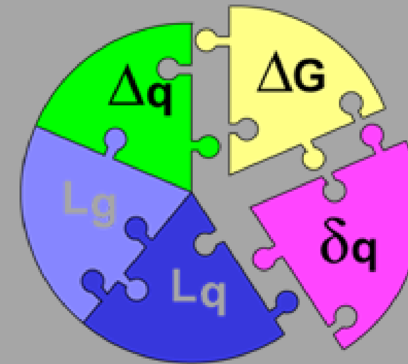
3D Partonic Structure of the Nucleon
Varenna 2011



Phenomenology

of Transvers Momentum Dependent

distributions

Alexei Prokudin
Jefferson Laboratory

- I will speak about phenomenology of Transverse Momentum Dependent distributions (TMDs) and expalin how knowledge of those helps us to reveal 3 dimensional structure of the nucleon. Understanding 3 dimensional dynamics of nucleons is a paramaunt goal of experimental studies at Jlab and other laboratories.

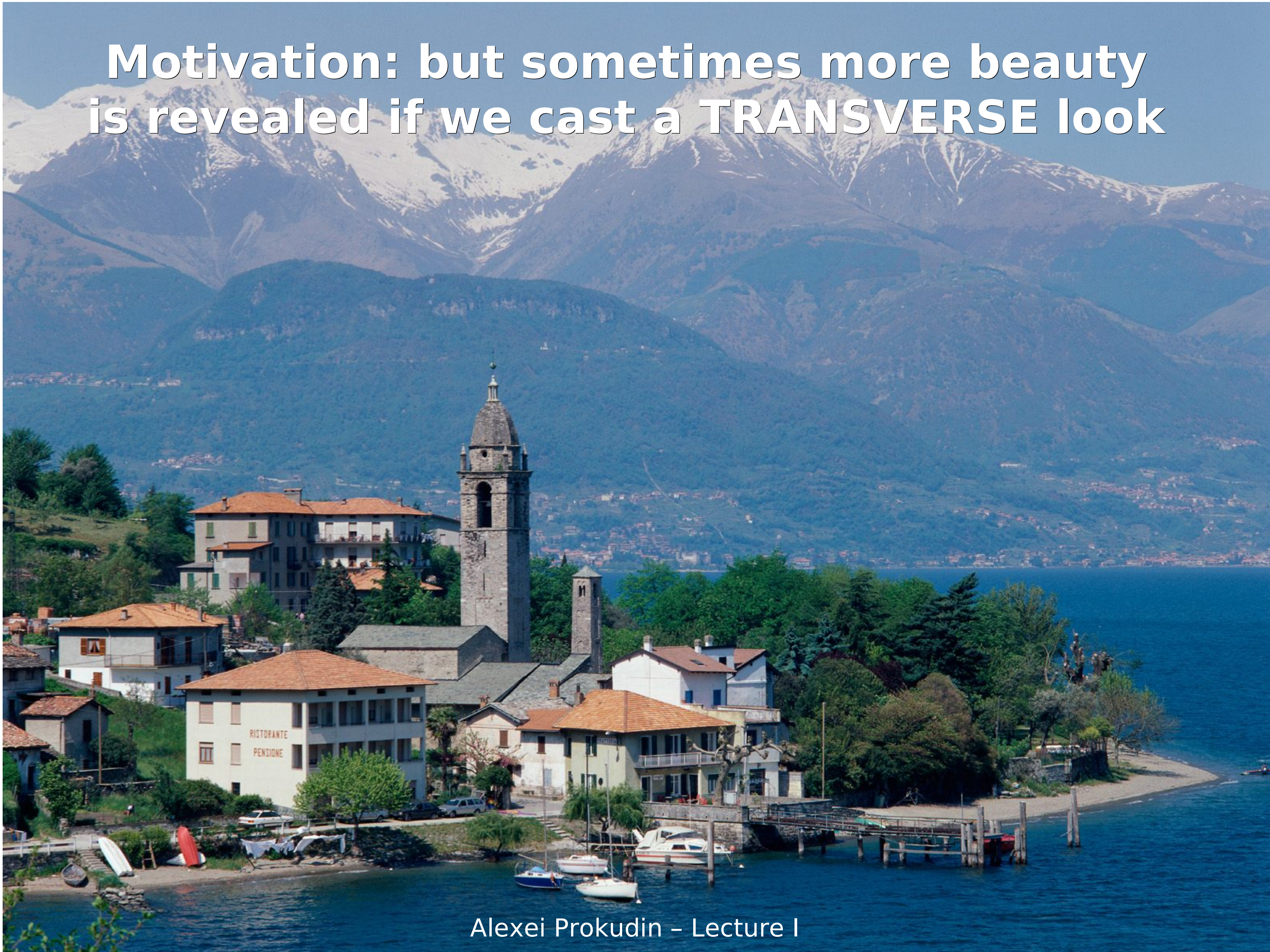
Motivation: we study structure of matter

Motivation: smaller probes reveal details

Motivation: smaller probes reveal details

Motivation: but sometimes more beauty is revealed if we cast a TRANSVERSE look

Motivation: but sometimes more beauty is revealed if we cast a TRANSVERSE look



Part I : Theory of **T**ransvers **M**omentum **D**ependent distributions

The Hilbert Space of State Vectors

See Taylor “Scattering Theory”

Wave functions

$$\int d^3x\, |\psi(\mathbf{x})|^2 < \infty$$

are coordinate projections of state vectors in so-called Hilbert space of state vectors describing particles $|\psi\rangle$

Scalar product is defined

$$\langle\psi|\phi\rangle = \int d^3x\, \psi(\mathbf{x})^* \phi(\mathbf{x})$$

We will use $|P;S\rangle$ to denote the Proton with momentum P and with some Spin vector S

Unitarity

See Taylor “Scattering Theory”

Probability of one state going to the other is described by S matrix

$$\mathbf{w}(\chi \leftarrow \mathbf{\Phi}) = |\langle\chi|\,\mathbf{S}\,|\mathbf{\Phi}\rangle|^{\mathbf{2}}$$

We can write

$$\mathbf{S_{ab}} = \delta_{\mathbf{ab}} + \mathbf{i}(\mathbf{2}\pi)^{\mathbf{4}}\delta^{(\mathbf{4})}(\mathbf{p_a} - \mathbf{p_b})\mathbf{T_{ab}}$$

Unitarity

See Taylor “Scattering Theory”

Probability of one state going to the other is described by S matrix

$$\mathbf{w}(\chi \leftarrow \mathbf{\Phi}) = |\langle\chi|\,\mathbf{S}\,|\mathbf{\Phi}\rangle|^{\mathbf{2}}$$

We can write

$$\mathbf{S_{ab}} = \delta_{\mathbf{ab}} + \mathbf{i}(\mathbf{2}\pi)^{\mathbf{4}}\delta^{(\mathbf{4})}(\mathbf{p_a} - \mathbf{p_b})\mathbf{T_{ab}}$$

No interaction

Unitarity

See Taylor "Scattering Theory"

Probability of one state going to the other is described by S matrix

$$\mathbf{w}(\chi \leftarrow \mathbf{\Phi}) = |\langle\chi|\,\mathbf{S}\,|\mathbf{\Phi}\rangle|^{\mathbf{2}}$$

We can write

$$\mathbf{S_{ab}} = \delta_{\mathbf{ab}} + \mathbf{i}(\mathbf{2}\pi)^{\mathbf{4}}\delta^{(\mathbf{4})}(\mathbf{p_a} - \mathbf{p_b})\mathbf{T_{ab}}$$

No interaction

Momentum conservation

Unitarity

See Taylor "Scattering Theory"

Probability of one state going to the other is described by S matrix

$$\mathbf{w}(\chi \leftarrow \boldsymbol{\Phi}) = |\langle\chi|\,\mathbf{S}\,|\boldsymbol{\Phi}\rangle|^{\mathbf{2}}$$

We can write

$$\mathbf{S_{ab}} = \delta_{\mathbf{ab}} + \mathbf{i}(\mathbf{2}\pi)^{\mathbf{4}}\delta^{(\mathbf{4})}(\mathbf{p_a} - \mathbf{p_b})\mathbf{T_{ab}}$$

No interaction

Momentum conservation

INTERACTION

Unitarity

See Taylor “Scattering Theory”

Probability is conserved

$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{S}^\dagger\mathbf{S} = \mathbf{1}$$

We can write

“X” all possible states
(complete set)

$$\mathbf{Im}\,\langle\chi|\,\mathbf{T}\,|\mathbf{\Phi}\rangle = \frac{1}{2}\sum_{\mathbf{X}}\langle\chi|\,\mathbf{T}\,|\mathbf{X}\rangle\,\langle\mathbf{X}|\,\mathbf{T}^\dagger\,|\mathbf{\Phi}\rangle\,(2\pi)^4\delta^{(4)}(\mathbf{P}_{\mathbf{\Phi}} - \mathbf{P}_{\mathbf{X}})$$

Momentum
conservation

INTERACTION

Unitarity

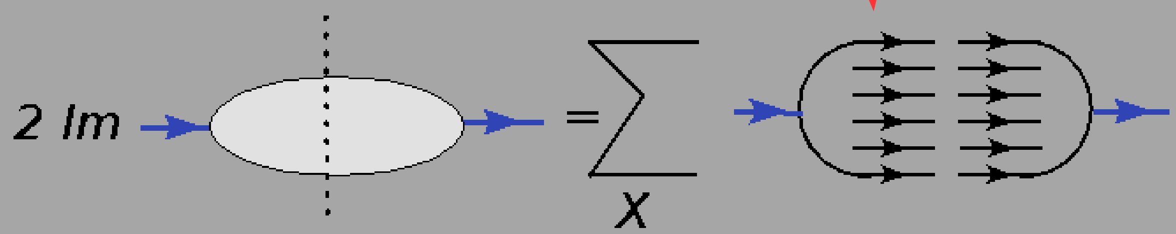
See Taylor “Scattering Theory”

Probability is conserved

$$\mathbf{S}\mathbf{S}^{\dagger} = \mathbf{S}^{\dagger}\mathbf{S} = \mathbf{1}$$

We can write

“X” complete set



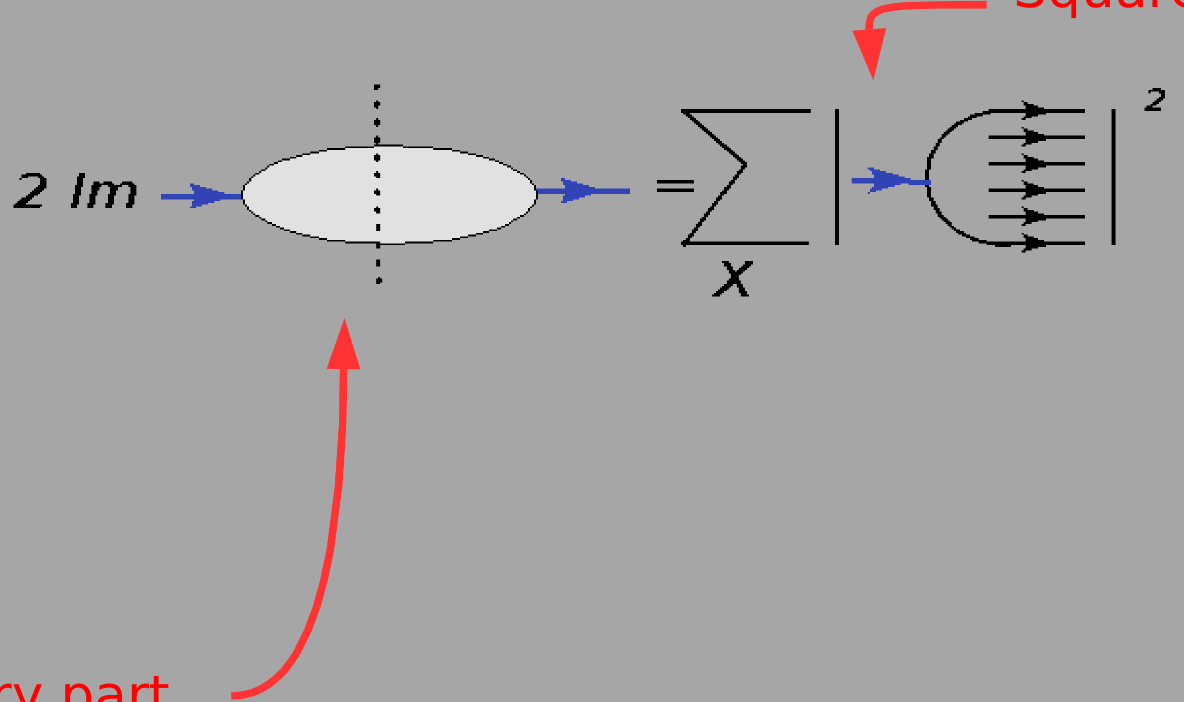
Imaginary part

Optical theorem

See Taylor "Scattering Theory"

Particular case for the identical states A and B is called
Optical theorem

Squared amplitude



Imaginary part

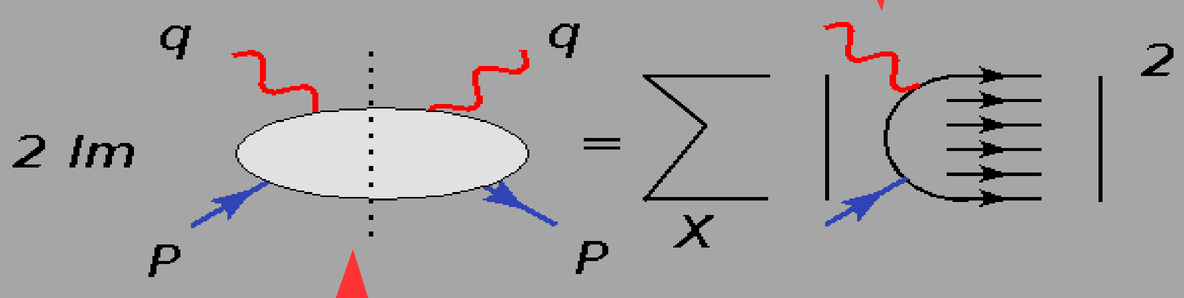
Optical theorem

See Taylor "Scattering Theory"

Optical theorem

We are interested in photon – proton interactions

Squared amplitude



Imaginary part

Cross-section

See Taylor “Scattering Theory”

Experimentally we do not measure probabilities, but cross-sections

Squared amplitude

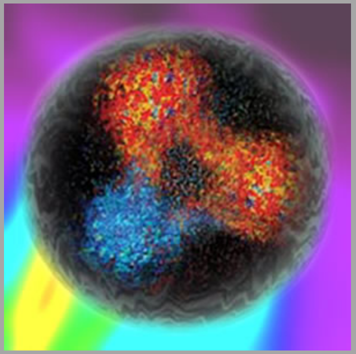
$$\sigma_{\Phi\to\Psi} = \frac{1}{F_\Phi} |A_{\Phi\to\Psi}|^2 \frac{d^3P_\Psi}{(2\pi)^3 2E_\Psi}$$

Flux of Φ

Phase space of Ψ

Wigner distribution

Our goal is to understand 3 dimentional distributions of partons,
How they move, there they are located inside a nucleon



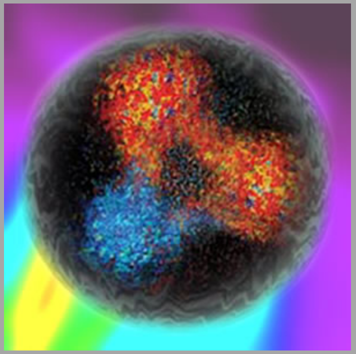
Wigner distribution (1933) is the answer

$$W(\mathbf{p},\mathbf{x}) = \int d^4\eta\, e^{i\,\mathbf{p}\eta}\psi^*(\mathbf{x}+\eta/\mathbf{2})\psi(\mathbf{x}-\eta/\mathbf{2})$$

It gives both position and momenta for quarks

Wigner distribution

Our goal is to understand 3 dimentional distributions of partons,
How they move, there they are located inside of a nucleon



Wigner (1933) distribution is the answer

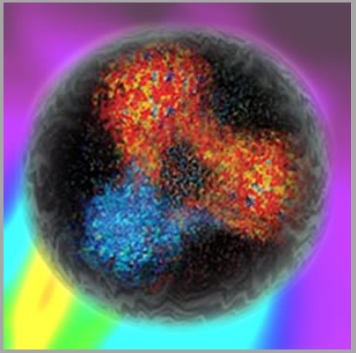
$$W(\mathbf{p},\mathbf{x}) = \int d^4\eta\, e^{i\,\mathbf{p}\eta}\psi^*(\mathbf{x}+\eta/\mathbf{2})\psi(\mathbf{x}-\eta/\mathbf{2})$$

It gives both position and momenta for quarks

Can it be measured?

Wigner distribution

Our goal is to understand 3 dimentional distributions of partons,
How they move, there they are located inside of a nucleon



Wigner (1933) distribution is the answer

$$W(\mathbf{p},\mathbf{x}) = \int d^4\eta\, e^{i\,\mathbf{p}\eta}\psi^*(\mathbf{x}+\eta/\mathbf{2})\psi(\mathbf{x}-\eta/\mathbf{2})$$

It gives both position and momenta for quarks

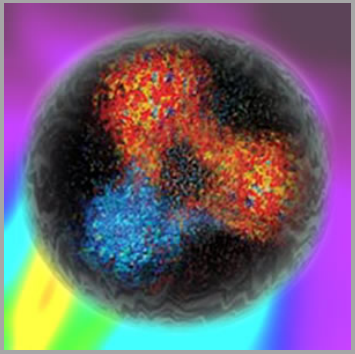
Can it be measured? NO! (as I know)

$$\Delta p \Delta x \geq \hbar/2$$

No simultaneous knowledge on position and momenta

Wigner distribution

Our goal is to understand 3 dimentional distributions of partons,
How they move, there they are located inside of a nucleon



Wigner (1933) distribution is the answer

$$W(\mathbf{p},\mathbf{x}) = \int d^4\eta \, e^{i\,\mathbf{p}\eta} \psi^*(\mathbf{x}+\eta/\mathbf{2})\psi(\mathbf{x}-\eta/\mathbf{2})$$

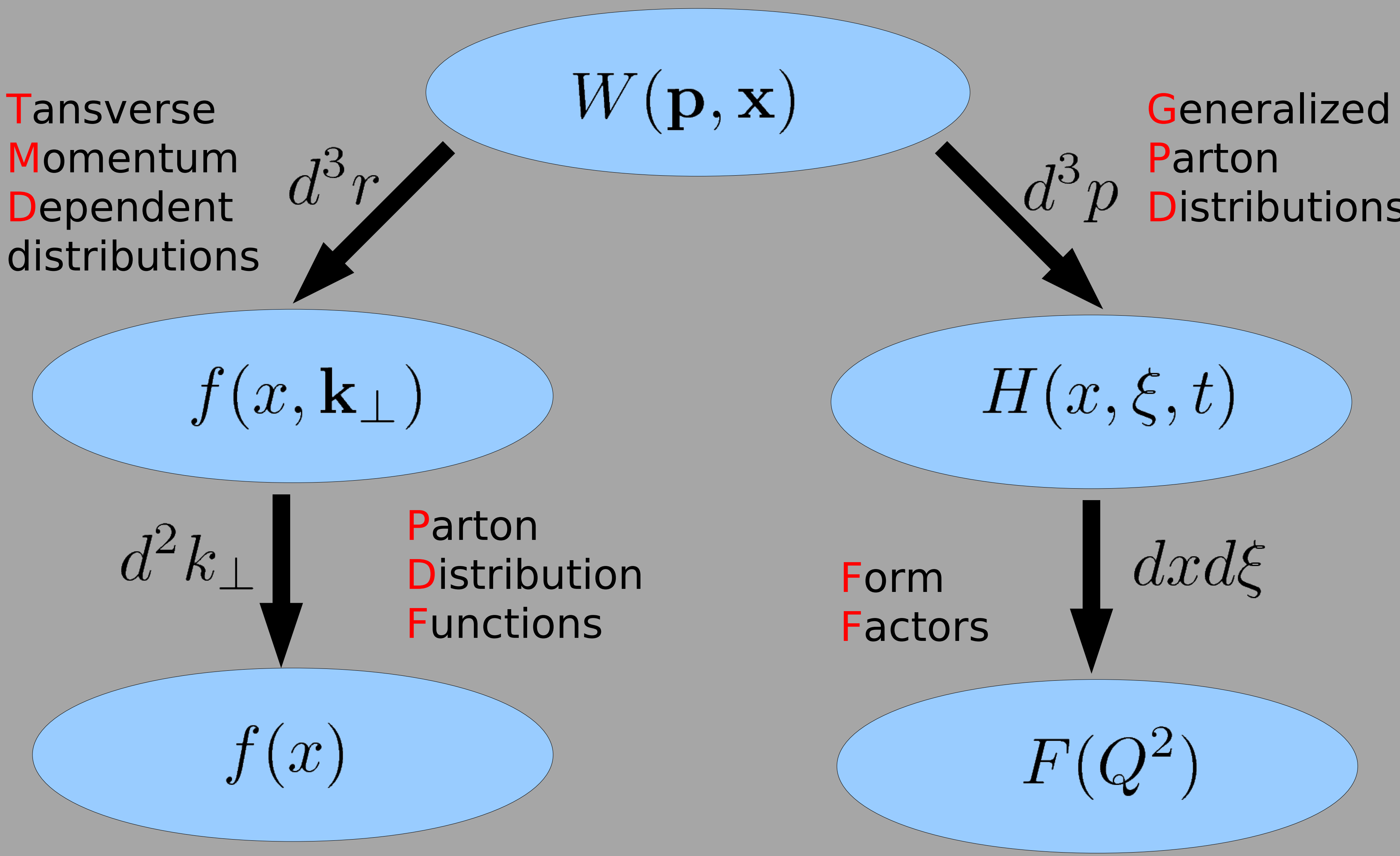
It gives both position and momenta for quarks

BUT **its integrals can be measured!**

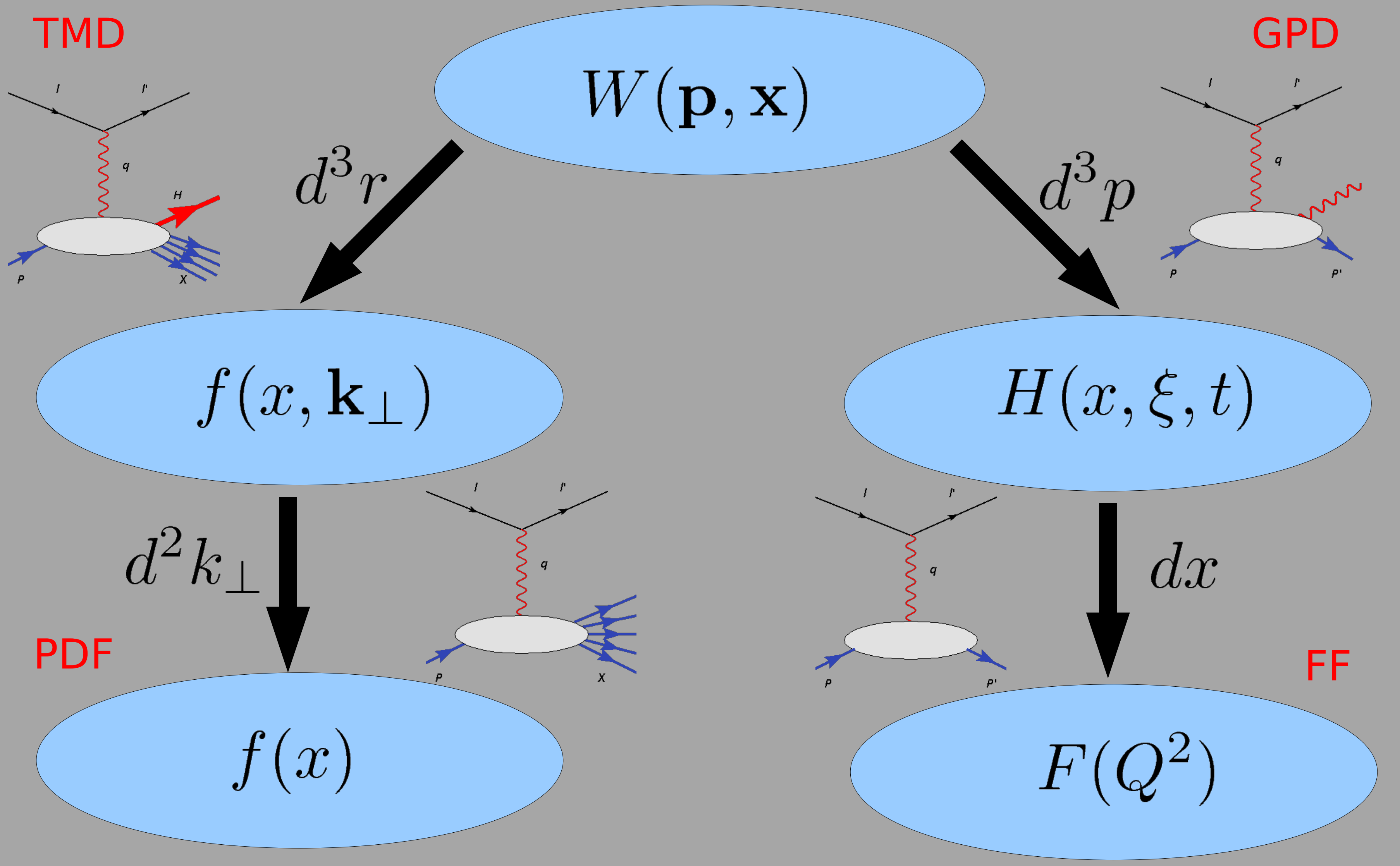
$$\Delta p_x \Delta y = 0$$

We should find those integrals

Wigner distribution



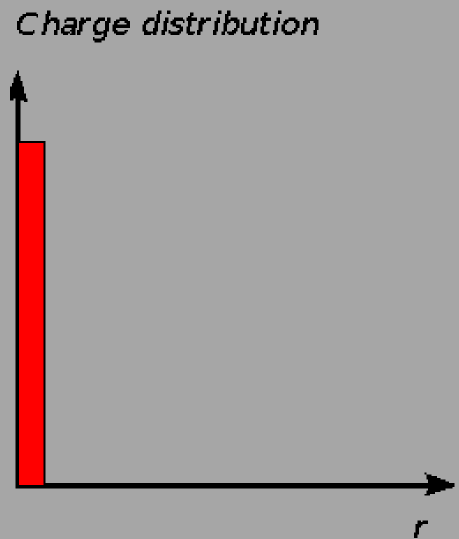
Wigner distribution

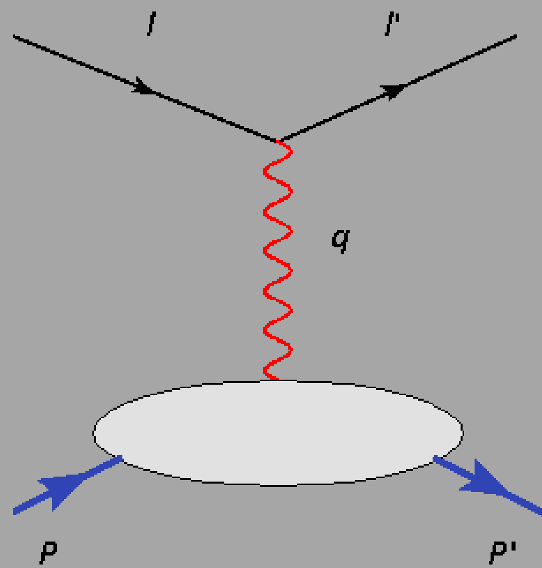


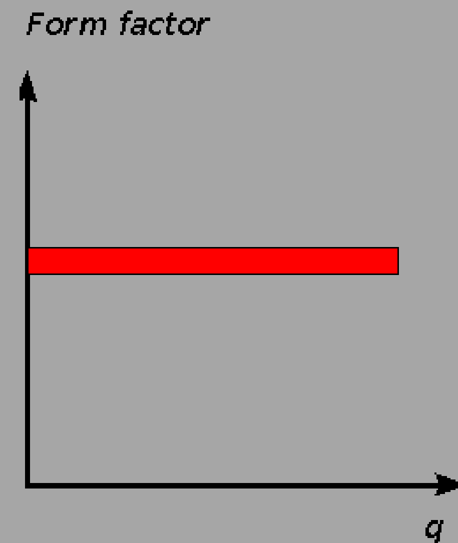
Form factors

Form factors measure charge distributions inside nucleon

Constant Form factor – pointlike object







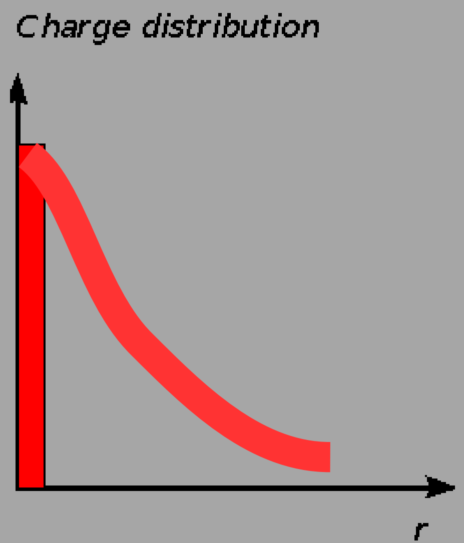
$$\mathbf{F}(\mathbf{q}^2) = \int \mathbf{d}^3\mathbf{r}\ \mathbf{e}^{\mathbf{iqr}} \rho(\mathbf{r})$$

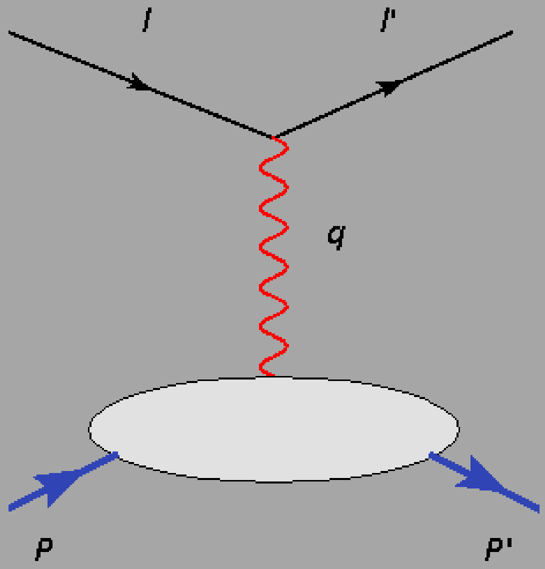
Position and momentum space are related by Fourier transform

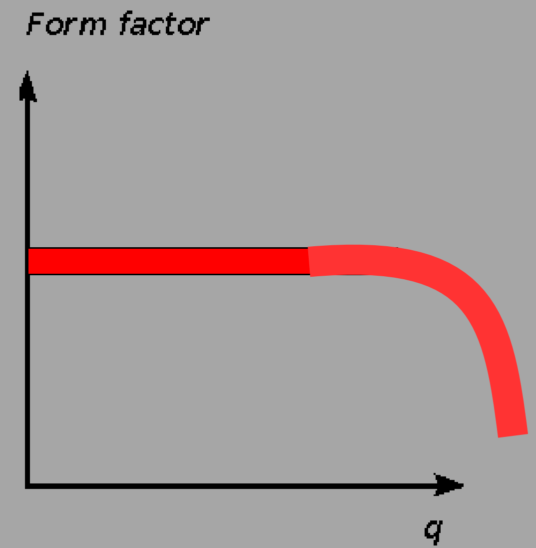
Form factors

Form factors measure charge distributions inside nucleon

Non constant Form factor – non pointlike object





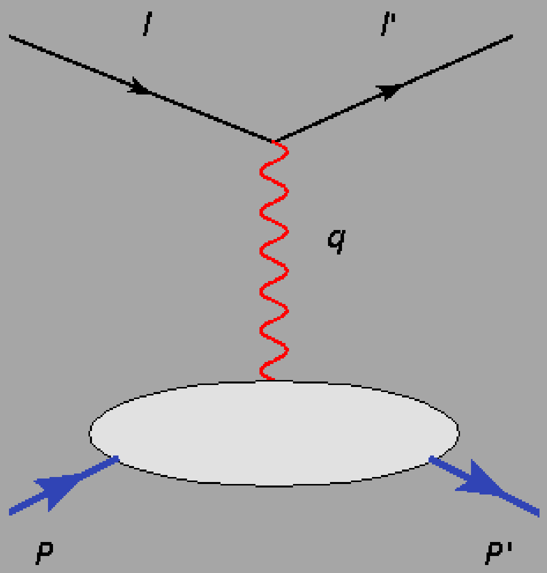


The proton remains intact, “elastic” stattering

We know that proton unlike electron is not a pointlike object!

Form factors

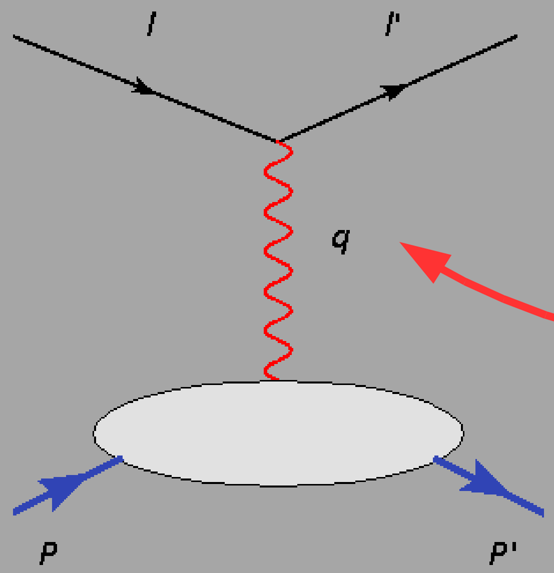
Let us write the square of scattering amplitude



$$|\mathbf{A}|^2 \propto \frac{1}{\mathbf{Q^4}} \mathbf{L}^{\mathbf{em}}_{\mu\nu} \mathbf{W}^{\mu\nu}_{\mathbf{hadr}}$$

Form factors

Let us write the square of scattering amplitude

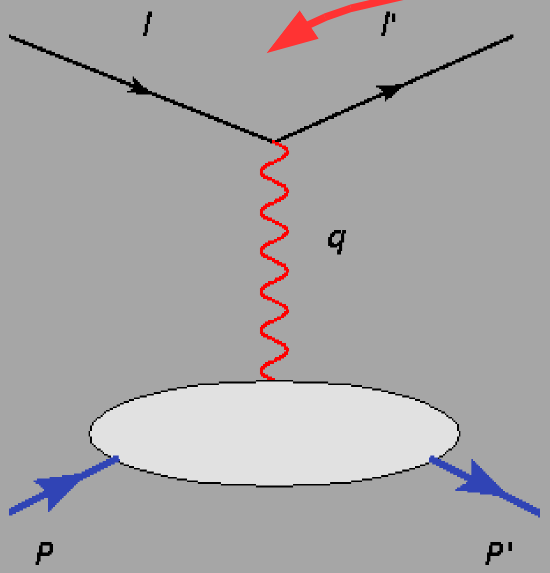


$$|\mathbf{A}|^2 \propto \frac{1}{\mathbf{Q^4}} \mathbf{L}^{\mathbf{em}}_{\mu\nu} \mathbf{W}^{\mu\nu}_{\mathbf{hadr}}$$

Photon propagator squared

Form factors

Let us write the square of scattering amplitude

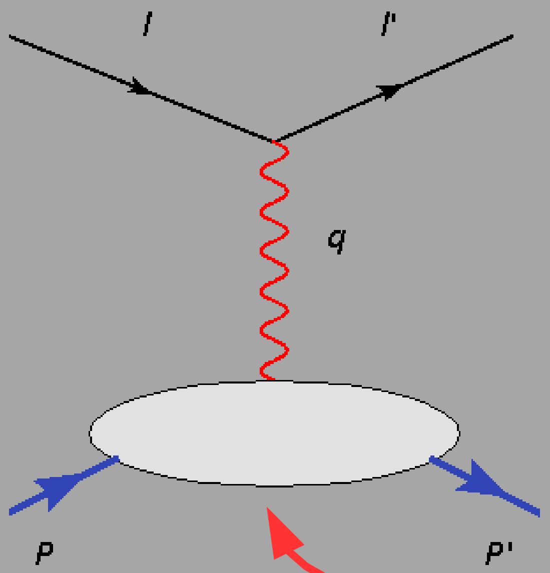


Leptonic tensor

$$|A|^2 \propto \frac{1}{Q^4} L^{em}_{\mu\nu} W^{\mu\nu}_{hadr}$$

Form factors

Let us write the square of scattering amplitude



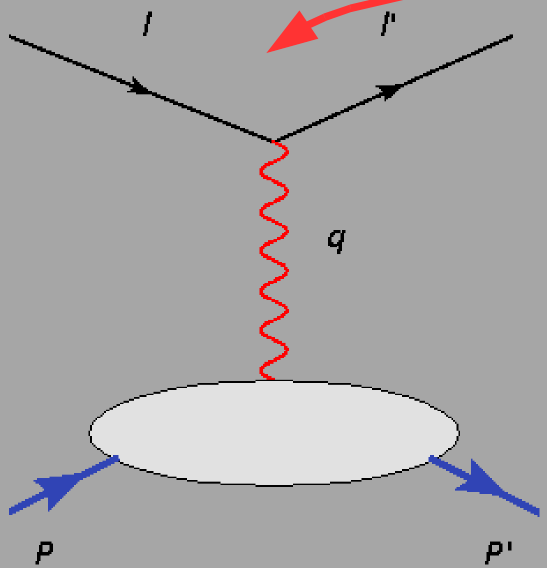
$$|\mathbf{A}|^2 \propto \frac{1}{\mathbf{Q^4}} \mathbf{L}^{\mathbf{em}}_{\mu\nu} \mathbf{W}^{\mu\nu}_{\mathbf{hadr}}$$

Hadronic tensor

Form factors

Form factors measure charge distributions inside nucleon

Leptonic tensor

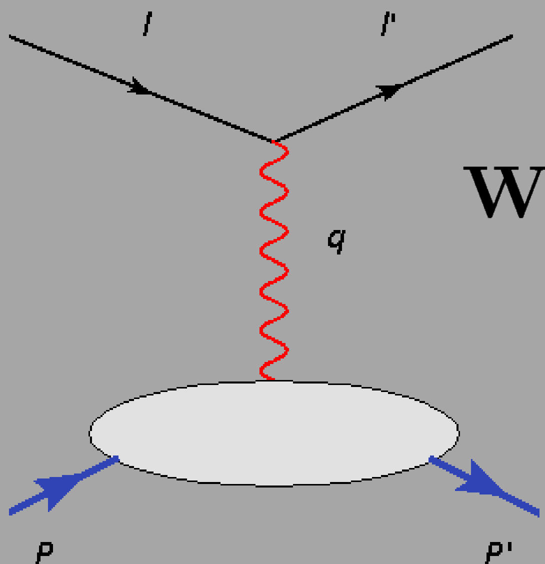


$$|\mathbf{A}|^2 \propto \frac{\mathbf{1}}{\mathbf{Q^4}} \mathbf{L}^{\mathbf{em}}_{\mu\nu} \mathbf{W}^{\mu\nu}_{\mathbf{hadr}}$$

$$L_{\mu\nu} = 4\delta_{\lambda\lambda'}(l_\mu l'_\nu + l_\nu l'_\mu - (l \cdot l')g_{\mu\nu} + i\lambda\epsilon_{\mu\nu\alpha\beta} l^\alpha q^\beta)$$

Form factors

Hadronic tensor can be expanded in terms of momenta and metric tensor. We consider symmetric part only



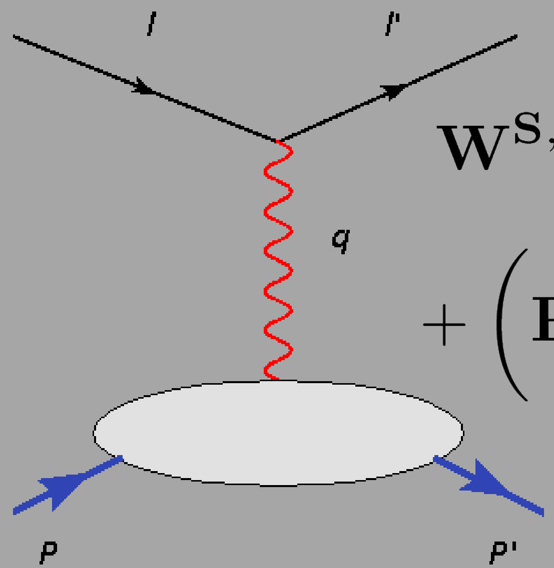
$$\mathbf{W}^{\mu\nu}(\mathbf{q},\mathbf{P}) = \mathbf{a}\mathbf{g}^{\mu\nu} + \mathbf{b}\mathbf{P}^{\mu}\mathbf{P}^{\nu} + \mathbf{c}\mathbf{P}^{\mu}\mathbf{q}^{\nu} + \mathbf{d}\mathbf{P}^{\nu}\mathbf{q}^{\mu} + \mathbf{e}\mathbf{q}^{\mu}\mathbf{q}^{\nu}$$

“**a,b,c,d,e**” are possible form factors. From electromagnetic current conservation $\partial_\mu j^\mu(x) = 0$ we obtain

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$$

Form factors

Hadronic tensor can be expanded in terms of momenta and metric tensor. We consider symmetric part only



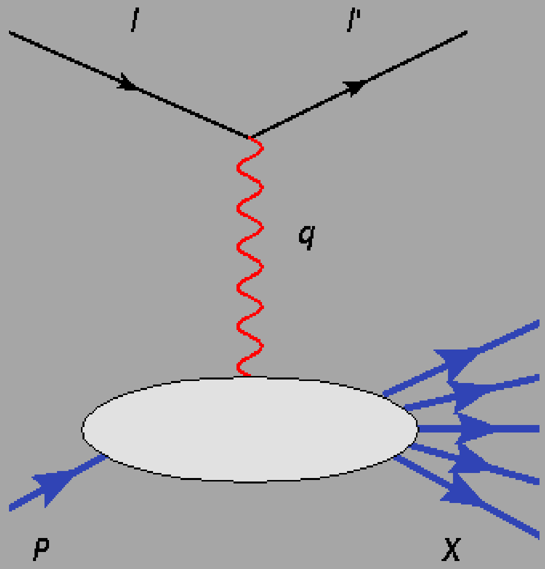
$$\mathbf{W}^{\mathbf{S},\mu\nu}(\mathbf{q},\mathbf{P}) = \left(-\mathbf{g}^{\mu\nu} - \frac{\mathbf{q}^{\mu}\mathbf{q}^{\nu}}{\mathbf{Q}^{2}}\right)\mathbf{F}_{1}(\mathbf{Q}^{2})$$
$$+ \left(\mathbf{P}^{\mu} + \frac{(\mathbf{P}\cdot\mathbf{q})\mathbf{q}^{\mu}}{\mathbf{Q}^{2}}\right)\left(\mathbf{P}^{\nu} + \frac{(\mathbf{P}\cdot\mathbf{q})\mathbf{q}^{\nu}}{\mathbf{Q}^{2}}\right)\mathbf{F}_{2}(\mathbf{Q}^{2})$$

Only two form factors remain!

Show it! Use $q_{\mu}W^{\mu\nu} = 0$, $W^{\mu\nu} = W^{\nu\mu}$

From Form factors to Distributions

In order to measure **distributions** we change the process we study: from elastic we go to deep inelastic scattering



The proton is destoyed

Bjorken limit is

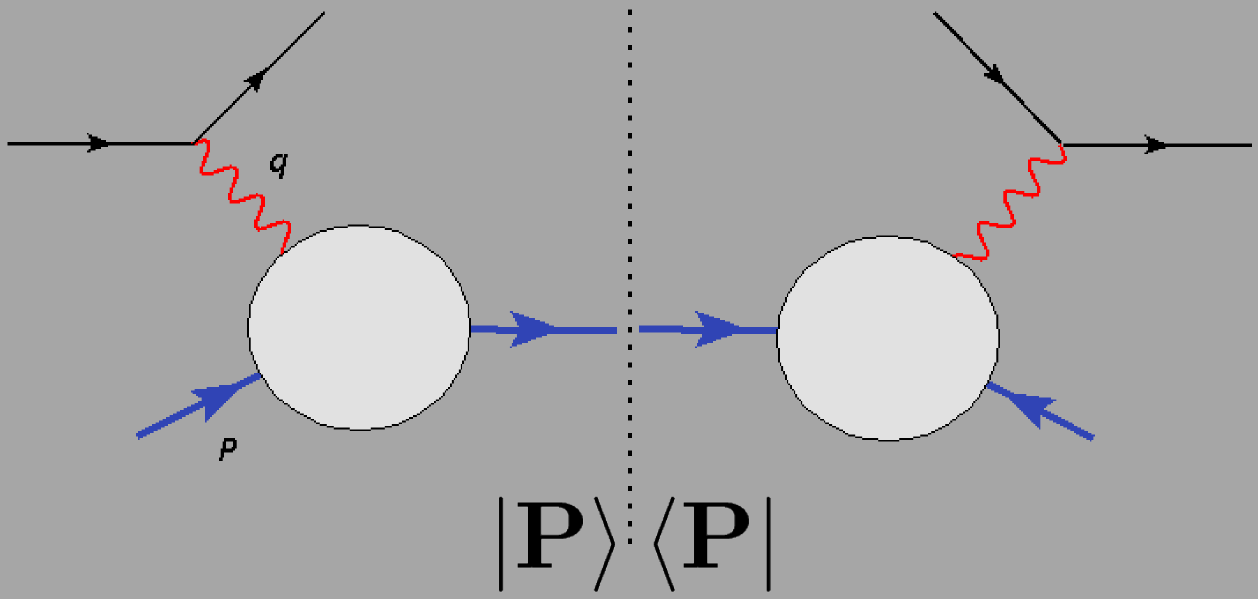
$$\mathbf{Q^2 \to \infty}$$

$$\mathbf{P \cdot q \to \infty}$$

$$\mathbf{x_{Bj} \equiv \frac{Q^2}{2P \cdot q} \to const}$$

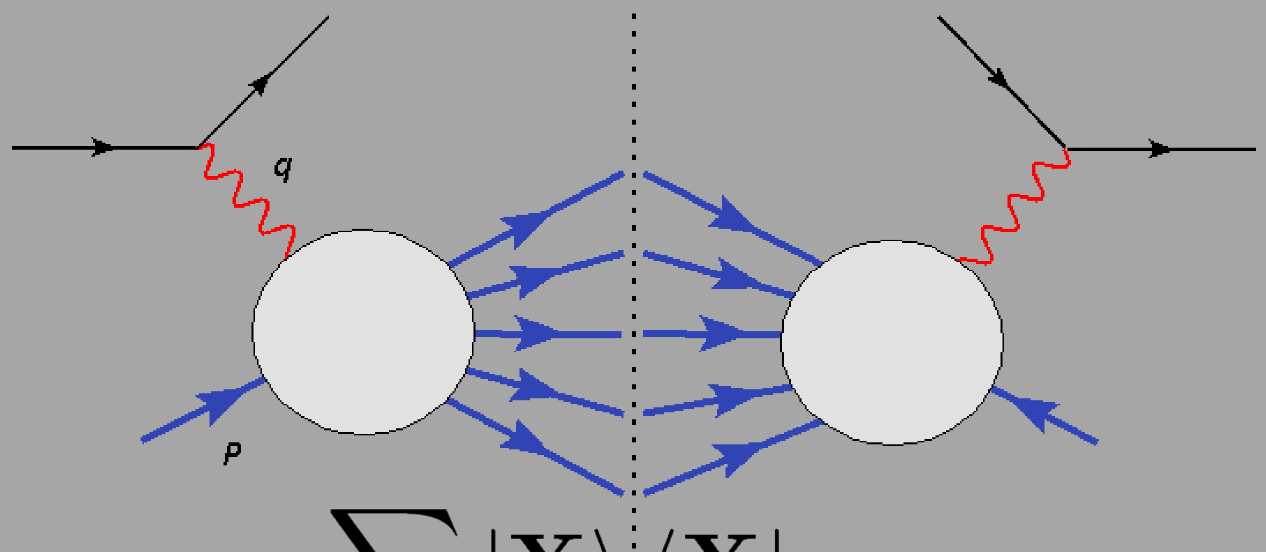
From Form factors to Distributions

Form factors are measured in elastic scattering



From Form factors to Distributions

Distributions are measured in deep inelastic scattering

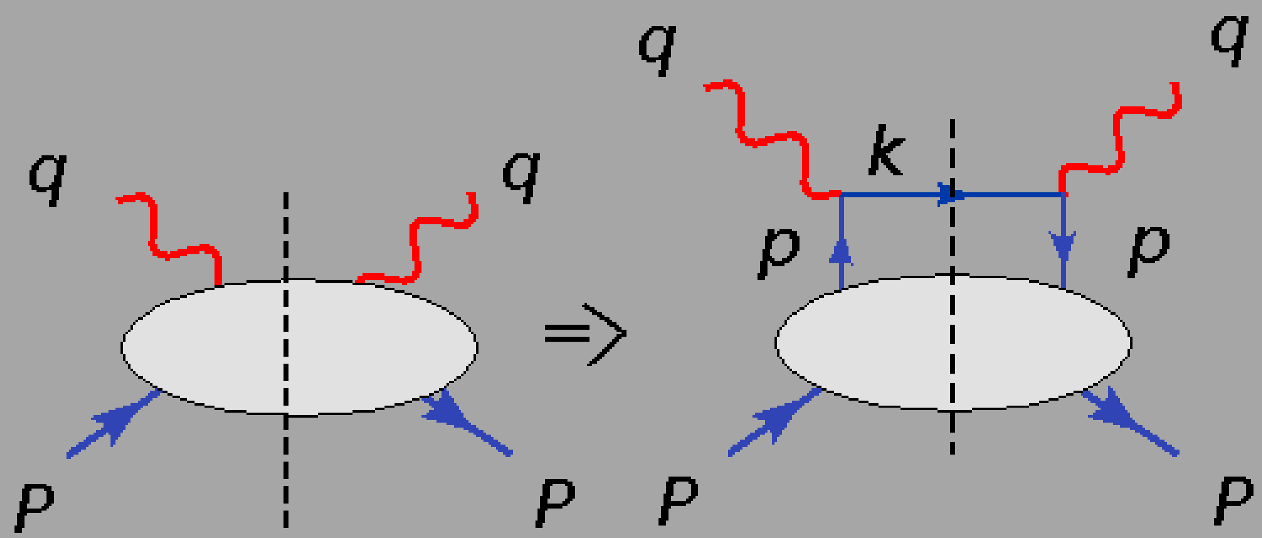


$$\sum_{\mathbf{X}} |\mathbf{X}\rangle\langle\mathbf{X}|$$

This sum makes it sensitive to parton structure!

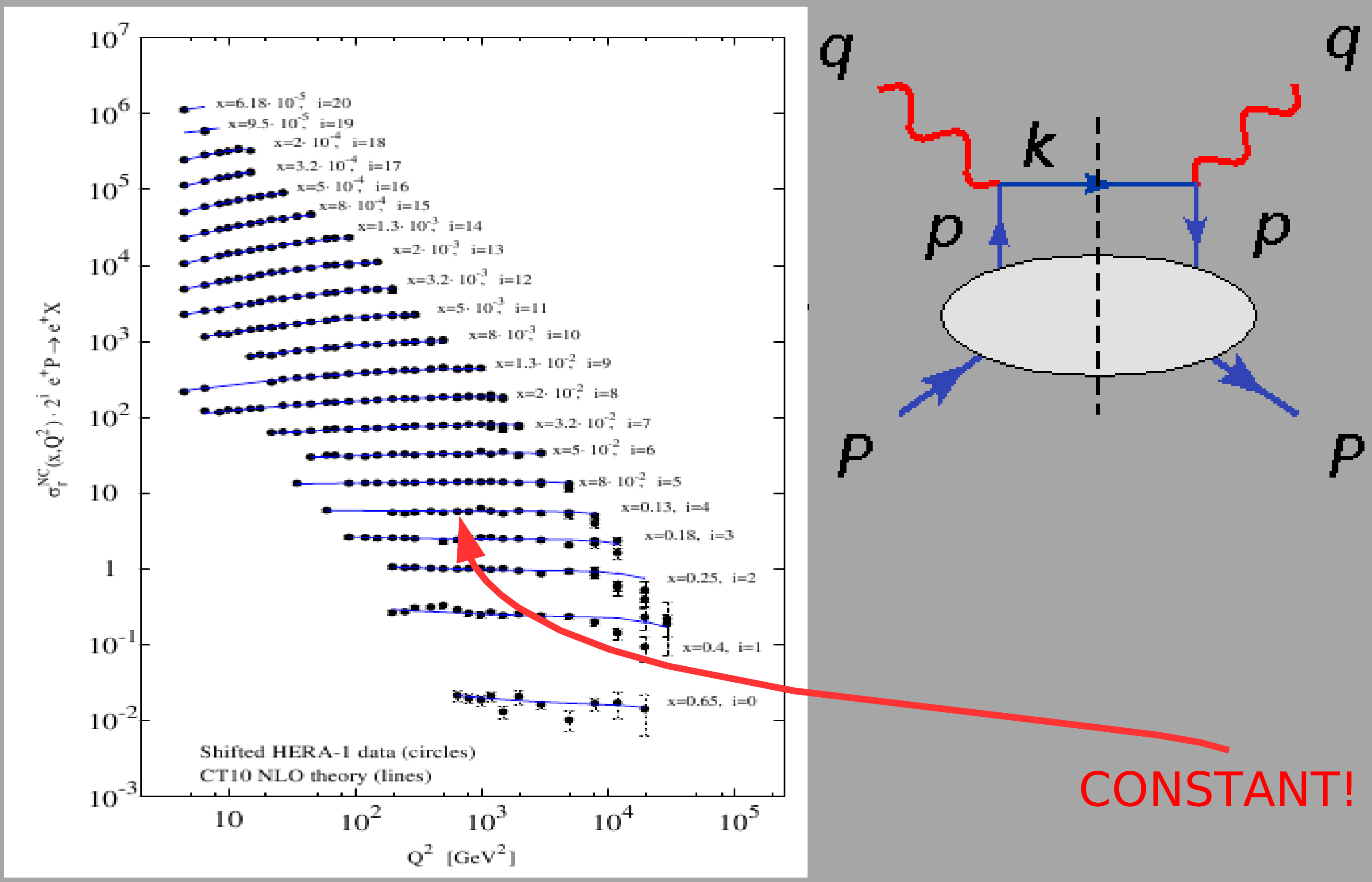
Distributions and parton model

Parton model is a logical step, we will see that partons are pointlike, so photon ineracts with them incoherently



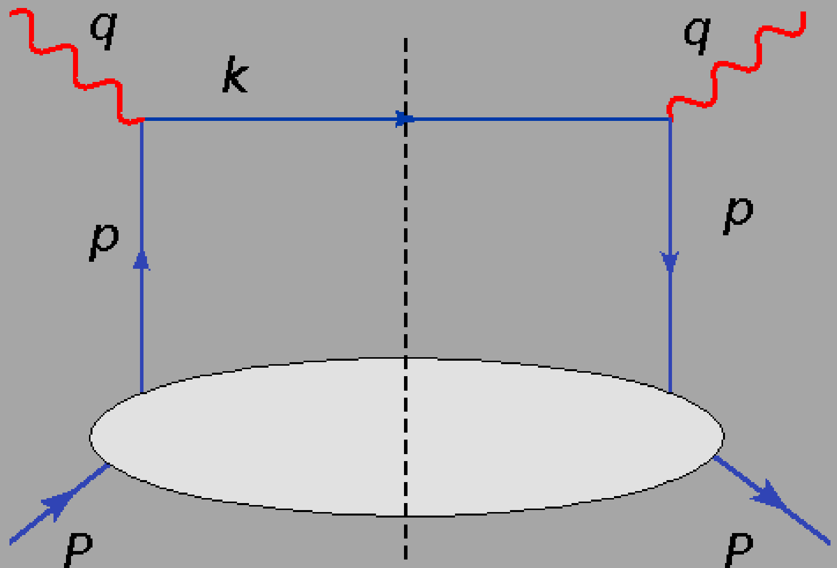
Distributions and parton model

Parton model is a logical step, we will see that partons are pointlike, so photon ineracts with them incoherently



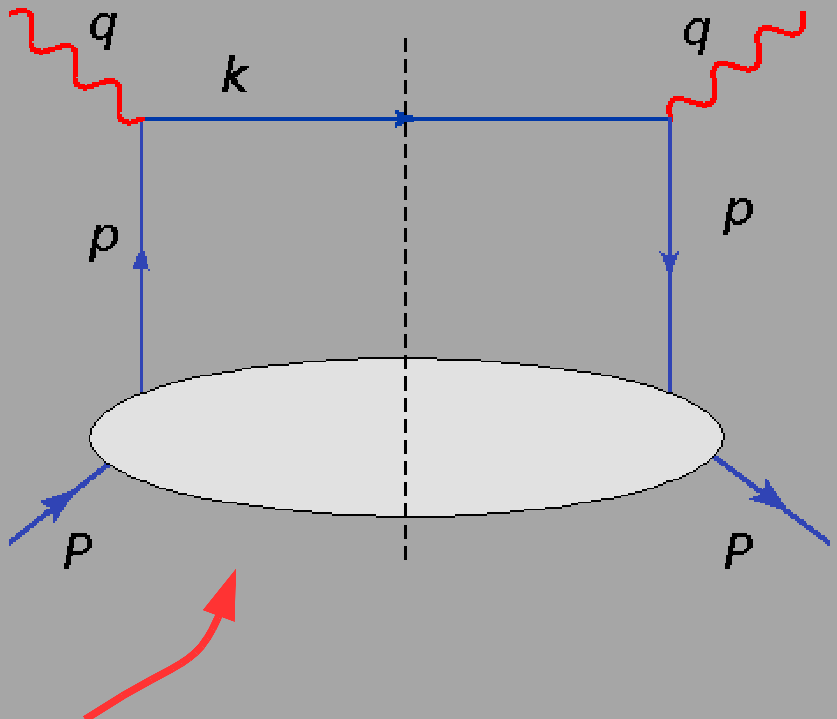
Distributions and parton model

This diagram is called “handbag diagram”



Distributions and parton model

This diagram is called “handbag diagram”

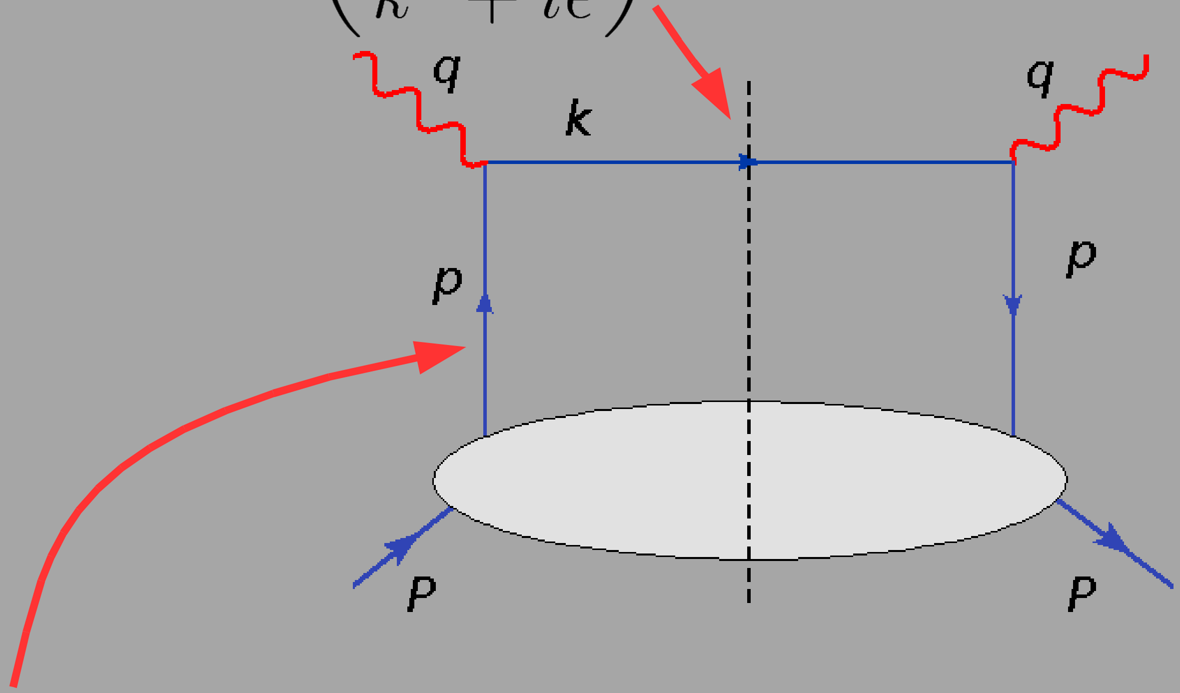


$\mathbf{\Phi(p, P)}$ - parton distribution

Distributions and parton model

Why quarks are on mass-shell?

$$Im\left(\frac{1}{k^2+i\epsilon}\right) = \pi\delta(k^2) \quad \Rightarrow \; k^2 \approx 0$$



This one is virtual! However the main contribution comes from

$$\int d^4p \left(\frac{1}{p^2+i\epsilon}\right)\left(\frac{1}{p^2-i\epsilon}\right) \quad \Rightarrow \; p^2 \approx 0$$

Distributions and parton model

What do we know about quark momentum? Suppose that proton is moving along Z direction with a high momentum, then

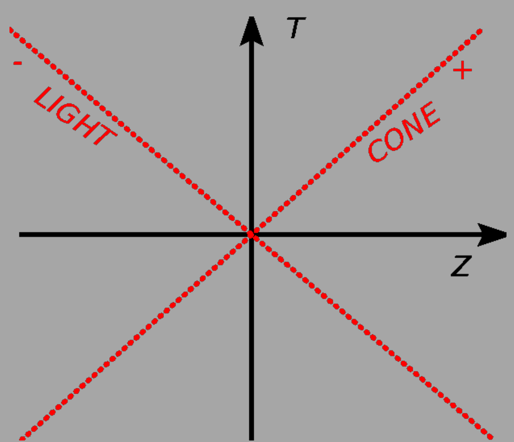
$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big" component $\sim Q$

$x = p^+/P^+$ is a new variable called lightcone momentum fraction

$$P^+ = \frac{1}{\sqrt{2}}\left(P^0 + P^z\right)$$

$$P^- = \frac{1}{\sqrt{2}}\left(P^0 - P^z\right)$$



Distributions and parton model

What do we know about quark momentum?

$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big" component $\sim Q$

"Small" component $\sim 1/Q$

Distributions and parton model

What do we know about quark momentum?

$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

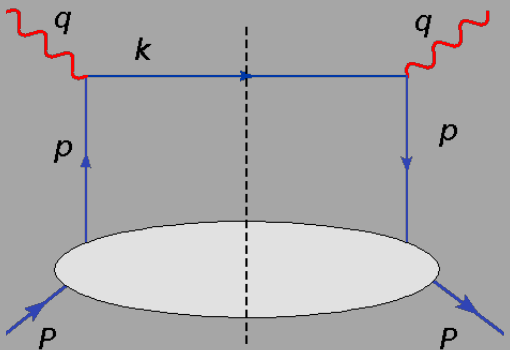
"Big" component $\sim Q$

"Small" component $\sim 1/Q$

"Transverse" component $\sim \Lambda_{QCD}$

Distributions and parton model

What do we know about hadronic tensor?



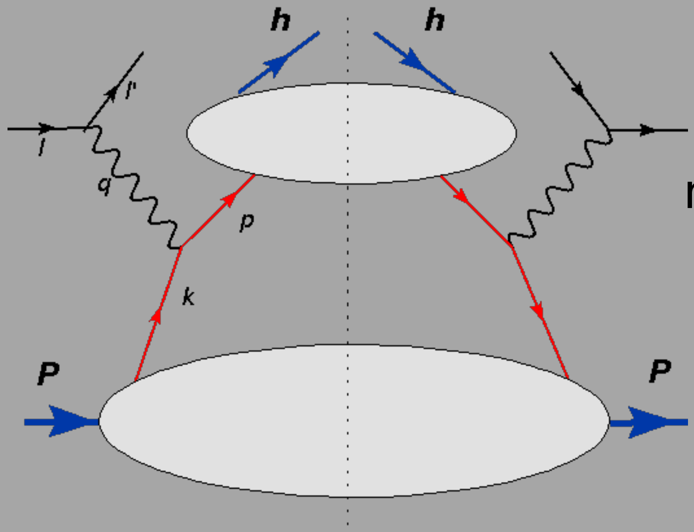
$$W^{\mu\nu} = \sum_q e_q^2 \int \frac{d^4p}{(2\pi)^4} Tr(\gamma^\mu(\not{p} + \not{q})\gamma^\nu \Phi(P,p))\delta((p+q)^2)$$

$$\delta((p+q)^2) \approx \delta(-Q^2 + 2xP\cdot q) = \frac{1}{2P\cdot q}\delta(x_{Bj} - x)\ ,$$

Quarks are “probed” at exactly value of x_{Bj}

How can we observe quark transverse momentum?

SIDIS

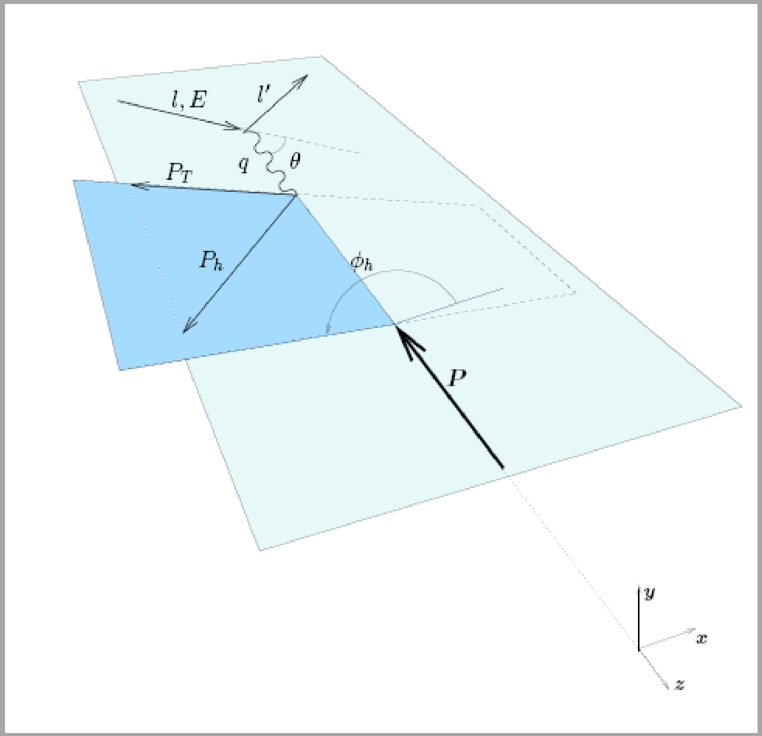


$$\mathbf{l} + \mathbf{P} \rightarrow \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

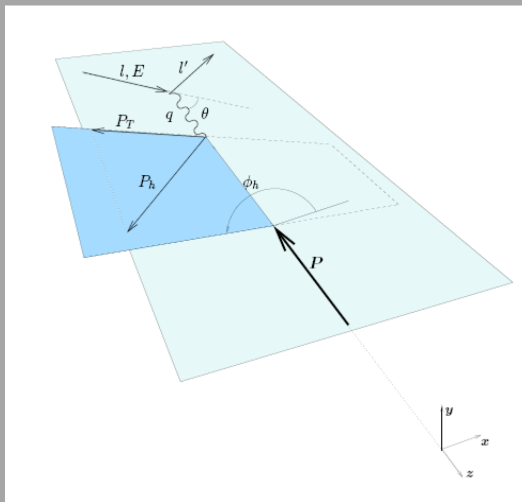
If produced hadron has transverse momentum

$$P_{hT} \sim \Lambda_{QCD}$$

it will be sensitive to quark transverse momentum $p_\perp$



$\mathbf{l} + \mathbf{P} \to \mathbf{l}' + \mathbf{h} + \mathbf{X}$ SIDIS: variables



$$d\sigma = \frac{1}{4P \cdot l} |M|^2 \frac{d^3 l'}{(2\pi)^3 2E'} \frac{d^3 P_h}{(2\pi)^3 2P_h}$$

6 variables

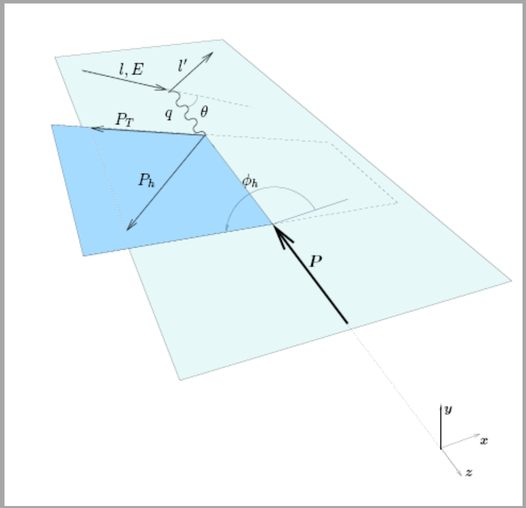
$$x_{Bj} \; Q^2 \; \mathbf{P}_{hT} \; z_h \; \Phi_S$$

Analogue of Bjorken x for fragmenting quark

Orientation of the spin of the proton

$$\mathbf{l} + \mathbf{P} \to \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

SIDIS: variables



$$d\sigma = \frac{1}{4P\cdot l}|M|^2 \frac{d^3 l'}{(2\pi)^3 2E'} \frac{d^3 P_h}{(2\pi)^3 2P_h}$$

$$s = (P+l)^2$$ cm energy

$$Q^2$$

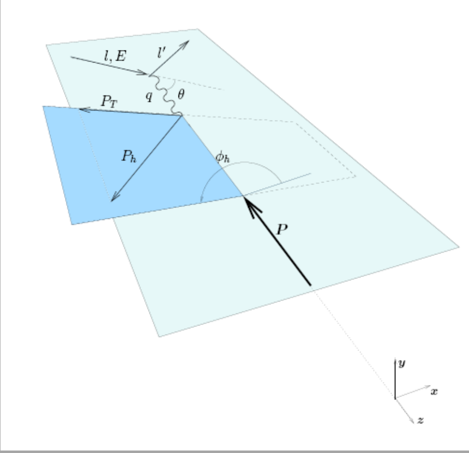
$$x_{Bj} = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot l}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q}$$

Relation (show it!)

$$Q^2 = sxy$$



SIDIS: fixed target vs collider

$$s = (P + l)^2$$ cm energy

Fixed target

$$\ell = (P_{lab}, 0, 0, -P_{lab})$$

$$P = (M_p, 0, 0, 0)$$

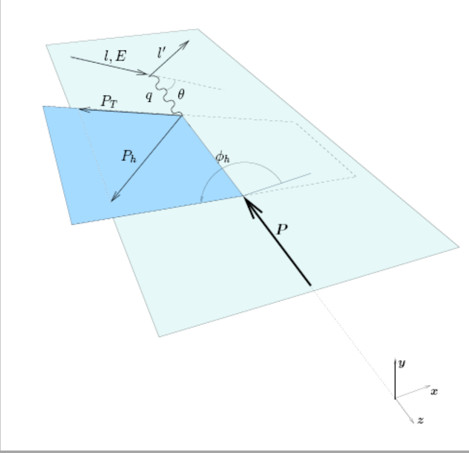
$$s = (P + l)^2 \simeq 2 M_p P_{lab}$$

Collider

$$\ell = (E_\ell, 0, 0, -E_\ell)$$

$$P = (E_P, 0, 0, E_P)$$

$$s = (P + l)^2 \simeq 4 E_P E_\ell$$



SIDIS: fixed target vs collider

$$s = (P + l)^2$$ cm energy

Fixed target

$$\ell = (P_{lab}, 0, 0, -P_{lab})$$

$$P = (M_p, 0, 0, 0)$$

$$s = (P + l)^2 \simeq 2M_p P_{lab}$$

Collider

$$\ell = (E_\ell, 0, 0, -E_\ell)$$

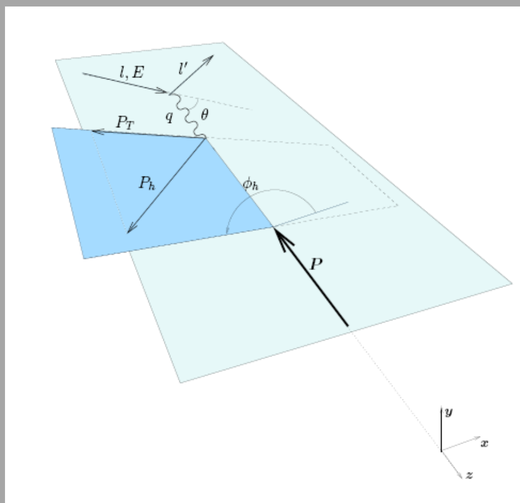
$$P = (E_P, 0, 0, E_P)$$

$$s = (P + l)^2 \simeq 4E_P E_\ell$$

Obvious advantage of a collider - energy can be much bigger

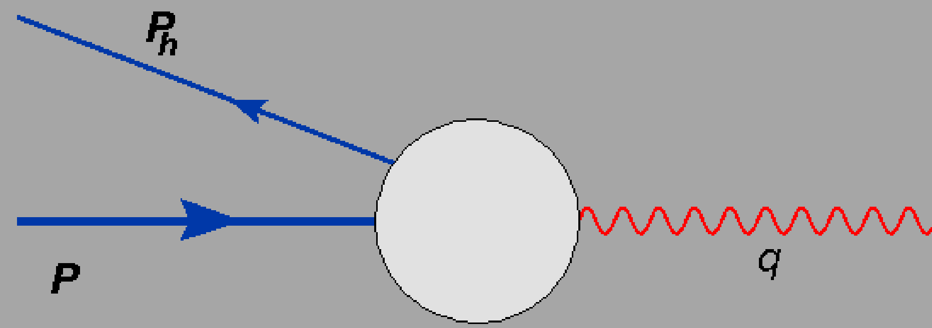
$$\mathbf{l + P \to l' + h + X}$$

SIDIS: frames

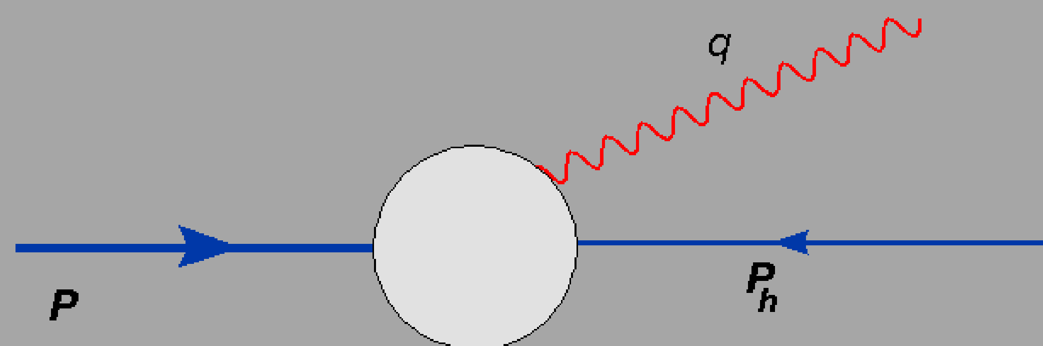


There are two convenient frames to study SIDIS

- $\gamma^* P$ frame



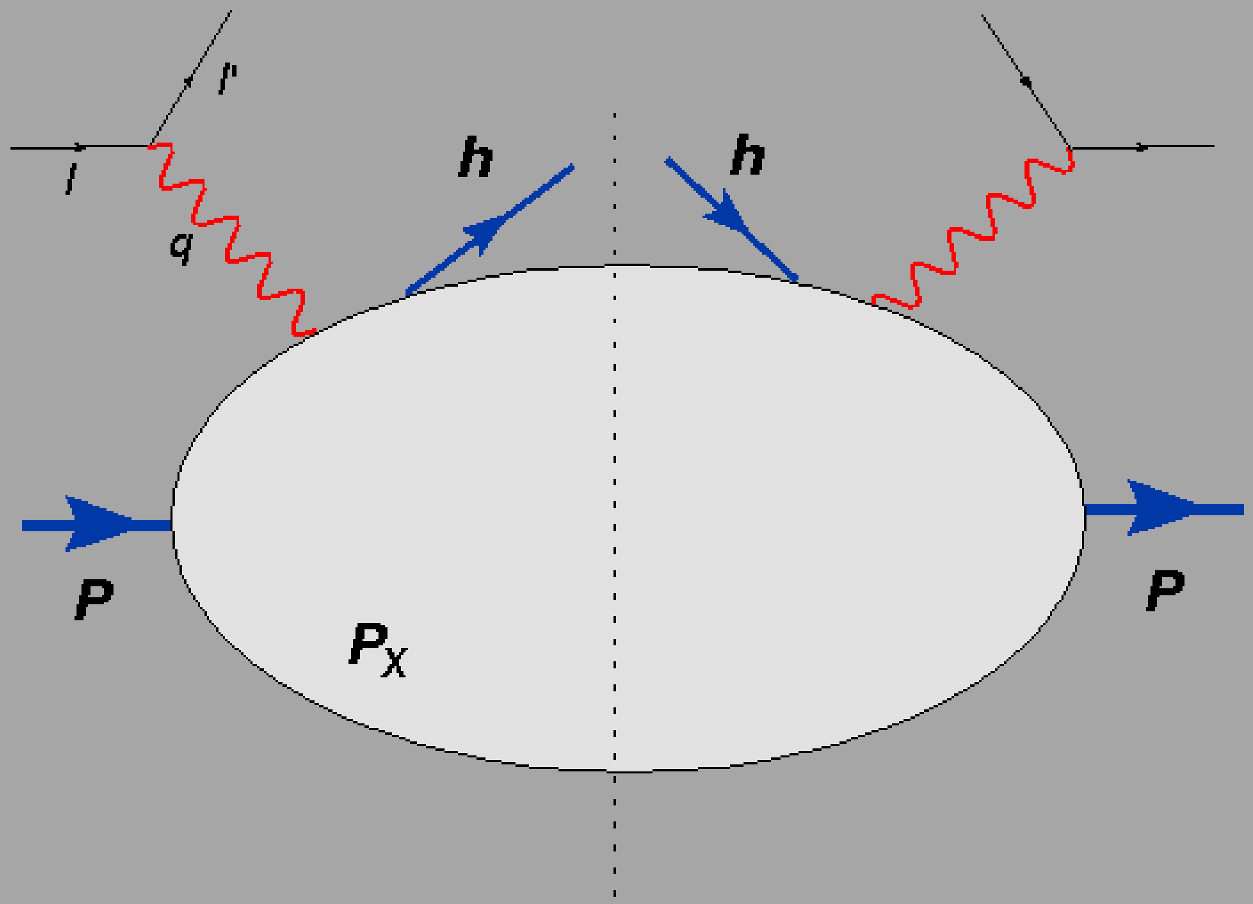
- $P_h P$ frame



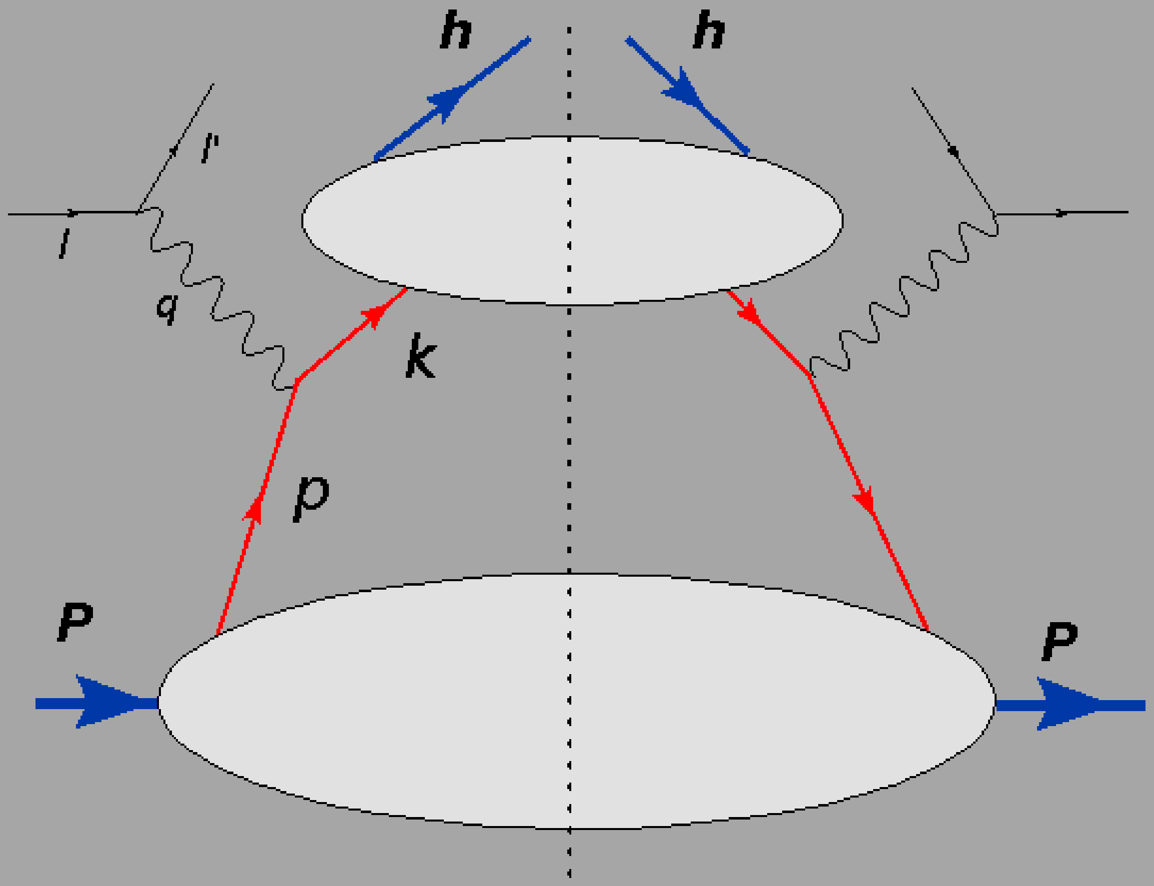
Relation:

$$\mathbf{q}_T \simeq -\frac{P_{hT}}{z_h}$$

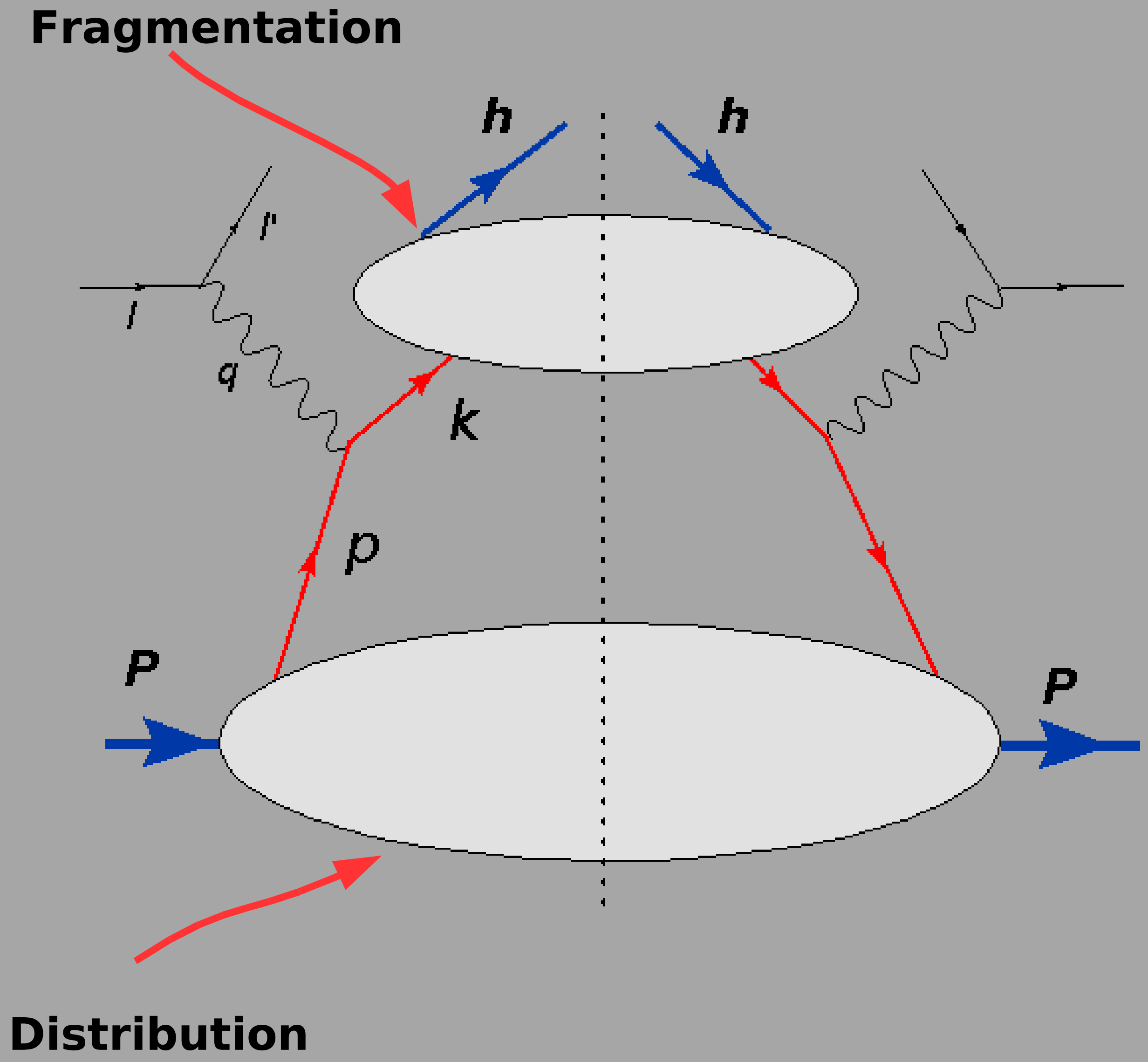
Experimentally we measure



Theoretically we assume



Theoretically we assume



$$\sigma_{\mathbf{SIDIS}} = \mathbf{D_{q/h}} \otimes \hat{\sigma}_{\mathbf{lq \to l'q'}} \otimes \mathbf{f_{q/P}}$$

TMDs

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading twist)