

# Progress on length based SAM

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$$M2_i = \frac{\sum_j \frac{dR}{dt} N_j \frac{\varphi_{ji}}{\varphi_j}}{N_i \omega_i}$$

A collection of colorful mathematical symbols including an integral sign, Greek letters, a square root, a plus sign, a delta function, an equals sign, a set of curly braces, a chi-squared symbol, a sigma symbol, and an exclamation point.

# Outline

- Background: SAM and time-varying selectivity
- Challenges in time-varying length based selectivity.
- Template Model Builder
- State-space Length structured ASsessment MOdel (SLASMO)

## Focus here on fishing mortality $F_{a,y}$ (and later $F_{l,y}$ )

- $F_{a,y}$  is the instantaneous fishing mortality rate for age group  $a$  in year  $y$
- Define the selectivity as:

$$S_{a,y} = \frac{F_{a,y}}{\sum_a F_{a,y}},$$

Often the selectivity is scaled to have a maximum of one in each year.

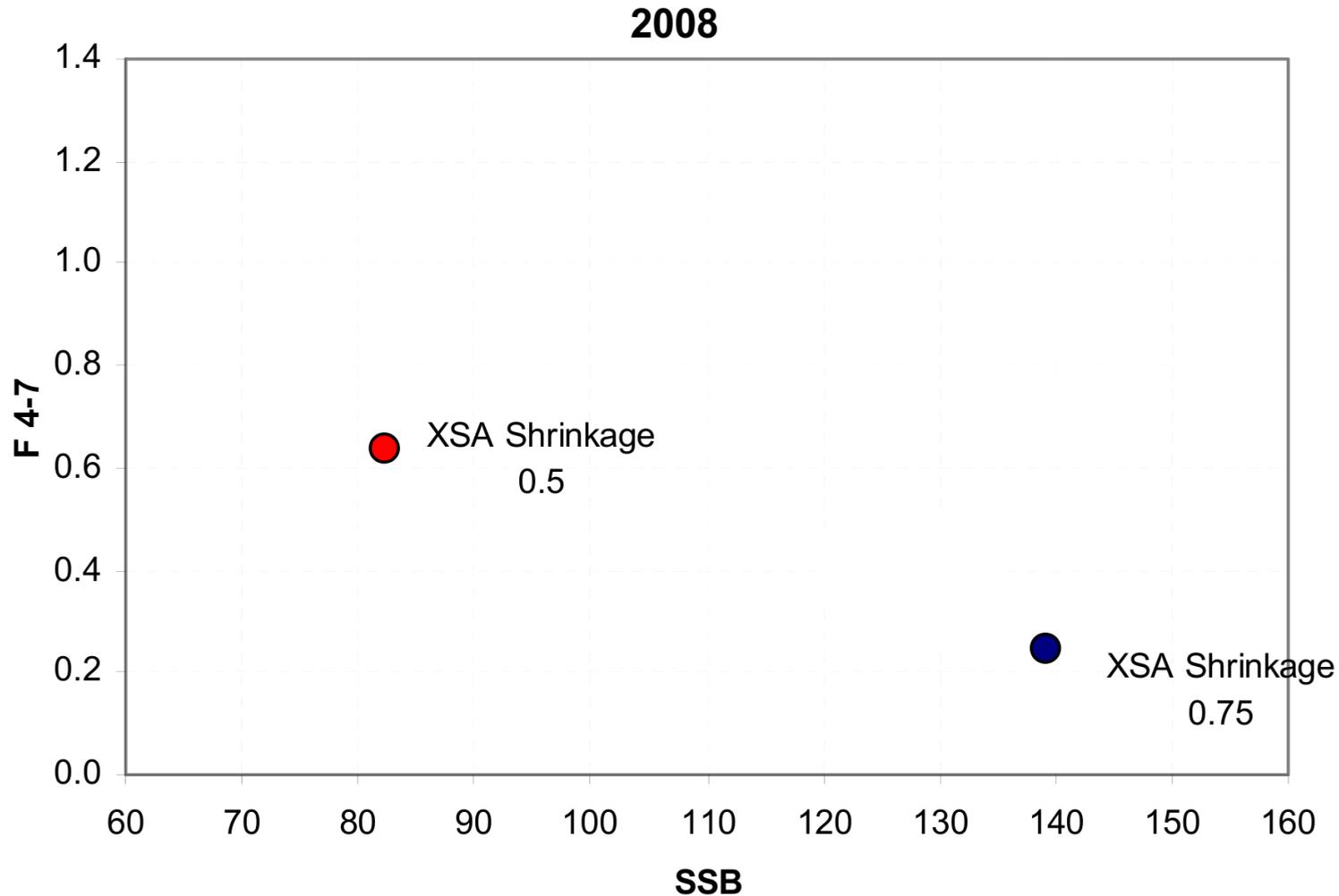
- There are many other interesting aspects in stock assessment models.
  - Natural mortality
  - Effort
  - Survey design
  - Stock recruitment model
  - Prediction ability
  - Growth
  - ...

# Approaches used to model $F$

	Year e.g. 1963–2014							
Age e.g. 1–7								
				$F_{a,y}$				

- Deterministic (Catch-at-age assumed known without error)
  - Extremely flexible  $F_{a,y}$  model.
  - Ad-hoc smoothing often added
  - No quantification of uncertainties
- Full parametric. E.g:
  - $F_{a,y} = S_a f_y$  (S may be a parametrized function)
  - $F_{a,y} = S_a f_y$  with separate  $S_a$  in time blocks
  - Splines with fixed degree of smoothness
  - Penalized deviances with fixed penalty

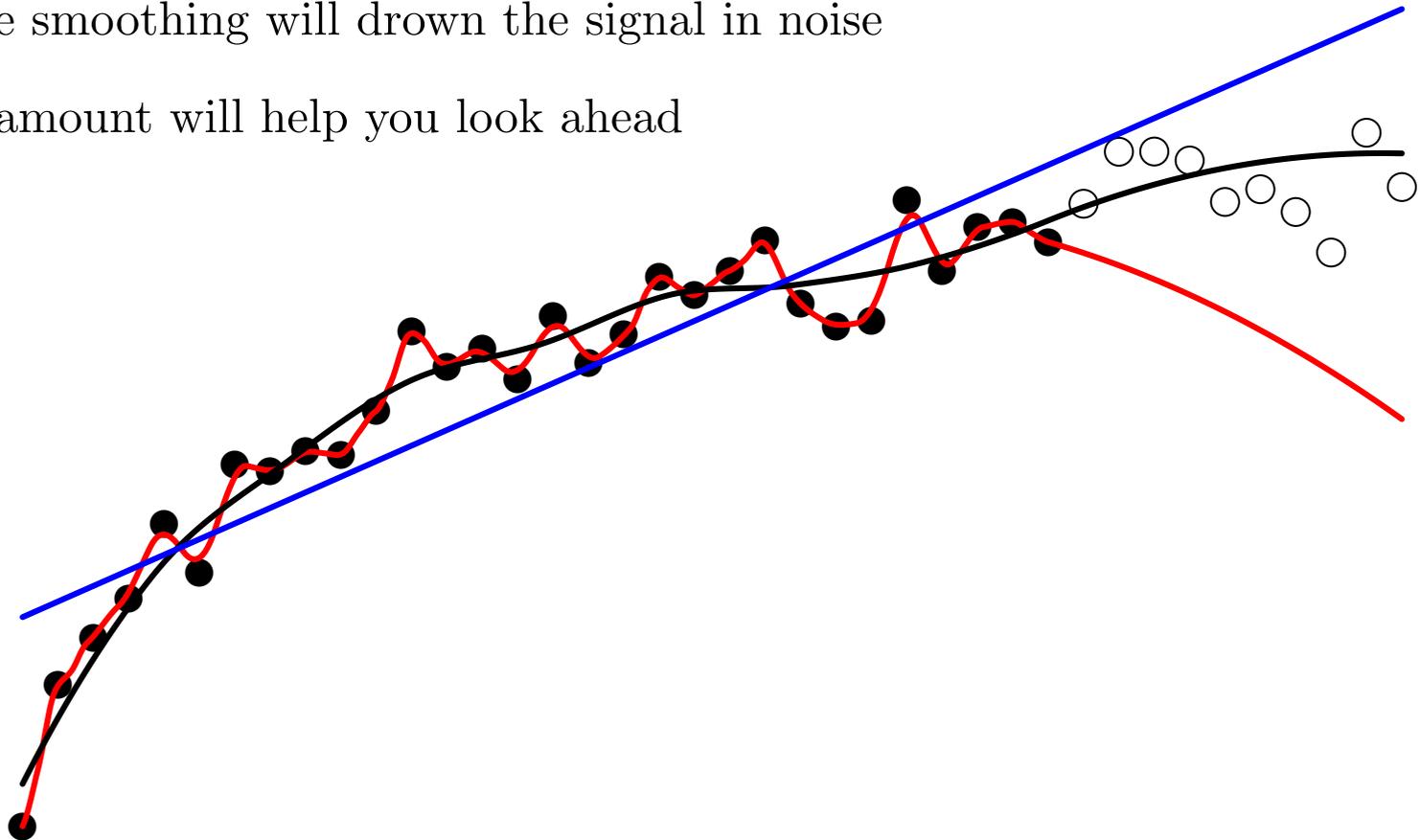
# These choices are important



- These differences are not small and theoretical
- There are no objective way to choose between these two deterministic approaches
- There should really be an objective criteria. A statistical framework.

# Further we need to predict

- Too much smoothing will bias the signal
- Too little smoothing will drown the signal in noise
- Correct amount will help you look ahead



- Correct amount should not be subjective.

# State-space assessment models

- Allow us to include things like:

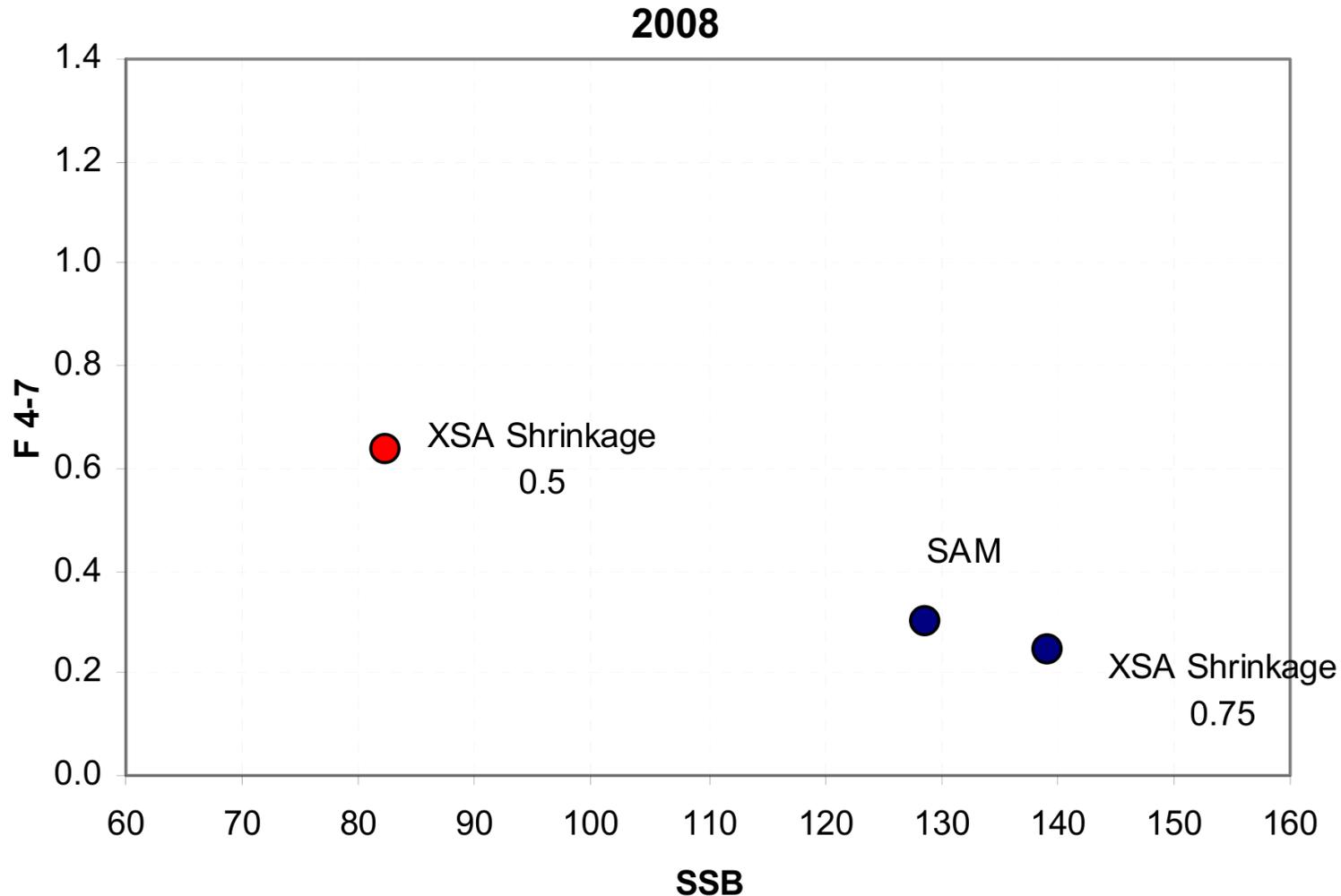
$F_{3,y}$  is a random walk with step variance  $\sigma^2$

- Importantly  $\sigma^2$  is a model parameter estimated in the model.
- This model class<sup>a</sup> is used in most other quantitative fields
- It is a very useful extension to full parametric statistical models.
- Introduced for stock assessment by Gudmundsson (1987,1994) and Fryer (2001).
- The reason state-space models have not been more frequently used in stock assessment is that software to easily handle these models has not been available. **It is now!**
- Has recently received increased attention (e.g. Brinch et al., 2011; Gudmundsson and Gunnlaugsson, 2012; Berg et al., 2013; Nielsen and Berg 2014)
- Can give very **flexible** models with low number of model parameters

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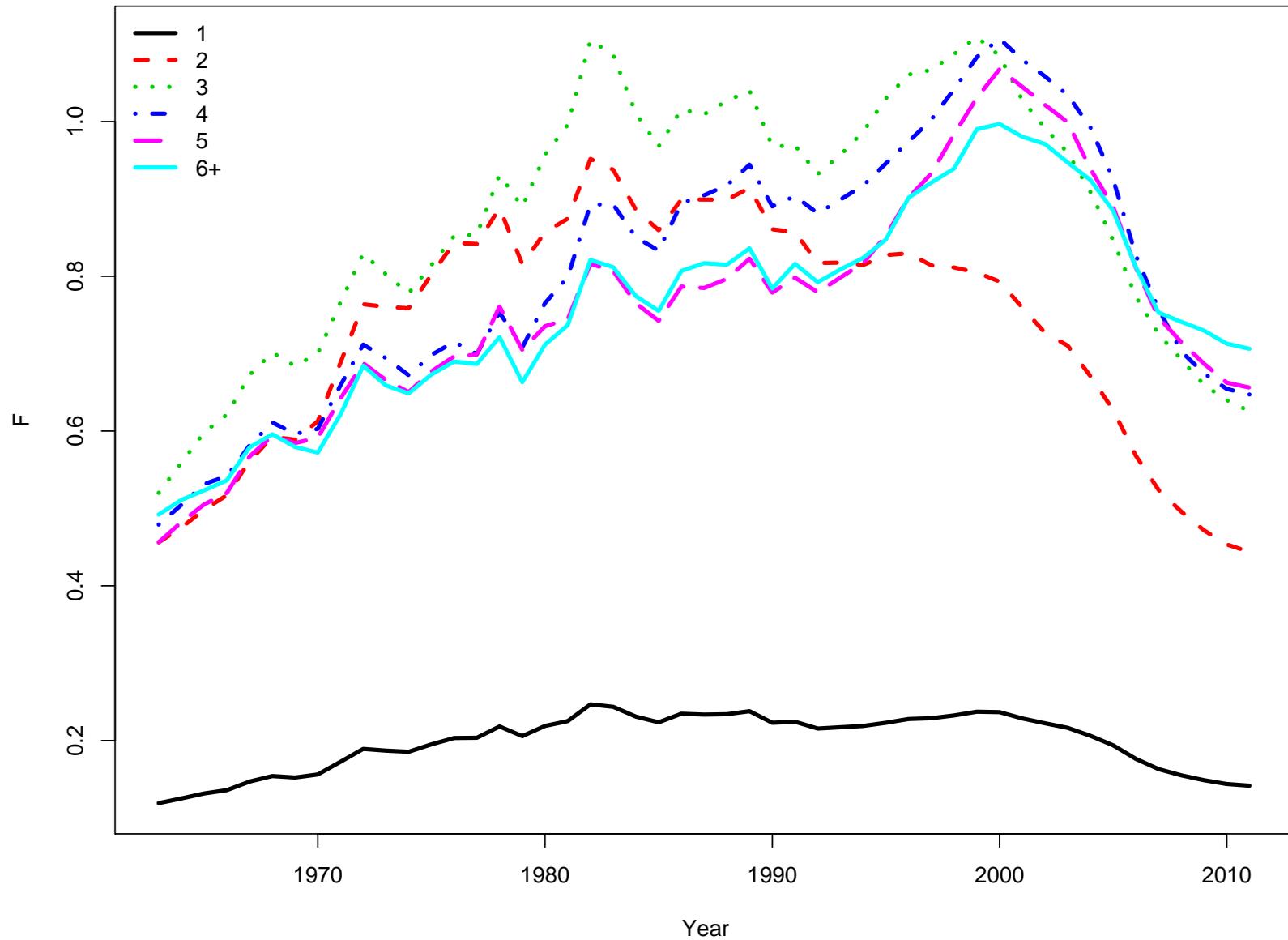
<sup>a</sup>a.k.a. **random effects models**, **mixed models**, **latent variable models**, **hierarchical models**, ...

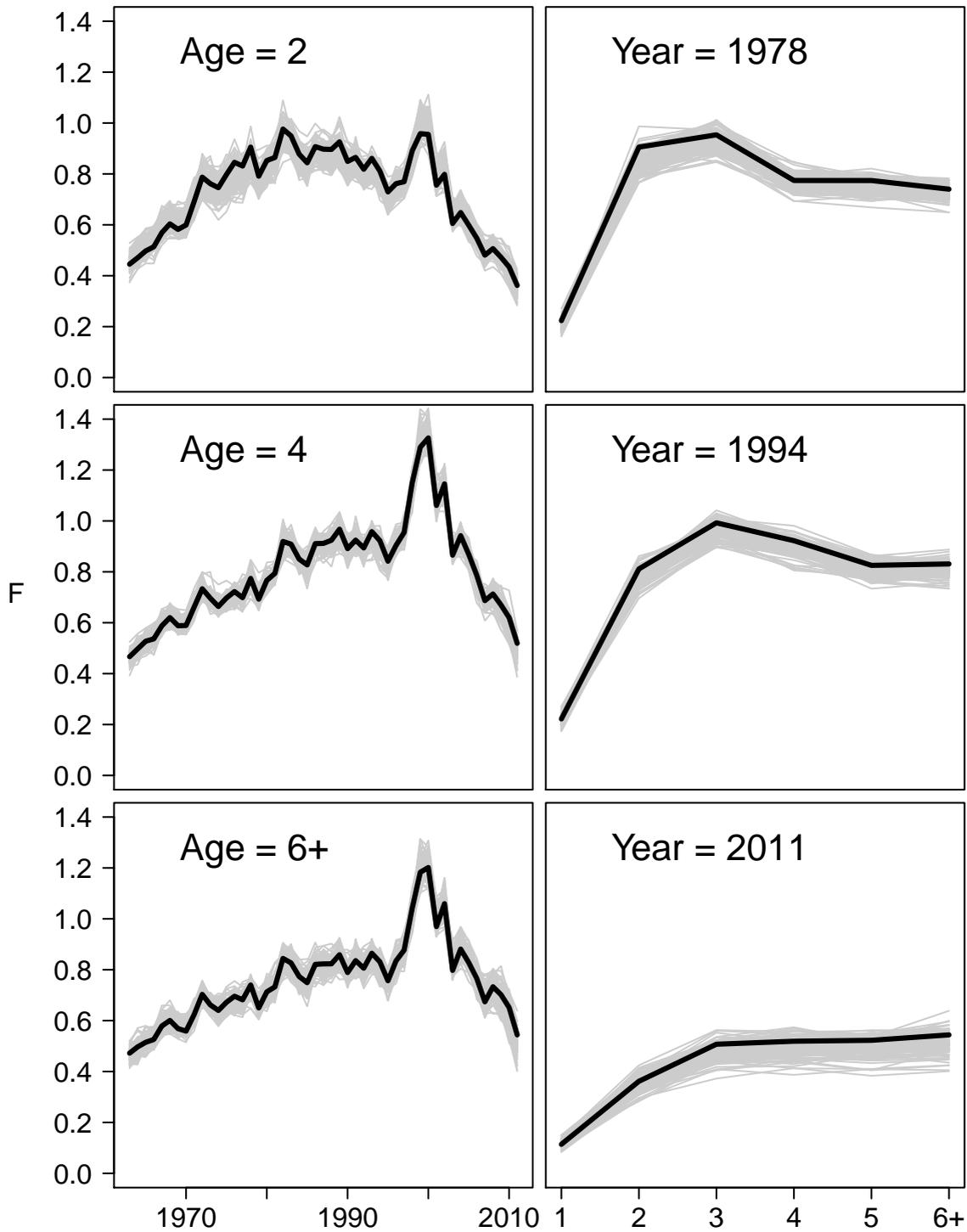
# Avoiding ad-hoc choices — Eastern Baltic Cod



- Using the State-space Assessment Model (SAM) gives us an objective criteria

# Evolving selectivity — North Sea Cod





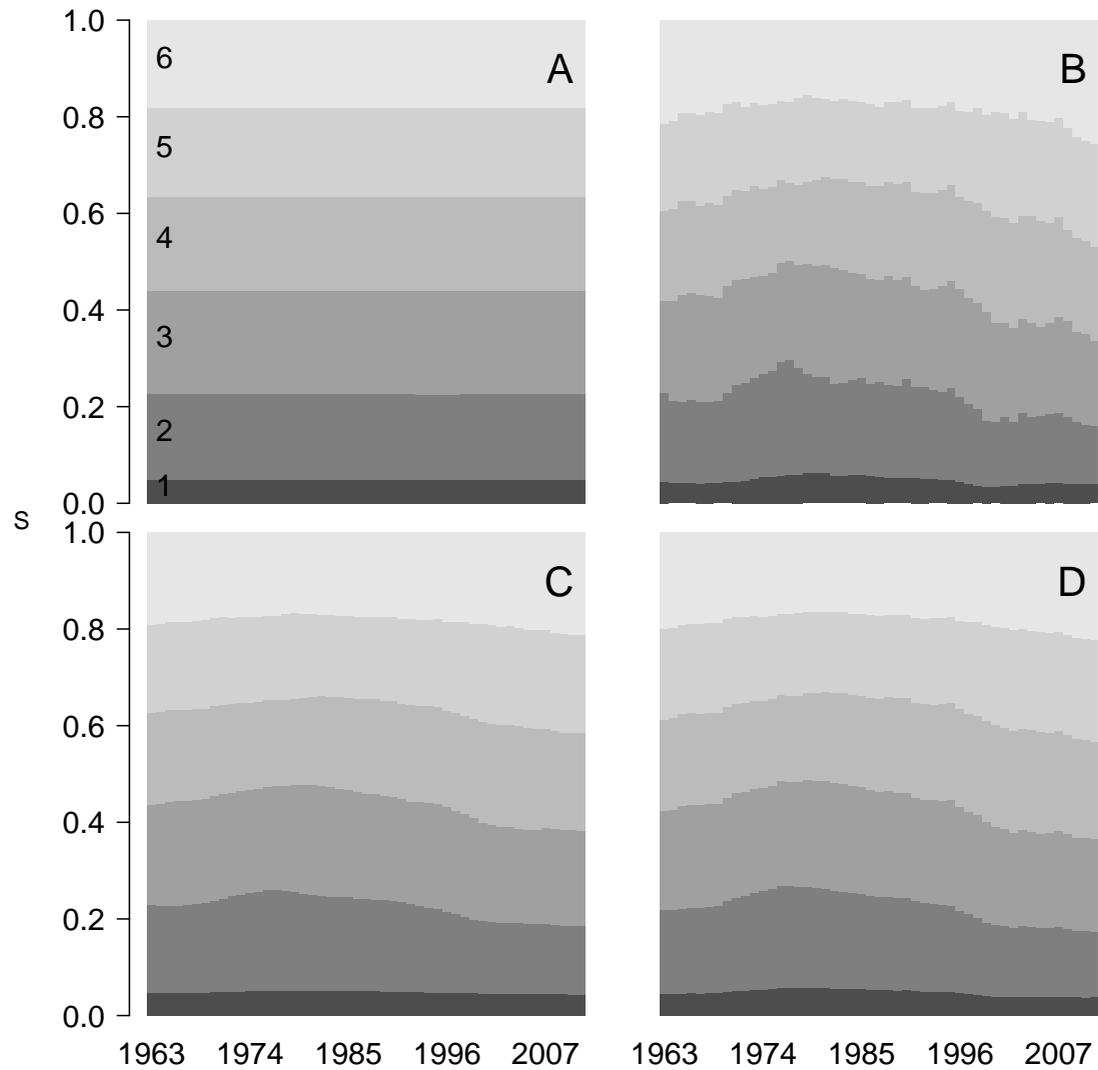
# Model selection ex: Correlated Random Walks

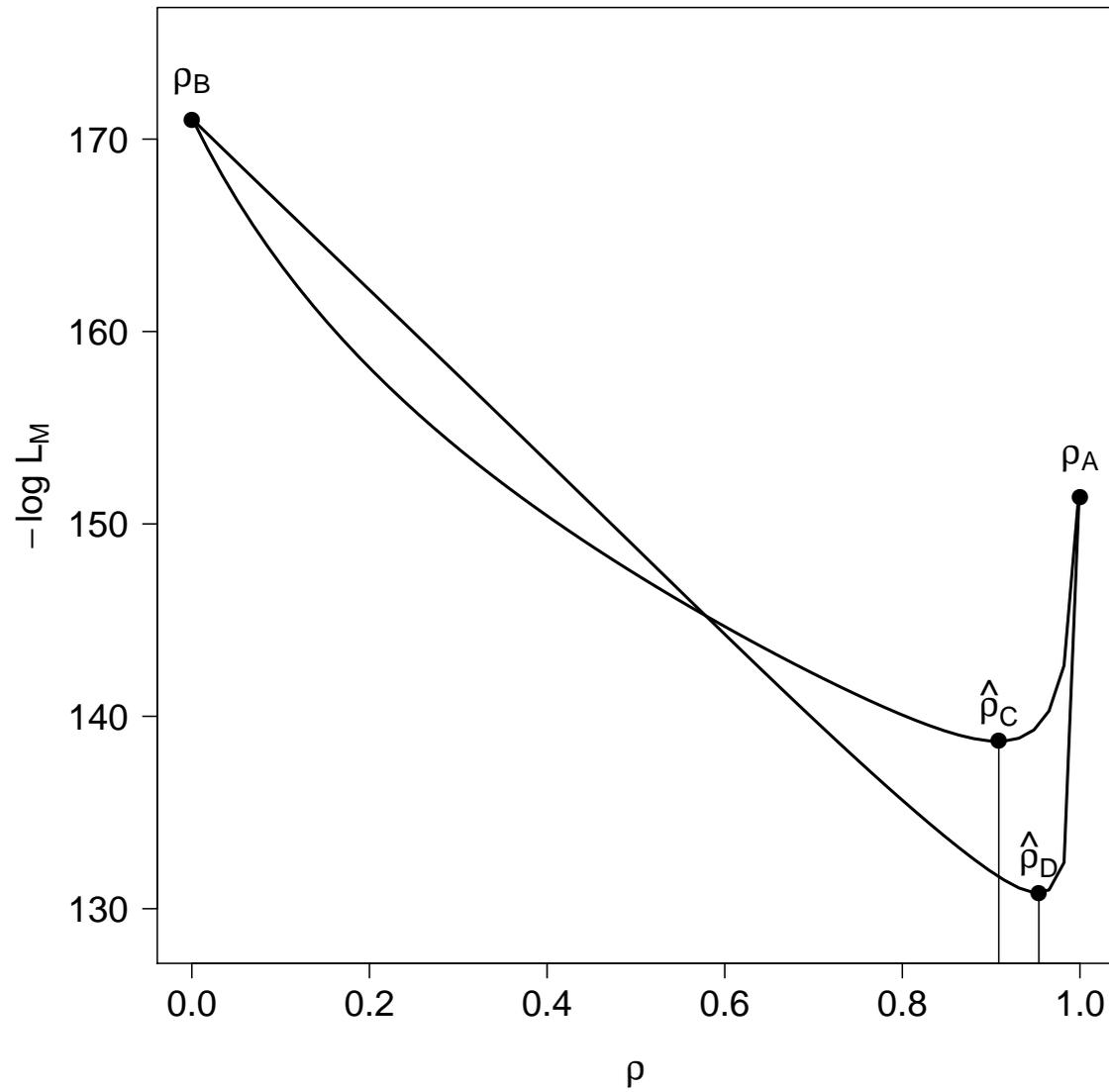
- Instead of independent random walks for  $F$  at different ages, we can allow those random walks to be correlated. Define  $\Delta \log F_y = \log F_y - \log F_{y-1}$ , then:

$$\Delta \log F_y \sim \mathcal{N}(0, \Sigma)$$

- For all combination of ages ( $a \neq \tilde{a}$ ):
  - A)** Parallel:  $\Sigma_{a,\tilde{a}} = \sqrt{\Sigma_{a,a}\Sigma_{\tilde{a},\tilde{a}}}$
  - B)** Independent:  $\Sigma_{a,\tilde{a}} = 0$
  - C)** Compound symmetry:  $\Sigma_{a,\tilde{a}} = \rho\sqrt{\Sigma_{a,a}\Sigma_{\tilde{a},\tilde{a}}}$
  - D)** AR(1):  $\Sigma_{a,\tilde{a}} = \rho^{|a-\tilde{a}|}\sqrt{\Sigma_{a,a}\Sigma_{\tilde{a},\tilde{a}}}$

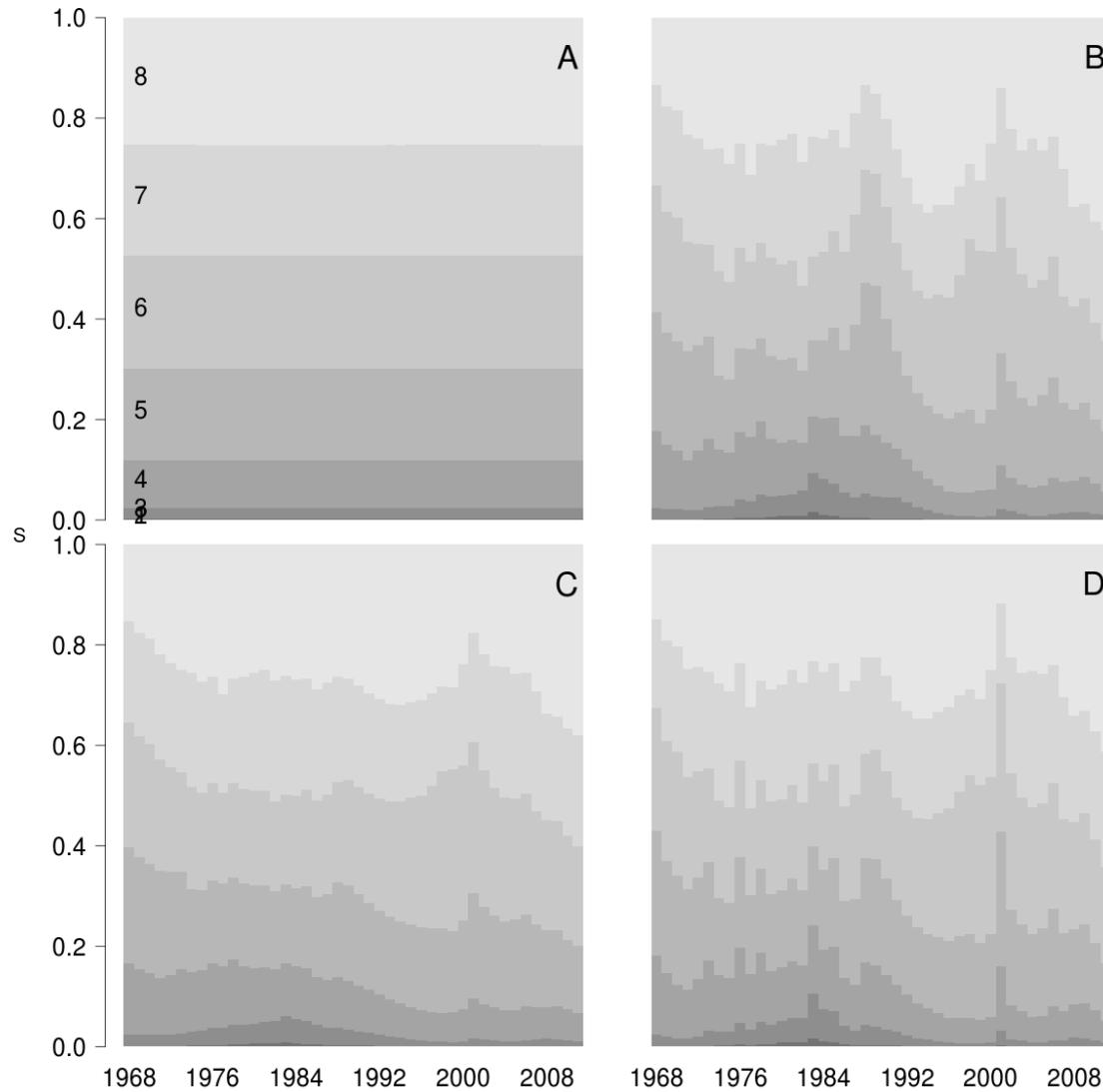
# Selectivities: North Sea cod



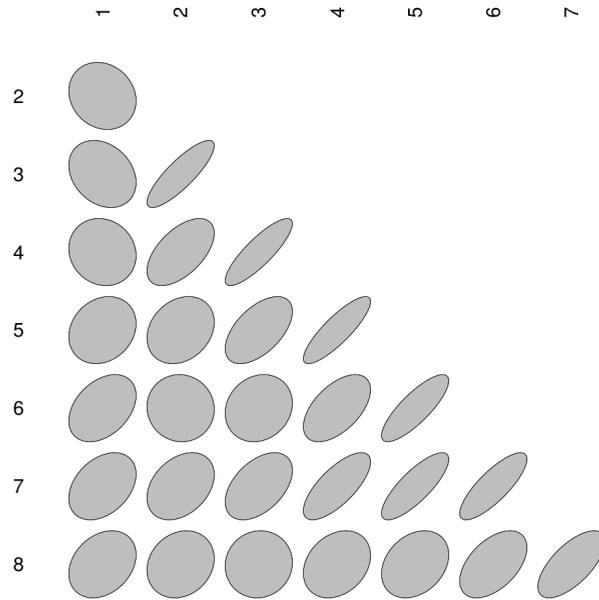


North Sea cod: Profile likelihood for the  $\rho$ -parameter for models C and D,  $\rho = 1$  corresponds to model A, and  $\rho = 0$  corresponds to model B.

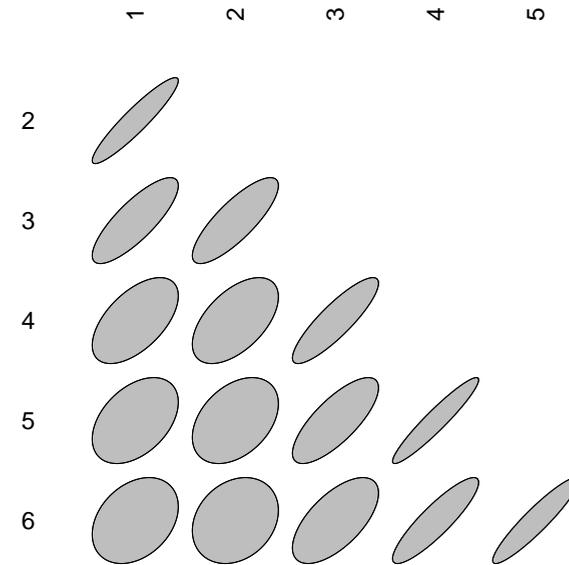
# Preliminary run: Namibian hake



## Namibian Hake



## North Sea Cod

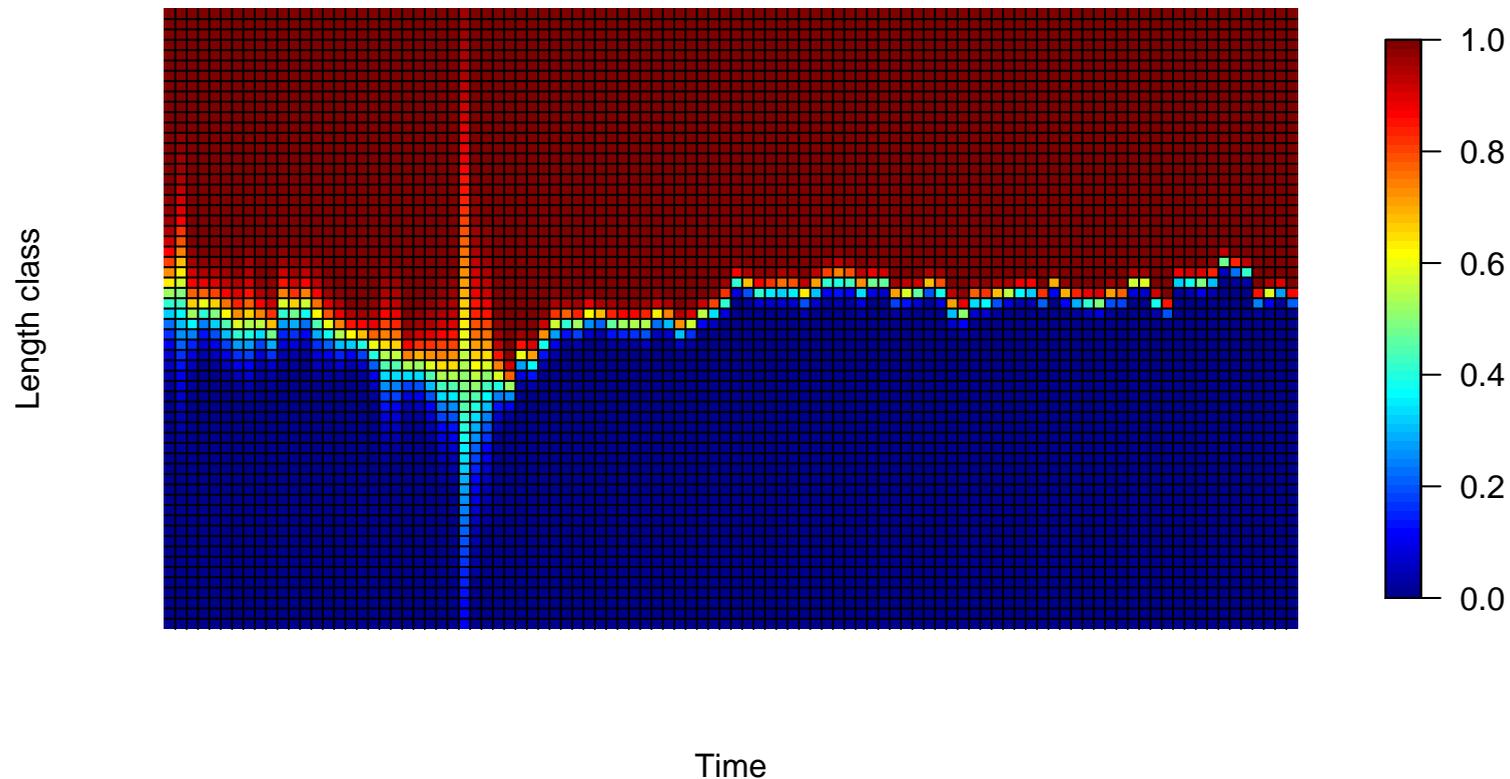


Name	Model			
	A	B	C	D
$\sigma_F$	0.30	0.41	0.38	0.50
$\sigma_R$	1.16	0.69	0.81	0.77
$\sigma_S$	0.67	0.44	0.53	0.53
$\sigma_C$	0.40	0.33	0.32	0.15
$\sigma_{I,1}$	0.95	0.99	0.98	0.98
$\sigma_{I,2}$	0.51	0.62	0.56	0.54
$\sigma_{I,3}$	0.27	0.38	0.32	0.31
$\sigma_{I,4}$	0.55	0.61	0.56	0.59
$\rho$			0.62	0.83
AIC	1898.23	1901.69	1866.18	<b>1816.04</b>

Name	Model			
	A	B	C	D
$\sigma_F$	0.11	0.10	0.11	0.11
$\sigma_R$	0.51	0.52	0.50	0.51
$\sigma_S$	0.14	0.09	0.10	0.10
$\sigma_{C_{a=1}}$	0.80	0.69	0.76	0.71
$\sigma_{C_{a=2}}$	0.28	0.16	0.21	0.20
$\sigma_{C_{a \geq 3}}$	0.08	0.11	0.09	0.09
$\sigma_{I_{a=1}}$	0.61	0.64	0.63	0.64
$\sigma_{I_{a \geq 2}}$	0.27	0.28	0.28	0.28
$\rho$			0.91	0.95
AIC	333.44	372.04	309.40	<b>293.63</b>

# Challenges in time-varying length based selectivity.

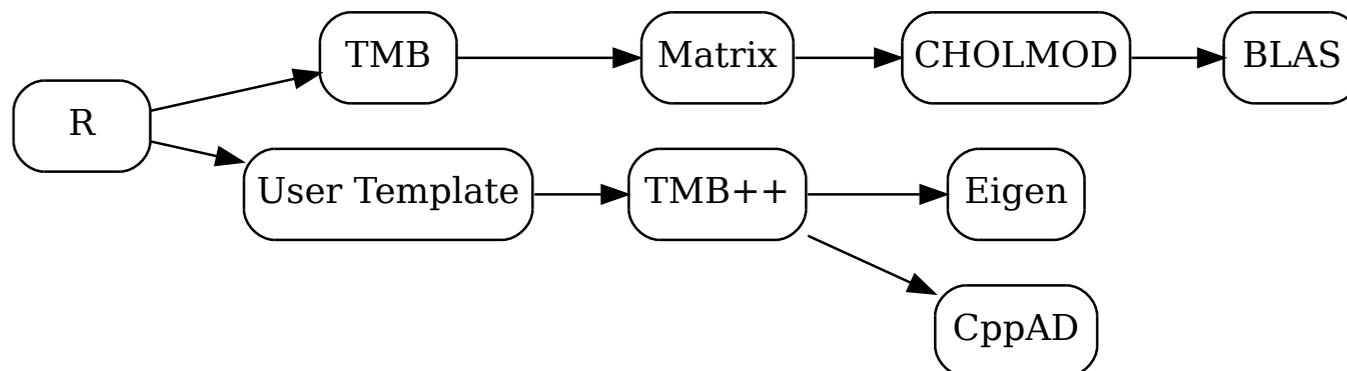
- The dimension is typically one order of magnitude higher
- More time steps are often needed
- Even more important to get correlation structure right



All of the above means longer run times — what can be done about that?

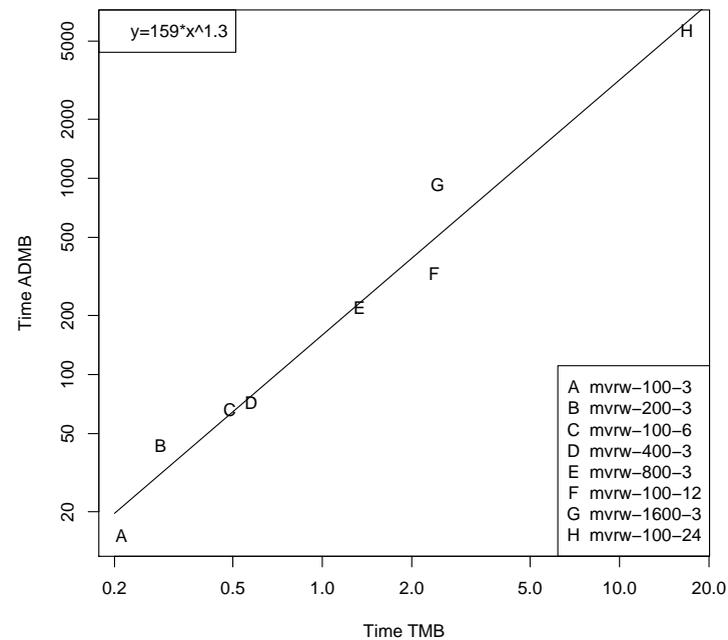
# Template Model Builder (TMB):

- Developed by Kasper Kristensen, DTU
- ADMB inspired R-package
- Combines external libraries: CppAD, Eigen, CHOLMOD
- Continuously developed since 2009, < 10000 lines of code
- Implements Laplace approximation for random effects
- C++ Template based
- Automatic sparseness detection
- Parallel computing supported on three levels: BLAS, parallel templates, and R.



# Timings!

Example	Time (TMB)	Speedup (TMB vs ADMB)
longlinreg	11.3	0.9
mvrw	0.3	97.9
nmix	1.2	26.2
orange_big	5.3	51.3
<b>sam</b>	<b>3.1</b>	<b>60.8</b>
sdv_multi	11.8	37.8
socatt	1.6	6.9
spatial	8.3	1.5
thetalog	0.3	22.8



# TMB: summary

- TMB combines automatic differentiation and the Laplace approximation for estimation in complex non-linear latent variable models (state-space, hierarchical models, Gaussian Markov random fields)
- Can handle very high dimensional problems ( $\sim 10^6$  random effects) through automatic detection and exploitation of sparseness structure wrt. random effects
- Orders of magnitude faster than other well-known methods (ADMB, BUGS)
- Good support for parallel computations
- R package and examples available at <http://tmb-project.org>

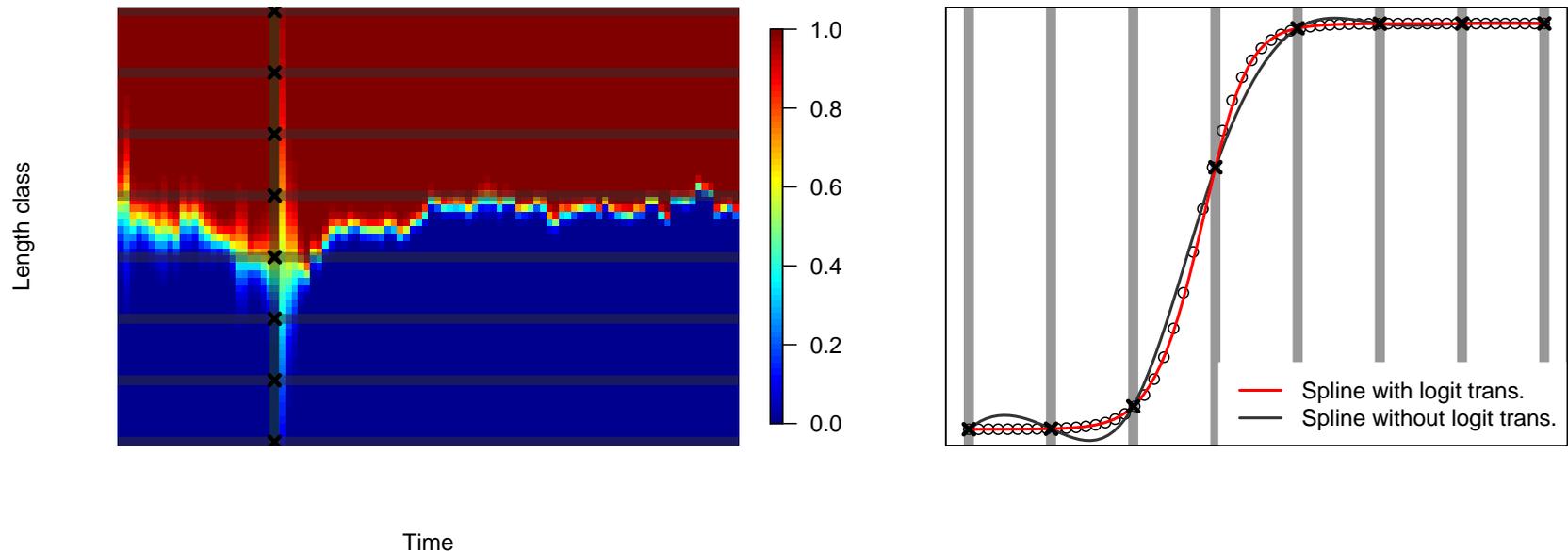
# Reduce number of processes

- Too slow to have a process for each length class
- Proposed solution:
  - Select a number of lengths  $\tilde{l}_i$  (e.g.  $\tilde{l}_1, \dots, \tilde{l}_8$  equidistant)
  - Define a multivariate  $\beta_t$  process as:

$$\beta_{t+1} = \beta_t + \eta_t, \text{ with } \eta_t \sim \mathcal{N}(0, \Sigma)$$

- $F$  at other lengths are calculated from a spline interpolation in the length direction

$$F_{l,t} = \psi \text{logit}^{-1}(\mathcal{S}_{(\tilde{l}_1, \beta_{1,t}), \dots}(l))$$



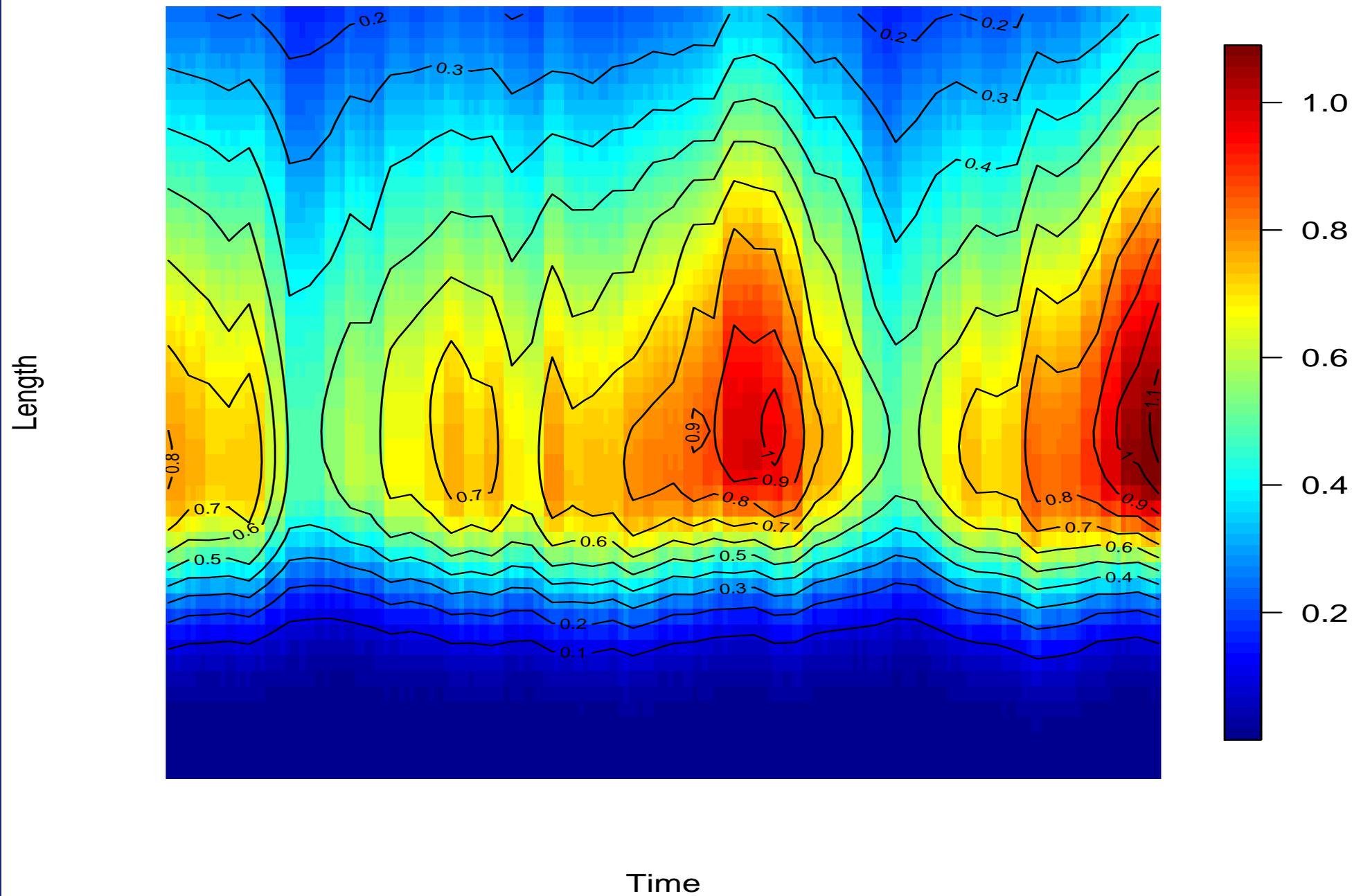
# Simulation study: Purely length-based approach

- Lengths in the dynamics
  - Follow the population length distribution directly
  - The dynamics is using Von Bertalanffy growth  $dl_t = k(L_\infty - l_t)dt$
  - Separate process for recruitment (four steps per year)
- Lengths in the observations
  - $F_{lt}$  is simulated as  $f_t S_l$  where  $S_l$  is a time-varying double normal selectivity
  - $F_{lt}$  is estimated via the non-parametric F-at-length field (prev. slide)
  - Catch-at-length are predicted as:

$$C_{lt} = \frac{F_{lt}}{F_{lt} + M_l} (1 - e^{-(F_{lt} + M_l)}) N_{lt}$$

- Data range
  - 50 years of data simulated
  - total catch-at-length quarterly
  - three length-based surveys

# Simulated vs. estimated F





# Summary

- State-space formulation is a natural representation of fish stock assessment models
- Makes it possible to avoid many ad-hoc tweaks
- Allows both process and measurement noise
- Selectivity (and other parameters) can be time-varying
- The number of model parameters can be kept reasonable low
- Prediction is straight-forward
- We now have the tool to handle these models (TMB)
- High dimension of length based selectivity is a numerical challenge
- Non-parametric F-field (random effects interpolated) is a promising approach
- Simulation study demonstrated that approach is identifiable and fast.
- More info at: <http://stockassessment.org> and <http://tmb-project.org>