Assessment of the South African Sardine Resource

C.L. de Moor*

Two population dynamics models were used for the base case South African sardine resource (Cunningham and Butterworth 2007, with further undocumented updates). The data used in these assessments are listed in Cunningham *et al*. 2007. The first assessment, detailed in Appendix A, uses the commercial proportion-at-length data together with a cohort-dependent two-straight line growth curve to estimate commercial selectivity at age, where this varied by quarter for ages 1 and 2. Selectivity at age 0 is estimated biennially. Given the lack of age data in this assessment, the model struggled to estimate realistic selectivity values and thus a constraint was included in the model so that selectivities at ages 2, 3 and 4 would not differ drastically (see page 10). Given that:

i) a large number of parameters were required in the estimation of the growth curve and selectivities,

ii) the commercial length data were not very informative, leading to estimation problems, and

iii) only selectivities at ages 1 to 5+ are required for OMP testing,

it was decided to use the average (over quarters) selectivities-at-age output from this assessment model as fixed inputs to a more stable assessment which excluded the commercial length data altogether. This assessment is detailed in Appendix B and was used to provide the operating models for OMP testing to avoid likely estimation problems with the former assessment procedure for MCMC evaluations of posteriors. In this model, bycatch is assumed to comprise 0 year old fish only, and directed catch 1+ year old fish only.

For both assessments an informative prior was used for the multiplicative bias in the November acoustic survey with the prior distributions for the remaining estimated parameters chosen to be relatively uninformative.

References

Cunningham, C.L., and Butterworth, D.S. 2007. Base Case Assessment of the South African Sardine Resource. MCM Document MCM/2007/SEP/SWG-PEL/06. 30pp.

Cunningham, C.L., van der Westhuizen, J.J., Durholtz D. and Coetzee, J. 2007. A Record of the Generation of Data Used in the Sardine and Anchovy Assessments. MCM Document MCM/2007/SEPT/SWG-PEL/03. 28pp.

* MARAM (Marine Resource Assessment and Management Group), Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch, 7701, South Africa. Email: c.l.demoor@telkomsa.net.

Appendix A: Bayesian Assessment Model for the South African Sardine Resource, Including Catch at Length Data

Base Case Model Assumptions

1) All fish have a theoretical birthdate of 1 November.
2) Sardine spawn for the first time (and are called adult sardine) when they turn two years old.
3) A plus group of age five is chosen.
4) Two surveys are held each year: the first takes place in November (known as the November survey) and surveys the adult stock; the second is in May/June (known as the recruit survey) and surveys juvenile sardine (also called recruits or 0-year-old sardine).
5) The November survey provides a relative index of abundance of known bias.
6) The recruit survey provides a relative index of abundance of unknown bias.
7) The survey strategy is such that it results in surveys of invariant bias over time.
8) Pulse fishing occurs four times a year, in the middle of each quarter after the birthdate.
9) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

Population Dynamics

Numbers-at-age at 1 November

$$N^S_{y,a} = \left(\left(\left(\left((N^S_{y-1,a-1}e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1})e^{-M^S_{a-1}/4}\right) - \hat{C}^S_{y,2,a-1}\right)e^{-M^S_{a-1}/4} - \hat{C}^S_{y,3,a-1}\right)e^{-M^S_{a-1}/4} - \hat{C}^S_{y,4,a-1}\right)e^{-M^S_{a-1}/8}$$

$$y = 1984,\ldots,2006\,,\ a = 1,\ldots,4$$

$$N^S_{y,5+} = \left(\left(\left(\left((N^S_{y-1,4}e^{-M^S_4/8} - \hat{C}^S_{y,1,4})e^{-M^S_4/4}\right) - \hat{C}^S_{y,2,4}\right)e^{-M^S_4/4} - \hat{C}^S_{y,3,4}\right)e^{-M^S_4/4} - \hat{C}^S_{y,4,4}\right)e^{-M^S_4/8}$$
$$+\left(\left(\left(\left((N^S_{y-1,5+}e^{-M^S_{5+}/8} - \hat{C}^S_{y,1,5+})e^{-M^S_{5+}/4}\right) - \hat{C}^S_{y,2,5+}\right)e^{-M^S_{5+}/4} - \hat{C}^S_{y,3,5+}\right)e^{-M^S_{5+}/4} - \hat{C}^S_{y,4,5+}\right)e^{-M^S_{5+}/8}$$

$$y = 1984,\ldots,2006 \qquad \text{(A.1)}$$

where

$N^S_{y,a}$ is the number (in billions) of sardine of age *a* at the beginning of November in year *y*;

$\hat{C}^S_{y,q,a}$ is the estimated number (in billions) of sardine of age *a* caught during quarter q of year y ($q=1$ for November $y-1$ to January y, $q=2$ for February to April y, $q=3$ for May to July y and $q=4$ for August to October y);

M^S_a is the rate of natural mortality (in year^{-1}) of sardine of age a.

Biomass associated with the November survey

$$\hat{B}^S_{y,N} = k^S_N \sum_{a=1}^{5+} N^S_{y,a} w^S_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(A.2)}$$

where

$\hat{B}^S_{y,N}$ is the biomass (in thousand tonnes) of adult sardine at the beginning of November in year *y*, associated with the November survey;

k^S_N is the constant of proportionality (multiplicative bias) associated with the November survey; and

$w^S_{y,a}$ is the mean mass (in grams) of sardine of age *a* sampled during the November survey of year *y*.

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB^S_{y,N} = \sum_{a=2}^{5+} N^S_{y,a} w^S_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(A.3)}$$

Catch

The catch at age by number is calculated using Pope's approximation (Pope 1984):

$$\hat{C}^S_{y,1,a} = N^S_{y-1,a} e^{-M^S_a/8} S_{y,1,a} F_{y,1}, \; a = 0,\ldots,5+$$

$$\hat{C}^S_{y,2,a} = \left(N^S_{y-1,a} e^{-M^S_a/8} - \hat{C}^S_{y,1,a} \right) e^{-M^S_a/4} S_{y,2,a} F_{y,2}, \; a = 0,\ldots,5+$$

$$\hat{C}^S_{y,3,a} = \left(\left(N^S_{y-1,a} e^{-M^S_a/8} - \hat{C}^S_{y,1,a} \right) e^{-M^S_a/4} - \hat{C}^S_{y,2,a} \right) e^{-M^S_a/4} S_{y,3,a} F_{y,3}, \; a = 0,\ldots,5+$$

$$\hat{C}^S_{y,4,a} = \left(\left(\left(N^S_{y-1,a} e^{-M^S_a/8} - \hat{C}^S_{y,1,a} \right) e^{-M^S_a/4} - \hat{C}^S_{y,2,a} \right) e^{-M^S_a/4} - \hat{C}^S_{y,3,a} \right) e^{-M^S_a/4} S_{y,4,a} F_{y,4}, \; a = 0,\ldots,5+ \qquad \text{(A.4)}$$

where

$S_{y,q,a}$ is the commercial selectivity at age a during quarter q of year y, which is assumed to be year-independent for ages 1+ (age 0 landings are mostly bycatch which vary year-to-year independent of the fishing mortality on the older fish in the directed fishery); and

$F_{y,q}$ is the fished proportion in quarter q of year y for a fully selected age class a.

In the above equations the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion is estimated by:

$$F_{y,1} = \frac{C^{ObsTon}_{y,1}}{\sum_{a=0}^{5+}\left(\frac{7}{8}w^S_{y-1,a} + \frac{1}{8}w^S_{y,a+1}\right)N^S_{y-1,a}e^{-M^S_a/8}S_{y,1,a}}$$ [1]

$$F_{y,2} = \frac{C^{ObsTon}_{y,2}}{\sum_{a=0}^{5+}\left(\frac{5}{8}w^S_{y-1,a} + \frac{3}{8}w^S_{y,a+1}\right)\left(N^S_{y-1,a}e^{-M^S_a/8} - \hat{C}^S_{y,1,a}\right)e^{-M^S_a/4}S_{y,2,a}}$$

$$F_{y,3} = \frac{C^{ObsTon}_{y,3}}{\sum_{a=0}^{5+}\left(\frac{3}{8}w^S_{y-1,a} + \frac{5}{8}w^S_{y,a+1}\right)\left(\left(N^S_{y-1,a}e^{-M^S_a/8} - \hat{C}^S_{y,1,a}\right)e^{-M^S_a/4} - \hat{C}^S_{y,2,a}\right)e^{-M^S_a/4}S_{y,3,a}}$$

$$F_{y,4} = \frac{C^{ObsTon}_{y,4}}{\sum_{a=0}^{5+}\left(\frac{1}{8}w^S_{y-1,a} + \frac{7}{8}w^S_{y,a+1}\right)\left(\left(\left(N^S_{y-1,a}e^{-M^S_a/8} - \hat{C}^S_{y,1,a}\right)e^{-M^S_a/4} - \hat{C}^S_{y,2,a}\right)e^{-M^S_a/4} - \hat{C}^S_{y,3,a}\right)e^{-M^S_a/4}S_{y,4,a}} \quad \text{(A.5)}$$

where

$C^{ObsTon}_{y,q}$ is the observed catch tonnage for quarter q of year y from the RLFs.

Given the predicted proportion-at-age in the quarterly commercial catch

$$\hat{p}^{com}_{y,q,a} = \frac{\hat{C}^S_{y,q,a}}{\sum_{a=0}^{5+}\hat{C}^S_{y,q,a}} \quad \text{(A.6)}$$

the predicted proportion-at-length is then estimated using a two-line growth equation[2]:

$$\hat{p}^{com}_{y,q,l} = \sum_{a=0}^{5+}\hat{p}^{com}_{y,q,a}A^{com}_{y,q,a,l} \qquad y = 1984,\ldots,2006 \quad \text{(A.7)}$$

where

$A^{com}_{y,q,a,l}$ is the proportion of sardine catch-at-age *a* that fall in the length group l (thus $\sum_{l=1}^{6}A_{y,q,a,l} = 1$ for all ages, quarters and years) in quarter q of year y (the quarterly catch-at-length distributions are split into at most 6 length groups).

The matrix A^{com} is calculated under the assumption that length-at-age is normally distributed about a mean given by a two-line growth equation, (Brandão *et al*. 2002):

$$L^{com}_{y,q,a} \sim N\left(L^{Mean}_{y,q,a}, \vartheta^2_a\right) \quad \text{(A.8)}$$

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[1] As no survey weight-at-age is available for 1983, it is assumed that $w^S_{1983,a} = w^S_{1984,a}$, and further that $w^S_{y,0} = 0$.

[2] Initial testing of the model used a von Bertalanffy equation, but this did not allow for satisfactory fits to the observed proportions-at-length, particularly for the older age classes.

where $L^{Mean}_{y,q,a} = \begin{cases} \frac{L_{Diff}}{2}\left(a+0.125+(q-1)/4\right)+L_{Mean0,y-a} & \text{if } a<2 \\ m_{2+}\left(a+0.125+(q-1)/4-2\right)+L_{Mean0,y-a}+L_{Diff} & \text{if } a\geq 2 \end{cases}$

The inflection point at age 2 was chosen after initial testing of the model with independent annual growth curves revealed relatively fast growth prior to age 2 and little growth after age 2. As selectivity is used to calculate quarterly catch which is assumed to be taken in the middle of each quarter, 0.125 is added to age *a*. Here

$L_{Mean0,y}$ denotes the mean length at age 0 in year y;

L_{Diff} denotes the difference between the mean length at age 2 (inflection point) and age 0;

m_{2+} denotes the slope of the growth curve for ages 2+; and

ϑ^2_a denotes the variance about the mean length for age a.

Recruitment

For the base case assessment a Hockey Stick stock-recruitment curve is assumed for all years outside of the "peak" years, during which a constant recruitment (i.e. in respect of distribution median and independent of spawning biomass) is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve. Thus recruitment in November is given by:

$$N^S_{y,0} = \begin{cases} a^S e^{\varepsilon^S_y} & \text{,if } \mathrm{SSB}^S_{y,N} \geq b^S \\ \frac{a^S}{b^S} SSB^S_{y,N} e^{\varepsilon^S_y} & \text{,if } \mathrm{SSB}^S_{y,N} < b^S \end{cases} \qquad y = 1984,\ldots,1999,2005,2006$$

$$N^S_{y,0} = c^S e^{\varepsilon^S_y} \qquad y = 2000,\ldots,2004 \quad \text{(A.9)}$$

where

a^S is the maximum recruitment (in billions) (i.e. median of the distribution in question);

b^S is the spawner biomass above which there should be no recruitment failure risk in the hockey stick model;

c^S is the constant recruitment (distribution median) during the "peak" years of 2000 to 2004; and

ε^S_y is the annual lognormal deviation of sardine recruitment.

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$\hat{N}^S_{y,r} = k^S_r \left(\left((N^S_{y-1,0} e^{-M^S_0/8} - \hat{C}^S_{y,1,0}) e^{-M^S_0/4} - \hat{C}^S_{y,2,0} \right) e^{-0.5t^S_y \times M^S_0/12} - \tilde{C}^S_{y,0bs} \right) e^{-0.5t^S_y \times M^S_0/12} \quad y = 1984,\ldots,2006 \quad \text{(A.10)}$$

where

$\hat{N}^S_{y,r}$ is the number (in billions) of juvenile sardine at the time of the recruit survey in year y;

k^S_r is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}^S_{y,0bs}$ is the observed number (in billions) of juvenile sardine caught between 1 May and the day before the start of the recruit survey, assuming a 15.5cm cut-off length; and

t^S_y is the time lapsed (in months) between 1 May and the start of the recruit survey in year y.

Fitting the Model to Observed Data (Likelihood)

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions. Thus the contribution of the survey abundance data to the negative log-likelihood function is given by:

$$\begin{aligned} -\ln L^{surv} &= \tfrac{1}{2} \sum_{y=1984}^{2006} \left\{ \frac{\left(\ln B^S_{y,N} - \ln(\hat{B}^S_{y,N})\right)^2}{(\sigma^S_{y,Nov})^2 + (\lambda^S_N)^2} + \ln\left[2\pi\left((\sigma^S_{y,Nov})^2 + (\lambda^S_N)^2\right)\right] \right\} \\ &+ \tfrac{1}{2} \sum_{y=1985}^{2006} \left\{ \frac{\left(\ln N^S_{y,r} - \ln(\hat{N}^S_{y,r})\right)^2}{(\sigma^S_{y,rec})^2 + (\lambda^S_r)^2} + \ln\left[2\pi\left((\sigma^S_{y,rec})^2 + (\lambda^S_r)^2\right)\right] \right\} \end{aligned} \quad \text{(A.11)}$$

where

$B^S_{y,N}$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass from the November survey in year y, with associated CV $\sigma^S_{y,Nov}$;

$N^S_{y,r}$ is the acoustic survey estimate (in billions) of sardine recruitment numbers from the recruit survey in year y, with associated CV $\sigma^S_{y,rec}$; and

$(\lambda^S_{N/r})^2$ is the additional variance (over and above the survey sampling CV $\sigma^S_{y,Nov/rec}$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

The commercial proportions at length from the raised length frequencies are assumed to be lognormally distributed; their contribution to the negative log-likelihood function is given by:

$$-\ln L^{prop} = w_{com}\sum_{y=1984}^{2006}\sum_{q=1}^{4}\sum_{l=1}^{l\max(y,q)}\left\{\frac{p_{y,q,l}^{com}\left(\ln p_{y,q,l}^{com}-\ln \hat{p}_{y,q,l}^{com}\right)^2}{2\left(\sigma_{com}^{S}\right)^2}+\ln\left(\frac{\sigma_{com}^{S}}{\sqrt{p_{y,q,l}^{com}}}\right)\right\}^{3} \qquad \text{(A.12)}$$

where

$p_{y,q,l}^{com}$ is the observed proportion (by number) of the commercial catch in length group l of during quarter q ($q=1$ for Nov-Jan, $q=2$ for Feb-Apr, $q=3$ for May-Jul, $q=4$ for Aug-Oct) of year y;

w_{com} is the weighting applied to the commercial proportion at length data;

σ_{com}^{S} is the standard deviation associated with the proportion-at-length data in the commercial catch, which is estimated in the fitting procedure by:

$$\sigma_{com}^{S} = \sqrt{\sum_{y=1984}^{2006}\sum_{q=1}^{4}\sum_{l=1}^{l\max(y,q)} p_{y,q,l}^{com}\left(\ln p_{y,q,l}^{com}-\ln \hat{p}_{y,q,l}^{com}\right)^2 \Big/ \sum_{y=1984}^{2006}\sum_{q=1}^{4}\sum_{l=1}^{l\max(y,q)}1} \ .$$

No proportion-at-length was fitted for the fourth quarter in 1984, 1985, 1986 and 1989 as the tonnage landed during this quarter was less than 4% of that for the year. The raw data are recorded by 0.5cm length classes from 3.5cm to 23cm. The data were combined to form six length groups: a minus group of 10.49cm, 10.5cm – 13.99cm, 14.0cm – 17.49cm, 17.5cm – 18.49cm, 18.5cm – 19.49cm and a plus group of 19.5cm. In some quarters, the proportion-at-length in these length groups was small (<2%) so that some length groups were further combined (see Cunningham *et al*. 2007).

<u>Fixed Parameters</u>

Six parameters were fixed externally in this assessment:

$M_{ju}^{S}=1$ and $M_{ad}^{S}=0.8$

$w_{com}=0.05$ There were four data points per year for commercial compared to one per year for the survey data. A higher weighting on the commercial proportion-at-length resulted in an unacceptably poor fit to the November survey, which is considered the most reliable source of information.

$L_{Mean0,2005}=5.32$ and $L_{Mean0,2006}=\frac{1}{21}\sum_{y=1984}^{2004}L_{Mean0,y}$, as there were insufficient data to estimate these parameters precisely. The value for $L_{Mean0,2005}$ was obtained from initial fits to the model for which overall convergence could not be confirmed,

To remove the confounding with fishing proportion F in equation (A.4), analyses set $S_4=1$, with $S_{q,a}=S_a$ for *a* = 3, 4 and 5+ and $q=1,\ldots,4$ to stabilize the estimation of selectivity.

[3] Although strictly there may be bias in the proportions of commercial length-at-age (as for the survey length-at-

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. The prior pdfs for the recruitment residuals are given by:

$$\varepsilon_y^S \sim N\left(0,\left(\sigma_r^S\right)^2\right) \quad , \qquad y = 1984,...,2005$$

A probability density function (pdf) for the overall bias in the November survey was calculated by drawing ten thousand samples from the individual pdfs for each source of error (I. Hampton pers. comm.), see Table A.1 and Figure A.1 below. In the last assessment, target strength was included as a source of error. Given that the new target strength expression has been used in the survey data, target strength was removed as a source of error from this bias, substantially narrowing the pdf (Figure A.1). There may, however, still be systematic errors relating to the target strength that are unaccounted for in this pdf. These are taken into account through sensitivity tests using alternative k_N^S values. A normal distribution, using the mean and standard deviation of the pdf was used as a prior for k_N^S, i.e. $k_N^S \sim N\left(0.722, 0.078^2\right)$.

Table A.1. Individual error factors for hydroacoustic surveys of sardine spawner biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to predicted biomass) in this document.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Random[4]
(Beam factor)	0.75	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Random
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

age), no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

[4] Note that for the purposes of this simulation, 'random' and 'variable' factors are treated in the same manner.

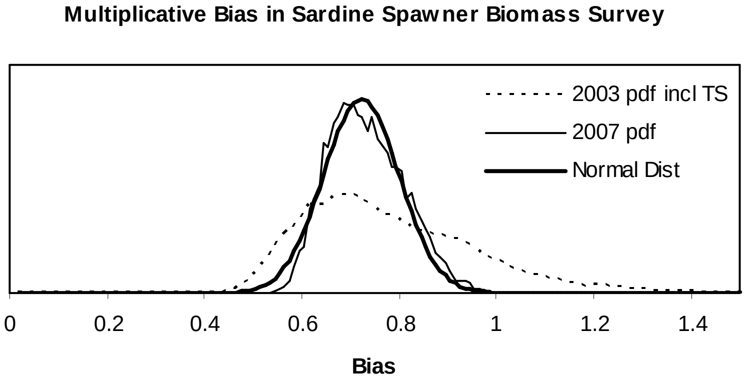

Figure A.1. The probability density function for the overall bias in the sardine November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of error. The normal distribution used as a prior for the bias is also shown. The pdf calculated in 2003 including target strength as a source of error is shown for comparison.

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$\log(k_r^S) \sim U(-100,0.4)$ (upper bound corresponding to $k_r^S = 1.5$)

$(\lambda_N^S)^2 \sim U(0,10)$

$(\lambda_r^S)^2 \sim U(0,10)$

$\log(a^S) \sim U(0,8)$ (given the lack of *a priori* information on the magnitude of a^S, a log-scale was used)

$\log(c^S) \sim U(0,8)$

$b^S / K^S \sim U(0,1)$

$(\sigma_r^S)^2 \sim U(0.4,10)$

$N_{1983,a}^S = N_{1983} \times Nprop_a$, where $N_{1983} \sim U(0,50)$ billion and $Nprop_a \sim U(0,1)$ for $a = 0,\ldots,2$ and $Nprop_3 = Nprop_4 = \frac{1}{2}\left(1 - \sum_{a=0}^{2} Nprop_a\right)$.

$S_{y,1,0} \sim U(0,1)$, with $S_{y,2,0} = S_{y,1,0}$, $y = 1984,\ldots,2006$

$S_{y,3,0} \sim U(0,1)$, with $S_{y,4,0} = S_{y,3,0}$, $y = 1984,\ldots,2006$

$S_{q,1} \sim U(0,1)$, $q = 1,\ldots,4$

$S_{q,2} \sim U(0,1)$, $q = 1,\ldots,4$

$S_3 \sim U(0,1)$, with $S_{q,3} = S_3$ for $q = 1,\ldots,4$

$S_4 \sim U(0,1)$, with $S_{q,4} = S_4$ for $q = 1,\ldots,4$

$S_{5+} \sim U(0,1)$, with $S_{q,5+} = S_{5+}$ for $q = 1,\ldots,4$

Initial testing of the model revealed almost no difference in selectivity by quarter for ages 3+ so that a single selectivity for each age over all quarters was estimated.

$L_{Mean0,y} \sim U(3,12)$ cm , $y = 1984,\ldots,2004$

$L_{Diff} \sim U(0,15)$ cm

$m_{2+} \sim U(0,1)$ cm.age^{-1}

$\vartheta_a^2 \sim U(0,10)$ cm, for $a = 0,1$

$\vartheta_a^2 \sim U(0,4)$ cm, for $a = 2,\ldots,5+$

One further penalty function was required to stabilize the estimates of selectivity at ages 2 and 3 by smoothing the age dependence towards a quadratic form:

$0.1 * \left(S_{q,2} - 2S_{q,3} + S_{q,4}\right)^2,\ q = 1,\ldots,4$

<u>Further Outputs</u>

Recruitment serial correlation:

$$s_{cor}^S = \frac{\sum_{y=1984}^{2004} \varepsilon_y \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{2004} \varepsilon_y^2\right)\left(\sum_{y=1984}^{2004} \varepsilon_{y+1}^2\right)}} \tag{A.13}$$

and the standardised recruitment residual value for 2005:

$$\eta_{2005}^S = \frac{\varepsilon_{2005}^S}{\sigma_r^S} \tag{A.14}$$

are also required as input into the OM.

A separate carrying capacity, K^S (essentially the B_N^S value where the replacement line and the stock recruit function intersect) is calculated representing the period of peak abundance (2000 – 2004) to that for the remaining years:

$$K_{normal}^S = a^S e^{\frac{1}{2}\left(\sigma_r^S\right)^2} \left[\sum_{a=1}^{4} \overline{w}_{normal,a}^S e^{-\sum_{a=0}^{a=1} M_a^S} + \overline{w}_{normal,5+}^S e^{-\sum_{a=0}^{4} M_a^S} \frac{1}{1 - e^{-M_{5+}^A}} \right] \tag{A.15}$$

$$K_{peak}^S = c^S e^{\frac{1}{2}\left(\sigma_r^S\right)^2} \left[\sum_{a=1}^{4} \overline{w}_{peak,a}^S e^{-\sum_{a=0}^{a=1} M_a^S} + \overline{w}_{peak,5+}^S e^{-\sum_{a=0}^{4} M_a^S} \frac{1}{1 - e^{-M_{5+}^A}} \right] \tag{A.16}$$

(calculated assuming maximum recruitment in the absence of fishing) where

$\overline{w}_{normal,a}^S$ is the mean mass (in grams) of sardine of age *a* sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available outside of the peak years (i.e. 1993, 1994, 1996 and 2006).

$\overline{w}^{S}_{peak,a}$ is the mean mass (in grams) of sardine of age *a* sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available during the peak period (i.e. 2001 - 2004).

The $e^{\frac{1}{2}\left(\sigma_r^S\right)^2}$ factor in the above equation is a bias correction factor, needed given the assumption that recruitment is log-normally distributed about an underlying stock-recruit curve.

Appendix B: Bayesian Assessment Model for the South African Sardine Resource, Excluding Catch At Length Data

Base Case Model Assumptions

These are identical to the assumptions listed in Appendix A.

Population Dynamics

Numbers-at-age at 1 November

$$N^S_{y,a} = \left(\left(\left(\left((N^S_{y-1,a-1}e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1})e^{-M^S_{a-1}/4}\right) - \hat{C}^S_{y,2,a-1}\right)e^{-M^S_{a-1}/4} - \hat{C}^S_{y,3,a-1}\right)e^{-M^S_{a-1}/4} - \hat{C}^S_{y,4,a-1}\right)e^{-M^S_{a-1}/8}$$

$$y = 1984,\ldots,2006, \quad a = 1,\ldots,4$$

$$N^S_{y,5+} = \left(\left(\left(\left((N^S_{y-1,4}e^{-M^S_4/8} - \hat{C}^S_{y,1,4})e^{-M^S_4/4}\right) - \hat{C}^S_{y,2,4}\right)e^{-M^S_4/4} - \hat{C}^S_{y,3,4}\right)e^{-M^S_4/4} - \hat{C}^S_{y,4,4}\right)e^{-M^S_4/8}$$
$$+\left(\left(\left(\left((N^S_{y-1,5+}e^{-M^S_{5+}/8} - \hat{C}^S_{y,1,5+})e^{-M^S_{5+}/4}\right) - \hat{C}^S_{y,2,5+}\right)e^{-M^S_{5+}/4} - \hat{C}^S_{y,3,5+}\right)e^{-M^S_{5+}/4} - \hat{C}^S_{y,4,5+}\right)e^{-M^S_{5+}/8}$$

$$y = 1984,\ldots,2006 \qquad \text{(B.1)}$$

where

$N^S_{y,a}$ is the number (in billions) of sardine of age *a* at the beginning of November in year *y*;

$\hat{C}^S_{y,q,a}$ is the estimated number (in billions) of sardine of age *a* caught during quarter q of year y ($q=1$ for November $y-1$ to January y, $q=2$ for February to April y, $q=3$ for May to July y and $q=4$ for August to October y);

M^S_a is the rate of natural mortality (in year^{-1}) of sardine of age a.

Biomass associated with the November survey

$$\hat{B}^S_{y,N} = k^S_N \sum_{a=1}^{5+} N^S_{y,a} w^S_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(B.2)}$$

where

$\hat{B}^S_{y,N}$ is the biomass (in thousand tonnes) of adult sardine at the beginning of November in year *y*, associated with the November survey;

k^S_N is the constant of proportionality (multiplicative bias) associated with the November survey; and

$w^S_{y,a}$ is the mean mass (in grams) of sardine of age *a* sampled during the November survey of year *y*.

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB_{y,N}^{S}=\sum_{a=2}^{5+}N_{y,a}^{S}w_{y,a}^{S} \qquad y=1984,\ldots,2006 \qquad \text{(B.3)}$$

Catch

The catch at age by number is calculated using Pope's approximation for ages 1+ (Pope 1984) and directly from the catch tonnage for age 0:

$$\hat{C}_{y,1,0}=\frac{C_{y,1}^{ObsTon0}}{\frac{7}{8}w_{y-1,0}^{S}+\frac{1}{8}w_{y,1}^{S}}$$

$$\hat{C}_{y,1,a}^{S}=N_{y-1,a}^{S}e^{-M_a^S/8}S_aF_{y,1}\,,\ a=1,\ldots,5+$$

$$\hat{C}_{y,2,0}=\frac{C_{y,2}^{ObsTon0}}{\frac{5}{8}w_{y-1,0}^{S}+\frac{3}{8}w_{y,1}^{S}}$$

$$\hat{C}_{y,2,a}^{S}=\left(N_{y-1,a}^{S}e^{-M_a^S/8}-\hat{C}_{y,1,a}^{S}\right)e^{-M_a^S/4}S_aF_{y,2}\,,\ a=1,\ldots,5+$$

$$\hat{C}_{y,3,0}=\frac{C_{y,3}^{ObsTon0}}{\frac{3}{8}w_{y-1,0}^{S}+\frac{5}{8}w_{y,1}^{S}}$$

$$\hat{C}_{y,3,a}^{S}=\left(\left(N_{y-1,a}^{S}e^{-M_a^S/8}-\hat{C}_{y,1,a}^{S}\right)e^{-M_a^S/4}-\hat{C}_{y,2,a}^{S}\right)e^{-M_a^S/4}S_aF_{y,3}\,,\ a=1,\ldots,5+$$

$$\hat{C}_{y,4,0}=\frac{C_{y,4}^{ObsTon0}}{\frac{1}{8}w_{y-1,0}^{S}+\frac{7}{8}w_{y,1}^{S}}$$

$$\hat{C}_{y,4,a}^{S}=\left(\left(\left(N_{y-1,a}^{S}e^{-M_a^S/8}-\hat{C}_{y,1,a}^{S}\right)e^{-M_a^S/4}-\hat{C}_{y,2,a}^{S}\right)e^{-M_a^S/4}-\hat{C}_{y,3,a}^{S}\right)e^{-M_a^S/4}S_aF_{y,4}\,,\ a=1,\ldots,5+ \qquad \text{(B.4)}$$

where

S_a is the commercial selectivity at age a, which is assumed to be year-independent for ages 1+; when $S_a=1$ the age-class a is said to be fully selected;

$F_{y,q}$ is the fished proportion of ages 1+ in quarter q of year y for an age class for which S_a is set equal to 1; and

$C_{y,q}^{ObsTon}$ is the estimated catch tonnage for age 0 in quarter q of year y; this is calculated using the predicted proportion of the observed catch tonnage that is 0 year olds from the assessment in Appendix A.

In the above equations the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion is estimated by:

$$F_{y,1}=\frac{C^{ObsTon1}_{y,1}}{\sum_{a=1}^{5+}\left(\frac{7}{8}w^S_{y-1,a}+\frac{1}{8}w^S_{y,a+1}\right)N^S_{y-1,a}e^{-M^S_a/8}S_a}$$ [5]

$$F_{y,2}=\frac{C^{ObsTon1}_{y,2}}{\sum_{a=1}^{5+}\left(\frac{5}{8}w^S_{y-1,a}+\frac{3}{8}w^S_{y,a+1}\right)\left(N^S_{y-1,a}e^{-M^S_a/8}-\hat{C}^S_{y,1,a}\right)e^{-M^S_a/4}S_a}$$

$$F_{y,3}=\frac{C^{ObsTon1}_{y,3}}{\sum_{a=1}^{5+}\left(\frac{3}{8}w^S_{y-1,a}+\frac{5}{8}w^S_{y,a+1}\right)\left(\left(N^S_{y-1,a}e^{-M^S_a/8}-\hat{C}^S_{y,1,a}\right)e^{-M^S_a/4}-\hat{C}^S_{y,2,a}\right)e^{-M^S_a/4}S_a}$$

$$F_{y,4}=\frac{C^{ObsTon1}_{y,4}}{\sum_{a=1}^{5+}\left(\frac{1}{8}w^S_{y-1,a}+\frac{7}{8}w^S_{y,a+1}\right)\left(\left(\left(N^S_{y-1,a}e^{-M^S_a/8}-\hat{C}^S_{y,1,a}\right)e^{-M^S_a/4}-\hat{C}^S_{y,2,a}\right)e^{-M^S_a/4}-\hat{C}^S_{y,3,a}\right)e^{-M^S_a/4}S_a} \quad \text{(B.5)}$$

where

$C^{ObsTon}_{y,q}$ is the estimated catch tonnage for ages 1+ in quarter q of year y. This is calculated using the predicted proportion of the observed catch tonnage that is 1+ year olds from the assessment in Appendix A.

Recruitment

For the base case assessment a Hockey Stick stock-recruitment curve is assumed for all years outside of the "peak" years, during which a constant recruitment (i.e. in respect of distribution median and independent of spawning biomass) is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve. Thus recruitment in November is given by:

$$N^S_{y,0}=\begin{cases} a^S e^{\varepsilon^S_y} & ,\text{if } \mathrm{SSB}^S_{y,N}\geq b^S \\ \frac{a^S}{b^S}SSB^S_{y,N}e^{\varepsilon^S_y} & ,\text{if } \mathrm{SSB}^S_{y,N}<b^S \end{cases} \qquad y=1984,\ldots,1999,2005,2006$$

$$N^S_{y,0}=c^S e^{\varepsilon^S_y} \qquad y=2000,\ldots,2004 \quad \text{(B.6)}$$

where

a^S is the maximum recruitment (in billions) (i.e. median of the distribution in question);

b^S is the spawner biomass above which there should be no recruitment failure risk in the hockey stick model;

c^S is the constant recruitment (distribution median) during the "peak" years of 2000 to 2004; and

[5] As no survey weight-at-age is available for 1983, it is assumed that $w^S_{1983,a}=w^S_{1984,a}$, and further that $w^S_{y,0}=0$.

ε_y^S is the annual lognormal deviation of sardine recruitment (see section on prior distributions).

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$\hat{N}_{y,r}^S = k_r^S\left(\left((N_{y-1,0}^S e^{-M_0^S/8} - \hat{C}_{y,1,0}^S)e^{-M_0^S/4} - \hat{C}_{y,2,0}^S\right)e^{-0.5t_y^S \times M_0^S/12} - \tilde{C}_{y,0bs}^S\right)e^{-0.5t_y^S \times M_0^S/12} \quad y = 1984,\ldots,2006 \quad \text{(B.7)}$$

where

$\hat{N}_{y,r}^S$ is the number (in billions) of juvenile sardine at the time of the recruit survey in year *y*;

k_r^S is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}_{y,0bs}^S$ is the observed number (in billions) of juvenile sardine caught between 1 May and the day before the start of the recruit survey, assuming a 15.5cm cut-off length; and

t_y^S is the time lapsed (in months) between 1 May and the start of the recruit survey in year *y*.

Fitting the Model to Observed Data (Likelihood)

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions. Thus the contribution of the survey abundance data to the negative log-likelihood function is given by:

$$\begin{aligned} -\ln L^{surv} &= \tfrac{1}{2}\sum_{y=1984}^{2006}\left\{\frac{\left(\ln B_{y,N}^S - \ln(\hat{B}_{y,N}^S)\right)^2}{(\sigma_{y,Nov}^S)^2 + (\lambda_N^S)^2} + \ln\left[2\pi\left((\sigma_{y,Nov}^S)^2 + (\lambda_N^S)^2\right)\right]\right\} \\ &+ \tfrac{1}{2}\sum_{y=1985}^{2006}\left\{\frac{\left(\ln N_{y,r}^S - \ln(\hat{N}_{y,r}^S)\right)^2}{(\sigma_{y,rec}^S)^2 + (\lambda_r^S)^2} + \ln\left[2\pi\left((\sigma_{y,rec}^S)^2 + (\lambda_r^S)^2\right)\right]\right\} \end{aligned} \quad \text{(B.8)}$$

where

$B_{y,N}^S$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass from the November survey in year y, with associated CV $\sigma_{y,Nov}^S$;

$N_{y,r}^S$ is the acoustic survey estimate (in billions) of sardine recruitment numbers from the recruit survey in year *y*, with associated CV $\sigma_{y,rec}^S$; and

$(\lambda_{N/r}^S)^2$ is the additional variance (over and above the survey sampling CV $\sigma_{y,Nov/rec}^S$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

Fixed Parameters

Seven parameters were fixed externally in this assessment:

$M_{ju}^{S} = 1$ and $M_{ad}^{S} = 0.8$

$S_1 = 0.43$ (average over all quarters from the output from the assessment of Appendix A)

$S_2 = S_3 = S_4 = S_5 = 1$ (chosen to smooth the output from the assessment of Appendix A, which was an average of 1 for age 2, 0.99 for age 3, 1 for age 4 and 2.6 for age 5+)

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. The prior pdfs for the recruitment residuals are given by:

$$\varepsilon_y^S \sim N\left(0, \left(\sigma_r^S\right)^2\right) \quad , \qquad y = 1984,...,2005$$

A probability density function (pdf) for the overall bias in the November survey was calculated by drawing ten thousand samples from the individual pdfs for each source of error (I. Hampton pers. comm.), see Table A.1 and Figure A.1 below. In the last assessment, target strength was included as a source of error. Given that the new target strength expression has been used in the survey data, target strength was removed as a source of error from this bias, substantially narrowing the pdf (Figure A.1). There may, however, still be systematic errors relating to the target strength that are unaccounted for in this pdf. These are taken into account through sensitivity tests using alternative k_N^S values. A normal distribution, using the mean and standard deviation of the pdf was used as a prior for k_N^S, i.e. $k_N^S \sim N\left(0.722, 0.078^2\right)$.

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$\log\left(k_r^S\right) \sim U(-100, 0.4)$ (upper bound corresponding to $k_r^S = 1.5$)

$\left(\lambda_N^S\right)^2 \sim U(0,10)$

$\left(\lambda_r^S\right)^2 \sim U(0,10)$

$\log\left(a^S\right) \sim U(0,8)$ (given the lack of *a priori* information on the magnitude of a^S, a log-scale was used)

$\log\left(c^S\right) \sim U(0,8)$

$b^S / K^S \sim U(0,1)$

$\left(\sigma_r^S\right)^2 \sim U(0.4,10)$

$N^S_{1983,a} = N_{1983} \times Nprop_a$, where $N_{1983} \sim U(0{,}50)$ billion and $Nprop_a \sim U(0{,}1)$ for $a = 0,\ldots,2$ and $Nprop_3 = Nprop_4 = \frac{1}{2}\left(1 - \sum_{a=0}^{2} Nprop_a\right)$.

Further Outputs

Recruitment serial correlation:

$$s^S_{cor} = \frac{\sum_{y=1984}^{2004} \varepsilon_y \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{2004} \varepsilon_y^2\right)\left(\sum_{y=1984}^{2004} \varepsilon_{y+1}^2\right)}} \tag{B.9}$$

and the standardised recruitment residual value for 2005:

$$\eta^S_{2005} = \frac{\varepsilon^S_{2005}}{\sigma^S_r} \tag{B.10}$$

are also required as input into the OM.

A separate carrying capacity, K^S (essentially the B^S_N value where the replacement line and the stock recruit function intersect) is calculated representing the period of peak abundance (2000 – 2004) to that for the remaining years:

$$K^S_{normal} = a^S e^{\frac{1}{2}\left(\sigma^S_r\right)^2} \left[\sum_{a=1}^{4} \overline{w}^S_{normal,a} e^{-\sum_{a=0}^{a=1} M^S_a} + \overline{w}^S_{normal,5+} e^{-\sum_{a=0}^{4} M^S_a} \frac{1}{1 - e^{-M^A_{5+}}} \right] \tag{B.11}$$

$$K^S_{peak} = c^S e^{\frac{1}{2}\left(\sigma^S_r\right)^2} \left[\sum_{a=1}^{4} \overline{w}^S_{peak,a} e^{-\sum_{a=0}^{a=1} M^S_a} + \overline{w}^S_{peak,5+} e^{-\sum_{a=0}^{4} M^S_a} \frac{1}{1 - e^{-M^A_{5+}}} \right] \tag{B.12}$$

(calculated assuming maximum recruitment in the absence of fishing) where

$\overline{w}^S_{normal,a}$ is the mean mass (in grams) of sardine of age *a* sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available outside of the peak years (i.e. 1993, 1994, 1996 and 2006).

$\overline{w}^S_{peak,a}$ is the mean mass (in grams) of sardine of age *a* sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available during the peak period (i.e. 2001 - 2004).

The $e^{\frac{1}{2}\left(\sigma^S_r\right)^2}$ factor in the above equation is a bias correction factor, needed given the assumption that recruitment is log-normally distributed about an underlying stock-recruit curve.