An alternative approach that may help resolve the data conflicts observed in the model presented in MARAM/IWS/DEC15/Hake/P1

A. Ross-Gillespie

The preliminary results presented in MARAM/IWS/DEC15/Hake/P1 show that the problem whereby the model battles to fit both the proportion of hake in diet of *M. paradoxus* predators and the historical trend information is still evident. This may in part be due to the fact the if the fit to the model-predicted proportion of hake in diet is adequate, then the *M. paradoxus* mortality rate is (unrealistically?) high, leading to a quick turn-over in the population trajectory that does not fit the historical trend data well.

There may, however, be a conflict in how the observed proportions from the diet data are calculated with the model-predicted proportions.

1. The diet data used are counts of non-empty stomachs that contain $> 50\%$ hake (by prey weight) against the total number of non-empty stomachs.
2. The model proportion of hake in diet is calculated by the total mass of hake consumed divided by the total mass of prey consumed.

For (1), the size of the prey is not taken into account. In other words, whether a predator eats 0, 1 or 2 year old hake does not matter — each hake prey item adds to the count. For (2), the size of prey does matter. Whether a predator eats a zero year old hake or a 2 year old hake will make a substantial difference to the proportion of hake in diet when the mass of the prey is taken into account.

In other words, the "target" proportion of hake in diet that the model so often battles to fit to is not in fact comparable to the model output, and the real proportion of hake in diet by mass may be quite different to the current assumed trends by numbers - and perhaps substantially lower.

There are two options that could be pursued:

Option (A): Leave the diet data as are, and calculate the model-predicted proportion of hake in diet by numbers.

Option (B): Leave the model proportions, and weight the diet data by prey length to give more weight to larger prey items.

A potential problem with (A) is that working with numbers may be less stable than with mass. A potential problem with (B) is that this might bias the data in ways that cannot be quantified. Further there are stomach content samples where the prey length is not available and these would have to be excluded from the data set. It seems to make sense to explore Option (A) as a first priority, and to achieve this some adjustments would would need to be made to equations in MARAM/IWS/DEC15/Hake/P1, as highlighted below.

1 Hake prey

$$V_{saym}^{s_p a_p} = N_{ym}^{s_p a_p} \gamma_{sa}^{s_p a_p} \frac{\nu_s^{s_p} \theta^{s_p a_p}}{1 + \sum_{s,a} \tilde{\nu}_s^{s_p} N_{saym} \gamma_{sa}^{s_p a_p} + \tilde{\nu}_{other}^{s_p} O_{other}^{s_p a_p}} \tag{1.1}$$

$$O_{other}^{s_p, a_p} = o_a^{s_p} e^{-(o_b^{s_p}) a_p} \tag{1.2}$$

where $o_a^{s_p}$ and $o_b^{s_p}$ are estimable parameters.

The mass of hake prey consumed in year y by predators of species s_p and age a_p is then given by

$$Q_{sym}^{s_p a_p, mass} = \frac{V_{saym}^{s_p a_p}}{Z_{saym}} N_{saym} w_{sa} \left(1 - e^{-Z_{saym}}\right) \tag{1.3}$$

The number of hake prey fish consumed in year y by predators of species s_p and age a_p is then given by

$$Q_{sym}^{s_p a_p, num} = \frac{V_{saym}^{s_p a_p}}{Z_{saym}} N_{saym} \left(1 - e^{-Z_{saym}}\right) \tag{1.4}$$

2 Other prey

$$V_{other,ym}^{s_p a_p} = N_{ym}^{s_p a_p} \frac{\nu_{other}^{s_p} \theta^{s_p a_p}}{1 + \sum_{s,a} \tilde{\nu}_s^{s_p} N_{saym} \gamma_{sa}^{s_p a_p} + \tilde{\nu}_{other}^{s_p} O_{other}^{s_p a_p}} \tag{2.1}$$

The mass of other prey consumed in year y by predators of species s_p and age a_p is then given by

$$Q_{other,ym}^{s_p a_p, mass} = \frac{V_{other,ym}^{s_p a_p}}{Z_{other,ym}} O_{other}^{s_p, a_p} w_{other}^{s_p} \left(1 - e^{-Z_{other,ym}}\right) \tag{2.2}$$

where $O_{other}^{s_p, a_p}$ is as above (note that in MARAM/IWS/DEC15/Hake/P1, the product of $O_{other}^{s_p, a_p} w_{other}^{s_p}$ is defined as a separate estimable quantity).

The number of other prey fish consumed in year y by predators of species s_p and age a_p is then given by

$$Q_{other,ym}^{s_p a_p, num} = \frac{V_{other,ym}^{s_p a_p}}{Z_{other,ym}} O_{other}^{s_p, a_p} \left(1 - e^{-Z_{other,ym}}\right) \tag{2.3}$$

3 Proportion of hake in diet

The model-predicted proportion of hake in diet in year y is calculated using $Q_{sym}^{s_p a_p, num}$, the **number** of hake consumed.

$$\hat{\rho}_y^{s_p l_p} = \left(\sum_m \sum_{a_p \neq 0} A^{s_p l_p a_p} \sum_s Q_{sym}^{s_p a_p, num} \right) \Big/ \left(\sum_m \sum_{a_p \neq 0} A^{s_p l_p a_p} \left(\sum_s Q_{sym}^{s_p a_p, num} + Q_{other,ym}^{s_p a_p, num} \right) \right) \tag{3.1}$$

4 Daily ration

Daily ration as a percentage of body mass, $\hat{\delta}_{ym}^{s_p a_p}$, is defined as before.

$$\hat{\delta}_{ym}^{s_p a_p} = \frac{\sum_s Q_{sym}^{s_p a_p, mass} + Q_{other,ym}^{s_p a_p, mass}}{\sum_{\tilde{a}_p = 12 a_p}^{12 a_p + 11} \tilde{\tilde{N}}_{y,m}^{s_p, \tilde{a}_p} w^{s_p \tilde{a}_p}} * 12/365 * 100 \tag{4.1}$$