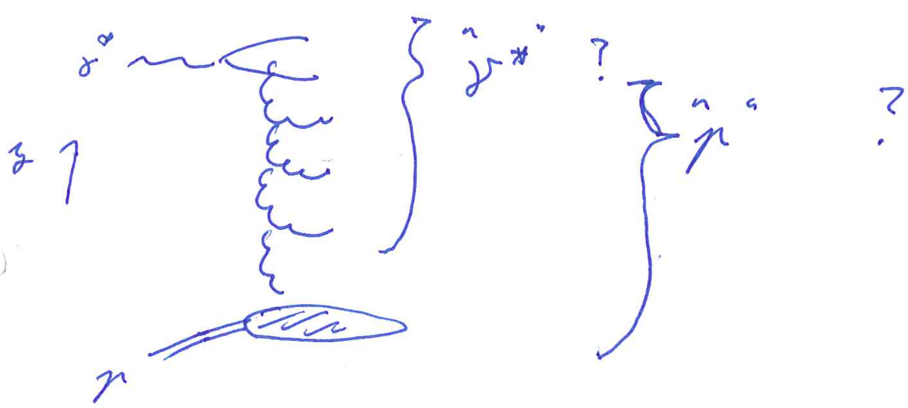
8-1 L13, 28.2.-11 $\mapsto$ 24.2.-11

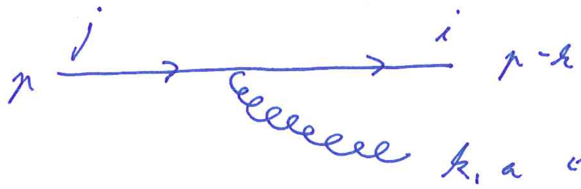
More on DIS in dipole picture

1. gluon radiation
2. DIS off classical color field

We argued that at small x, DIS can be seen as a $q\bar{q}$-dipole scattering eikonally off the target. Now at high energy there is a large rapidity separation between the dipole and the proton, so just like for quark-quark scattering there are higher order corrections that go like $\alpha_s \ln\frac{1}{x}$, $\alpha_s^2 \ln^2\frac{1}{x}$, ... to the process. They should then be resummed. One now has to distinguish between two descriptions: are these to be seen as corrections to the γ^* wavefunction or to the proton wavefunction?



Let us first try to understand them as corrections to the γ^* wavefunction, because that is something we can calculate. First we need the equivalent of an eikonal vertex in LC quantization. Then we will combine two of these to see what soft gluon radiation ($k_g^+ \ll k_{q\bar{q}}^+ = q_{\gamma^*}^+ \to$ MRK!) does to the dipole.



$\leftarrow \lambda = \pm 1$ Only physical transverse polarization, this is an on-shell external gluon

$k^+ = z p^+$

MRK: $z << 1$

$$\varepsilon^\mu = \left(0, \frac{\underset{\sim}{k} \cdot \underset{\sim}{\varepsilon}}{k^+}, \underset{\sim}{\varepsilon}\right)$$

$A^+ = 0$ - gauge $\quad$ $\underbrace{k^+}$ ~~large~~ small, this dominates over transverse part of ε^μ (The equivalent of an eikonal vertex)

$$\psi_{q\to qg}(\underset{\sim}{k}, z) = \sqrt{p^+} \frac{\bar{u}(p-k)}{\sqrt{(2\pi)^3 2(p^+-k^+)}} \frac{t^a_{ij}\, g\, \not{\varepsilon}(k)}{\sqrt{(2\pi)^3 2k^+}} \frac{u(p)}{\sqrt{(2\pi)^3 2p^+}} \underbrace{\frac{1}{p^- - \frac{\underset{\sim}{k}^2}{2k^+} - \frac{(\underset{\sim}{p}-\underset{\sim}{k})^2}{2(p^+-k^+)}}}$$

Using Pauli review hep-ph/0103106 $\qquad \approx \dfrac{-2k^+}{\underset{\sim}{k}^2}$, k^+ small

$$\bar{u}(p-k)\gamma^+ u(p) = \sqrt{2p^+\, 2(p^+-k^+)}\ \delta_{s,s'}$$

$\leftarrow$ eikonal interaction, conserves spin

$$\psi_{q\to qg}(\underset{\sim}{k}, z) = -\frac{\sqrt{2}\, g}{\sqrt{(2\pi)^3}}\, t^a_{ij} \frac{1}{\sqrt{z}} \frac{\underset{\sim}{k}\cdot\underset{\sim}{\varepsilon}}{\underset{\sim}{k}^2}\, \delta_{s,s'}$$

Note: probability to emit soft gluon:

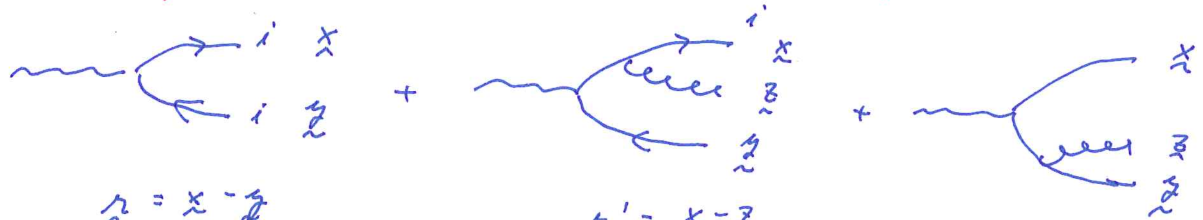
$$|\psi|^2 \sim \frac{dP}{dz\, d^2\underset{\sim}{k}} \sim \frac{1}{z}\frac{1}{\underset{\sim}{k}^2}$$

$\to$ typical gauge theory divergence (IR) $\frac{1}{z}$ "soft", $\frac{1}{\underset{\sim}{k}^2}$ "collinear"

In coordinate space, recall $\psi(\underset{\sim}{r}) = \int \frac{d^2 \underset{\sim}{k}}{\sqrt{(2\pi)^2}} e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}} \psi(\underset{\sim}{k})$

$$\psi_{q\to qg}(\underset{\sim}{r}, z) = -i \frac{\sqrt{2}\, g}{\sqrt{(2\pi)^3}}\, t^a_{ij} \frac{1}{\sqrt{z}} \frac{\underset{\sim}{r}\cdot\underset{\sim}{\varepsilon}}{\underset{\sim}{r}^2}\, \delta_{s,s'}$$

Now we use this result to get the contribution of radiating an extra gluon from the dipole:



$\mathbf{r} = \mathbf{x}-\mathbf{y}$ $\qquad$ $\mathbf{r}' = \mathbf{x}-\mathbf{z}$ $\qquad$ $\mathbf{z}-\mathbf{y} = \mathbf{r}-\mathbf{r}'$

To get the signs right in the following one probably needs to go via the k-space formulation:

$$|\gamma^*\rangle = |\gamma^*\rangle_0 + \int dz\, d^2\mathbf{r}\, \psi_{\gamma^*\to q\bar q}(z,\mathbf{r}) \frac{1}{\sqrt{N_c}} C(\mathbf{r})\, |q_i(\mathbf{x})\bar q_i(\mathbf{y})\rangle_0$$

$$+ \int dz\,dz' \int d^2\mathbf{r}\, d^2\mathbf{r}'\, \frac{1}{\sqrt{N_c}} \frac{-i\sqrt{2}\,g}{\sqrt{(2\pi)^3}} \frac{t^a_{ij}}{\sqrt{z'}}\, \psi_{\gamma^*\to q\bar q}(\mathbf{r},z)\left[\frac{\mathbf{r}'\cdot\boldsymbol\varepsilon}{(\mathbf{r}')^2} + \frac{(\mathbf{r}-\mathbf{r}')\cdot\boldsymbol\varepsilon}{(\mathbf{r}-\mathbf{r}')^2}\right]$$

$$\times\, |q_i(\mathbf{x})\,\bar q_j(\mathbf{y})\, g^a(\mathbf{z})\rangle$$

$\frac{1}{\sqrt{N_c}}$: so that $\langle\gamma^*|\gamma^*\rangle$ has $\frac{1}{N_c}\sum_{i=1}^{N_c} = 1$

We inserted a constant $C(\mathbf r) = 1 + O(\alpha_s)$ to keep the wavefunction normalized to the same thing it was without the g-radiation correction. Remember: the free states $|q\bar q\rangle_0$, $|q\bar q g\rangle_0$ are normalized to 1 (i.e. delta functions).

$$|C(\mathbf r)|^2 = 1 - \underbrace{\frac{2g}{(2\pi)^3}\frac{1}{N_c}\, t^a_{ij}\, t^a_{ji}}_{\frac{\alpha_s}{2\pi^2}\frac{N_c^2-1}{N_c}}\; \underbrace{\int\frac{dz'}{z'}}_{\Delta y}\; \int d^2\mathbf r'\; \underbrace{\sum_{\lambda=\pm1}\left|\left(\frac{\mathbf r'}{(\mathbf r')^2}+\frac{\mathbf r-\mathbf r'}{(\mathbf r-\mathbf r')^2}\right)\cdot\boldsymbol\varepsilon^{(\lambda)}\right|^2}_{=\left(\frac{\mathbf r'}{(\mathbf r')^2}+\frac{\mathbf r-\mathbf r'}{(\mathbf r-\mathbf r')^2}\right)^2 = \frac{\mathbf r^2}{(\mathbf r')^2(\mathbf r-\mathbf r')^2}}$$

$$= 1 - \frac{\alpha_s}{2\pi^2}\frac{N_c^2-1}{N_c}\,\Delta y\int d^2\mathbf r'\, \frac{\mathbf r^2}{(\mathbf r')^2(\mathbf r-\mathbf r')^2}$$ ← UV divergent, but we'll worry about that later → actually not a problem in the end

The cross section is (recall optical model) $\sigma = 2\int d^2\mathbf b\, N(\mathbf b,\mathbf r)$ and the correction to the scattering amplitude from the soft gluon is

$$N^{y+\Delta y}_{q\bar q}(\mathbf r) - N^{y}_{q\bar q}(\mathbf r) = \frac{\alpha_s}{2\pi^2}\frac{N_c^2-1}{N_c}\,\Delta y\int d^2\mathbf r'\, \frac{\mathbf r^2}{(\mathbf r')^2(\mathbf r-\mathbf r')^2}\Big[\overbrace{N_{q\bar q g}(\mathbf r,\mathbf r')}^{\text{real}} - \overbrace{N_{q\bar q}(\mathbf r)}^{\text{virtual}}\Big]$$

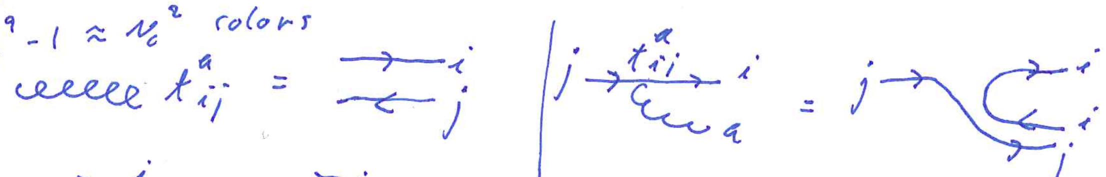
{ Real = radiate extra gluon
Virtual = correct wavefunction normalization to account for extra contribution

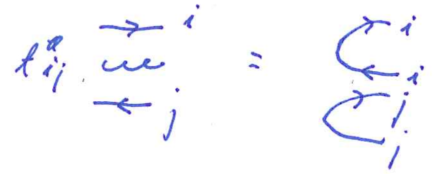
8-4 26.2.-11

So far we have not assumed anything about how the dipole scatters off the target. We will do so shortly, but first we discuss a general relation between dipoles and the gluons:

large N_c => gluon = $q\bar q$ pair $\left(t^a_{ij} t^a_{ji} = \frac{N_c^2-1}{2} \approx \frac{1}{2}\delta_{ii}\delta_{jj} = \frac{N_c^2}{2} \right)$

$N_c^2 - 1 \approx N_c^2$ colors





$\rightarrow$ i.e. $q\bar q g$ = 2 $q\bar q$-dipoles

$$t^a_{ij}\, |q_i(\underset{\sim}{x})\bar q_j(\underset{\sim}{y}) g^a(\underset{\sim}{z})\rangle \approx |q_i(\underset{\sim}{x})\bar q_i(\underset{\sim}{z})\, q_j(\underset{\sim}{z})\bar q_j(\underset{\sim}{y})\rangle$$

This makes it possible to relate the (eikonal) scattering of a $q\bar q g$-system to the dipole scattering amplitude.

$$S = 1 - N = P(\text{no scatter})$$

$$\Rightarrow S_{q\bar q g}(\underset{\sim}{x},\underset{\sim}{y},\underset{\sim}{z}) \underset{N_c\to\infty}{\approx} S_{q\bar q}(\underset{\sim}{x}-\underset{\sim}{z})\, S_{q\bar q}(\underset{\sim}{y}-\underset{\sim}{z})$$ (no scattering = neither dipole scatters)

$$\Rightarrow N_{q\bar q g}(\underset{\sim}{r}', \underset{\sim}{r}-\underset{\sim}{r}') \underset{N_c\to\infty}{\approx} \underbrace{N_{q\bar q}(\underset{\sim}{r}')}_{\text{dipole 1 scatters}} + \underbrace{N_{q\bar q}(\underset{\sim}{r}-\underset{\sim}{r}')}_{\text{dipole 2 scatters}} - \underbrace{N_{q\bar q}(\underset{\sim}{r}')N_{q\bar q}(\underset{\sim}{r}-\underset{\sim}{r}')}_{\text{both scatter, remove double counting}}$$

$\frac{N_c^2-1}{N_c} \approx N_c$

$$\boxed{\Rightarrow \frac{d}{dy} N^y_{q\bar q}(\underset{\sim}{r}) = \frac{\alpha_s N_c}{2\pi^2} \int d^2\underset{\sim}{r}' \frac{\underset{\sim}{r}^2}{(\underset{\sim}{r}')^2(\underset{\sim}{r}-\underset{\sim}{r}')^2}\left[\overbrace{N_{q\bar q}(\underset{\sim}{r}') + N_{q\bar q}(\underset{\sim}{r}-\underset{\sim}{r}') - N_{q\bar q}(\underset{\sim}{r}')N_{q\bar q}(\underset{\sim}{r}-\underset{\sim}{r}')}^{\text{real}} \overbrace{- N_{q\bar q}(\underset{\sim}{r})}^{\text{virtual}}\right]}$$

BK equation, 1995 Balitsky - Kovchegov

This is the <u>holy grail</u> of small x physics, simple and powerful. Some remarks:

- drop nonlinear term $\rightarrow$ get BFKL
- divergence at $\underset{\sim}{r}' \to 0$ canceled by $N_{q\bar q}(\underset{\sim}{r}) \to 0$, $\underset{\sim}{r} \to 0$, color neutrality
- enforces $N \leq 1$, BFKL does not
- can be seen as an evolution of dipole, target or amplitude, depending on frame / point of view; $N \sim$ unintegrated gluon distribution of target.

8-5 | 28.2.-11

DIS off classical color field

$$\sigma^{\gamma^* p} \sim \begin{Bmatrix} \frac{1}{x}^{0.3} & (\text{pheno}) \\ \frac{1}{x}^{\frac{\alpha_s N_c}{\pi} 4 \ln 2} & (\text{BFKL}) \end{Bmatrix} \sim x g(x, Q^2) \sim x^{-\lambda}$$

The scattering cross section grows at small x, ~~and~~ it is proportional to the gluon density (σ_L explicitly, σ_T after some argumentation: sea quark distribution $\sim \alpha_s \times$ gluon distribution).

Thus occupation numbers of gluonic states in the target wavefunction become large. This is the regime where a classical approximation to QM is valid. This idea is at the heart of what is known as the Color Glass Condensate (the exact definition of CGC means different things to different people).

CGC: small x gluons = classical color field

Now, what does DIS off such ~~an~~ a field look like? Consider an incoming quark with large p^+. From the eikonal vertex, or from the ε_μ-vectors in $\frac{1}{\varepsilon \cdot k}$ soft gluon radiation we remember that it couples most strongly to the A^--component of the color field. To understand the scattering we solve the Dirac equation

$$(i\not{\partial} - g\not{A})\psi = 0, \quad \text{eikonal approx } \psi = \chi(x) e^{-ip\cdot x} u(p)$$

$$\not{A} = \gamma^+ A^- = \gamma^+ A^-_a t_a \Rightarrow \partial_+ \chi = -igA^- \chi$$

$$\chi(x^+, x^-, \underset{\sim}{x}) = P e^{-ig\int_{-\infty}^{x^+} dy^+ A^-(y^+, x^-, \underset{\sim}{x})}$$

x^- - dependence negligible compared to p^+x^-

Wilson line

P: path ordering = exponential of a matrix is defined as a power series $e^M = 1 + M + \frac{1}{2}M^2 + \ldots$. In this power series one should always ~~[illegible]~~ order the terms with the largest x^+ on the left (= path order) Note: conventions differ!

Recall the (nonrelativistic) eikonal approximation from the early part of this course. This case is fully analogous. Now for a quark

$$|q\, i\rangle_{out} = U_{ij}(\underline{x})\, |q\, j\rangle_{in} \qquad U(\underline{x}) = P e^{-ig\int_{-\infty}^{\infty} dx^+ A^-(\underline{x}, x^+)}$$

and for a dipole

$$|q(\underline{x})_i\; \bar{q}(\underline{y})_k\rangle_{out} = U_{ij}(\underline{x})\, U^\dagger_{jk}(\underline{y})\, |q(\underline{x});\, \bar{q}(\underline{y})_{jk}\rangle$$

$\frac{1}{N_c}\delta_{ik}$ projects out only color neutral dipoles out

(For the optical theorem we need the final state to be in the same color state as the initial one, this is the elastic forward scattering amplitude)

$$N_{q\bar{q}}(\underline{x} - \underline{y}) = 1 - \frac{1}{N_c}\left\langle \mathrm{tr}\, U(\underline{x})\, U^\dagger(\underline{y})\right\rangle_{target}$$

There needs to be some averaging over the target wavefunction. In the CGC this is a probabilistic average.

Note: $U(x)$ is a unitary matrix.

Identically $\frac{1}{N_c}\mathrm{Re}\,\mathrm{tr}\, UU^\dagger \leq 1 \rightarrow N \geq 0$

BD limit $N=1$ is $\mathrm{tr}\, UU^\dagger = 0$; this is what one gets if matrices $U(\underline{x})$ and $U(\underline{y})$ are fully uncorrelated and evenly distributed on the SU(3) group manifold.