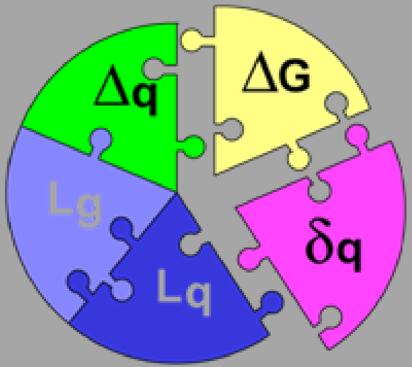
3D Partonic Structure of the Nucleon
Varenna 2011



Phenomenology

of Transvers Momentum Dependent

distributions

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Download

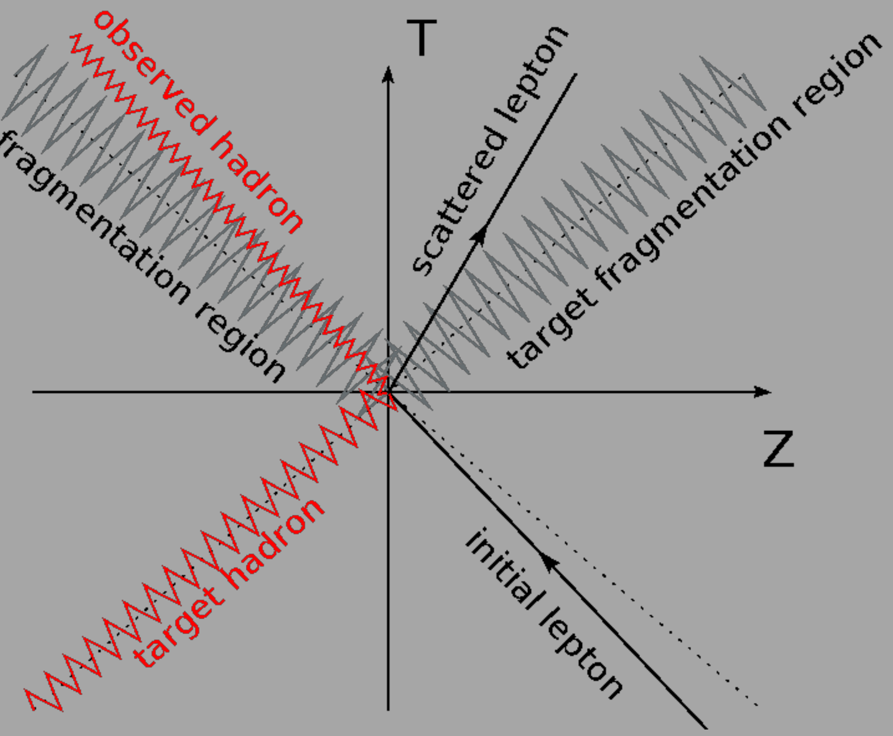
http://www.to.infn.it/~prokudin/varenna/SIDIS.nb

http://www.to.infn.it/~prokudin/varenna/Dipole.nb

$$\mathbf{l + P \to l' + h + X}$$

SIDIS: experiment

The Proton moves
along Z in space



Number of particles for opposite polarizations of the target is counted

$N^{\uparrow}$ $N^{\downarrow}$

$$\mathbf{l + P \to l' + h + X}$$

SIDIS: experiment

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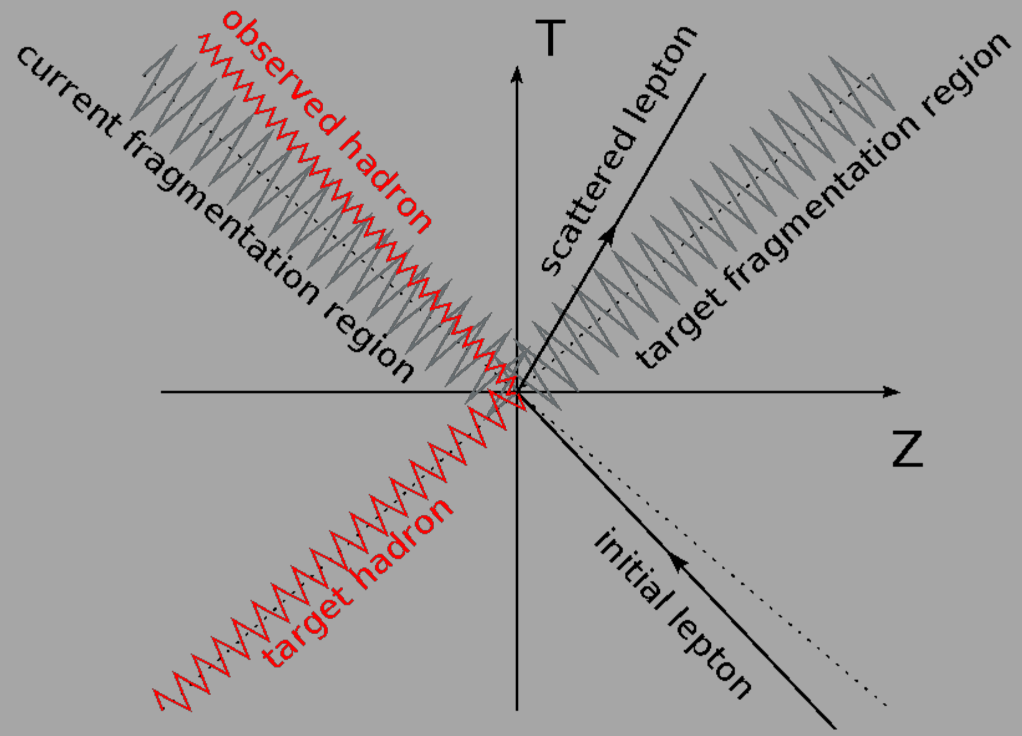


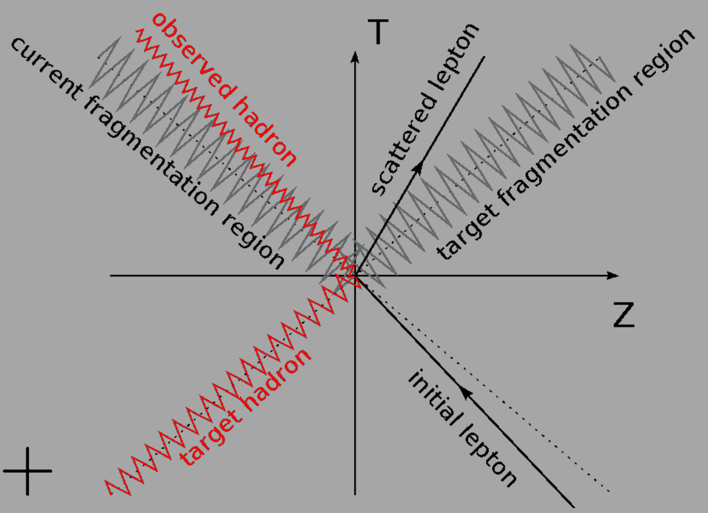
Single Spin Asymmetry is measured

$$A \propto \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}}$$

$$\mathbf{l} + \mathbf{P} \to \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

SIDIS: experiment

Different TMDs are sensitive to spin direction



$$N^{\uparrow} - N^{\downarrow} \propto$$

$$\underbrace{f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)}_{\text{Sivers effect}} +$$

$$+ \underbrace{h_1 \otimes \Delta\hat{\sigma}^{\uparrow} \otimes H_1^{\perp} \sin(\phi_h + \phi_S)}_{\text{Collins effect}} + \ldots$$

Angular dependence allows to disentangle different contributions

Sivers function

Let's consider unpolarised quarks inside
transversely polarised nucleon

General distribution

$$f(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{[\mathbf{p}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{p}_T^2)$$

Usual unpolarised distribution

This one is called SIVERS function

Sivers function

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General distribution

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$[\mathbf{p}_T \times P] \cdot S_T$ Is the only allowed combination as spin is a pseudovector and we need another pseudovector

$$[\mathbf{p}_T \times P]$$

Parity and all that

Parity transformation $\mathbf{P}:\vec{X}\rightarrow -\vec{X}$

We observe squares, thus two parity states

$$\mathbf{P}\Phi(\vec{X}) = \pm 1\ \Phi(\vec{X})$$

Spin vector is P-even: $\mathbf{P}:\vec{S}\rightarrow \vec{S}$

Momentum is P-odd: $\mathbf{P}:\vec{p}\rightarrow -\vec{p}$

Vector product is P-even: $\mathbf{P}:[\mathbf{p}_T\times P]\rightarrow [\mathbf{p}_T\times P]$

QCD is invariant under parity transformation

$[\mathbf{p}_T\times P]\cdot S_T$ is the only allowed combination for unpolarized distribution

Time reversal

Time reversal $$\mathbf{T} : t \longrightarrow -t$$

We observe squares, thus two time reversal states

$$\mathbf{T}\Phi(\vec{t}) = \pm 1 \; \Phi(\vec{t})$$

Spin vector is T-odd: $$\mathbf{T} : \vec{S} \longrightarrow -\vec{S}$$

Momentum is T-odd: $$\mathbf{T} : \vec{p} \longrightarrow -\vec{p}$$

Vector product is T-even: $$\mathbf{T} : [\mathbf{p}_T \times P] \longrightarrow [\mathbf{p}_T \times P]$$

QCD is invariant under time reversal

$$\mathbf{T} : [\mathbf{p}_T \times P] \cdot S_T \longrightarrow -[\mathbf{p}_T \times P] \cdot S_T$$

T-odd, thus Sivers function is T-odd

Sivers function

$$f(x,\mathbf{p}_T,S) = f_1(x,\mathbf{p}_T^2) - \frac{[\mathbf{p}_T \times \hat{P}]\cdot S_T}{M} f_{1T}^{\perp}(x,\mathbf{p}_T^2)$$

This function gives access to 3D imaging

Spin-orbit correlation

Physics of gauge links is represented

Requires Orbital Angular Momentum

Access to 3D imaging

$$f(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{[\mathbf{p}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{p}_T^2)$$

Symmetric part $\quad f_1(x, \mathbf{p}_T^2) = f(x, \mathbf{p}_{xT}^2 + \mathbf{p}_{yT}^2)$

Sivers function $\quad \hat{P} = (0, 0, 1) \qquad S_T = (0, 1, 0)$

$$\frac{[\mathbf{p}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{p}_T^2) = \frac{\mathbf{p}_{xT}}{M} f_{1T}^{\perp}(x, \mathbf{p}_{xT}^2 + \mathbf{p}_{yT}^2)$$

Dipole deformation

Access to 3D imaging

$$f(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{[\mathbf{p}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{p}_T^2)$$

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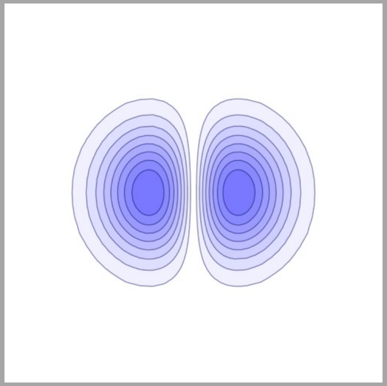

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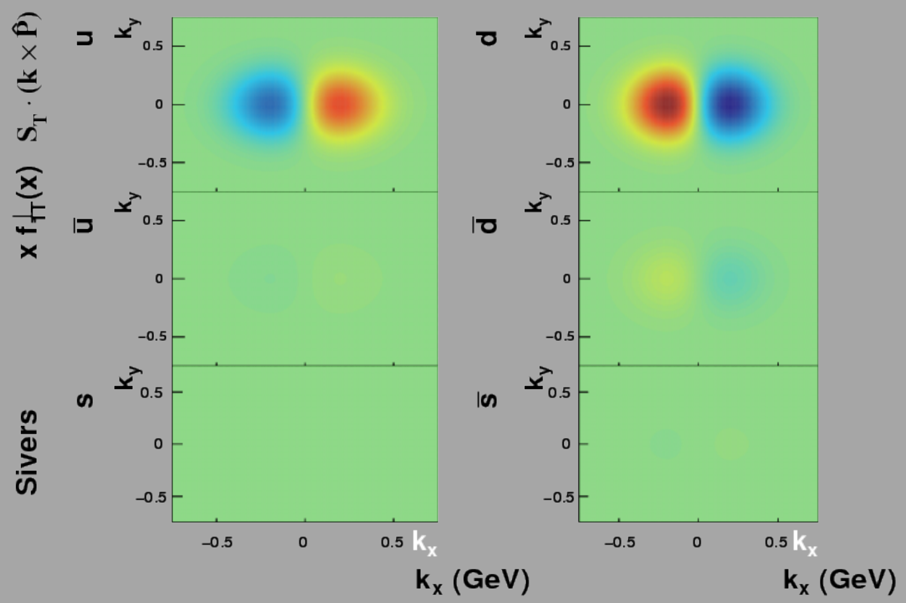


Sivers function from experimental data HERMES and COMPASS

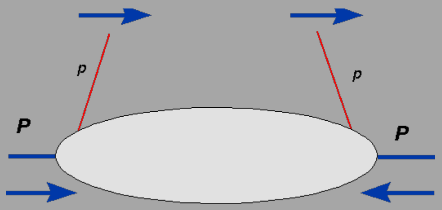
Dipole deformation

Spin orbit correlation and OAM

Sivers function requires proton helicity flip

$$f(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{[\mathbf{p}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{p}_T^2)$$

$$f_{1T}^{\perp}(x, \mathbf{p}_T^2) \propto f_1(x, \mathbf{p}, S) - f_1(x, \mathbf{p}, -S)$$



In terms of wave functions it means interference

between states with $\mathbf{L}_z$ and $\mathbf{L}_z = \mathbf{L}_z \pm 1$

Thus Sivers function requires OAM of quarks

OAM and Spin Crisis

The Spin of the proton can be decomposed as

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + < L_z^{q,\bar{q}} > + < L_z^G >$$

Experimentally $\quad \Delta\Sigma = \sum_{q,\bar{q}} \Delta q \simeq 0.3$

Spin Crisis – only 30% of the spin of the proton is carried by quarks, not almost 100% as expected!

Elliot Leader, Mauro Anselmino
"A Crisis In The Parton Model: Where, Oh Where Is The Proton's Spin?" 1988

OAM and Spin Crisis

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Spin Crisis – only 30% of the spin of the proton is carried by quarks

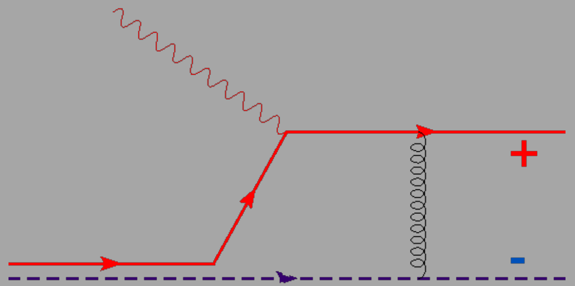
Can ΔG be big? Experimentally $\Delta G \sim 0$

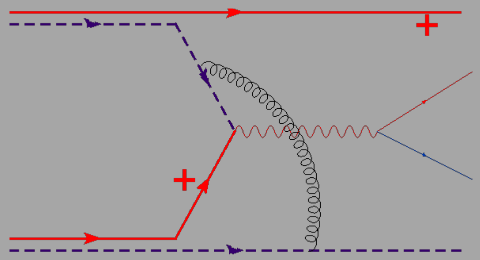
Orbital motion of partons is important.
Sivers function encodes this motion!

Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance technically implemented by Wilson lines - gauge links.

Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell Yan)



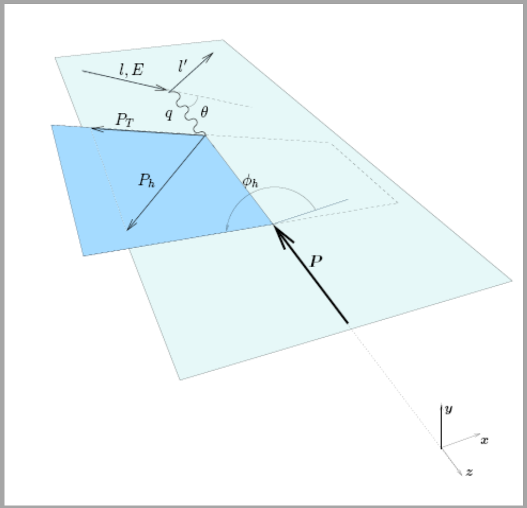


$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

Sivers function would be zero if gluons were absent

How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} \propto \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$



Unpolarised electron beam
Transversely polarised proton

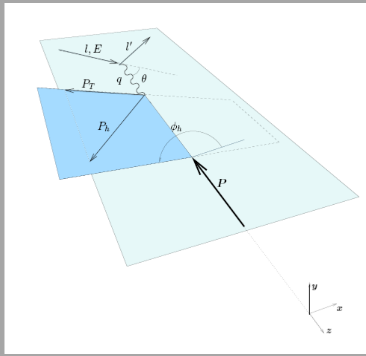
$$\sigma^{\uparrow} - \sigma^{\downarrow} = f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)$$

$$\sigma^{\uparrow} + \sigma^{\downarrow} = f_1 \otimes d\hat{\sigma} \otimes D_{h/q}$$

How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = 2\frac{\int d\Phi_S d\Phi_h \sin(\Phi_h-\Phi_S)(\sigma^{\uparrow}-\sigma^{\downarrow})}{\int d\Phi_S d\Phi_h (\sigma^{\uparrow}+\sigma^{\downarrow})}$$

Unpolarised electron beam
Transversely polarised proton



$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = -\frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

What symbol $\otimes$ means?

How do we measure Sivers function?

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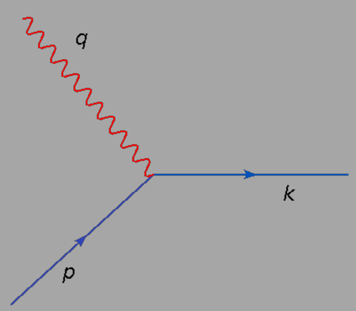
What symbol $\otimes$ means?

$$\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q} \equiv F_{UT}^{\sin(\Phi_h-\Phi_S)} =$$

$$x \sum_q e_q^2 \int d^2\mathbf{p}_T\, d^2\mathbf{k}_T\, \delta^{(2)}\left(z\mathbf{p}_T + \mathbf{k}_T - \mathbf{P}_{hT}\right) \frac{\mathbf{p}_T \cdot \hat{P}}{M}\, f_{1T}^{\perp q}(x, \mathbf{p}_T^2)\, D_{q/h}(z, \mathbf{k}_T^2)$$

Convolution

Momentum conservation



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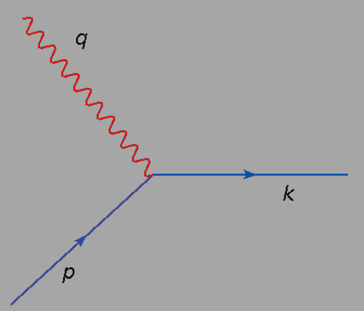


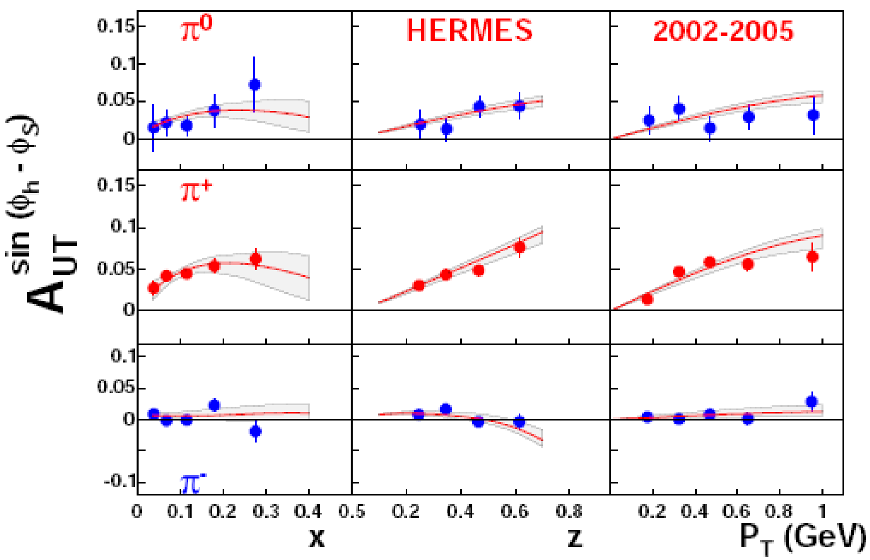
Calculate using Mathematica!

How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = -\frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

HERMES

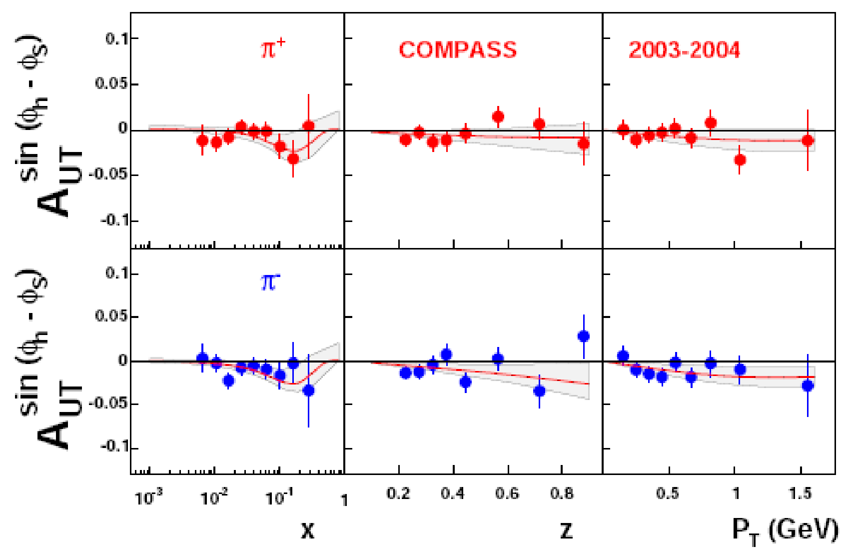
$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.



Anselmino et al 2010 in preparation

COMPASS

$\mu D \to \mu\pi X$, $p_{lab} = 160$ GeV.



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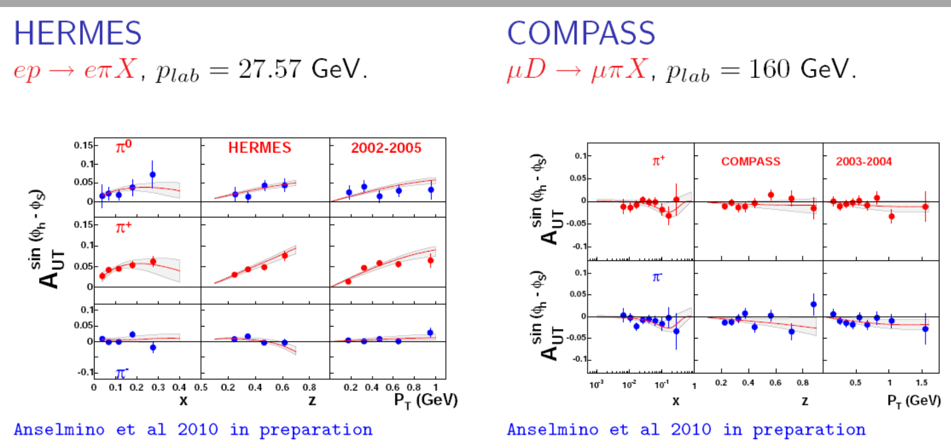

COMPASS

Deuteron: proton+neutron

$$P(u,u,d) \quad N(d,d,u)$$

$$f_{u/P} = f_{d/N} \quad f_{d/P} = f_{u/N}$$

$$f_{u/D} = f_{u/P} + f_{d/P} \qquad f_{d/D} = f_{d/P} + f_{u/P}$$

$$f_{1T}^{\perp u/D} = f_{1T}^{\perp u} + f_{1T}^{\perp d} \simeq 0$$

How do we measure Sivers function?

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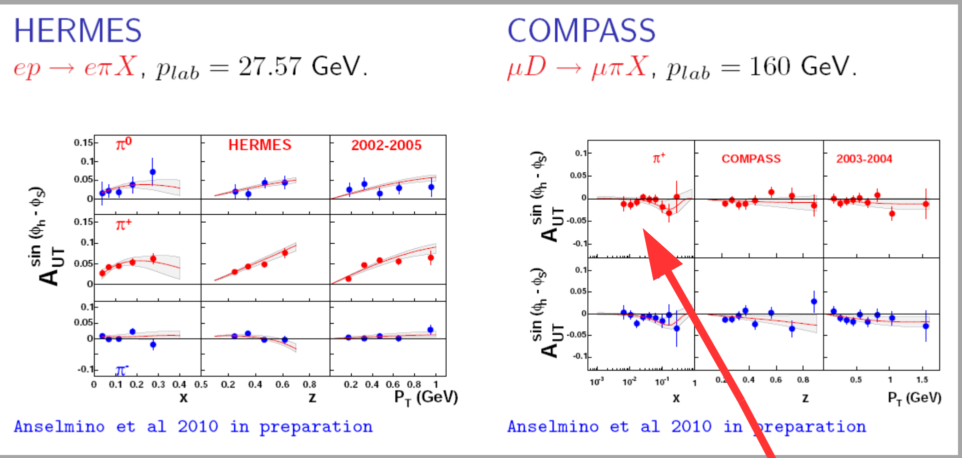

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HERMES
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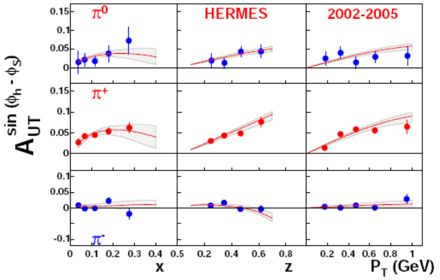

Anselmino et al 2010 in preparation

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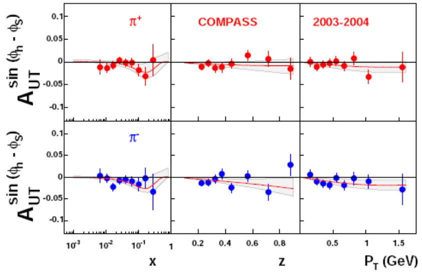


Anselmino et al 2010 in preparation

HERMES

Proton

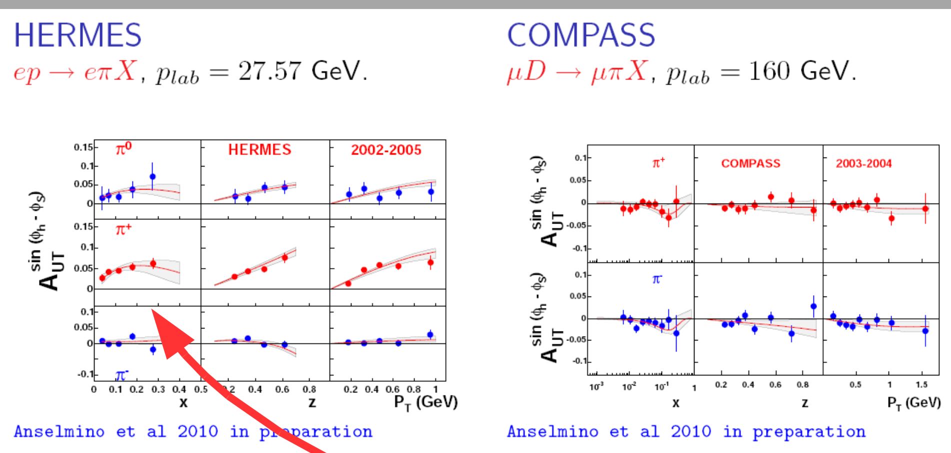
$$-\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}$$

$$-\frac{4}{9} f_{1T}^{\perp u} \otimes d\hat{\sigma} \otimes D_{u/\pi^+} + \frac{1}{9} f_{1T}^{\perp d} \otimes d\hat{\sigma} \otimes D_{d/\pi^+}$$

$$f_{1T}^{\perp u} < 0$$

How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = - \frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$



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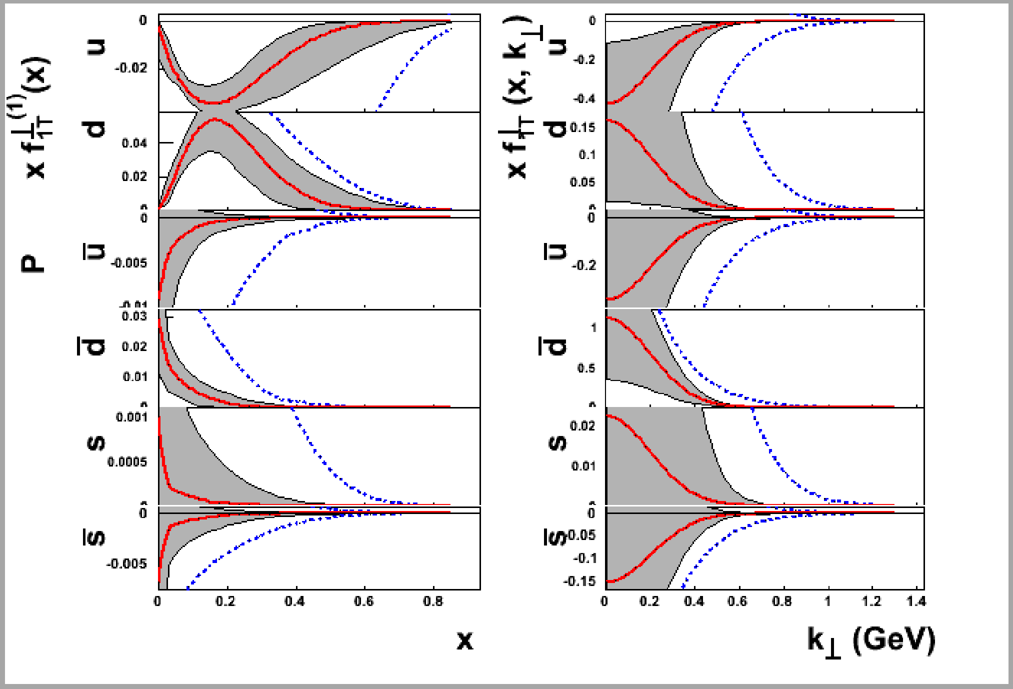

Anselmino et al 2008

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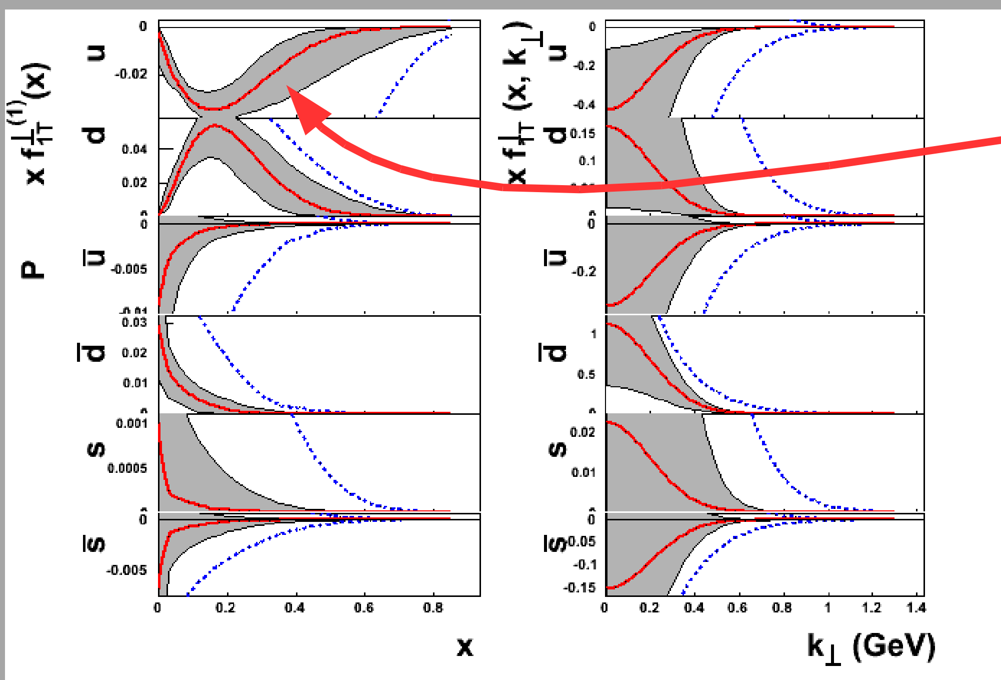

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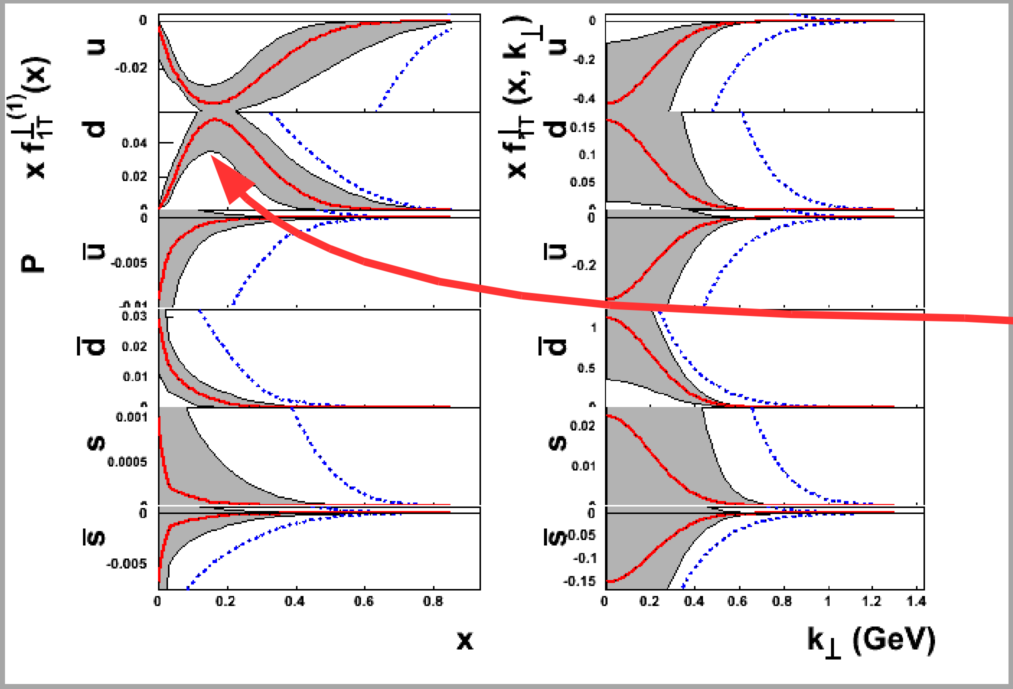

Anselmino et al 2008

$$f_{1T}^{\perp u} < 0$$

$$f_{1T}^{\perp d} \simeq -f_{1T}^{\perp u}$$

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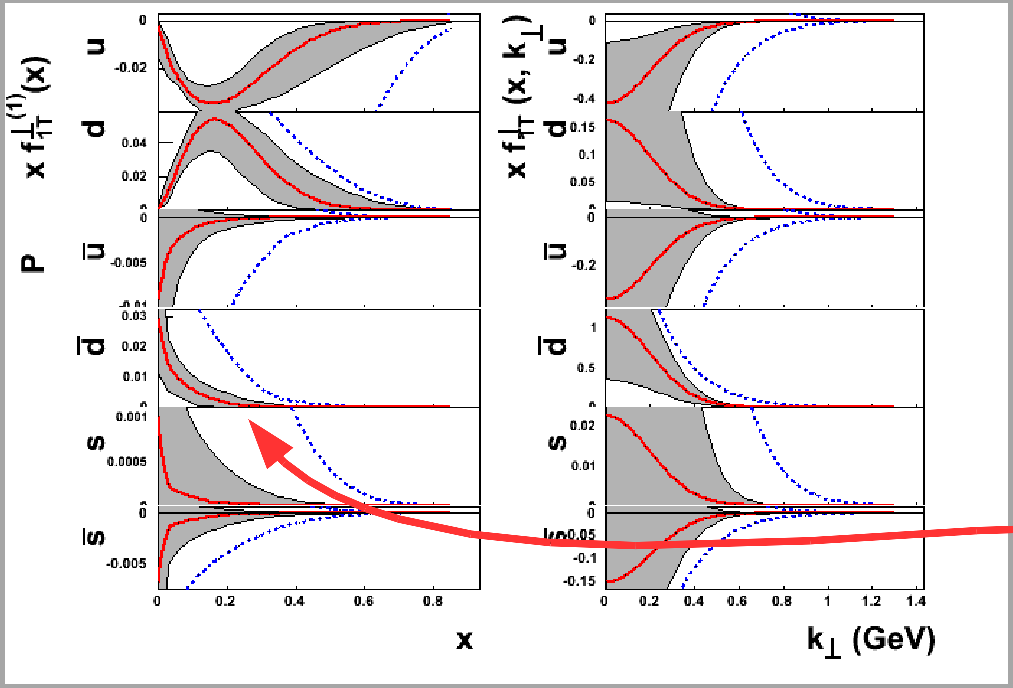


Anselmino et al 2008

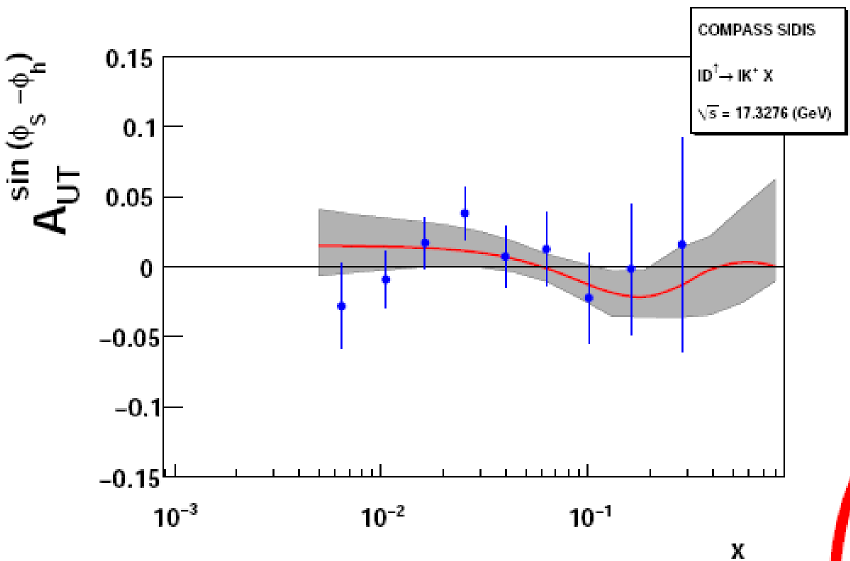
$$f_{1T}^{\perp u} < 0$$

$$f_{1T}^{\perp d} \simeq -f_{1T}^{\perp u}$$

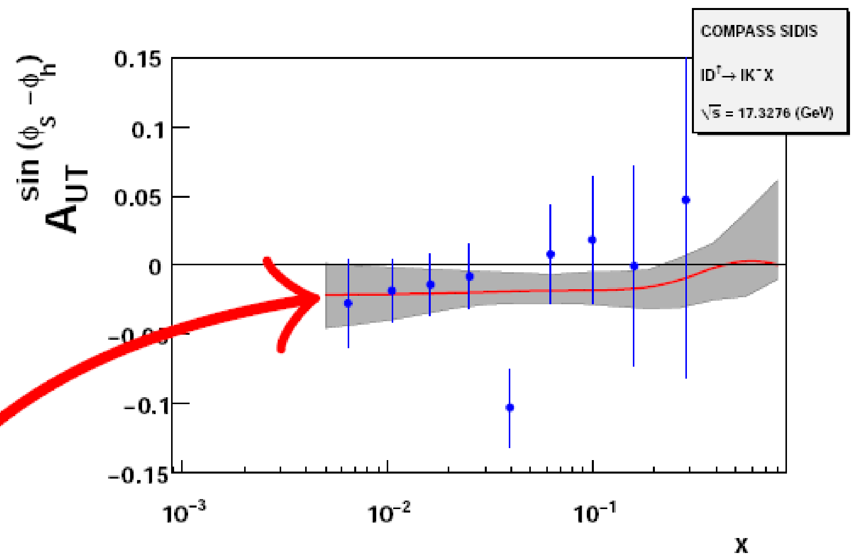
Hints on non zero sea Sivers functions

How do we "see" sea quarks in the data?

$K^+(u\bar{s})$



$K^-(\bar{u}s)$



COMPASS with sea contributions

COMPASS with sea contributions

$$\Delta^N f_{\bar{u}/p^\uparrow} + \Delta^N f_{\bar{d}/p^\uparrow} < 0$$

$$\Rightarrow \Delta^N f_{\bar{d}/p^\uparrow} < 0$$

How do we fit TMDs?

$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = -\frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

Choose parametrization

$$f_{1T}^{\perp}(x, \mathbf{p}_T^2) \sim x^{\alpha}(1-x)^{\beta} \frac{1}{\pi\sigma^2} e^{-\frac{\mathbf{p}_T^2}{\sigma^2}}$$

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Fit parameters to experimental data

How do we fit TMDs?

$$A_{UT}^{\sin(\Phi_h - \Phi_S)} = - \frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

Chi-square function is formed

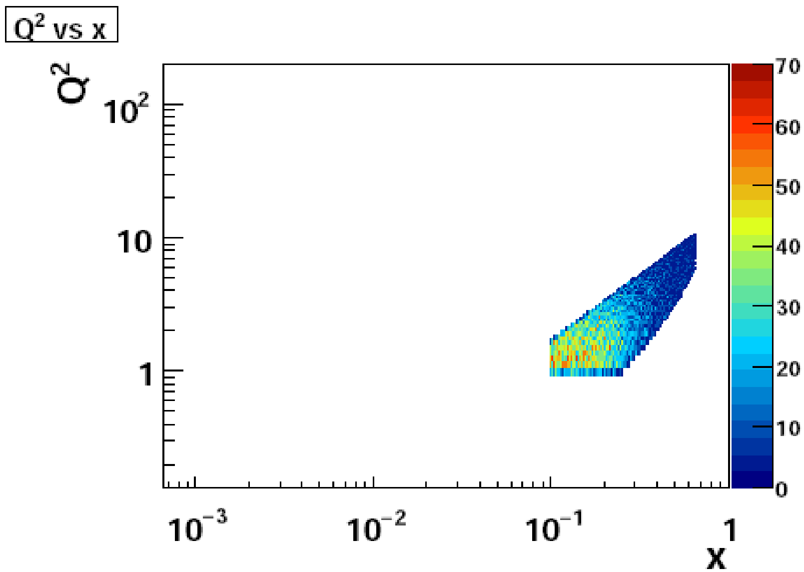
$$\chi^2 = \sum_{n=1}^{N_{data}} \left(\frac{theory_n - experiment_n}{experimental\ error_n} \right)^2$$

Chi-square function is minimized and values of parameters are obtained

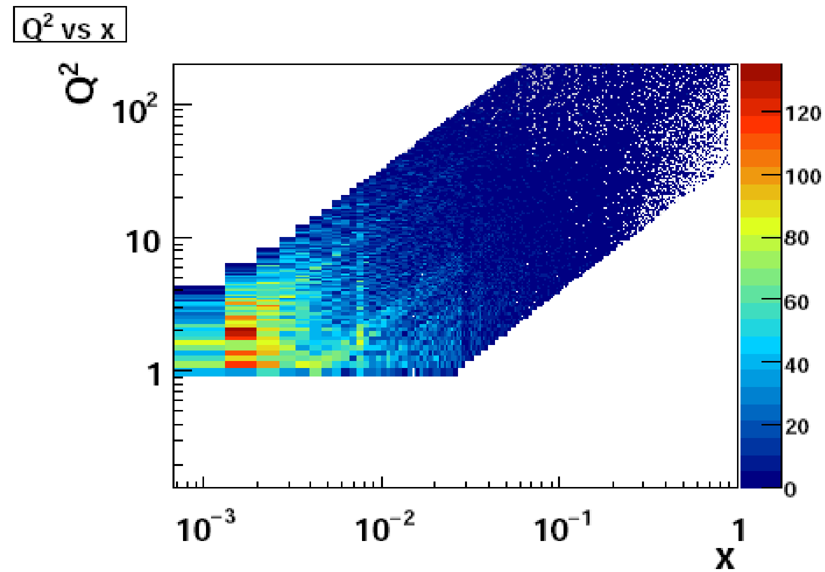
Future of Sivers function studies

JLab will operate at 12 GeV in 2015
Electron Ion Collider is proposed for 2025

JLab 12 $\sqrt{s} = 4.63$ GeV



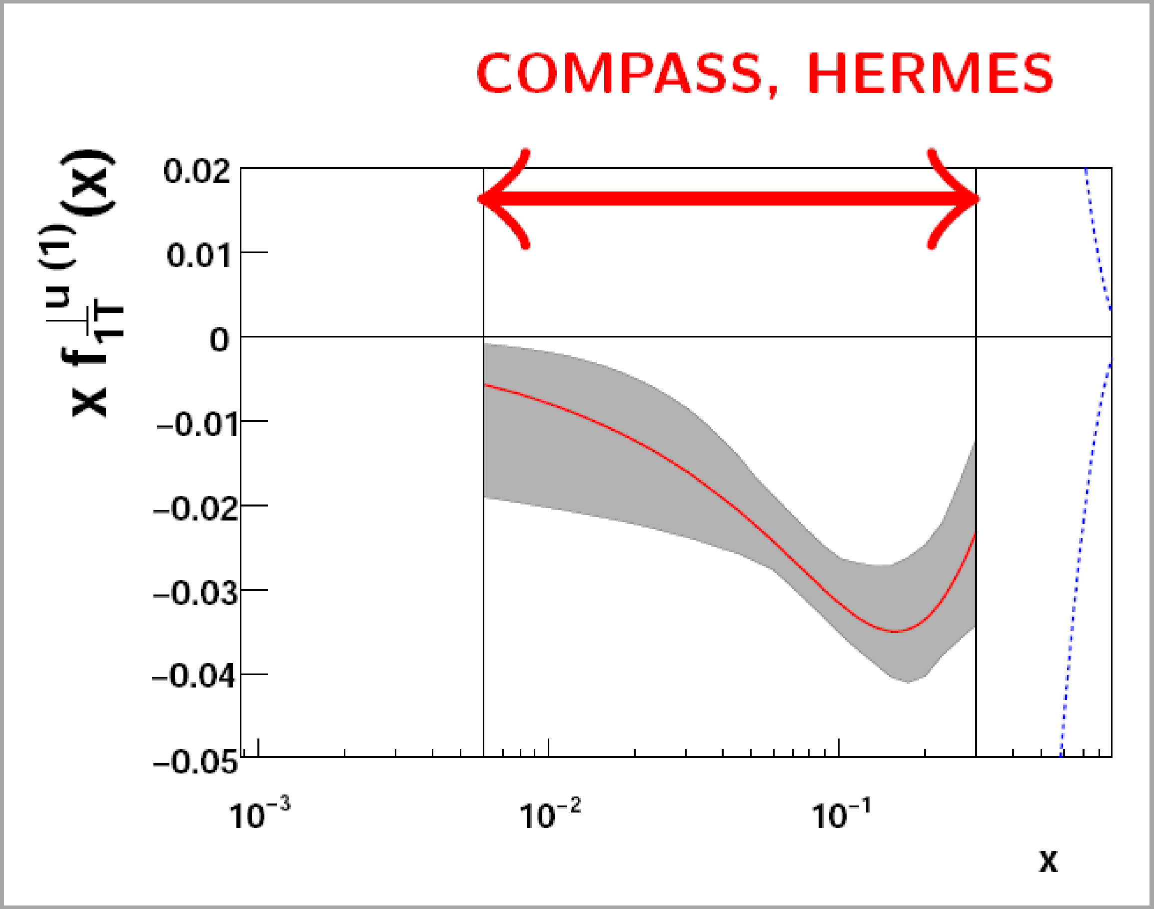
EIC $\sqrt{s} = 65$ GeV



- JLab 12 will extend owr knowledge of distributions at high-x region.
- EIC will explore low-x region.

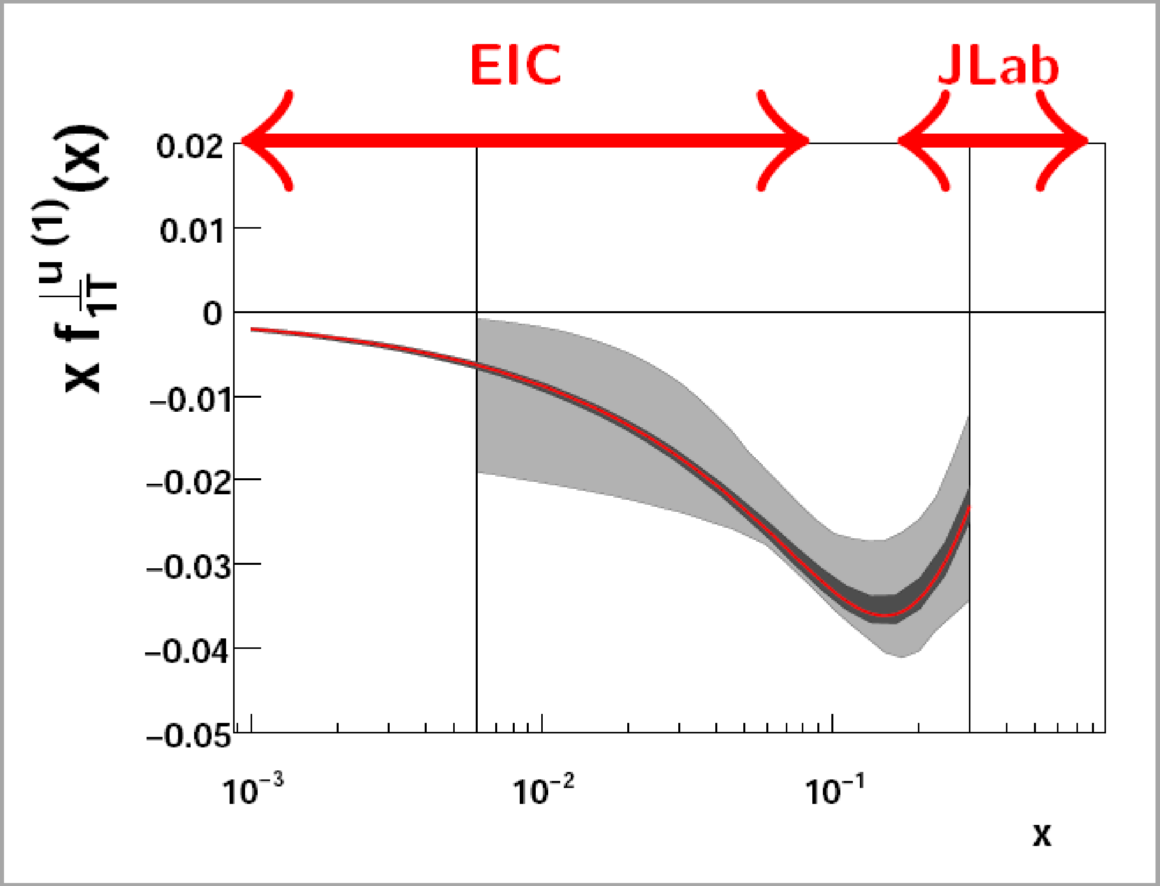
Future of Sivers function studies

Present knowledge:



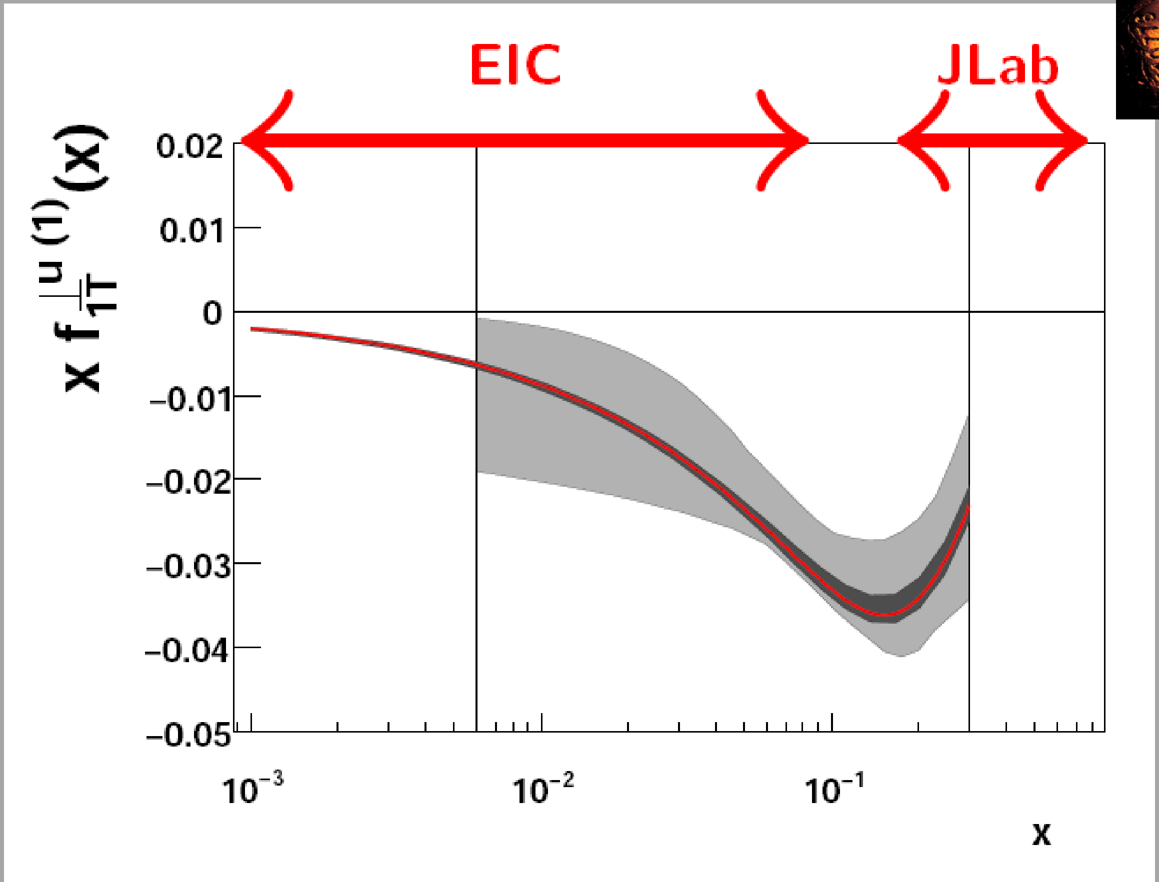
Future of Sivers function studies

Future knowledge:



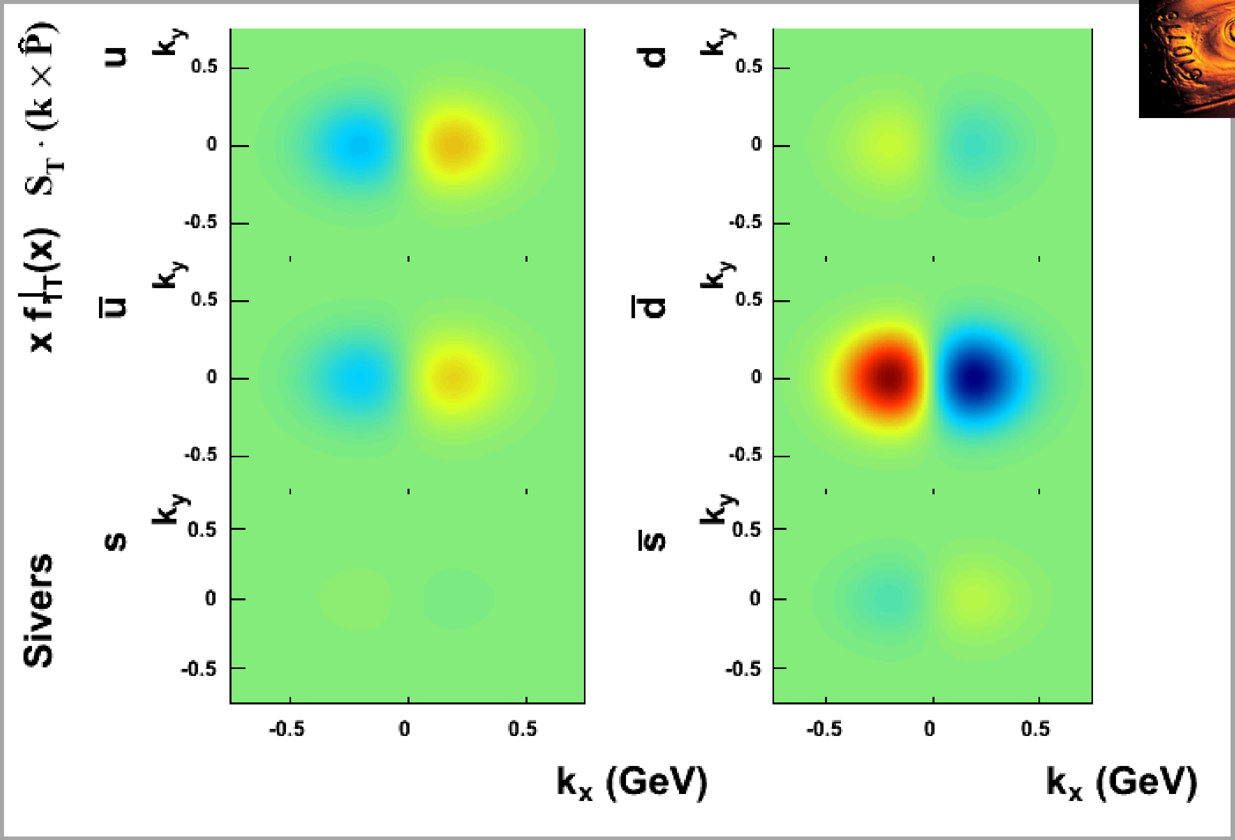
Future of Sivers function studies

Future knowledge:



Future of Sivers function studies

Future knowledge: x =0.01



Literature

Taylor “ Scattering Theory”

Halzen, Martin “ Quarks & Leptons”

Feynman “Photon-Hadron Interactions”

Collins “Foundations of perturbative QCD”

Barone, Drago, Ratcliffe http://arxiv.org/pdf/hep-ph/0104283

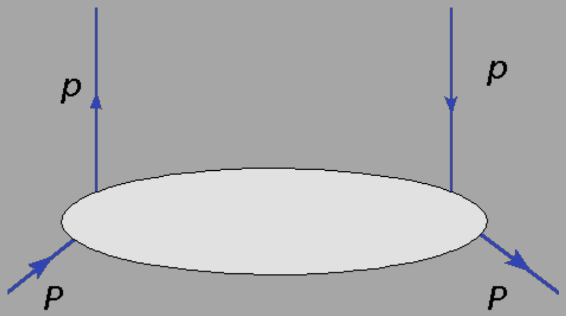

Thank you
Danke
Xie xie
Khawp khun
Mahalo
Salamat
Obrigada
Spacibo
Arigato

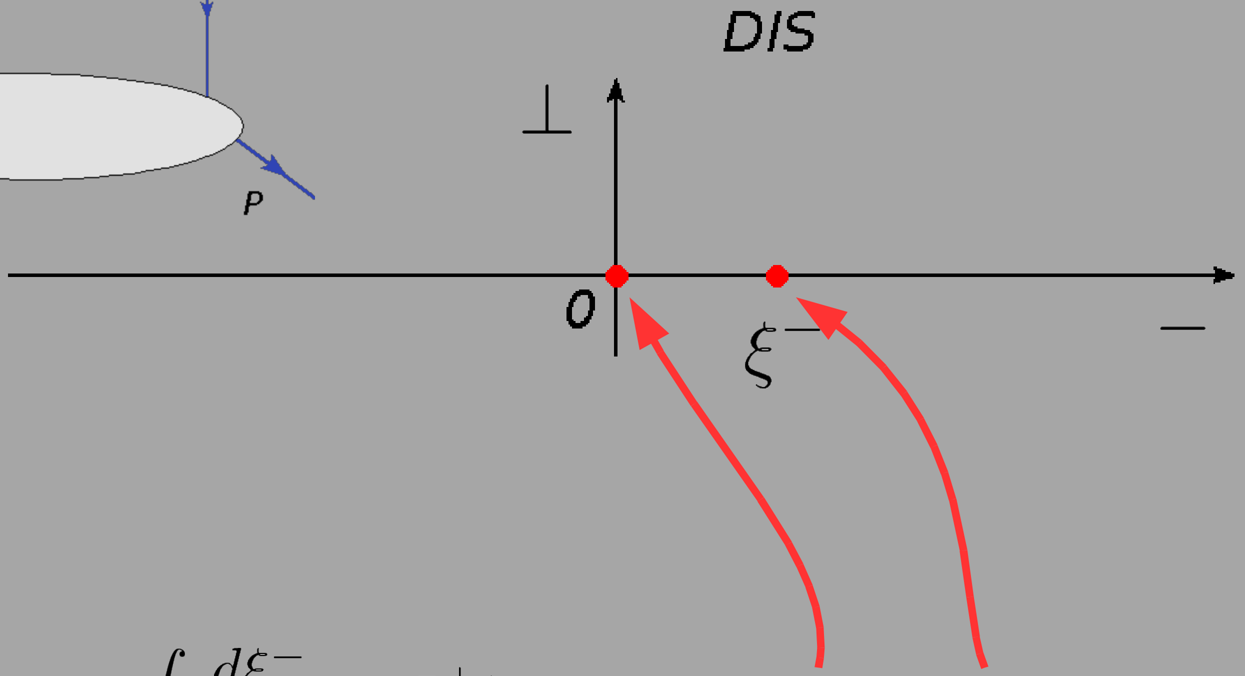
THANK YOU!

Back up slides

Distributions and parton model

What do we know about distributions?





$$\Phi_{ij}(x,P) = \int \frac{d\xi^-}{(2\pi)}\, e^{ixP^+\xi^-}\, \langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi^-)|P, S_P\rangle|_{\xi^+=0,\xi_{\mathbf{T}}=0}$$

Gauge invariance

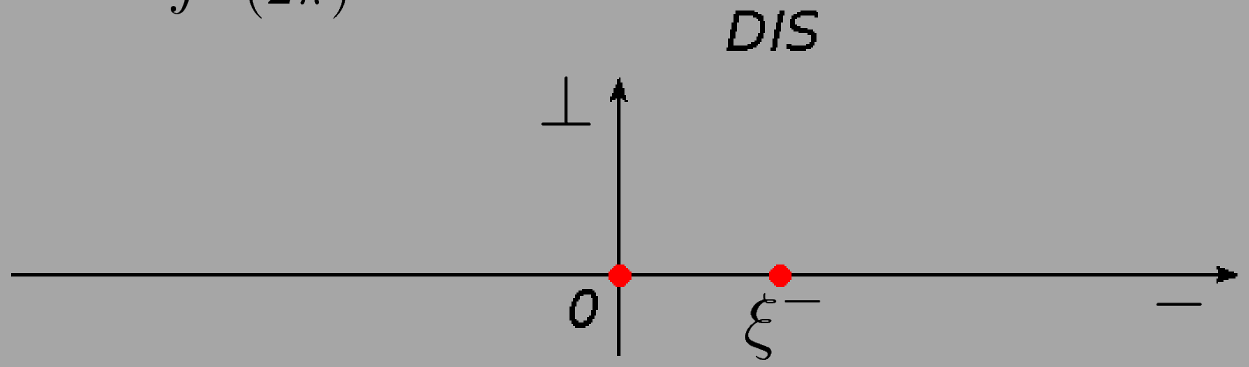
QCD is invariant under gauge transformations

$$\psi(x) \to \psi'(x) = \mathcal{U}(x)\psi(x) \qquad \bar{\psi}(x) \to \bar{\psi}'(x) = \bar{\psi}(x)\mathcal{U}^\dagger(x)$$

$$\mathcal{U}^\dagger(x)\mathcal{U}(x) = \mathbf{1}$$

It means that all observables are also gauge invariant

$$\Phi_{ij}(x,P) = \int \frac{d\xi^-}{(2\pi)}\, e^{ixP^+\xi^-}\, \langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi^-)|P,S_P\rangle|_{\xi^+=0,\xi_{\mathbf{T}}=0}$$



$$\bar{\psi}_j(0)\psi_i(\xi^-) \to \bar{\psi}_j(0)\mathcal{U}^\dagger(0)\mathcal{U}(\xi_-)\psi_i(\xi^-)$$

Gauge invariance

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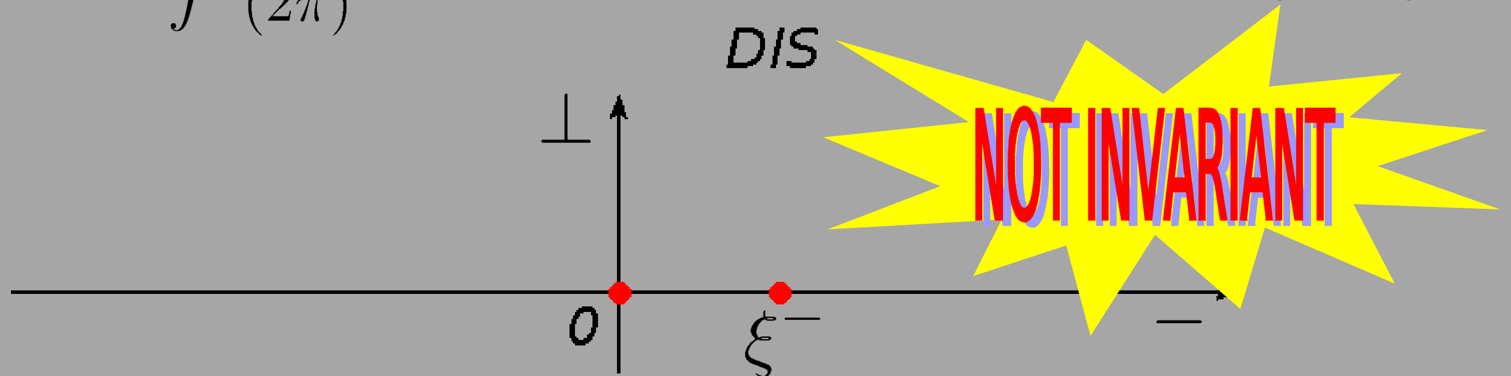
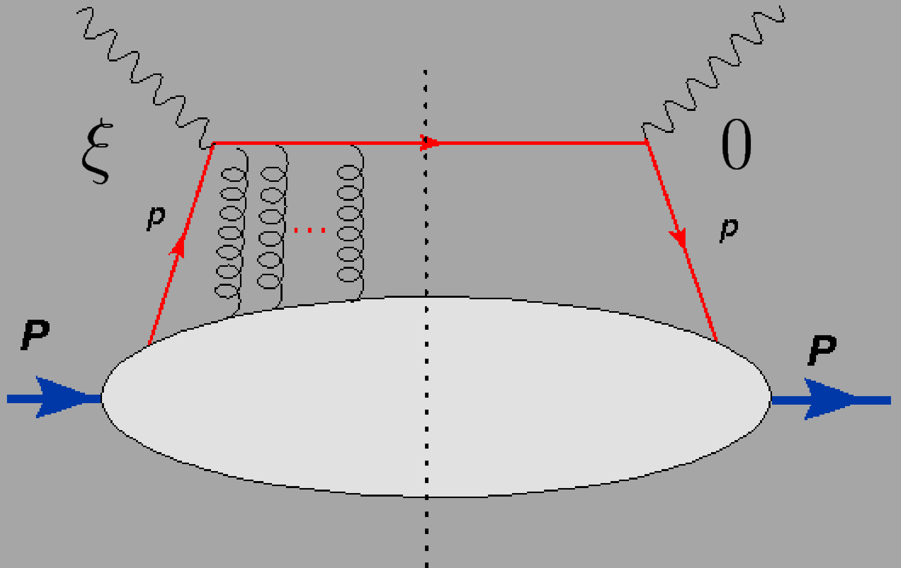


$$\bar\psi_j(0)\psi_i(\xi^-) \to \bar\psi_j(0)\mathcal{U}^\dagger(0)\mathcal{U}(\xi_-)\psi_i(\xi^-)$$

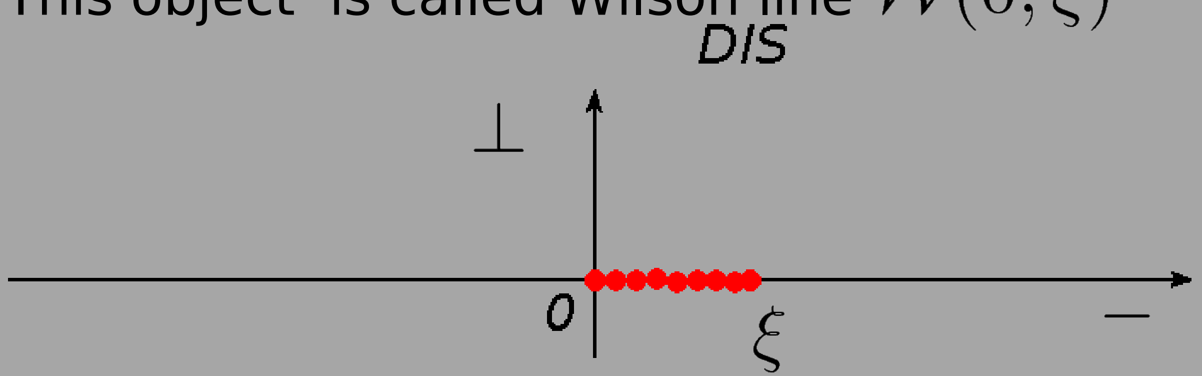
$$\mathcal{U}^\dagger(0)\mathcal{U}(\xi_-) \neq 1$$

What we forgot?

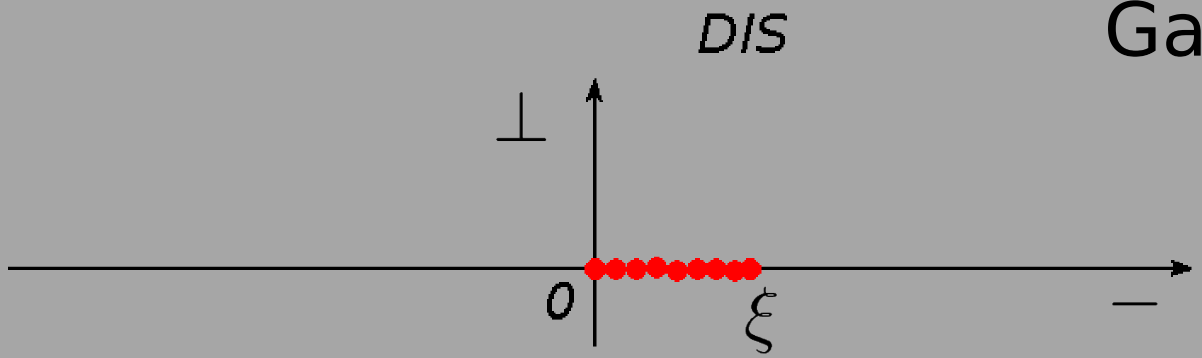
We forgot that quark and remnant are colored thus they interact via gluon exchanges!



This object is called Wilson line $\mathcal{W}(0,\xi)$



Gauge link



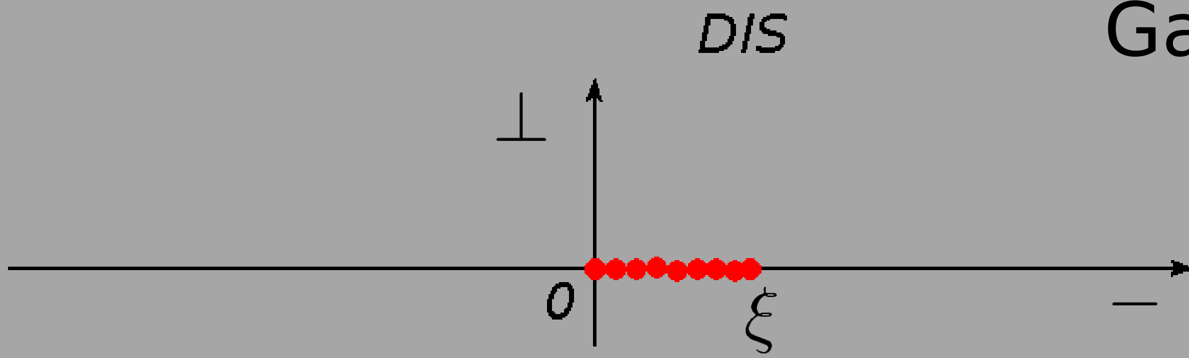
Wilson line restores gauge invariance!

$$\mathcal{W}(0,\xi) \to \mathcal{W}'(0,\xi) = \mathcal{U}(0)\mathcal{W}(0,\xi)\mathcal{U}^\dagger(\xi)$$

so that

$$\bar{\psi}_j(0)\mathcal{W}(0,\xi)\psi_i(\xi^-) \to \bar{\psi}_j(0)\mathcal{U}^\dagger(0)\mathcal{U}(0)\mathcal{W}(0,\xi)\mathcal{U}^\dagger(\xi_-)\mathcal{U}(\xi_-)\psi_i(\xi^-)$$

Gauge link



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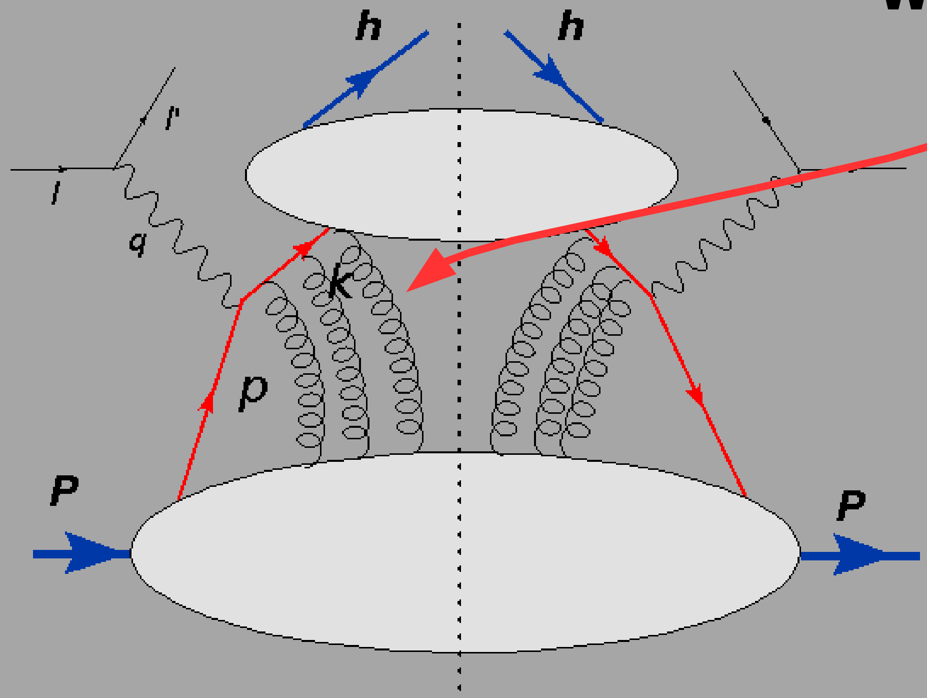
so that

$$\bar{\psi}_j(0)\mathcal{W}(0,\xi)\psi_i(\xi^-) \to \bar{\psi}_j(0)\mathcal{U}^\dagger(0)\mathcal{U}(0)\mathcal{W}(0,\xi)\mathcal{U}^\dagger(\xi_-)\mathcal{U}(\xi_-)\psi_i(\xi^-)$$

$$= \bar{\psi}_j(0)\mathcal{W}(0,\xi)\psi_i(\xi^-)$$

Gauge invariance

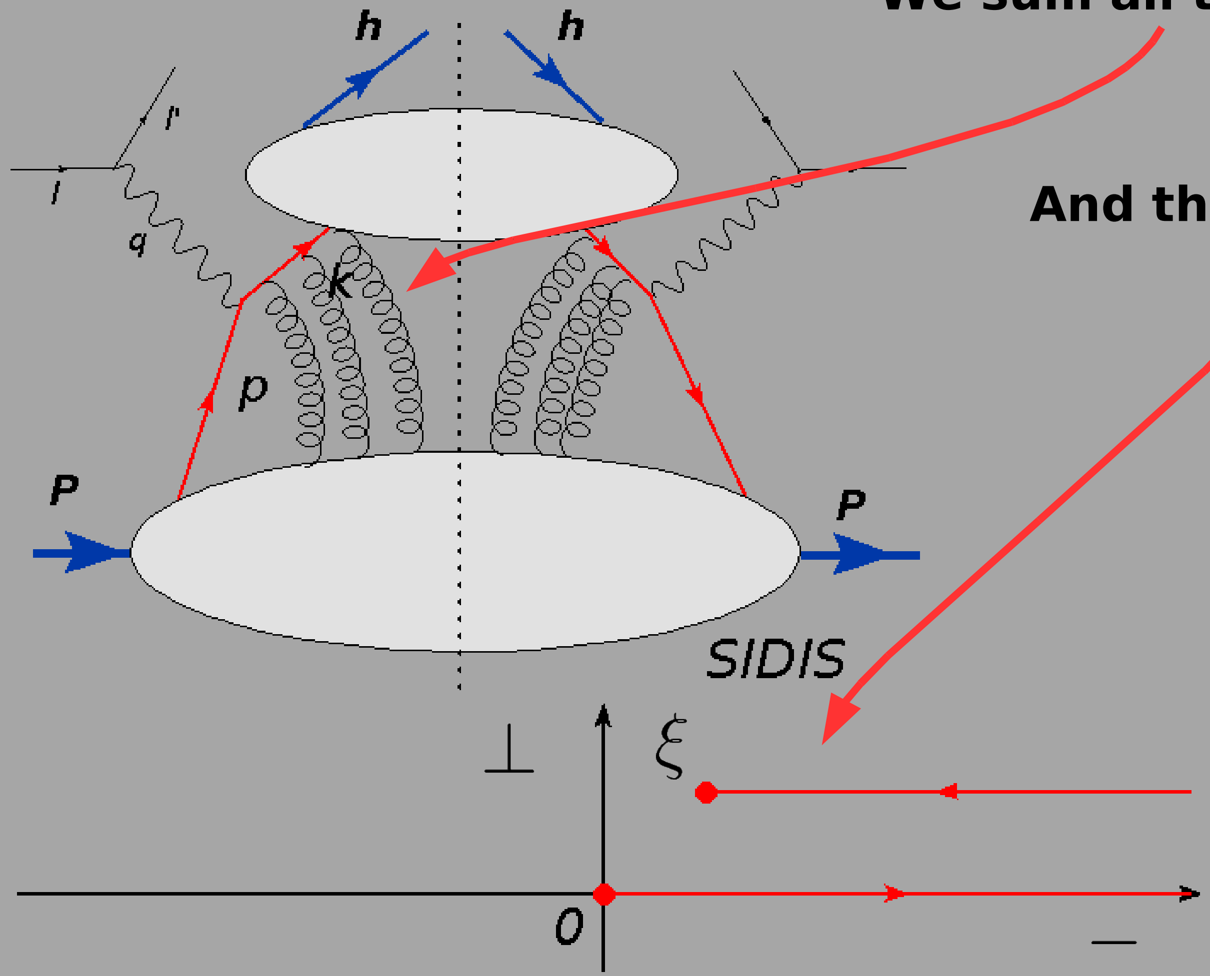
We sum all these gluons



Gauge invariance

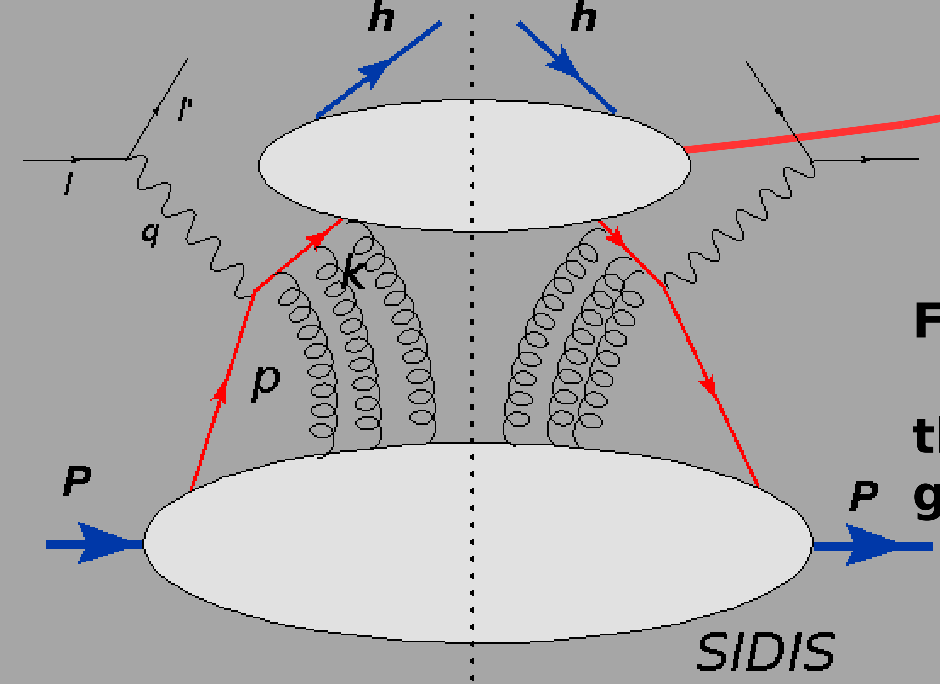
We sum all these gluons

And the gauge link is now



Gauge invariance

We sum all these gluons



Fields are not only separated in $-$ direction, but also in $\perp$ this makes TMDs sensitive to gauge invariance