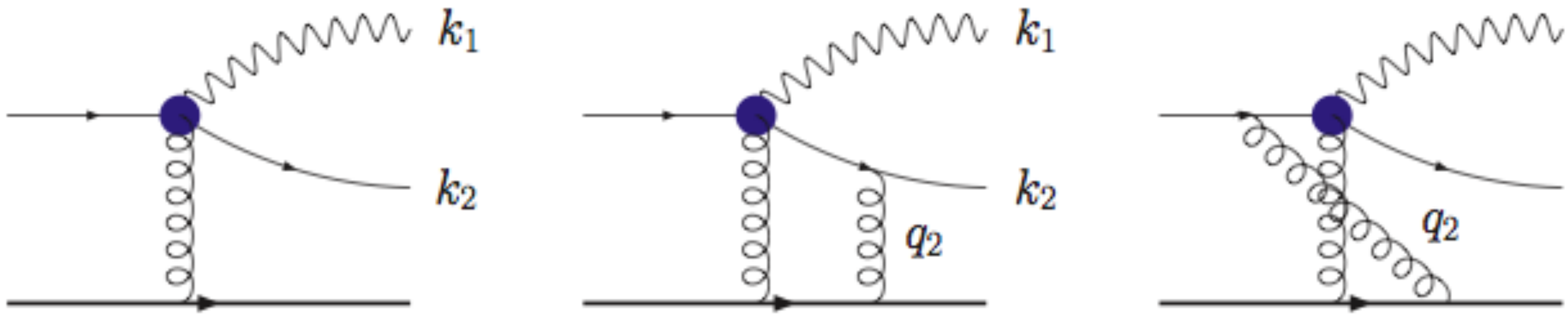
Dominguez,Marquet,Xiao,Yuan (2011)

$$\frac{d\sigma^{\gamma^*_{\mathrm{T,L}}A\to q\bar{q}X}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x}\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})} e^{ik_{2\perp}\cdot(y-\bar{y})} \left[1+Q(x,y;\bar{y},\bar{x}) - D(x,y) - D(\bar{y},\bar{x})\right]$$

$$D(x,y) = \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y \qquad Q(x,y;\bar{y},\bar{x}) = \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_{\bar{x}}^\dagger V_{\bar{y}} V_y^\dagger)\rangle_Y$$

Cannot be further simplified a priori even in the large N_c limit

Universality: Di-jets in p/d-A collisions



Jalilian-Marian, Kovchegov (2004)
Marquet (2007)
Dominguez,Marquet,Xiao,Yuan (2011)

$$\frac{d\sigma^{qA\to qgX}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x},\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})} e^{ik_{2\perp}\cdot(y-\bar{y})} \left[S_6(x,y,\bar{x},\bar{y}) - S_4(x,y,v) - ...\right]$$

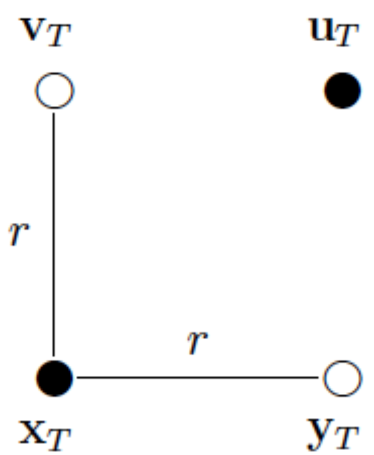
$$\frac{N_c}{2\,C_F}\left\langle Q(x,y\,\bar{y},\bar{x})D(y,\bar{y}) - \frac{D(x,\bar{x})}{N_c}\right\rangle \qquad \frac{N_c}{2\,C_F}\left\langle D(x,y)D(\bar{y},\bar{x}) - \frac{D(x,\bar{x})}{N_c}\right\rangle$$

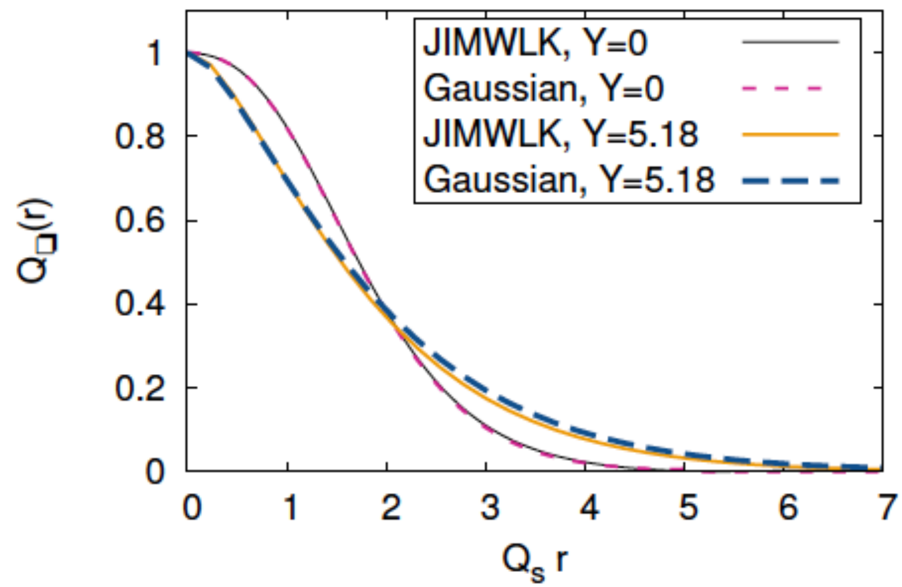
Fundamental ingredients are the universal dipoles and quadrupoles

For finite N_c, anticipate higher multipole contributions

Gaussian approximation to JIMWLK

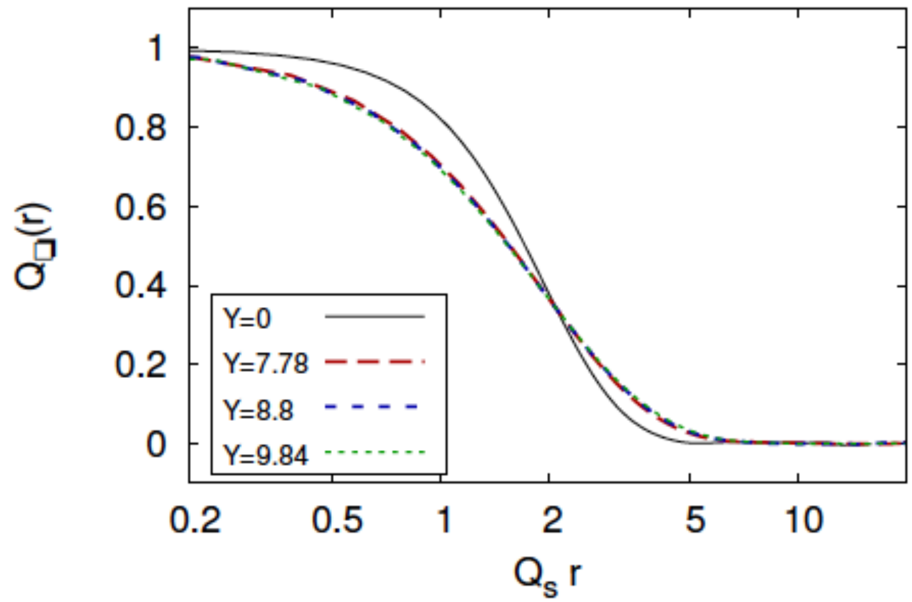
Dumitru,Jalilian-Marian,Lappi,Schenke,RV



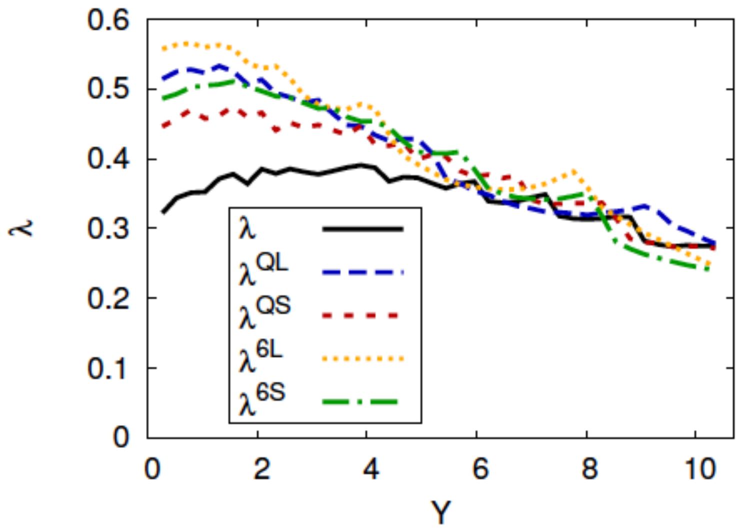


Gaussian approximation to JIMWLK

Dumitru,Jalilian-Marian,Lappi,Schenke,RV



Geometrical scaling



RG evolution of multipole correlators