On assessment of the Bering-Chukchi-Beaufort Seas stock of bowhead whales (*Balaena mysticetus*) using a Bayesian approach

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ABSTRACT

This paper explores a number of issues surrounding the current assessment of the Bering-Chukchi-Beaufort (B-C-B) Seas stock of bowhead whales and provides a 'preferred' set of specifications for this assessment. A Bayesian approach appears to be preferable. However, the Bayesian Synthesis method is subject to the Borel paradox. Reverting to a 'standard' Bayesian approach which places all 'indirect' information in priors (rather than representing this information as likelihoods) would overcome this problem. The basis for the prior distributions used should be documented clearly, and the sources of information for the B-C-B bowhead stock divided into 'indirect' and 'direct'. Simulation results and 'in principle' arguments support the choice of a current population size rather than the pre-exploitation equilibrium size for the parameter to scale the population size (i.e. a 'backwards' rather than a 'forwards' approach). Arguments are presented that the most appropriate choice for a productivity-related parameter, for which a prior has to be specified, is the maximum steady rate of increase. A method for treating the N_4/P_4 estimates as relative indices of abundance, allowing for prior information about the relationship between absolute abundance and those estimates, and accounting for the correlation among the indices of relative abundance derived from the N_4 and P_4 data is developed. Two 'preferred approaches' for assessing the resource both lead to estimates for the lower 5th percentile of the replacement yield that are greater than the current annual strike limit of 67 for the B-C-B stock.

KEYWORDS: BOWHEAD WHALE; POPN ASSESSMENT; TRENDS; BIOLOGICAL PARAMETERS; MODELLING; WHALING-ABORIGINAL; ARCTIC

INTRODUCTION

Bowhead whales *(Balaena mysticetus)* of the Bering-Chukchi-Beaufort Seas (B-C-B) stock are subject to subsistence whaling in Alaska and Chukotka. Thus the assessment of this stock is important for providing management advice to the International Whaling Commission (IWC), the intergovernmental body that establishes catch limits. The present regulations state that the total number of landed whales for seasons 1998-2002 shall not exceed 280, with no more than 67 struck in anyone year (IWC, 1999).

Recent assessments of this stock have been conducted using both conditioned maximum likelihood (e.g. Butterworth and Punt, 1992; 1995; Punt and Butterworth, 1996; 1997a) and Bayesian methods (e.g. Givens *et al.*, 1995; Givens and Thompson, 1996). The Bayesian assessments have been based on Bayesian Synthesis (e.g. Raftery *et al.*, 1995a) and standard Bayesian methods (e.g. Punt and Butterworth, 1997a; Breiwick, 1997). These Bayesian analyses involve the development of a coherent joint posterior distribution for seven population model parameters: the total (1+) pre-exploitation size of the resource, K_{1+}; *MSYR; MSYL;* the age-at-maturity, a_m; the survival rate of adults in the absence of exploitation, $S_{adult} = \exp(-M_{adult})$; the survival rate of juveniles in the absence of exploitation, $S_{juv} = \exp(-M_{juv})$; and the greatest age at which juvenile natural mortality applies, *a*. The assessment conducted by the IWC Scientific Committee (hereafter 'Scientific Committee') at its 1994 meeting (IWC, 1995) used pre-model distributions [1] for each of these parameters, as well as pre-model distributions for the recent rate of population increase *(ROI),* the 1988 (1+) population size *(P1988),* the maximum pregnancy rate $(f^{max})^2$, and the proportion of mature animals and calves in the population from 1985 to 1992.

There are three main reasons for using a Bayesian approach for stock assessment: (a) it provides a relatively straightforward means to represent the full range of uncertainty (both parameter uncertainty and model-structure uncertainty); (b) information based on 'expert opinion' and inferences about other stocks/species can be incorporated explicitly into the stock assessment within a statistically defensible framework; and (c) the output of the analysis is exactly the information needed to parameterise operating models for evaluating alternative candidate management procedures (viz. the probability of alternative states of nature). Thus, unlike the situation for maximum likelihood approaches, it is not necessary to argue that the joint distribution obtained for parameter estimates can be assumed to represent these probabilities, because it is exactly these probabilities which a Bayesian approach provides.

The principles underlying Bayesian Synthesis have been criticised as this method is subject to the Borel paradox (Wolpert, 1995; Bravington, 1996). Put simply, the Borel paradox arises because there are (through the relationships provided by the population dynamics model) two different prior distributions for the same quantity (Raftery and Givens, 1997). Concern has also been expressed within the Scientific Committee about some of the prior distributions selected for the 1994 assessment (IWC, 1995) - see

[1] The term 'pre-model distribution' will be reserved in this paper for references to Bayesian Synthesis applications. The more common terms 'prior' and 'likelihood' will be used when discussing issues related to standard Bayesian assessments.

[2] The term 'pregnancy rate' refers to the fraction of females past the age-at-first-parturition that give birth in a year (Punt, 1996; 1999). This definition differs from usage in some other earlier papers (e.g. de la Mare, 1989; Punt and Butterworth, 1991) in that it applies to births of both sexes rather than to females only.

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Butterworth (1995), Punt and Butterworth (1996; 1997a), and discussion of points raised therein in the section 'Priors and Likelihoods' below.

The information used in a Bayesian assessment can be obtained from both 'direct' and 'indirect' sources (Bravington, 1996). 'Direct' information means observations from the population being assessed (e.g. the 'proportion' data for the B-C-B bowhead stock). 'Indirect' information involves inferences that do not depend on the population being assessed (e.g. inferences concerning natural mortality from estimates for other baleen whale species and stocks). The difference between these two sources of information is critical to an understanding of Bayesian stock assessments, as the two need to be treated quite differently within such analyses.

The parameters for which prior (pre-model) distributions were specified in the 1994 B-C-B bowhead assessments (IWC, 1995) are conventional inputs for HITTER-FITTER (de la Mare, 1989) with its underlying BALEEN II model (Punt, 1996; 1999). However, when specifying prior distributions, it is often better to select 'natural parameterisations'. The choice of parameterisation should be made to ease the specification of the priors. Some choices for parameters are simpler for scientists to relate to practical experience, and should therefore be preferred. For example, we argue later that specifying a prior for the average number of years an animal lives after reaching maturity may be more 'natural' than doing this for an adult natural mortality rate.

Punt and Hilborn (1997) advocate using parameters that do not depend on a separate parameter that scales the population size. This is because such parameters are then comparable among stocks/species, making it considerably easier to construct priors on the basis of inferences for other stocks/species. This practice also means that the biological parameters are independent of the parameter that scales the population size. The parameters chosen for the 1994 B-C-B bowhead stock assessment (IWC, 1995) confonn to this suggestion.

In several instances, the use of an 'uninformative' prior is advocated. This is perhaps somewhat misleading (and perhaps even unhelpful) because it is not always clear to what extent a particular prior is uninformative. For example, the selection of a uniform prior for *MSYR* may be 'uninformative' with respect to *MSYR,* but it will certainly not be uninformative with respect to the current replacement yield (a possibly more important quantity from the management viewpoint). In most cases where we advocate that a prior be chosen to be uninformative, we suggest that it should be uniform. However, several alternatives exist (e.g. uniform on a log-scale). The selection of a metric for uninformative priors can have a substantial impact on assessment results. There is therefore a need for the metric to be explicitly considered when specifying uninformative priors. For example, if comparing the options of uniform on the given or a log-scale, the key question to be addressed is on which of the two scales do intervals of equal length correspond to equally likely ranges of possible values.

This paper first considers the appropriate framework (Bayesian Synthesis or 'standard' Bayesian) for conducting the assessment of the B-C-B bowhead stock. It then compares the 'backwards' and 'forwards' approaches[3] by means of simulation and considers the information about each model parameter and data type in turn to suggest how these should be treated in a Bayesian assessment of the B-C-B bowhead stock. Finally, the results for two 'preferred' variants are presented and discussed.

[3] The 'backwards' approach effectively projects population trajectories backwards from a population estimate generated from a prior for a population size in a recent year, whereas the 'forwards' methodology generates a population size for the year in which exploitation started from a prior for the pre-exploitation equilibrium population *K,* and projects this forwards in time by means of the population model.

BA YES IAN SYNTHESIS OR BAYESIAN ANALYSIS

Bayesian stock assessment and risk analysis methods have been applied in the fisheries field for several years (e.g. Walters and Hilborn, 1976; Bergh and Butterworth, 1987; Sainsbury, 1988; Collie and Walters, 1991; Thompson, 1992; Hilborn *et al.,* 1994; McAllister *et al.,* 1994; Walters and Ludwig, 1994; Walters and Punt, 1994). The assessment method applied to the South African fur seal population by Butterworth *et al.* (1987) and more recently by Givens *et al.* (1993; 1995) for the B-C-B bowhead stock differs from other Bayesian assessments because it is based on a Bayesian Synthesis approach rather than a standard Bayesian analysis. As such, these assessments are subject to the Borel paradox (Wolpert, 1995).

Bravington's (1996) appraisal of Bayesian Synthesis highlights the Borel paradox and suggests that sensitivity to this paradox can be explored through relabelling of model inputs and outputs. This suggestion is both sensible and adequate but it is required only if the assessment has been provided with more priors than are actually needed. (As detailed in the following section, one of the two sources of the Borel paradox in the 1994 B-C-B assessment was removed by the Scientific Committee's decision in 1997 not to include a prior on S_{juv} (IWC, 1998c).)

Bayesian analysis deals with priors and likelihoods in different ways. However, the 1994 B-C-B bowhead assessment treats some priors (e.g. that for the maximum pregnancy rate) as likelihoods. This practice is dangerous and can readily be shown to lead to erroneous results (e.g. Bravington, 1996). Raftery and Poole (1997) and Poole and Raftery (1998) provide suggestions on how to combine priors in a manner that overcomes this problem. However, for the B-C-B stock of bowhead whales, the most obvious solution to this problem is to place all of the 'indirect' information into the prior distributions and to represent all of the data for the B-C-B bowhead stock in the fonn of a likelihood function. In this situation (which we will refer to as a 'standard' Bayesian assessment), the Borel paradox is not a concern provided the joint prior is of the same dimension as the parameter vector. Naturally, one cannot use a 'standard' Bayesian assessment if there really is 'indirect' information about both model inputs and outputs. However, we will argue below that the basis for some of the priors used in the 1994 B-C-B bowhead assessment is so weak that it is perhaps better to ignore certain of these priors and thus be able to take advantage of adopting a 'standard' Bayesian approach.

THE 'REFERENCE' ANALYSIS

In 1997, the Scientific Committee specified a 'reference case' for comparing alternative approaches to the assessment of the B-C-B bowhead stock (IWC, 1998b and see Tables 1 and 2). The 1994 B-C-B bowhead assessment (IWC, 1995) incorporated priors for $MSYR_{mat}$, $MSYL_{mat}$, a_m, a, S_{adult}, S_{juv} and f_{max}. However, given values for any six of these seven parameters, the value for the seventh can be derived from the BALEEN II population dynamics model (Punt, 1999). The use of all seven priors therefore leads to an instance of the Borel paradox. The specifications of the 'reference case' resolve this problem as no prior is placed on S_{juv} and instead the values for the parameters S_{adult}, a_m, a, *MSYR*, *MSYL* and f_{max} and the relationships within BALEEN II are used to compute a value for S_{juv}. For ease of presentation, the analyses presented in this paper are all variants of this 'reference case'. The results of the assessments are summarised by eight management-related quantities, the first seven of which were identified by IWC (1998c).

Table 1

Historical catches for the Bering-Chukchi-Beaufort Seas stock of bowhead whales (source: J.M. Breiwick, pers. comm.).

Year	Catch	Year	Catch	Year	Catch	Year	Catch
1848	18	1886	168	1924	41	1962	20
1849	573	1887	240	1925	53	1963	15
1850	2,067	1888	160	1926	35	1964	24
1851	898	1889	127	1927	14	1965	14
1852	2,709	1890	136	1928	30	1966	24
1853	807	1891	284	1929	30	1967	12
1854	166	1892	346	1930	17	1968	27
1855	2	1893	180	1931	32	1969	32
1856	0	1894	234	1932	27	1970	48
1857	78	1895	117	1933	21	1971	25
1858	461	1896	118	1934	21	1972	44
1859	372	1897	130	1935	15	1973	51
1860	221	1898	309	1936	24	1974	42
1861	306	1899	234	1937	53	1975	32
1862	157	1900	148	1938	36	1976	74
1863	303	1901	55	1939	18	1977	72
1864	434	1902	162	1940	20	1978	17
1865	590	1903	116	1941	38	1979	23
1866	554	1904	86	1942	26	1980	38
1867	599	1905	105	1943	14	1981	26
1868	516	1906	69	1944	8	1982	14
1869	382	1907	96	1945	23	1983	16
1870	637	1908	123	1946	20	1984	16
1871	138	1909	61	1947	21	1985	14
1872	200	1910	37	1948	8	1986	22
1873	147	1911	48	1949	11	1987	29
1874	95	1912	39	1950	23	1988	28
1875	200	1913	23	1951	23	1989	25
1876	76	1914	61	1952	11	1990	41
1877	270	1915	23	1953	41	1991	47
1878	80	1916	23	1954	9	1992	46
1879	266	1917	35	1955	36	1993	51
1880	480	1918	27	1956	11	1994	38
1881	435	1919	33	1957	5	1995	57
1882	242	1920	33	1958	5	1996	45
1883	42	1921	9	1959	2	1997	62
1884	160	1922	39	1960	33		
1885	377	1923	12	1961	17		

Table 2

The prior distributions and data assumed when conducting the 'reference case' assessment of the Bering-Chukchi-Beaufort Seas stock of bowhead whales (IWC, 1998c) - see text for definition of the parameters listed. (t_n is a t random variable with n degrees of freedom.)

Parameter	Prior distribution/likelihood
Prior	
S_{adult}	$N(0.99; 0.02^2)$ $S_{juv} \le S_{adult} \le 0.995$
Maximum pregnancy rate, f_{max}	$1/f_{max} \sim U[2.5; 4]$
Transition age, a	1,2, … , 9 equally likely
Age-at-maturity, a_m	$N(20;3^2)$ $13.5 \le a_m \le 26.5$
K_{1+}	$\ell n K_{1+} \sim U[\ell n 7000, \ell n 31000]$
$MSYL_{1+}$	U[0.4; 0.8]
$MSYR_{1+}$	U[0.01; 0.07]
Data source	
1993 population size, P_{1993}	$N(8200, 564^2)$
1978-93 *ROI*[1]	$\exp(0.0319+0.0076t_8) - 1$
Propn of calves[2] 1985-92	$0.052 + 0.0164t_5$
Propn of matures[2] 1985-92	$0.411 + 0.0286t_5$

[1] J.E. Zeh (pers. comm.). [2] Givens *et al.* (1995).

- K_{1+} - the pre-exploitation size of the 1+ component of the population.
- P^{1+}_{1998}/K_{1+} - the ratio (expressed as percentage) of the size of the 1+ component of the population at the start of 1998 to K_{1+}.
- P^f_{1998}/K^f - the ratio (expressed as percentage) of the size of the mature female component of the population at the start of 1998 to the corresponding pre-exploitation size.
- $P^{1+}_{1998}/MSYL_{1+}$ - the ratio (expressed as percentage) of the size of the 1+ component of the population at the start of 1998 to *MSYL*.
- $MSYR_{1+}$ - *MSYR* for uniform selectivity harvesting of the 1+ component of the population, expressed as a percentage.
- *RY* (1998) - the replacement yield for 1998.
- Q_0 (1998) - the value of the quantity Q_0 (Wade and Givens, 1997) for 1998:

$$Q_0(1998) = \begin{cases} 0.9\,MSY_{1+} & \text{if } P^{1+}_{1998} / K_{1+} > MSYL_{1+} \\ \min(RY(1998) \\ -1, 0.9\,MSY_{1+}) & \text{otherwise} \end{cases}$$

where $MSY_{1+} = MSYR_{1+}\, MSYL_{1+}\, K_{1+}$

- *Slope* - the annual rate of increase of the 1+ population from 1978 to 1993, expressed as a percentage.

The posterior distribution is approximated numerically using a variant of the Sampling-Importance-Resampling (SIR) algorithm. This involves drawing Z_1 sets of parameter values from the joint prior distribution[4] and then calculating the likelihood corresponding to each vector. The likelihood is set equal to zero if the value for S_{juv} is greater than that for S_{adult} or if the population is rendered extinct. The posterior is then based on $Z_2 = 5{,}000$ draws (with replacement) from the Z_1 sets of parameter values, where the probability of selecting a given parameter set is proportional to its likelihood. The maximum weight (the ratio of the likelihood for most likely set of parameter values to the total likelihood over all sets of parameter values) is used to assess whether the SIR algorithm has converged adequately to the posterior distribution.

The results for the 'reference case'

Table 3 lists post-model-pre-data and posterior distributions for the 'backwards' and 'forwards' approaches. Results are shown in Table 3 for the 'forwards' approach for Z_1 = 1,000,000 ('reference case') and Z_1 =2,500,000. The maximum weight for anyone draw for the reference case 'backwards' analysis (0.00066) suggests that Z_1 = 250,000 is more than sufficient to obtain an adequate numerical representation of the posterior. In contrast, the maximum weight for the reference case 'forwards' analysis (0.02286) is perhaps larger than desirable. Increasing Z_1 from 1,000,000 to 2,500,000 decreases the maximum weight to 0.00894, which seems adequate. The results for these two choices of Z_1, however, differ only marginally (Table 3).

The posterior distributions differ markedly from the post-model-pre-data distributions (both in terms of precision and central tendency). The post-model-pre-data distributions for 'backwards' are more similar to the posteriors because the 'backwards' projections include the prior information

[4] Unless stated otherwise, z_1 for the analyses of this paper is 250,000 for the 'backwards' analyses and 1,000,000 for the 'forwards' analyses.

Table 3

Estimates of eight management-related quantities for the Bering-Chukchi-Beaufort Seas stock of bowhead whales. The point estimates given are posterior medians, followed by posterior means in round parentheses. Posterior 90% credibility intervals are given in square parentheses. Results are shown for the 'forwards' and 'backwards' approaches to implementing the method (see text for details).

	K_{1+}	RY (1998)	Q_0 (1998)	P^{1+}_{1998}/K_{1+}	P^{f}_{1998}/K^{f}	$P^{1+}_{1998}/MSYL_{1+}$	$MSYR_{1+}$	*Slope*
Post-model-pre-data								
Forwards	20953 (21406)	45 (54)	235 (260)	98.5 (92.7)	95.6 (88.9)	137.2 (135.7)	1.86 (2.00)	0.00 (0.17)
	[13834 30399]	[20 140]	[110 566]	[55.9 100.0]	[43.8 101.8]	[87.9 173.5]	[1.07 3.66]	[-0.07 1.24]
Backwards	14956 (14897)	154 (159)	149 (156)	55.7 (57.4)	39.8 (40.5)	84.0 (84.7)	1.72 (1.87)	1.60 (1.74)
	[11401 18328]	[95 258]	[91 262]	[38.4 87.8]	[28.1 59.0]	[55.3 122.1]	[1.06 3.48]	[0.92 3.39]
Forwards								
Reference case	13995 (14223)	180 (180)	171 (174)	64.1 (63.9)	43.5 (43.7)	94.5 (93.0)	2.04 (2.05)	1.94 (1.95)
	[11547 17918]	[110 250]	[104 249]	[47.5 85.4]	[36.1 55.8]	[69.9 119.3]	[1.12 3.35]	[0.99 3.10]
Z_1=2,500,000	13967 (14170)	183 (181)	174 (175)	64.0 (63.8)	43.6 (43.7)	94.0 (92.7)	2.04 (2.07)	1.96 (1.96)
	[11758 17847]	[110 250]	[106 250]	[47.8 85.4]	[35.9 54.4]	[69.4 115.7]	[1.13 3.40]	[1.01 3.15]
With plus-group	13363 (13553)	168 (168)	168 (169)	65.5 (66.4)	41.9 (42.4)	97.0 (96.5)	2.04 (2.10)	1.87 (1.91)
	[11078 17411]	[106 242]	[104 259]	[48.6 90.7]	[35.0 55.3]	[73.0 125.8]	[1.16 4.17]	[0.99 3.33]
S_{max} = 0.999	13523 (13794)	195 (191)	188 (185)	65.6 (65.8)	43.7 (44.3)	95.1 (94.7)	2.20 (2.23)	2.14 (2.14)
	[11425 17670]	[115 268]	[109 265]	[48.5 85.1]	[36.4 54.0]	[72.0 117.3]	[1.18 3.37]	[1.06 3.29]
Alt Preg constraint	13870 (14092)	176 (175)	172 (173)	64.4 (65.2)	43.8 (44.9)	96.2 (96.4)	2.07 (2.09)	1.94 (1.94)
	[11577 17823]	[110 246]	[105 254]	[48.1 90.5]	[36.3 60.7]	[72.1 127.1]	[1.15 3.60]	[1.02 3.13]
Backwards								
Reference case	12631 (12863)	211 (209)	209 (207)	71.2 (70.9)	45.9 (46.1)	100.2 (99.5)	2.59 (2.58)	2.49 (2.48)
	[10924 16531]	[141 273]	[136 279]	[53.5 90.1]	[37.8 57.9]	[76.8 122.8]	[1.51 3.78]	[1.39 3.67]
With plus-group	11833 (12055)	193 (191)	206 (204)	75.2 (74.9)	43.9 (44.4)	104.9 (104.1)	2.67 (2.68)	2.49 (2.48)
	[10158 15829]	[132 253]	[137 270]	[56.5 93.3]	[36.3 56.3]	[81.4 125.5]	[1.58 3.97]	[1.44 3.61]
S_{max} = 0.999	12500 (12719)	215 (212)	215 (212)	72.1 (71.8)	45.8 (46.2)	101.5 (100.9)	2.67 (2.66)	2.59 (2.56)
	[10843 16409]	[145 275]	[139 281]	[54.6 91.1]	[38.0 58.3]	[78.5 124.1]	[1.55 3.89]	[1.43 3.74]
Alt Preg constraint	12547 (12775)	206 (204)	208 (206)	71.6 (71.4)	46.2 (46.6)	101.9 (101.7)	2.62 (2.61)	2.50 (2.47)
	[10710 16468]	[137 270]	[133 277]	[54.0 91.0]	[38.3 59.4]	[78.8 126.4]	[1.54 3.89]	[1.40 3.68]

about P_{1993} whereas the 'forwards' projections do not. Similarly to previous studies that have compared results from 'backwards' and 'forwards' (e.g. Butterworth and Punt, 1995; Givens and Thompson, 1996), the results for 'backwards' are more optimistic in terms of stock productivity that those for 'forwards'. This is reflected by higher values for all of the quantities (except K_{1+} which shows a compensating decrease). It is noteworthy that the posterior for *Slope* for 'backwards' is closer to the estimate inferred from the data (see Table 2) than that for 'forwards'. Both variants suggest that the population is close to the *MSY* level (in terms of the 1+ component of the population) and both analyses suggest that the lower 5%iles of the distributions for Q_0 and the current replacement yield are substantially larger than the current annual strike limit for the B-C-B stock of 67 animals.

Previous analyses based on 'backwards' and 'forwards' have obtained posteriors for *Slope* that are closer to the actual data (e.g. Punt and Butterworth, 1997a provides 'reference case' median estimates of 2.81 and 2.28% respectively). Three sensitivity tests were conducted to examine which of the changes made in IWC (1998c) to the prior distributions might have led to the change to the posterior for the *Slope* statistic. These three sensitivity tests involved (i) dropping the specification that the survival rate is zero for animals aged 100 years ('With plus-group'), (ii) increasing the upper limit for adult survival, S_{max}, from 0.995 to 0.999 ('S_{max} = 0.999'), and (iii) decreasing the lower limit of the prior for the maximum calving interval from 2.5 to 2 years ('Alt preg constraint'). The first and third of these changes have little impact on *Slope* (Table 3). Increasing S_{max} from 0.995 to 0.999 brings the results in closer agreement with those from previous analyses, but there are still notable differences between the posterior for *Slope* for the 'S_{max} = 0.999' sensitivity test and the posteriors from previous analyses, so that the reasons for these differences from the previous results are not immediately obvious.

COMPARING ALTERNATIVE STOCK ASSESSMENT METHODS

Punt and Butterworth (1997a) evaluated the relative performances of three alternative estimation procedures (two maximum likelihood methods and the 'forwards' Bayesian Synthesis approach) for the B-C-B bowhead stock by means of a Monte Carlo simulation exercise. The evaluation involved generating 100 sets of artificial abundance and 'proportion' data, applying each estimation approach to each data set, and then comparing point estimates (posterior medians for these Bayesian methods) with true values. The results of the simulation trials were summarised in Punt and Butterworth (1997a) by the biases and root-mean-square errors (RMSEs) (expressed in relative terms) of four quantities (indicated by Q below) of interest to management:

$$\hat{Q}^U = (1+\beta)Q^{True,U} + \varepsilon^U \qquad (1)$$

and

$$\mathrm{RMSE}(Q) = \sqrt{\tfrac{1}{100}\sum_{U=1}^{100}\left(\hat{Q}^U / Q^{True,U} - 1\right)^2} \qquad (2)$$

where $Q^{True,U}$ is the true value of quantity Q in simulation U,
$\hat{Q}^U$ is the estimate of Q in simulation U,
β is the relative bias, and
ε is assumed to be a normally distributed random variate.

The analyses of this paper involve applying the simulation testing framework developed by Punt and Butterworth (1997a) to compare the estimation ability of the 'backwards' and 'forwards' approaches to Bayesian analysis[5]. Punt and Butterworth (1997a) considered eleven trials, the first nine of which involved fixed values for the biological parameters. Here we consider only the remaining two trials, which involved generating true values for the biological parameters from the posterior distributions obtained from either the 'forwards' or the 'backwards' variants of the Bayesian assessment.

IWC (1997) noted that previous simulation evaluations had made no attempt to compare estimation methods for the B-C-B bowhead stock with respect to their estimates of precision. Both estimation procedures are Bayesian, and so can readily be applied to provide comparable 90% credibility intervals. The intervals are compared for each management quantity using three measures of performance: (a) the probability that the 90% credibility interval includes the true value, (b) the probability that the true value is smaller than the lower 90% limit and (c) the probability that the true value is larger than the upper 90% limit. If the estimation procedure performed 'perfectly', the values for these quantities would be 0.90, 0.05 and 0.05.

Table 4 lists the relative biases and RMSEs for the 'forwards' and 'backwards' approaches for six quantities of interest to management (K_{1+}, $MSYR_{1+}$, Q_0 (1998), P^f_{1998}/K^f, $P^{+1}_{1998}/MSYL_{1+}$, and RY (1998). Figs 1 and 2 plot the actual and estimated values for four of the six quantities for the two trials. Results are not shown in these figures for *RY* (1998) and $p^{1+}_{1998}/MSYL_{1+}$ because they are qualitatively the same as those for Q_0 (1998) and P^f_{1998}/K^f respectively. Not surprisingly, the performance of the 'forwards' estimation approach is better when 'forwards' rather than 'backwards' is used to generate the true data, although it remains poor in both cases. But importantly, whichever approach is used to generate the data, the 'backwards' estimation approach outperforms its 'forwards' counterpart in terms of both RMSEs and the absolute size of the bias. Both approaches tend to provide 'conservative' (i.e. negatively biased) estimates of the management quantities upon which catch limits would be based (Table 4; Figs 1 and 2).

In terms of coverage probability, the 'backwards' approach again performs better than the 'forwards' approach (Table 5 on p. 60). The poor performance of the 'forwards' approach is attributable to the fact that the estimate of the upper 90% credibility value is far too low for all of the quantities except K_{1+} for which the lower limit is too high.

This comparison overestimates the confidence to be placed in the Bayesian credibility intervals because all the estimators assume the exact form of the true population dynamics model, and further because the assumption of deterministic dynamics made by all the estimation procedures is correct. Had the simulations allowed for process error effects (such as variation in the juvenile survival rate or uncertainty about historical catches), it is likely that the credibility intervals would have been shown to be overly narrow. Punt and Butterworth (1993) demonstrate that coefficients of variation estimated using bootstrap procedures for hake assessments can be negatively biased by some tens of percent when observation errors (but not process errors) are taken into account.

Table 4

Percentage biases and root mean square errors (in parentheses) for two estimators and two management-related quantities (see text for details).

	Estimation procedure	
Simulation trial	Forwards	Backwards
Backwards		
K_{1+}	15.9 (21.5)	4.7 (12.6)
$MSYR_{1+}$	-37.0 (36.1)	-17.2 (25.2)
Q_0 (1998)	-31.1 (31.7)	-13.5 (21.0)
P^f_{1998}/K^f	-14.8 (16.5)	-8.3 (11.9)
$P^{1+}_{1998}/MSYL_{1+}$	-15.6 (18.0)	-7.7 (13.6)
RY (1998)	-29.5 (30.6)	-13.6 (20.4)
Forwards		
K_{1+}	8.2 (15.7)	-1.2 (10.2)
$MSYR_{1+}$	-28.3 (28.8)	-7.6 (27.3)
Q_0 (1998)	-23.6 (26.5)	-5.5 (24.9)
P^f_{1998}/K^f	-9.8 (12.9)	-4.1 (10.1)
$P^{1+}_{1998}/MSYL_{1+}$	-10.8 (15.1)	-4.0 (13.2)
RY (1998)	-22.9 (25.5)	-7.1 (22.3)

PRIORS AND LIKELIHOODS

The analyses presented above illustrate the need to identify those quantities for which some of the available information comes from 'indirect' sources (e.g. inferences from data for other stocks/species), and those for which all of the information comes from 'direct' sources. In a 'standard' Bayesian assessment, the former quantities must be included as priors, while the information contained in the latter should form part of the likelihood[6]. In the 1994 B-C-B bowhead assessment (IWC, 1995), two of the prior distributions (those for f_{max} and P_{1988}) were treated as 'data' rather than priors in the analysis even though they were based, in part, on 'indirect' evidence. The contribution of P_{1988} to the likelihood function included 'indirect' information (for example, about whale numbers and behaviour - e.g. Raftery and Zeh, 1991) as well as information from the 'direct' count data collected at Point Barrow, Alaska. The likelihood contribution for f_{max} was not based on any direct information about the pregnancy rate of bowhead whales at very low population size, but rather on inferences about what this rate might be, taking account of perceptions/observations for other baleen whale species (IWC, 1992; 1995).

The first step needed in this process is to list the various sources of information which could contribute to the assessment, and then to clarify which are data (and so should be incorporated into the likelihood function) and which constitute 'indirect' information (and must therefore form part of the (joint) prior distribution). Sainsbury *et al.* (1998) highlight the point that this step in the process of conducting a Bayesian assessment has often been missing

> 'there should be much more careful documentation of the steps involved in successive updating (i.e. the initial definition of the prior, the information used to calculate a posterior that in turn is the prior for the next iteration of the analysis)'.

[6] It is often computationally more efficient to update priors based solely on 'indirect' data with the 'direct' data for the parameters concerned - this does not impact the final results at all.

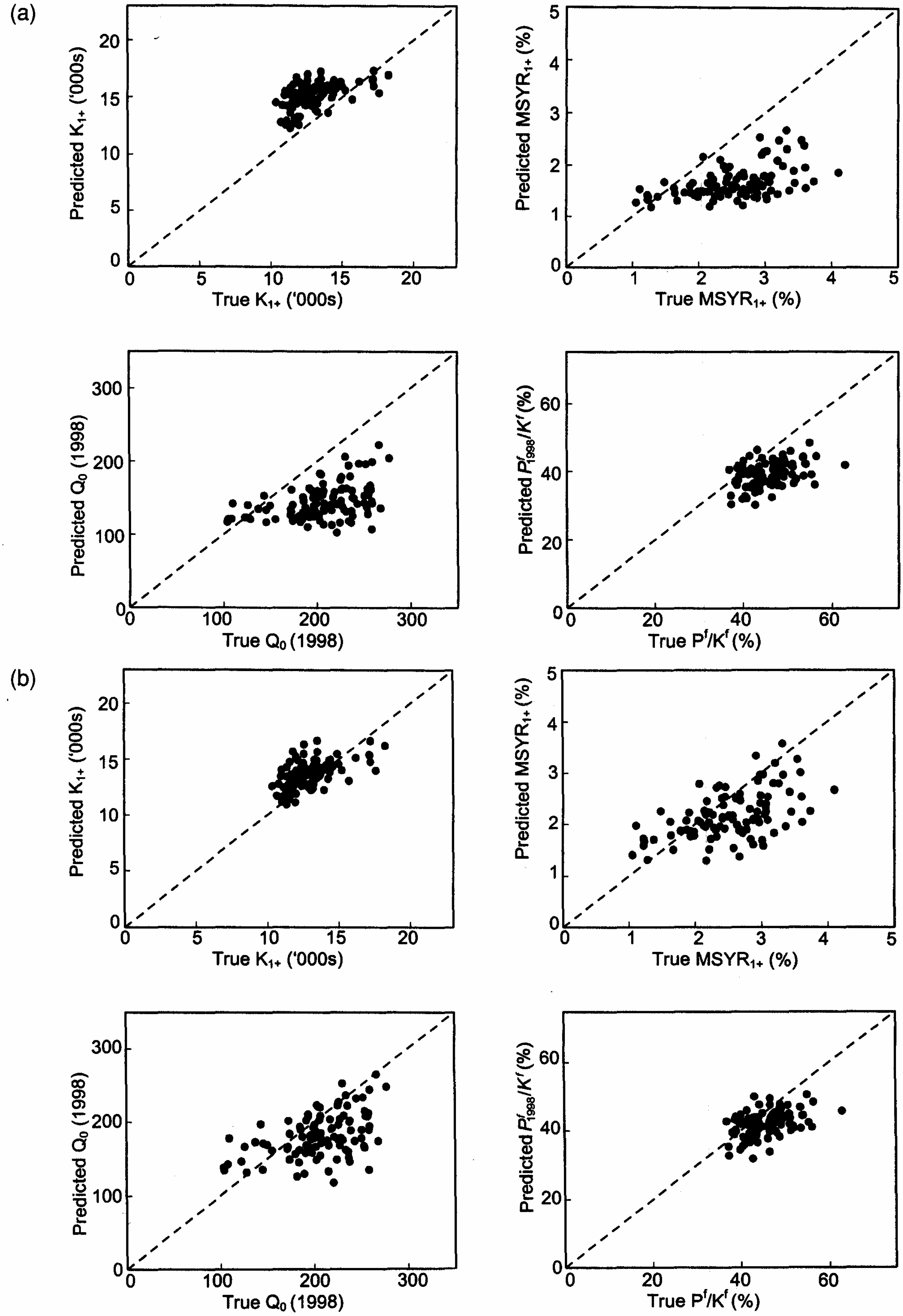

Fig. 1. Plots of predicted and true values for K_{1+}, $MSYR_{1+}$, P^f_{1998}/K^f, and Q_0 (1998) for the 'backwards' simulation trial. Results are shown for (a) the 'forwards' and (b) the 'backwards' estimation approaches.

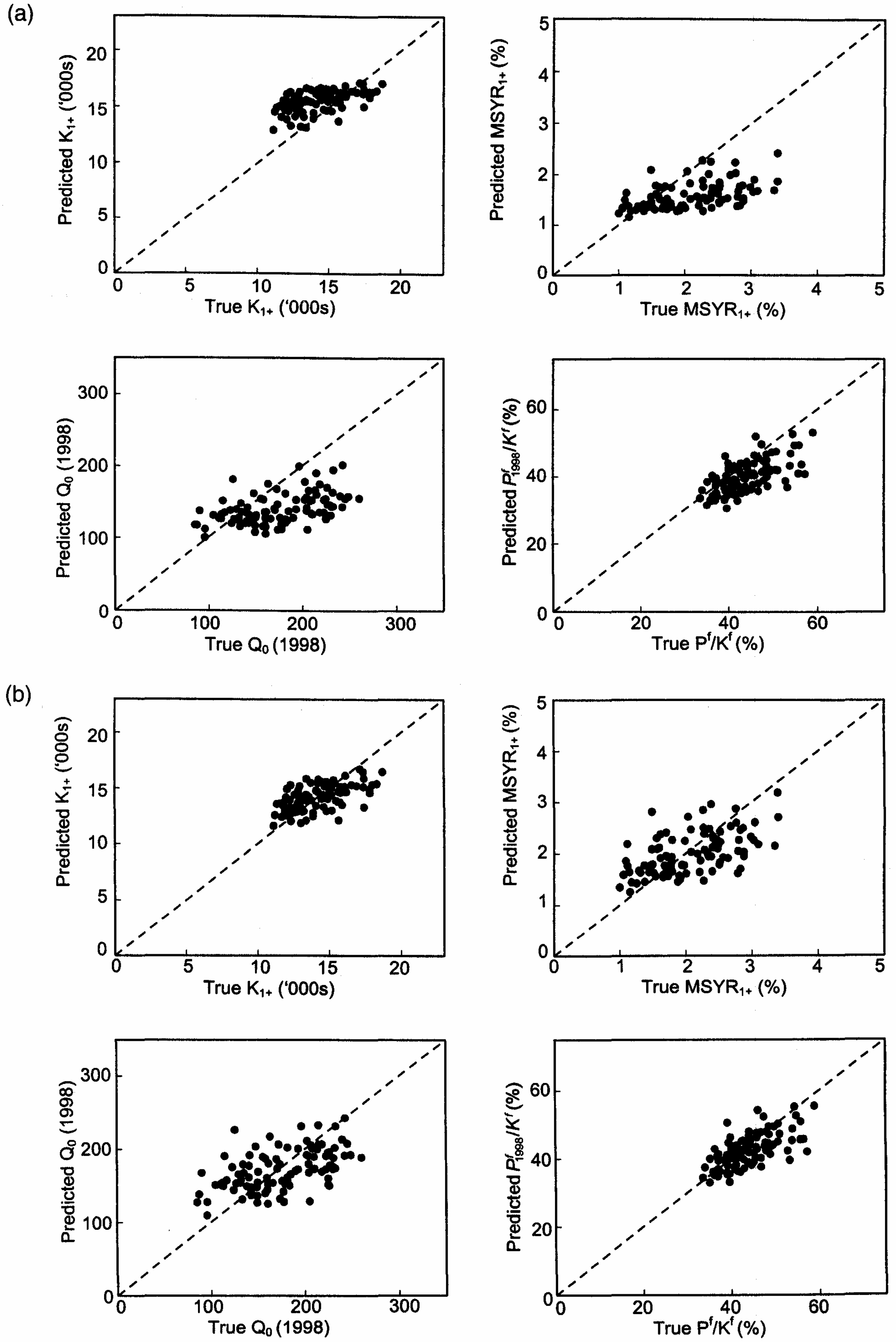

Fig. 2. Plots of predicted and true values for K_{1+}, $MSYR_{1+}$, P^f_{1998}/K^f, and Q_0 (1998) for the 'forwards' simulation trial. Results are shown for (a) the 'forwards' and (b) the 'backwards' estimation approaches.

Table 5

Performance statistics to measure the quality of 90% credibility intervals for six management quantities obtained using two Bayesian estimation procedures. The three statistics for each trial/estimator are: the probability that the 90% credibility interval includes the true value, the probability that the true value is smaller than the lower 90% limit and the probability that the true value is larger than the upper 90% limit.

Simulation trial	Estimation procedure					
	Forwards			Backwards		
Backwards						
K_{1+}	0.62	0.38	0.00	0.86	0.12	0.02
$MSYR_{1+}$	0.59	0.00	0.41	0.86	0.01	0.13
Q_0 (1998)	0.63	0.00	0.37	0.91	0.00	0.09
P^f_{1998}/K^f	0.65	0.00	0.35	0.89	0.00	0.11
$P^{1+}_{1998}/MSYL_{1+}$	0.69	0.01	0.30	0.91	0.01	0.08
RY (1998)	0.66	0.00	0.34	0.89	0.00	0.11
Forwards						
K_{1+}	0.81	0.17	0.02	0.94	0.00	0.06
$MSYR_{1+}$	0.81	0.01	0.18	0.94	0.06	0.00
Q_0 (1998)	0.80	0.03	0.17	0.92	0.05	0.03
P^f_{1998}/K^f	0.80	0.00	0.20	0.94	0.01	0.05
$P^{1+}_{1998}/MSYL_{1+}$	0.84	0.01	0.15	0.97	0.01	0.02
RY (1998)	0.81	0.02	0.17	0.91	0.04	0.05

The 1994 application of the Bayesian Synthesis method (IWC, 1995) was based on seven input pre-model (prior) distributions and five terms in the likelihood function. The basis for each of the prior distributions based on 'indirect' data is discussed below, including comments on some of the updates reflected by the 'reference case' specifications of 'IWC (1998c). This section does not deal in detail with the derivation of the 'direct' data (e.g. the 'proportion' data) because they were derived using standard statistical procedures, but does comment on alternative approaches to including the abundance data in the likelihood function.

Absolute abundance

All stock assessments must incorporate a parameter that scales the overall abundance. Punt and Hilborn (1997) note that this parameter is of particular importance in most assessments, but that data for other stocks/species can rarely be used to construct an informative prior for it. In many stock assessments, this parameter is chosen to be K (the pre-exploitation equilibrium biomass), although it is possible to select the biomass/numbers/exploitation rate in any year as this scaling parameter. For stock assessment methods based on (conditional) maximum likelihood estimation, the choice of this parameter (whether, for example, K or the current biomass) is irrelevant because the likelihood is invariant to transformations of the model parameters. However, for a Bayesian approach, this choice can be very important and the results may be highly sensitive to it because a Bayesian assessment is not invariant to such transformations.

Two approaches to the B-C-B bowhead Bayesian Synthesis assessment have been a focus for discussion. The 'forwards' approach requires a prior distribution for K_{1+} and projects population trajectories forwards from realisations generated from this prior distribution. In contrast, the 'backwards' approach uses realisations from a prior distribution for a recent estimate of absolute abundance to essentially extrapolate trajectories back to 1848; thus it provides an implicit distribution for K_{1+}, and so avoids the need for an explicit specification of a prior distribution for this parameter. Punt and Bu,tterworth (1997a) show that the results of the 'backwards' approach are not notably sensitive to the year (within the period of the past two decades) for which abundance estimates are available which is selected to provide the recent estimate of abundance.

Considerable attention has been directed towards identifying the reason for the difference in the results for these two approaches (see Table 3). Much of the debate initially centred on the justification of the basis used to provide the 'direct' component of the prior distribution for K_{1+}. For the 1994 assessment (IWC, 1995), this was based on an application of the DeLury (1947) estimation procedure to historical (1849-1870) catch per unit effort (CPUE) data. Butterworth and Punt (1995) criticised the derivation of this prior because the DeLury method effectively assumes that $MSYR$ = 0 and because the general acceptance of CPUE as an index of abundance has proved problematic in the past in the Scientific Committee (IWC, 1988, p.35; IWC, 1989). Punt and Butterworth (1996) and Givens and Thompson (1996) show, however, that including the 'direct' component of the K_{1+} prior in the likelihood when applying the 'backwards' method has virtually no impact on the results. Subsequently, Raftery and Poole (1997) showed that the reason for the differences between the results for 'forwards' and 'backwards' is attributable to differences in the joint region of support for P1993, K_{1+}, and $MSYR$ for the two approaches

This is a case in which there are two priors for the same quantity (the parameter that scales the population). However, although the full pooling approach of Raftery and Poole (1997) and Poole and Raftery (1998) removes the associated problem of the Borel paradox, in doing so it introduces a new one, namely how to choose the pooling weight that is to be placed on the two priors (K_{1+} and P_{1993}) when conducting full pooling[7]. Clearly results are sensitive to the weight chosen, as demonstrated by the differences in results for the two extreme choices for this weight (corresponding to 'backwards' and 'forwards') which are shown in Table 3. In addition, the assumption of *a priori* independence between K_{1+} and $MSYR$ underlying the inclusion of 'forwards' in full-pooling is violated because, prior to inclusion of the data in the assessment, some combinations of K_{1+} and $MSYR$ can be rejected as implausible (see below for further details). Rather than attempting to combine these two priors, we prefer instead to choose the more appropriate of the two.

This choice initially seems rather arbitrary because both seem reasonable *a priori*. However, the simulation results of Table 4 are available to guide a choice in this matter. These suggest that the assumption underlying the 'backwards' approach is more appropriate as it leads to lower MSEs and less biased 90% credibility intervals. There are also two 'in principle' reasons for preferring a current rather than a historical population size as the parameter which scales the

[7] Logarithmic pooling, which is the approach generalised in full pooling, given two priors p_1 and p_2 for the same parameter, provides a pooled prior proportional to $p_1^{\alpha}p_2^{(1-\alpha)}$, where the pooling weight α reflects the relative reliability accorded to the two sources of information that underlie the two priors specified. Raftery and Poole (1997) argue for $\alpha = 0.5$ when pooling priors for P_{1993} and K_{1+} on the basis of invariance under relabeling of inputs and outputs (initial and current population sizes), suggesting also that two priors that are agreed by the same 'expert' (the IWC Scientific Committee) should accordingly be deemed equally reliable. We disagree with these views, judging for the various reasons put forward in this paper that the (or indeed any) prior advanced for K_{1+} in the particular case of the B-C-B bowhead population is much less reliable than that for P_{1993}.

population size. The first is that there is no need when applying the 'backwards' approach to specify that the population is not currently extinct, because this is incorporated implicitly in the prior for current population size. If a 'forwards' approach is taken, it is necessary to place a prior distribution on current depletion (or current population size) and incorporate it as a bound in the likelihood function. The second reason is that, in the absence of data, the 'backwards' approach does not update any of the prior distributions (i.e. the post-model-pre-data distributions for *MSYR*, *MSYL*, a_m, a and S_{adult} are identical to their priors[8]). In contrast, the 'forwards' approach updates the joint prior distribution substantially (*inter alia* because combinations of low K_{1+} and low *MSYR* correspond to extinction and can thus be excluded). Intuitively, it would seem undesirable to update prior distributions in the absence of direct data. The greater difference between the post-model-pre-data distribution and the prior for $MSYR_{1+}$ for 'backwards' than 'forwards' (contrast the prior in Table 2 with post-model-pre-data distributions in Table 3) would seem to contradict this. However, there are two factors that determine the post-model-pre-data distribution. The first is the impact of rejecting parameter combinations that give rise to juvenile survival rates exceeding S_{adult}, and the second is the impact of the effect just discussed. The post-model-pre-data distribution for $MSYR_{1+}$ for 'backwards' reflects only the first while that for 'forwards' reflects both. As expected, the post-model-pre-data distribution for 'forwards' gives greater probability to higher values for *MSYR*.

Another seemingly undesirable property of the 'forwards' approach is that once a single abundance estimate becomes available, the joint distribution for the biological parameters (including *MSYR*) is updated. This seems intuitively undesirable because a single abundance estimate does not provide any information about *MSYR*. The past application of the 'Hitting with fixed MSYR' methodology by the Scientific Committee constitutes specific concurrence with this assertion. Even though an estimate of absolute abundance is available, all values for *MSYR* remain equally likely[9] because there is always some combination of *MSYR* and K_{1+} which 'hits' the estimate of abundance exactly. As a consequence, the likelihood is the same for all values of *MSYR*, but the associated prior is updated in the Bayesian integration under the 'forwards' approach. This is because the non-linearity of the relationship between *MSYR* and K_{1+} (given a single estimate of abundance) means that equal intervals on the K_{1+} axis do not correspond to equal intervals on the *MSYR* axis (Butterworth and Punt, 1997, illustrate what amounts to this point). In contrast to this situation for the 'forwards' approach, the effect of a single estimate of abundance on the 'backwards' approach is to update the prior for current abundance without having any impact on the distribution for *MSYR*.

A concern with the 'backwards' approach is its use of the N(7800; 1300^2) prior for P_{1993}. The basis for this prior is unclear because there is no obvious independent information that could be used to construct an informative prior for P_{1993}. However, the 'backwards' approach can be applied using an 'uninformative' U[0, ∞) prior for P_{1993}. As the likelihood for P_{1993} would be very informative compared to any sensible 'uninformative' prior for P_{1993}, the results are unlikely to be very sensitive to the choice of an 'uninformative' prior for P_{1993}.

MSYR

The second parameter that all stock assessments have to incorporate is one that determines the overall productivity of the resource. In the BALEEN II population dynamics model, this is the resilience parameter (A). The 1994 B-C-B bowhead assessment (IWC, 1995) placed a prior distribution on $MSYR_{mat}$ rather than on A (presumably because scientists are familiar with values for MSYR, which facilitates comparison among stocks/species, unlike the situation for A). Other possible choices for the productivity parameter include the increase rate at low population size, λ_{max}, and the current rate of increase, *ROI* (Punt, 1999).

The U[1%; 7%] prior for $MSYR_{mat}$ selected by the Scientific Committee (IWC, 1995) is consistent with that used in the development of the *Catch Limit Algorithm* for commercial whaling. Some concern has been expressed over the validity of the approach taken and its consistency in previous discussions about $MSYR_{mat}$ (IWC, 1995, p.148). Butterworth and Punt (1995) point out that the upper 2.5%ile of the posterior for $MSYR_{mat}$ under the 'backwards' approach suggests that values of $MSYR_{mat}$ higher than the upper bound of 7% permitted by its prior above are not incompatible with the other information available. Gelman *et al.* (1995) suggest that all plausible values for the model parameters should be assigned non-zero prior probability. One reason for this is that if the prior assigns zero probability to the value of some parameter, this value is always assigned zero probability in the posterior distribution. Punt and Butterworth (1996) argue that any prior for $MSYR_{mat}$ for the B-C-B bowhead stock should be viewed as 'uninformative', because the dissimilarities of bowheads and other baleen whale species render inferences for bowheads drawn from those other species questionable. Consequently, they advocate that the prior be chosen to be uniform and over a wider range than specified by IWC (1995). This suggestion was implicitly accepted by IWC (1998c) where a U[1%, 7%] prior for $MSYR_{1+}$ (corresponding to an upper bound for $MSYR_{mat}$ considerably larger than 7%) is specified (see Table 2).

Punt and Butterworth (1997a) developed an approach to Bayesian analysis ('less both') that ignores both of the priors for K_{1+} and *MSYR*. It involves generating values for current population size and the current rate of increase of the population (*ROI*) from prior distributions, and then selecting values for K_{1+} and *MSYR* to 'hit' the values generated for P_{1988} and *ROI* exactly. An assumption (not explicitly stated by Punt and Butterworth (1997a)) underlying this approach is that there is an 'indirect' prior for *ROI* which is U(-∞, ∞). The approach is thus equivalent to placing all of the direct information about *ROI* into the likelihood (for example, in the manner indicated in equation (3) following) and generating values for current population size from its prior and for *ROI* from U(-∞, ∞).

Punt (1999) outlines an approach for placing a prior on λ_{max} instead of on $MSYR_{1+}$ when conducting a Bayesian assessment. Best (1993) provides estimates of annual increase rates at low population size for a range of severely depleted stocks of baleen whales. Ignoring the estimate for the B-C-B bowhead stock (to avoid using the abundance data

[8] This is an oversimplification to better make the essential point, which relates in particular to the update of the *MSYR* prior under 'forwards'. The reason it is not exactly correct as stated, however, is that even before the BALEEN II population model trajectories are computed, certain combinations of these biological parameters are impossible because of incompatibility with the demographics underlying the BALEEN II model, so that this aspect alone converts the independent priors into a joint distribution with some non-zero (but typically small) covariances.

[9] For the purposes of simplicity of presentation, this argument has ignored the possibility of oscillatory trajectories.

for this stock twice in the analysis) and the estimate for the Eastern North Pacific gray whale (which is not currently at a small fraction of its pre-exploitation equilibrium size), and taking the lower rate of increase when more than one estimate is provided for a given population, leads to seven estimated rates of increase at low population size (Table 6). The mean of these estimated annual rates is 0.085 (SD 0.024).

Table 6

Estimated annual rates of increase (with 95% confidence intervals) for several severely depleted stocks of baleen whales (source: Best, 1993).

Stock	Point estimate	95% CI
South African Right	0.068	[0.048, 0.086]
Argentine Right	0.073	[0.038, 0.108]
W. Australian Right	0.127	[0.076, 0.178]
NW. Atlantic Humpback	0.094	[-0.12, 0.30]
W. Australian Humpback	0.088	[0.030, 0.146]
E. Australian Humpback	0.097	[0.06, 0.13]
NE. Atlantic Blue	0.051	[0.026, 0.076]

Some account needs to be taken of the likely difference in productivity between bowheads and other baleen whales when using the information in Table 6 to develop a prior for the maximum steady rate of increase for the bowhead stock. Accordingly a range of alternative prior distributions for λ_{max} are considered for the sensitivity tests of this paper. These prior distributions should bound most interpretations of the information.

(a) N(0.085, 0.024^2) – Using the empirical distribution as summarised by a normal distribution.
(b) U[0, 0.127] – A uniform distribution which covers the range of estimates and includes all non-negative values for λ_{max} lower than the largest value in Table 6.
(c) U[0, 0.051] – A uniform distribution with an upper bound equal to the lowest value in Table 6 – this reflects the perception that bowheads are among the least productive of the baleen whales.
(d) U[0.005, 0.051] - A uniform distribution with an upper bound equal to the lowest value in Table 6 and a lower bound chosen to exclude the possibility of a very unproductive stock.

Note that some of the estimates in Table 6 pertain to increase rates for stocks that are probably not currently at 'very low' levels (e.g. Best (1993) reports that the West Australian humpback population is currently 16 – 21% of its pre-exploitation equilibrium level). Use of such estimates therefore leads to the prior being biased towards low values.

Table 7 lists results for the 'reference case' 'backwards' analysis and the four sensitivity tests that place a prior on λ_{max} instead of on $MSYR_{1+}$. The results are notably sensitive to the choice of the prior for λ_{max}. This is not surprising because λ_{max} is closely related to $MSYR_{1+}$ and it is well known that the results of the B-C-B bowhead assessment are sensitive to the choice of the prior (particularly the choice of its upper bound) for $MSYR_{1+}$. The sensitivity test which places a $N(0.082;\ 0.024^2)$ prior on λ_{max} leads to more optimistic results (in terms of resource productivity) than the 'reference case', while the sensitivity tests which place an upper bound of 0.051 on λ_{max} lead to less optimistic results. It is notable, however, that the lower 5%iles of the *RY* and Q_0 distributions remain larger than 67, even for the most pessimistic assessment. The posteriors for the two most pessimistic cases suggest that the stock is most likely below *MSYL* in terms of the 1+ component of the population.

The prior distribution for $MSYR_{1+}$ upon which the 'reference case' is based was inferred from estimates of λ_{max}. The results in Table 7 suggest that considerable care needs to be taken in choosing species/stocks when constructing a prior for *MSYR* by inference because the results are very sensitive to which stocks/species are chosen.

With respect to the selection among *MSYR*, λ_{max}, and *ROI* as the parameter to choose (and with which probably to associate a uniform prior) to reflect the productivity of the resource, it should be noted that most of the estimates of *MSYR* for baleen whales that have been put forward (e.g., see summary in Butterworth and Punt, 1992) have been argued from inferences from increase rates at low population size. Such inferences depend implicitly on the values assumed for the biological parameters (such as *MSYL*) (Butterworth and Best, 1990). To avoid this need, we prefer here to place a prior on λ_{max} and let the population dynamics model make the link to *MSYR*, whose quantitative relationship to λ_{max} will vary across the ranges of values for the various biological parameters. On the other hand, bowheads have been argued to be dissimilar to other baleen whale species because of their unusually high age at maturity, which renders the defensibility of a prior on *MSYR* or λ_{max} based on those other species somewhat questionable. Furthermore, the current replacement yield (*RY*) is an output of the assessment of particular importance, and this is closely related to the product of the current population size and *ROI*. A uniform prior on *ROI* would be less informative about *RY* than would such a prior on *MSYR* or λ_{max}. However, there is no basis for

Table 7

Estimates of eight management-related quantities for the Bering-Chukchi-Beaufort Seas stock of bowhead whales based on the 'backwards' approach. The point estimates given are posterior medians, followed by posterior means in round parentheses. Posterior 90% credibility intervals are given in square parentheses. This table includes results for analyses that place a prior on λ_{max} rather than on *MSYR*.

	K_{1+}	*RY* (1998)	Q_0 (1998)	P^{1+}_{1998}/K_{1+}	P^f_{1998}/K^f	$P^{1+}_{1998}/MSYL_{1+}$	$MSYR_{1+}$	*Slope*
Reference case	12631 (12863)	211 (209)	209 (207)	71.2 (70.9)	45.9 (46.1)	100.2 (99.5)	2.59 (2.58)	2.49 (2.48)
	[10924 16531]	[141 273]	[136 279]	[53.5 90.1]	[37.8 57.9]	[76.8 122.8]	[1.51 3.78]	[1.39 3.67]
λ_{max} ~ N(0.082;0.024^2)	12178 (12375)	216 (214)	221 (217)	74.1 (73.8)	47.2 (47.6)	104.8 (104.2)	2.82 (2.81)	2.72 (2.69)
	[10687 15754]	[152 277]	[148 285]	[57.0 91.8]	[38.8 60.5]	[83.7 125.8]	[1.74 3.96]	[1.63 3.77]
λ_{max} ~ U[0, 0.127]	12631 (13109)	207 (203)	206 (201)	70.4 (69.8)	45.4 (45.8)	100.2 (99.2)	2.58 (2.52)	2.48 (2.42)
	[10893 18579]	[123 272]	[118 276]	[50.4 89.4]	[36.9 57.9]	[75.6 123.5]	[1.27 3.81]	[1.16 3.67]
λ_{max} ~ U[0, 0.051]	13767 (14412)	193 (185)	183 (176)	64.1 (62.7)	42.8 (42.6)	90.8 (89.3)	2.10 (2.01)	2.02 (1.92)
	[12365 21022]	[94 256]	[90 242]	[44.5 78.5]	[34.9 51.3]	[68.9 107.2]	[0.96 2.79]	[0.82 2.73]
λ_{max} ~ U[0.005, 0.051]	13755 (14326)	193 (187)	183 (177)	63.9 (62.9)	42.7 (42.6)	90.6 (89.3)	2.11 (2.02)	2.03 (1.93)
	[12386 20129]	[97 254]	[93 241]	[45.1 78.6]	[34.9 51.4]	[69.1 108.1]	[0.99 2.79]	[0.85 2.71]

specifying a prior on *ROI* (which depends on the current status of the resource) so on balance we advocate placing a prior on λ_{max}.

The choice of a prior for λ_{max} is complicated by a lack of data (see Table 6). We tentatively prefer a U[0, 0.127] prior. This prior implicitly acknowledges the perception that bowheads are likely to be relatively unproductive compared to other baleen whales by including values for λ_{max} lower than the lowest value in Table 6, but equally does not exclude higher values which are not incompatible with the data on the bowhead rate of increase.

Natural mortality

The derivation of the priors for S_{adult} and S_{juv} in 1994 (IWC, 1995) is poorly documented. The basis for the choice of the prior for S_{adult} appears to be inferences from the capture of a very large animal at Wainright in 1993 with two stone harpoons of a pattern generally out of use by the start of the 20th century, the age determination study by Nerini (1983), and estimates of natural mortality for adult right whales (IWC, 1995). IWC (1992) used values of $0.01yr^{-1}$ and $0.02yr^{-1}$ in Hitter-Fitter runs for the B-C-B bowhead stock primarily (it appears) because higher values were incompatible with estimates of the proportion of the population which is immature. Fig. 3 shows the marginal prior distribution assumed for S_{adult} by IWC (1995), as well as the corresponding distribution for the number of further years for which 5% of the population will survive after maturing. The upper tail of this distribution (5% point = 562yrs) is clearly unrealistic. The Scientific Committee could, in its reconsideration of priors for the B-C-B bowhead assessment, perhaps consider a prior on longevity rather than on adult natural mortality because this is arguably a more 'natural' parameter. Any prior on longevity would exclude unrealistically long lifespans and hence (effectively) place an upper bound on S_{adult}. In suggesting the 'reference' priors in Table 2, IWC (1998c) explicitly dealt with this issue by placing an upper bound of 0.995 on S_{adult} and by imposing a maximum age of 100 years.

Whitcher *et al.* (1996) and J.L. Laake (pers. commn) provide separate preliminary estimates of the survival rate of adult bowhead whales using data from aerial photographs of identified whales. These estimates are 0.970 (SD 0.054) [M-profile-1] and 0.995 (SD 0.055) [M-profile-2] respectively (see Fig. 4, upper panel). The prior distribution for S_{adult} selected by IWC (1995) (see Fig. 4, lower panel, for its marginal[10]) was not based (explicitly at least) on these data, so that this prior distribution and the aerial-photography-based distributions for the estimates can legitimately be considered to be independent. These aerial-photography-based distributions can therefore be included along with the abundance and 'proportion' information in the 'direct' data used when applying Bayesian methods. Naturally, because the Whitcher and Laake estimates are based on the same data, they cannot both be included in the same analysis.

The incorporation of these data can be achieved in two ways. The prior distribution of IWC (1995) for S_{adult} can be updated using Bayes' theorem, or this prior distribution can be left unchanged and an extra component added to the likelihood function. These two procedures will give identical results. The first is computationally more efficient and so has been applied here.

[10] This is a marginal distribution because IWC (1995) specified a joint pre-model distribution for S_{adult} and S_{juv}.

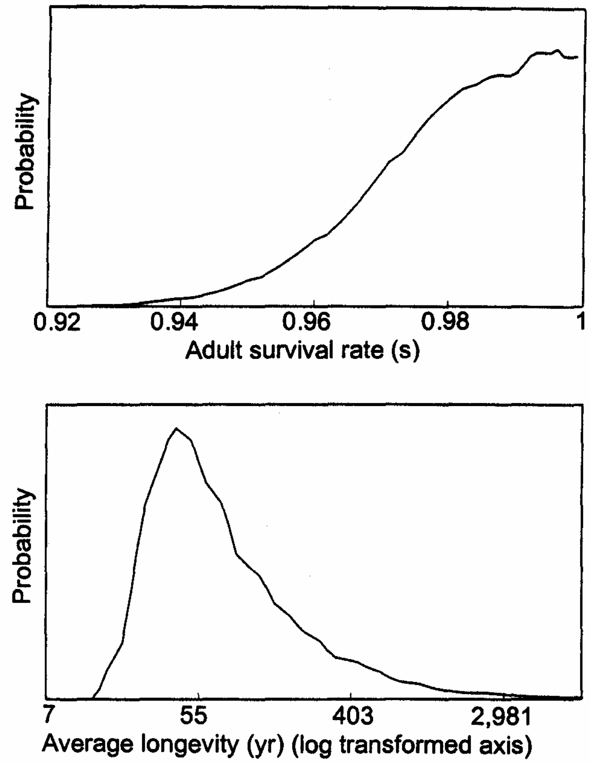

Fig. 3. Marginal prior distributions for adult survival rate S_{adult} (upper panel - from IWC (1995), see Table 2) and the number of years for which 5% of the population survive after reaching maturity (lower panel).

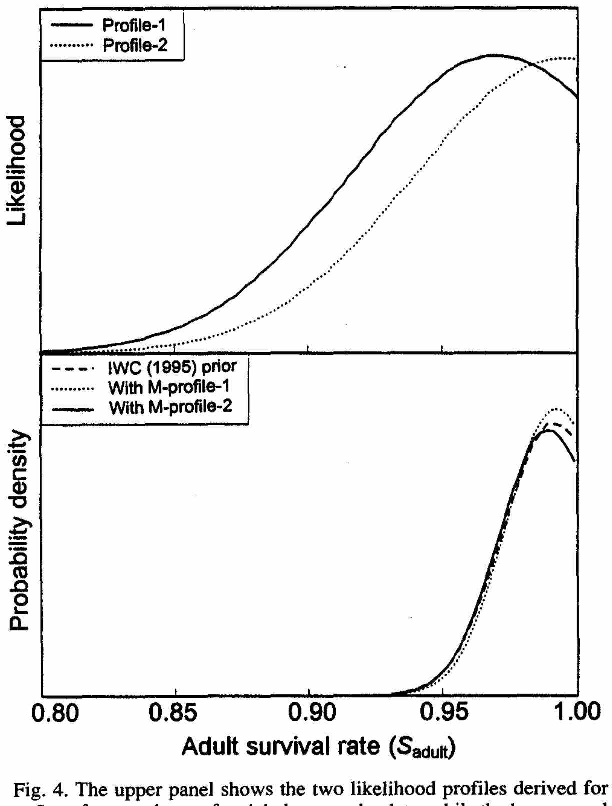

Fig. 4. The upper panel shows the two likelihood profiles derived for S_{adult} from analyses of aerial photography data, while the lower panel shows the marginal prior distribution for S_{adult} specified by IWC (1995) and two prior distributions for S_{adult} obtained by updating the IWC (1995) prior by means of the likelihood profiles in the upper panel.

Butterworth (1995) raises the issue that the priors for age-at-maturity (a_m), natural mortality (M) and maximum fecundity should be correlated because, *a priori*, one expects that a high value for a_m would be linked to low values for M (e.g. Gunderson and Dygert, 1988). One of the impacts of the independence of the priors for a_m and S_{adult} (under the assumptions of IWC (1995)) is that there is a greater prior probability than seems reasonable that the age-at-maturity lies in the upper tail of its prior (say at the maximum of 26 years) and simultaneously that the adult survival rate lies in the lower tail of its prior (corresponding, say, to an average age that a bowhead lives after maturity of only 8 years or less).

IWC (1992) notes that there is no direct evidence about S_{juv}. The prior selected by IWC (1995) is based on the (seemingly reasonable) assumption that juvenile natural mortality is less than adult natural mortality (although it should be noted that there are also arguments that the reverse applies for fur seals, at least, because of the stress placed on the adults by the cost of reproduction—I. Boyd, pers. commn). However, the actual prior for S_{juv} is essentially arbitrary. This is hardly surprising because there are precious few (if any) reliable estimates of S_{juv} for baleen whales. Punt and Butterworth (1996) and Wade (1999) propose methods that avoid the need for specifying priors for both of S_{adult} and S_{juv}. The method proposed by Wade (1999) involves generating values for *MSYR*, *MSYL*, S_{adult}, a, a_m and f_{max} from their priors and then calculating a value for S_{juv}. If the value for S_{juv} is greater than the value for S_{adult}, the set of parameters is assigned zero likelihood. This approach was adopted by IWC (1998c), thus avoiding the need to specify a prior for S_{juv} in Table 2. It also implicitly forces a relationship (and hence correlation) between a_m and natural mortality and hence partially resolves the problem caused by the *a priori* assumption that a_m and S_{adult} are independent.

Including the data on S_{adult} from analysis of aerial photographs of identified bowheads hardly impacts the results of the assessment. This is a consequence of the highly informative prior distribution assumed for S_{adult} and the comparatively uninformative nature of the data. The lower panel of Fig. 4 shows the distributions for S_{adult} obtained by updating its prior distribution using the likelihoods in the upper panel. The updated distributions are very similar to the original distribution, confirming the uninformative nature of the data given the prior assumed for S_{adult}.

George *et al.* (1998) provided estimates of age for 42 bowheads using the aspartic acid racemization technique. Four of the animals were estimated to be older than 100 years. These data should be used in future to update the prior for S_{adult}.

Age-at-recruitment

The assessment assumes that recruitment occurs at age 1 and that the historical harvest has been taken with uniform selectivity from the 1 + component of the population. This is equivalent to assuming a delta-function prior for the age-at-recruitment at age 1. However, the age-at-recruitment is likely to have changed over time because, during the early years of the commercial fishery, whalers presumably targeted large animals (IWC, 1992). Following the demise of this fishery, aboriginal exploitation targeted smaller animals (IWC, 1992). The current formulation of the Baleen II model cannot allow for changes to the age-at-recruitment explicitly. It can, however, divide catches into those from the mature component of the population and those from the recruited (in this case the 1 +) component. The available data should be examined to see if an appropriate division of the historical harvests can be made between those to be assumed to be taken uniformly from the 1+ component and those taken similarly from the mature population, for a better reflection of the historical reality.

Age-at-maturity

The length at maturity for bowheads (averaged over both sexes) is approximately 13m (Withrow and Angliss, 1992). The prior assumed for a_m by IWC (1995) is based on converting a length of 13m to age. Information from carbon isotope ageing of baleen plates suggests that bowheads reach 13m between 18 and 20 years of age (Schell *et al.*, 1989). The prior for age-at-maturity, a normal distribution with mean 20 and standard deviation 3, constrained to lie between 13.5 and 26.5, was chosen to encompass the best estimates of 18-20 years and to incorporate a minimum value of 14 years (the lower limit for age-at-maturity obtained by Schell *et al.*, 1989). It is therefore based primarily on 'direct' information from ageing. The prior is probably overly precise because no account was taken of uncertainty in ageing methods and of the assumption that the length at maturity is 13m. Such information would have to be reflected as 'indirect' information.

Age of transition from juvenile to adult natural mortality rate

The prior for the greatest age at which juvenile natural mortality is assumed to apply, a, is based on a suggestion by Givens *et al.* (1995). They argued that because nothing is known about this parameter, a discrete uniform distribution from 1-9 years would be appropriate. The upper limit for this prior was selected to be less than 10 years, the lower limit of the prior assumed by Givens *et al.* (1995) for a_m. When the Scientific Committee (IWC, 1995) selected a prior for a_m which differed from that suggested by Givens *et al.* (1995), no changes were made to the prior for a. The available data are uninformative about the value for this parameter (Givens *et al.*, 1995).

Maximum pregnancy rate

The prior for the maximum pregnancy rate in IWC (1995) (taken to be the maximum possible pregnancy rate by Punt and Butterworth (1996)) was modified by the Scientific Committee in 1997 (IWC, 1998c). The lower bound for f_{max} of 0.25 was supported by evidence from photographically identified B-C-B bowheads. IWC (IWC, 1998c) does not provide a basis for the assumption of a uniform distribution on $1/f_{max}$, nor for the upper bound of 0.4. IWC (1995, p.146) does, however, refer to a 3-4 year calving interval under optimal conditions, although it is not completely clear how this is intended to relate to the maximum pregnancy rate. The prior selected by IWC (1998c) is markedly more informative than that suggested by Givens *et al.* (1995), which had most of its mass between 0.14 and 0.5 but also had some mass between 0.5 and 1.

The BALEEN II population dynamics model assumes that density dependence acts on fecundity (de la Mare, 1989; Punt, 1999)[11]. It differentiates between a pregnancy rate, which is density-dependent, and a constant 0-year-old natural mortality rate that is pre-specified. It is unclear to what extent this has been considered in previous assessments.

[11] A variant of the BALEEN II model exists which allows for the assumption that density dependence acts on natural mortality rather than fecundity (Punt, 1996), and has been applied in an assessment of the B-C-B bowhead stock (Punt and Butterworth, 1996).

MSYL

The prior for this quantity is based on the arguments of Givens *et al.* (1995). It encompasses the range of values considered during the development of the *CLA* for commercial whaling. The prior chosen for $MSYL_{1+}$ (U[0.4; 0.8]) is centred on the Scientific Committee's choice in general past practice of *MSYL* = 0.6. This choice was based primarily on empirical evidence (e.g. Fowler, 1981) that the *per capita* growth rate of large mammal populations as a function of population size has a negative second derivative -see Butterworth and Best (1994) for a more detailed historical summary. However, the evidence and justification for this view has more recently been called into question (de la Mare, 1994; IWC, 1994; MacCall and Tatsukawa, 1994). IWC (1995) specified this prior for $MSYL_{mat}$, but this was changed to $MSYL_{1+}$ in IWC (1998c), in the light of arguments reflected in IWC (1998a).

Table 8

Data used in the construction of an index of abundance for the Bering-Chukchi-Beaufort Seas stock of bowhead whales (source: Zeh *et al.*, 1995). The N_4/P_4 estimates are based on analyses conducted by Zeh *et al.* (1995).

	N_4		P_4		N_4/P_4	
Year	Estimate	SE	Estimate	SE	Estimate	CV
1978	3,383	289	N/A		5,019	0.294
1980	2,737	488	N/A		4,061	0.336
1981	3,231	716	0.750	0.108	4,308	0.266
1982	4,612	798	N/A		6,843	0.333
1983	4,399	839	N/A		6,527	0.343
1985	3,134	583	0.519	0.131	6,039	0.317
1986	4,006	574	0.518	0.062	7,734	0.187
1987	3,615	534	N/A		5,364	0.320
1988	4,862	436	0.739	0.053	6,579	0.115
1993	7,249	505	0.933	0.013	7,770	0.071

Including the abundance data in the likelihood function

The data available for the assessment of the B-C-B bowhead stock are the estimates of the proportions of mature animals and calves in the population from 1985 to 1992 (the 'proportion' data) and the estimates of 1+ abundance from surveys conducted at Point Barrow, Alaska (the 'abundance' data).

The 'proportion' data are included in the likelihood function by assuming that the observations are *t* distributed with 5 degrees of freedom (IWC, 1995). The model-estimates are taken to be average of the predicted proportions for 1988 and 1989. Only the proportions of calves and of mature animals are included in the likelihood function and these proportions are assumed to be independent of each other.

Zeh *et al.* (1995) present a series of estimates of abundance for the B-C-B bowhead stock based on visual and acoustic counts of bowheads off Point Barrow (reproduced here as Table 8)[12]. Bayes Empirical Bayes (B-E-B) estimates of abundance are available for 1988 and 1993 (Raftery and Zeh, 1991; 1998; Zeh *et al.*, 1995). The B-E-B estimates are constructed from the data from the visual and acoustic surveys but also utilise prior information. For example, the B-E-B estimate for 1993 is based on a prior of 7,800 (SD 1,300) and a likelihood of approximately $N(8{,}293; 626^2)$. For ease of presentation, the former will be referred to as the B-E-B prior and the latter as the B-E-B likelihood. The following discussion deals only with the B-E-B likelihood because the B-E-B prior does not comprise part of the likelihood function[13].

Several alternative prescriptions are available to incorporate the abundance data in the likelihood function. Following past practice (IWC, 1995; 1998c), we assume that the N_4/P_4 estimates in Table 8 provide information on relative abundance while the B-E-B estimate for 1993 is an estimate of absolute abundance. Prescriptions (b) and (c) examine the implications of assuming that the N_4/P_4 estimates provide information on absolute abundance.

(a) The B-E-B likelihood for 1993 (or 1988) is assumed to provide information on absolute rather than relative abundance (IWC, 1992) and the N_4/P_4 estimates are used to obtain an estimate of the rate of increase in 1+ abundance. These two sources of information are then treated as being independent when constructing the likelihood function. The contribution of the abundance data to the negative of the logarithm of the likelihood function (excluding constants) in this case is given by:

$$-\ell nL = \frac{1}{2(\sigma_{1993}^{BEB})^2}\left(N_{1993}^{BEB} - \hat{N}_{1993}\right)^2 + \frac{9}{2}\ell n\left(1 + \frac{1}{8}(ROI^{obs} - \hat{ROI})^2 / \sigma_{ROI}^2\right) \quad (3)$$

where N_y^{BEB} is the B-E-B estimate of abundance for year y (assumed to apply to the 1+ component of the population and to be equal to the mode of the likelihood at 8,293 for $y = 1993$),
$\hat{N}_y$ is the estimate of the number of 1+ animals at the start of year y from the population model,
σ_y^{BEB} is the standard error of N_y^{BEB} (taken to be 626, the likelihood standard deviation, for $y = 1993$),
ROI^{obs} is the estimate of the 1978-93 rate of increase, which is assumed to have a t_8 distribution (see Table 2),
$\hat{ROI}$ is an estimate of ROI based on a regression of $\{\hat{N}_y; y = 1978, 79,...93\}$ on y, and
σ_{ROI} is the standard error of ROI^{obs}.

(b) The data in Appendix A are assumed to provide independent estimates of absolute abundance; in terms of this assumption, the B-E-B likelihood would provide exact duplicate information to that already contained in the corresponding entry in the survey series, and hence is ignored. The contribution of the abundance data to the negative of the logarithm of the likelihood function (excluding constants) in this case is given by[14]:

$$-\ell nL = \sum_y \frac{1}{2(\sigma_y^{surv})^2}\left(\ell n N_y^{surv} - \ell n\hat{N}_y\right)^2 \quad (4)$$

where N_y^{surv} is the estimate of abundance for year y based on the survey and acoustic data

[12] 'Additional variance' (IWC, 1997) will be ignored because Zeh *et al.* (1995) and Cooke (1996) report that these abundance estimates and their *CV*s are consistent with the assumption that any such variance is of negligible size.

[13] Except when applying the 'forwards' approach to Bayesian Synthesis, for which the analysis is subject to the Borel paradox.

[14] The choice of log-normal error is based on a suggestion by Buckland (1992).

(assumed to apply to the 1+ component of the population), and

σ_y^{surv} is the standard error of the logarithm of N_y^{surv}.

(c) As for (b), except that account is taken of the covariance among the estimates of abundance (see Appendix A). The contribution of the abundance data to the negative of the logarithm of the likelihood function (excluding constants) is then given by (in vector-matrix notation):

$$-\ell nL = \tfrac{1}{2}(\ell n\underline{N}^{surv} - \ell n\underline{\hat{N}})^{T}(\Sigma^{surv})^{-1}(\ell n\underline{N}^{surv} - \ell n\underline{\hat{N}}) \quad (5)$$

where Σ^{surv} is the variance-covariance matrix for the logarithms of the survey estimates.

(d) The survey data are assumed to provide independent indices of relative abundance, with the B-E-B likelihood ignored for the same reason as in (b). The contribution of the abundance data to the negative of the logarithm of the likelihood function (excluding constants) in this case is:

$$-\ell nL = \sum_y \frac{1}{2(\sigma_y^{surv})^2}\left(\ell nN_y^{surv} - \ell n(b\hat{N}_y)\right)^2 \quad (6)$$

where b is the survey bias.

For this case, it is necessary to specify a prior for b. In the absence of information about b, an uninformative prior $\ell nb \sim U[-\infty, \infty]$ is assumed for this parameter.

(e) As for (d), except that account is taken of the covariance among the estimates of abundance. The contribution of the abundance data to the negative of the logarithm of the likelihood function (excluding constants) in this case is given by:

$$-\ell nL = \tfrac{1}{2}(\ell n\underline{N}^{surv} - \ell n(b\underline{\hat{N}}))^{T}(\Sigma^{surv})^{-1} (\ell n\underline{N}^{surv} - \ell n(b\underline{\hat{N}})) \quad (7)$$

(f) The likelihood for the 1993 B-E-B estimate and the survey-based estimate for 1993 provide an estimate of the survey bias factor b of 0.936 (CV = 0.026)[15]. Following Butterworth *et al.* (1999) and Butterworth and Punt (1992), this estimate can be incorporated into the likelihood function as independent information about b. For the case in which the covariance among the estimates is ignored, this leads to the following negative log-likelihood:

$$-\ell nL = \frac{1}{2\sigma_b^2}(\ell n\tilde{b} - \ell nb)^2 + \sum_y \frac{1}{2(\sigma_y^{surv})^2}\left(\ell nN_y^{surv} - \ell n(b\hat{N}_y)\right)^2 \quad (8)$$

where $\tilde{b}$ is the estimate of the survey bias factor (0.936) and,

σ_b is the CV of $\tilde{b}$ (0.026).

(g) As for (f), except that account is taken of the covariance among the estimates of abundance. The contribution of the abundance data to the negative of the logarithm of the likelihood function (excluding constants) in this case is given by:

$$-\ell nL = \frac{1}{2\sigma_b^2}(\ell n\tilde{b} - \ell nb)^2 + \tfrac{1}{2}(\ell n\underline{N}^{surv} - \ell n(b\underline{\hat{N}}))^{T}(\Sigma^{surv})^{-1}(\ell n\underline{N}^{surv} - \ell n(b\underline{\hat{N}})) \quad (9)$$

[15] This *CV* follows from the *CV*s for the N_4/P_4 and B-E-B estimates for 1993 assuming that these estimates are uncorrelated.

Approach (a) above forms part of the 'reference case', while approaches (b)-(g) provide increasingly sophisticated treatments of the data. Equation (9) provides the most complete treatment of the data as it assumes: that the data in Appendix A provide an index of relative rather than absolute abundance; that those estimates are correlated; and that the likelihood for the B-E-B estimate provides information on absolute abundance. The likelihood for the 1993 B-E-B estimate is not explicitly included in Equation (9) as much of the information underlying this likelihood is already included in the survey estimate for 1993. Zeh and Givens (1997) illustrate that including both the likelihood of the B-E-B estimate and the information corresponding to the data on trend in an analysis can lead to severely biased estimates of quantities of importance to management. If data were available on the likelihood for the 1988 B-E-B estimate of abundance, Equation (9) could be extended by including a second term related to the associated estimate of survey bias.

Table 9 presents results for analyses based on the 'backwards' approach. Results are shown in this table for the 'reference' method for incorporating the abundance data in the likelihood function (Equation 3) and six alternative methods (see Equations 4 to 9). The results based on Equations 4-9 indicate a slightly less productive population and hence lower values for RY (1998) and Qo (1998). It should be noted that the abundance estimates for these latter analyses are not identical to those upon which ROI is based (contrast the estimates in Tables 8 and A.1). However, this is not the only reason for the differences in Table 9 because Punt and Butterworth (1996) show that incorporating the N_4/P_4 data in Table 8 into the assessment as absolute indices of abundance (cf. Equation 4) also leads to less optimistic results.

The results become slightly less optimistic if account is taken of the correlation among the estimates of abundance. Treating the abundance estimates as relative (Equations 6 and 7) rather than as absolute indices of abundance (Equations 4 and 5) or including a prior for the bias factor (Equations 8 and 9) increases the widths of the 90% credibility intervals slightly. However, the posterior means and medians are not impacted markedly by this change. Including a prior on the bias factor (our preferred approach) leads to results that are intermediate in terms of the widths of the 90% credibility intervals between those which treat all of the abundance estimates as absolute and those which treat all of the abundance data as relative. The posterior medians for Slope, RY (1998) and Q_0 (1998) for our preferred approach are also intermediate.

CONDUCTING THE POPULATION PROJECTION FOR RECENT YEARS ONLY

Assessments of the B-C-B bowhead stock have been conducted under the assumptions that, at the start of the catch series (1848), the population was at pre-exploitation equilibrium and that the carrying capacity of the bowhead population has not changed over time. An alternative to this set of assumptions is to assume instead that the population had a stable age-structure in some more recent year (see Punt (1999) for details of how this is implemented *for* the Baleen II model). The assessments of the Eastern North Pacific stock of gray whales are based on this latter assumption (Punt and Butterworth, 1997b; Wade, 1997; 1999).

One arguable advantage of this approach to conducting assessments of the B-C-B bowhead stock is that it becomes possible to place a (joint) prior distribution on K_{1+} and the $_{1+}$

Table 9

Estimates of eight management-related quantities for the Bering-Chukchi-Beaufort Seas stock of bowhead whales based on the 'backwards' approach. The point estimates given are posterior medians, followed by posterior means in round parentheses. Posterior 90% credibility intervals are given in square parentheses. The analyses in this table differ in how the abundance data are included in the likelihood function.

	K_{1+}	RY (1998)	Q_0 (1998)	P^{1+}_{1998}/K_{1+}	P^{f}_{1998}/K^{f}	$P^{1+}_{1998}/MSYL_{1+}$	$MSYR_{1+}$	*Slope*
Post-model-pre-data	14,956 (14,897)	154 (159)	149 (156)	55.7 (57.4)	39.8 (40.5)	84.0 (84.7)	1.72 (1.87)	1.60 (1.74)
	[11,401 18,328]	[95 258]	[91 262]	[38.4 87.8]	[28.1 59.0]	[55.3 122.1]	[1.06 3.48]	[0.92 3.39]
Reference case	12,631 (12,863)	211 (209)	209 (207)	71.2 (70.9)	45.9 (46.1)	100.2 (99.5)	2.59 (2.58)	2.49 (2.48)
	[10,924 16,531]	[141 273]	[136 279]	[53.5 90.1]	[37.8 57.9]	[76.8 122.8]	[1.51 3.78]	[1.39 3.67]
Eqn. 4 likelihood	12,776 (13,026)	210 (206)	203 (201)	64.7 (64.6)	41.1 (41.5)	92.0 (91.4)	2.51 (2.51)	2.47 (2.47)
	[10,900 16,577]	[127 277]	[122 280]	[47.0 86.6]	[34.1 53.0]	[68.2 117.1]	[1.43 3.90]	[1.33 3.80]
Eqn. 5 likelihood	13,137 (13,397)	203 (198)	195 (192)	62.9 (63.0)	40.5 (41.1)	90.0 (89.8)	2.36 (2.37)	2.31 (2.31)
	[11,062 17,125]	[114 275]	[111 274]	[45.3 84.8]	[33.7 52.2]	[66.3 115.6]	[1.27 3.68]	[1.15 3.67]
Eqn. 6 likelihood	12,838 (13,084)	202 (199)	201 (199)	65.5 (65.6)	42.2 (42.8)	92.7 (92.4)	2.49 (2.50)	2.41 (2.41)
	[10,836 16,816]	[124 279]	[121 282]	[44.3 91.7]	[29.6 61.4]	[64.4 124.0]	[1.39 3.91]	[1.26 3.82]
Eqn. 7 likelihood	13,184 (13,413)	194 (192)	193 (191)	63.6 (64.1)	41.8 (42.3)	90.7 (90.8)	2.34 (2.37)	2.25 (2.28)
	[10,952 17,286]	[117 276]	[113 280]	[43.0 91.3]	[29.2 60.7]	[62.8 123.1]	[1.28 3.83]	[1.16 3.75]
Eqn. 8 likelihood	12,854 (13,066)	208 (203)	204 (202)	68.3 (68.2)	44.0 (44.4)	96.3 (96.1)	2.49 (2.51)	2.42 (2.42)
	[10,749 16,878]	[127 269]	[122 285]	[49.1 91.0]	[36.0 56.9]	[71.7 121.4]	[1.38 4.04]	[1.28 3.82]
Eqn. 9 likelihood	13,195 (13,429)	200 (196)	195 (193)	66.2 (66.5)	43.4 (44.0)	94.9 (94.4)	2.33 (2.36)	2.25 (2.27)
	[10,981 17,327]	[119 267]	[114 279]	[47.8 89.8]	[35.7 56.4]	[69.4 121.8]	[1.27 3.85]	[1.15 3.71]

Table 10

Estimates of eight management-related quantities for the Bering-Chukchi-Beaufort Seas stock of bowhead whales based on the 'backwards' approach. The point estimates given are posterior medians, followed by posterior means in round parenthesis. Posterior 90% credibility intervals are given in square parenthesis. This table includes results for analyses that start the population trajectories from a more recent year (y_1) rather than from an assumed pre-exploitation equilibrium in 1848.

	K_{1+}	RY (1998)	Q_0 (1998)	P^{1+}_{1998}/K_{1+}	P^{f}_{1998}/K^{f}	$P^{1+}_{1998}/MSYL_{1+}$	$MSYR_{1+}$	*Slope*
Reference case	12,631 (12,863)	211 (209)	209 (207)	71.2 (70.9)	45.9 (46.1)	100.2 (99.5)	2.59 (2.58)	2.49 (2.48)
	[10,924 16,531]	[141 273]	[136 279]	[53.5 90.1]	[37.8 57.9]	[76.8 122.8]	[1.51 3.78]	[1.39 3.67]
y_1 = 1930	17,920 (18,710)	241 (245)	239 (245)	50.7 (54.1)	31.9 (34.9)	73.7 (76.8)	2.46 (2.47)	2.55 (2.53)
	[10,002 30,097]	[127 401]	[136 399]	[30.8 91.5]	[19.2 63.8]	[43.7 123.0]	[1.48 3.68]	[1.41 3.81]
y_1 = 1950	16,190 (16,482)	234 (238)	234 (239)	56.5 (59.3)	35.7 (38.4)	80.5 (83.4)	2.51 (2.52)	2.54 (2.53)
Max K_{1+} = 25,000	[9,637 24,453]	[118 397]	[132 393]	[37.7 93.2]	[23.5 67.2]	[52.7 124.4]	[1.49 3.69]	[1.35 3.79]
y_1 = 1950	18,541 (19,032)	244 (248)	242 (248)	49.3 (53.3)	30.9 (34.3)	71.8 (75.4)	2.48 (2.48)	2.57 (2.54)
Max K_{1+} = 31,000	[10,038 30,114]	[125 402]	[137 400]	[30.7 91.9]	[19.1 63.8]	[43.5 123.0]	[1.48 3.68]	[1.39 3.83]
y_1 = 1950	23,188 (25,191)	253 (256)	251 (256)	39.6 (44.8)	24.6 (28.7)	57.5 (63.9)	2.42 (2.42)	2.58 (2.56)
Max K_{1+} = 50,000	[10,276 48,051]	[134 411]	[141 409]	[19.7 90.2]	[12.2 62.4]	[28.6 121.4]	[1.44 3.62]	[1.44 3.77]
y_1 = 1960	18.415 (19.027)	247 (247)	243 (248)	49.5 (53.4)	30.9 (34.4)	72.0 (75.6)	2.48 (2.48)	2.57 (2.55)
	[9.847 30.148]	[124 401]	[135 398]	[30.7 92.3]	[19.0 65.1]	[43.5 123.0]	[1.50 3.66]	[1.39 3.82]
y_1 = 1970	18.417 (19.044)	247 (248)	244 (248)	49.6 (53.3)	31.1 (34.3)	72.0 (75.5)	2.49 (2.48)	2.58 (2.55)
	[9.921 30.202]	[124 401]	[136 399]	[30.8 91.8]	[19.0 64.4]	[43.5 123.0]	[1.49 3.65]	[1.37 3.79]

population size in 1993, P_{1993}[16]. Thus, the problem of having to choose between the 'backwards' and 'forwards' approaches is eliminated. Other arguable advantages are that results are no longer dependent on values for catches during the early period of the fishery, which have had to be estimated in the absence of specific records, and that the possibility of regime shifts (tantamount to changes in K over time) is admitted. Four alternative choices for the first year in the analysis, y_1, are considered (1930, 1950, 1960, and 1970). The prior for K_{1+} is taken to be that for the 'reference case', $\ell n K_{1+} \sim U[\ell n 7000, \ell n 31000]$. The sensitivity of the results to the choice for the upper end of the prior for K_{1+} is explored by changing it from 31,000 to 25,000 and to 50,000.

Table 10 lists results for six sensitivity tests that do not start the population projections from deterministic pre-exploitation equilibrium in 1848, but assume instead that the population had a stable age-structure in some more recent year. For comparability with the 'reference case', the sensitivity tests are based on the 'reference' likelihood of Equation 3. The results are not particularly sensitive to the choice of y_1. However, there are notable differences between the results for the 'reference case' and those for the six sensitivity tests. For example, the posterior distributions for Q_0 (1998) and RY (1998) have longer tails at high values and the posterior for the pre-exploitation size does not differ much from its prior. One consequence of the latter result is that the depletion- and population size-related results depend strongly on the upper bound of the prior assumed for K_{1+}. For example, the posterior median for P^{1+}_{1998}/K_{1+} drops from 56.5 to 39.6 (%) as the upper bound for K_{1+} is increased from 25,000 to 50,000. In contrast, the posterior medians for RY (1998), Q_0 (1998), *Slope*, and $MSYR_{1+}$ do not depend notably on the upper bound for K_{1+}.

Thus, not unexpectedly, dropping the assumption that the population was at its pre-exploitation level in 1848 (and that all the historical catches are known exactly) leads to much wider 90% credibility intervals for all quantities except *Slope* and $MSYR_{1+}$. This effect is perhaps most notable for RY (1998) for which the 90% credibility interval is (roughly) [125, 400] for the sensitivity tests compared to [141, 273] for the 'reference case'. It is perhaps notable that the lower 5%ile for Q_0 (1998) hardly differs among the analyses

[16] This needs to be a joint prior distribution because the *a priori* constraint that $P_{1993} < K$ must be imposed (there being negligible probability that possible oscillatory behaviour of the population trajectory as a result of time-lags in the dynamics could see the population above K in recent years).

although the upper 95%ile differs markedly between the 'reference case' and the sensitivity tests. The posterior median for the *Slope* statistic for the sensitivity tests is slightly closer to the 'observed' value in Table 2.

OUR PREFERRED ANALYSIS

The advantages of starting the population projections in some more recent year are that it is not necessary to assume that the carrying capacity of the bowhead population has remained unchanged over the last 150 years and that all the historical catches are known exactly. We believe that analyses should be conducted for both options, i.e. assuming that the population was at its pre-exploitation level in 1848 and assuming that it had a stable age-structure in some more recent year. Results are presented for the case $y_1 = 1950$ for the analyses that start the projections in a recent year as such results are not very sensitive to the choice for y_1. The results for this assessment are shown for the 'reference case' prior for K_{1+}. This choice is essentially arbitrary, so little confidence can be placed in the results for the depletion- and population size-related quantities because the posteriors for these quantities depend critically on the choice of a prior distribution for K_{1+}, which is barely updated by the data (Table 10). However, Table 10 does also show that the key management-related quantities *RY* (1998) and Q_0 (1998) are relatively insensitive to variations in the specification of a prior for K_{1+}, so that this approach retains utility.

We prefer the 'backwards' to the 'forwards' approach for three main reasons: the simulation test results show a clear preference for 'backwards', any assessment based on 'forwards' which also places a prior on the abundance in a recent year is subject to the Borel paradox, and 'forwards' updates the prior for *MSYR* before any data are included in the assessment. As noted above, although the full pooling approach of Raftery and Poole (1997) and Poole and Raftery (1998) resolves the Borel paradox, in doing so it introduces a new problem of how to choose pooling weights (i.e. how to specify their relative reliabilities of the priors for K_{1+} and P_{1993}). λ_{max} (the maximum rate of population increase, which occurs at low population size) is our preferred choice for the productivity parameter, because nearly all the data available from other whale species upon which to base a productivity prior constitute observations of this quantity. We tentatively suggest a U[0, 0.127] prior for λ_{max} for reasons discussed above.

We prefer the likelihood defined by Equation (9) because it treats the abundance estimates in Table A.1 as relative indices of abundance and because it incorporates the (independent) information about the bias factor explicitly in the likelihood. Equation (9) is preferred to Equation (8) because it takes account of the correlation among the abundance estimates.

Table 11 lists results for the 'reference case' 'backwards' analysis and the two 'preferred' variants. In addition to providing results for the eight quantities listed above, results are also shown for S_{juv}, S_{adult}, a_m, f_{max}, P_{1993}, $MSYL/K_{1+}$ $MSYR_{mat}$, and λ_{max}. The results for $MSYL/K_{1+}$, $MSYR_{mat}$ and λ_{max} are presented as percentages. As expected from the results of Table 9 which show that the use of the Equation 9 likelihood leads to notably lower estimates of productivity, the results for the 'preferred' analyses are less optimistic than those for the 'reference case'. The lesser productivity of the resource is reflected by lower posterior medians for *MSYR* and λ_{max}. This appears to be a reflection primarily of lower values for S_{juv} and $MSYL/K_{1+}$ - the posterior medians for which drop from 0.94 and 0.72 respectively for the 'reference case' to 0.93 and 0.70/0.69 for the two 'preferred' variants. Although the results for the population size- and depletion-related quantities from the $y_1 = 1950$ variant are unreliable owing to their sensitivity to the choice of a prior for K_{1+}, the posteriors for the biological parameters are remarkably similar for the two preferred analyses. The lower 5%iles for *RY* (1998) and Q_0 (1998), although insensitive to the choice of y_1, are notably lower than those for the 'reference case' (only about 80 compared to some 140). This is a consequence of a smaller current population size and lower productivity. However, these lower 5%iles are all larger than the current annual strike limit for the B-C-B stock of 67.

Table 11

Estimates of the eight management-related quantities considered previously and eight further biological parameters/variables for the Bering-Chukchi-Beaufort Seas stock of bowhead whales based on the 'backwards' approach. The point estimates given are posterior medians, followed by posterior means in round parentheses. Posterior 90% credibility intervals are given in square parentheses. Results are shown in this table for the reference case and the two 'preferred' analyses.

	Estimation procedure					
Quantity	'Backwards' reference case		Preferred - y_1=1848		Preferred - y_1=1950	
K_{1+}	12,631 (12,863)	[10,924 16,531]	13,353 (1,3935)	[10,998 20,881]	17,979 (18,660)	[9,692 30,322]
RY (1998)	211 (209)	[141 273]	195 (186)	[84 265]	207 (216)	[83 413]
Q_0 (1998)	209 (207)	[136 279]	189 (183)	[81 275]	206 (215)	[82 412]
P^{1+}_{1998}/K_{1+}	71.2 (70.9)	[53.5 90.1]	65.2 (64.4)	[41.0 88.5]	49.6 (52.7)	[29.5 91.5]
P^{f}_{1998}/K^{f}	45.9 (46.1)	[37.8 57.9]	42.9 (43.2)	[33.1 56.3]	32.6 (35.4)	[19.4 66.2]
$P^{1+}_{1998}/MSYL_{1+}$	100.2 (99.5)	[76.8 122.8]	94.1 (92.9)	[65.1 121.3]	73.5 (76.6)	[43.3 124.5]
$MSYR_{1+}$	2.59 (2.58)	[1.51 3.78]	2.27 (2.25)	[0.88 3.78]	2.22 (2.21)	[0.94 3.65]
Slope	2.49 (2.48)	[1.39 3.67]	2.19 (2.15)	[0.75 3.63]	2.24 (2.23)	[0.74 3.87]
$MSYL/K_{1+}$	0.72 (0.71)	[0.62 0.80]	0.70 (0.69)	[0.57 0.79]	0.69 (0.69)	[0.56 0.79]
S_{juv}	0.937 (0.926)	[0.827 0.985]	0.928 (0.917)	[0.810 0.983]	0.928 (0.917)	[0.813 0.983]
S_{adult}	0.988 (0.987)	[0.975 0.995]	0.986 (0.985)	[0.970 0.995]	0.986 (0.985)	[0.971 0.995]
a_m	20 (20)	[16 25]	20 (20)	[16 25]	20 (20)	[16 25]
f_{max}	0.31 (0.31)	[0.26 0.39]	0.31 (0.32)	[0.26 0.39]	0.30 (0.31)	[0.26 0.39]
λ_{max}	5.14 (5.16)	[3.63 6.95]	4.74 (4.78)	[2.95 7.09]	4.77 (4.77)	[3.00 6.88]
P_{1993}	8,212 (8,195)	[7,262 9,250]	7,984 (7,988)	[7,246 8,887]	8,009 (8,016)	[7,230 9,031]
$MSYR_{mat}$	4.86 (4.89)	[2.77 7.55]	4.21 (4.22)	[1.64 7.33]	4.12 (4.14)	[1.72 7.16]

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Appendix A

CONSTRUCTING INDICES OF ABUNDANCE FOR THE B-C-B BOWHEAD STOCK

The data available for the construction of indices of abundance for the B-C-B bowhead stock include estimates of N_4 (the observed number of bowheads passing within visual range) for ten years and estimates of P_4 (the observed proportion passing within visual range) for five of these years (Table 8). Previous analyses of these data (e.g. Raftery *et al.* (1995b); Zeh *et al.* (1995)) have assumed that the proportion passing within visual range for the years for which estimates of P_4 are not available is equal to the average over the years for which such estimates are available. Cooke (1996) criticised the methods used by Raftery *et al.* (1995b) and Zeh *et al.* (1995) because they treat the observation and process errors in an *ad hoc* manner. He proposed the following statistical model for the analysis of the data in Table 8:

$$N_{4y} = P_y p_y e^{\nu_y} \qquad \nu_y \sim N(0;\sigma^2_{\nu,y}) \qquad \text{(A.1)}$$

$$P_{4y} = p_y e^{\varepsilon_y} \qquad \varepsilon_y \sim N(0;\sigma^2_{\varepsilon,y}) \qquad \text{(A.2)}$$

$$p_y = \pi e^{\eta_y} \qquad \eta_y \sim N(0;\sigma^2_\eta) \qquad \text{(A.3)}$$

where N_{4y} is the estimate of N_4 for year y,
P_{4y} is the estimate of P_4 for year y,
P_y is the size of the 1+ component of the population in year y,
p_y is the proportion of the 1+ component of the population within visual range in year y,
π is the mean proportion of the 1+ component of the population within visual range (the actual proportion varies from year to year as a consequence of process error),
$\sigma^2_{\nu,y}$ is the variance of the logarithm of N_{4y} (reflecting observation error),
$\sigma^2_{\varepsilon,y}$ is the variance of the logarithm of P_{4y} (reflecting observation error), and
σ^2_η is the process error variance.

The parameters of this model are P_y and p_y for each of the ten years, π, and σ^2_η. The estimates of the 22 parameters of this model are obtained by maximum likelihood. This involves finding the values for the parameters that minimise the following negative log-likelihood (after removal of constants):

$$-\ell nL = \sum_y \frac{(\ell nN_{4y} - \ell n(P_y p_y))^2}{2\sigma^2_{\nu,y}} + \sum_y \frac{(\ell nP_{4y} - \ell np_y)^2}{2\sigma^2_{\varepsilon,y}} + \sum_y \frac{(\ell np_y - \ell n\pi)^2}{2\sigma^2_\eta} + \tfrac{1}{2}\ell n\left\|1 + \sigma_\eta U\right\|$$

where U is the reduced information matrix (which accounts for the impact of the estimation of the nuisance parameters - see Seber and Wild, 1989).

The first and third summations are over all ten years and the second summation is over the five years for which estimates of P_4 are available. The maximum likelihood estimate of p_y for those years for which estimates of P_4 are not available is π.

Table A.1 lists the estimates of the P_y, the estimates of the asymptotic standard errors of ℓnP_y and the correlation matrix for ℓnP_y that result from the implementation of Cooke' s approach.

Table A.1

Estimates, CVs (actually standard errors of the logarithms), and the correlation matrix for the indices of abundance for the Bering-Chukchi-Beaufort bowhead stock. Values are based on the estimation procedure described in Appendix 1.

Year	Estimate	CV	Correlation matrix									
1978	4,820	0.273	1.000									
1980	3,900	0.314	0.166	1.000								
1981	4,389	0.253	0.054	0.047	1.000							
1982	6,572	0.311	0.168	0.146	0.047	1.000						
1983	6,268	0.321	0.163	0.141	0.046	0.143	1.000					
1985	5,132	0.269	0.126	0.109	0.025	0.110	0.107	1.000				
1986	7,251	0.186	0.080	0.070	0.012	0.070	0.068	0.108	1.000			
1987	5,151	0.298	0.175	0.152	0.049	0.154	0.149	0.115	0.074	1.000		
1988	6,609	0.113	0.038	0.033	0.012	0.033	0.032	0.018	0.009	0.035	1.000	
1993	7,778	0.071	0.002	0.001	0.001	0.001	0.001	-0.002	-0.002	0.001	0.001	1.000