Results of sensitivity tests applied to the South African squid stock assessment model, including for harvesting strategies with a constant annual catch component for small scale fishers

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Introduction

Currently a Bayesian approach is applied in assessing the status of the South African squid resource, *Loligo reynaudii*. The underlying model is biomass-based and a year is split into two time periods to better model the dynamics of the stock and the two fisheries (jig and trawl) that exploit it. This paper reports results from various sensitivity tests conducted on the existing model.

Sensitivity tests

The model specifications are provided in Appendix A. The sensitivity tests conducted are as follows:

S1: Omit the trawl CPUE data from the model.
S2: Test alternative values to σ_R(input) (equation A.3). Alternative values of 0.2 and 0.6 were considered.
S3: Consider alternative values for C (equation A.15), the constant applied in the determination of the standard deviation associated with the abundance indices to ensure that no abundance index receives unrealistically high weight in the fitting process. Fixed values for C ranging from 0.1-0.6 in units of 0.1 were tested.
S4: Include the survey biomass estimates from the autumn and spring surveys that utilized the new trawl gear.
S5: Include a fixed annual catch for small scale fishers.

Methodology

The most recent assessment results for the Base Case model (specified in Appendix A) are reported in FISHERIES/2012/AUG/SWG-SQ/26. All results presented in this paper will be compared against those of that Base Case.

Sensitivity tests S1-S4 require changes to either the input data or assumptions made in the model, and therefore also require that the assessment model be re-run. Given the time-consuming nature of Bayesian analyses, stochastic projections 10 years into the future under different constant effort scenarios were carried out using the joint posterior mode as the starting point for these particular sensitivity tests. They are then compared to the Base Case model which has also been projected forward from its joint posterior mode to allow for a direct comparison.

Sensitivity test S5 does not require any changes to be made to the assessment model; hence the 5000 samples that were randomly selected from the chain to project forward from for the Base Case (Glazer and Butterworth, 2012) were used for this particular sensitivity test and are compared with the Base Case projections using the same 5000 samples.

Performance statistics reported for the various sensitivity tests, and compared to those for the Base Case, comprise the following:

- average annual catches
- average annual variation (AAV) in catch from one year to the next, where:

$$AAV = \frac{1}{20}\sum_{y=2012}^{y=2021} \left|C_y - C_{y-1}\right|/C_{y-1}$$

- $\frac{B^*_{2022}}{K}$
- $\frac{B^*_{lowest}}{K}$

Results and Discussion

Table 1 presents results for various model parameters at the joint posterior mode for sensitivity tests S1-S4. It should be noted that for S3 only results for C=0.1 are reported since it produces the best model fit within the range tested (Table 2).

The exclusion of the trawl CPUE indices from the model fit results in a larger R_0 and hence pristine biomass, and current stock status ($\frac{B^*_{2012}}{B^*_{1971}}$) is estimated to be about double the result for that of the Base Case. The estimate of η is also lower than that of the Base Case, implying that there is less dependence on the effect of jig disturbance on recruitment. This effect was originally postulated by Roel (1998) in order to fit the declining trend observed in the trawl CPUE indices better. A comparison of the Hessian based CVs for key parameters is shown in Table 3. It is notable that the CVs for most of these parameters increase substantially when the trawl CPUE indices are omitted from the model fit.

Assuming a σ_R(input) value of 0.6 improves the Base Case model fit to the jig CPUE and the January-March trawl CPUE, as well as to the spring survey, but the fit deteriorates for the other two indices. A σ_R(input) value of 0.2 results in the reverse effect. Only for the January-March trawl CPUE is the improvement relatively large.

Assuming C = 0.1 produces results in a much smaller pristine biomass being estimated than for the Base Case. Fits to the jig CPUE, spring survey and January-March trawl indices are improved, but deteriorate for the other two indices.

The inclusion of the survey abundance indices utilizing the new trawl gear (S4) produces key model parameter estimates similar to those for the Base Case, with an improvement in current stock status compared to the Base Case. For both autumn old and new gear surveys the catchability coefficients indicate that the autumn survey over-estimates squid abundance, and more so with the new gear than the old gear. Conversely, for both spring old and new gear surveys the catchability coefficients indicate that the spring survey under-estimates squid abundance, but less so with the new gear than the old gear. Since the spring new gear survey series comprises only 4 data points, the effect of omitting this series from the model fit was tested (see S4b in Table 1). A comparison of the Hessian based CVs is shown in Table 4 and indicates that the inclusion of the new gear survey indices does not substantially decrease CVs when compared to the Base Case.

Figure 1 shows median average annual jig catches, together with 90% probability intervals for the Base Case and sensitivity tests 1-4 where projections from 2012 to 2021 were conducted from the joint posterior mode as the starting point. Similarly, performance statistics related to AAV and B/K are presented in Figures 2 and 3 respectively. Results are shown for 4 constant effort scenarios, where the current target effort level in this fishery is 300 000 man-days. Most sensitivity tests show similar median catches to that of the Base Case (Figure 1), with the most notable exceptions being the model that excludes the trawl CPUE indices. However, as indicated in Table 3, the Hessian based CVs for most of the parameters for this model are substantially higher when compared to the Base Case. There is a relatively high level of variability in average annual catches: typically they can vary between 13% and 49% annually for the effort scenarios tested (Figure 2). It is clear from Figure 3 that effort in excess of 300 000 man-days results in what would probably be considered unacceptably high risks of biomass depletion across all models tested.

In order to allow for a fixed annual catch allocation to small scale fishers (S5) from 2013 in addition to the jig fishery continuing at a constant effort level, the biomass equations for the projection period were modified as follows:

$$B_y = B_y^* e^{-g/4} - C_y^{jig\ J-M} - C_y^{trawl\ J-M} - C_y^{small\ scale\ J-M}$$

$$B_{y+1}^* = B_y e^{-3g/4} + R_y - C_y^{jig\ A-D} - C_y^{trawl\ A-D} - C_y^{small\ scale\ A-D}$$

An assumption is required regarding the split of the small scale catch between the January-March and April-December periods. The average catch taken in the jig fishery over the last 3 years in each period was determined and the ratio of each to the total average catch was determined. As a result it has been assumed that 35% of any annual catch allocated to small scale fishers would be caught in the first period and 65% in the second period. Four scenarios for small scale catch were tested, namely 10%, 15%, 20% and 50% of the average jig catch over the last three years (9638 tons). This leads to the following scenarios:

- S5a (10% scenario): 337.3 tons in Jan-Mar and 626.5 tons in Apr-Dec
- S5b (15% scenario): 506.0 tons in Jan-Mar and 939.7 tons in Apr-Dec
- S5c (20% scenario): 674.7 tons in Jan-Mar and 1252.9 tons in Apr-Dec
- S5d (50% scenario): 1686.6 tons in Jan-Mar and 3132.4 tons in Apr-Dec

Given the possible allocation of a fixed catch to the small scale fishers, it is important to understand the level of risk involved. Table 5 therefore reports the effect on jig catches if jig effort is maintained at a constant level for various (additional) constant catch allocations made to the small scale fishers. These results are shown graphically in Figures 4-6. Figure 4 shows median average annual jig and total (jig+small scale) catches, together with 90% probability intervals for the Base Case and the various scenarios related to sensitivity test 5, with projections conducted using 5000 randomly selected samples, with replacement, from the MCMC chain. Similarly, performance statistics related to AAV and B/K are presented in Figures 5 and 6 respectively. It is clear that average jig catches decline as the amount of catch allocated to small scale fishers increases, and furthermore catch variability increases as effort increases for the scenarios tested (Figure 4). This is to be expected because the extra small scale catch reduces abundance on average, so that for the same effort the jig fishery catches less. Average annual variability in jig catches increases across the fixed small scale catch scenarios tested (Figure 5), and is highest particularly for the 50% scenario for each level of effort reported. It is evident from Figure 6 that even under the existing target level of effort (300 000 man-days) the 50% small scale catch scenario poses a risk of extinction. Furthermore, for effort levels in excess of 300 000 man-days catches in excess of 10% allocated to small scale fishers could potentially result in extinction of the squid resource.

Table 6 reports that for the same risk as for the Base Case for a particular level of jig effort, the amount by which this jig effort needs to be dropped to keep risk at a similar level when allowing for fixed catches for the small scale fishers. For these calculations, risk is defined by the 5th percentile of $\frac{B^*_{lowest}}{K}$. These results are shown graphically in Figures 7-9. Figure 7 shows median average annual jig and total (jig+small scale) catches, together with 90% probability intervals for the Base Case and the various scenarios related to sensitivity test 5, with projections again conducted using 5000 randomly selected samples, with replacement, from the MCMC chain. Similarly, performance statistics related to AAV and B/K are presented in Figures 8 and 9 respectively. It is clear that average jig catches decline across the constant small scale catch scenarios and that jig effort is also reduced substantially across these scenarios (Figure 7). Importantly, as the magnitude of the constant small scale catch introduced grows, the overall total average annual catch shows a declining trend for the lower starting effort levels, though the extent of this decline is large only for the lowest effort level considered of 300 000 man-days. It is evident from Figure 8 that average annual variability is less variable (compared to that shown in Figure 5 where jig effort levels are maintained). Reducing jig effort to levels that have similar lower 5th percentiles to that of the Base Case also effectively eliminates risk of the resource becoming extinct, as evident in Figure 9 (compared to that shown in Figure 6 where jig effort levels are maintained). It is also notable that cases where jig effort exceeds 300 000 man-days have what would likely be considered unacceptably low 5th percentiles for $\frac{B^*_{lowest}}{K}$ (ranging from 0.05-0.14).

Table 1: Parameter estimates at the joint posterior mode for the Base Case and various sensitivities thereof.

Model parameters	BC	S1	S2a	S2b	S3	S4a	S4b
	σ_R (input) = 0.3, C=0.2	Excl trawl CPUE	σ_R (input) = 0.2	σ_R (input) = 0.6	C=0.1	incl new gear surveys	incl autumn new gear, excl spring new gear
Key Model Parameters							
R_0 (initial recruitment)	24039	35893	25702	24955	17897	25123	24030
h	0.512	0.534	0.506	0.585	0.564	0.511	0.512
η	0.328	0.010	0.446	0.290	0.100	0.357	0.301
g	1.257	1.231	1.266	1.231	1.304	1.256	1.257
C	0.20	0.20	0.20	0.20	0.10	0.20	0.20
B*1971	33592	50698	35792	35252	24567	35121	33579
B*2012	7903	28016	9540	6783	5378	9946	9219
B*2012/B*1971	0.235	0.553	0.267	0.192	0.219	0.283	0.275
stock-recruit residuals							
σ_R (input)	0.30	0.30	0.20	0.60	0.30	0.30	0.30
σ_R (estimated)	0.23	0.15	0.18	0.29	0.28	0.23	0.23
CPUE jig Apr-Dec							
q	0.001604	0.000534	0.001413	0.001760	0.002167	0.001493	0.001578
σ*	0.222	0.217	0.234	0.213	0.128	0.231	0.227
CPUE trawl Jan-Mar							
q	0.000576		0.000499	0.000643	0.000850	0.000533	0.000568
σ*	0.242		0.269	0.223	0.116	0.243	0.242
CPUE trawl Apr-Dec							
q	0.000140		0.000124	0.000154	0.000193	0.000131	0.000139
σ*	0.253		0.259	0.256	0.214	0.252	0.253
Survey Autumn (old gear)							
q	1.210620	0.398911	1.061060	1.333640	1.686650	1.132700	1.195970
σ*	0.420	0.348	0.417	0.423	0.396	0.421	0.422
Survey spring (old gear)							
q	0.659763	0.227789	0.568025	0.737639	0.906217	0.619567	0.653361
σ*	0.332	0.293	0.343	0.328	0.293	0.333	0.333
Survey Autumn (new gear)							
q						1.572200	1.689690
σ*						0.300	0.305
Survey spring (new gear)							
q						0.702119	
σ*						0.296	
-lnL values							
jig A-D	-8.431	-9.075	-6.797	-9.713	-16.081	-7.200	-7.693
trawl J-M	-7.452		-3.779	-10.655	-24.200	-7.398	-7.474
Trawl A-D	-5.909		-5.054	-5.489	-5.065	-5.980	-5.924
autumn (old gear)	6.121	2.777	6.014	6.237	6.452	6.179	6.205
spring (old gear)	1.360	-0.415	1.772	1.189	1.337	1.390	1.372
autumn (new gear)						-0.030	0.106
spring (new gear)						-0.102	
S/R residuals	-0.007	-6.238	-11.633	21.602	6.463	-0.125	0.032
penalties	-1.143	-1.307	-1.027	-1.325	-0.829	-1.144	-1.143
total	-15.461	-14.260	-20.504	1.845	-31.923	-14.411	-14.520

Table 2: Total $-\ell nL$ for fixed values of C (a parameter relating to the maximum weight given to an abundance index – see equation A.16) ranging from 0.1 – 0.6 in steps of 0.1.

Model	C=0.1	C=0.2 (Base Case)	C=0.3	C=0.4	C=0.5	C=0.6
$-\ell nL$	-31.923	-15.461	0.137	14.338	27.048	38.402

Table 3: Hessian based CVs associated with the estimable parameters for the Base Case and S1 (the model that excludes the trawl CPUE indices from the model fit).

Parameter	Base Case	S1
ℓnX	0.04	0.08
h	0.07	0.27
η	1.82	2.58
g	0.08	0.08
σ_R(estimated)	0.10	0.21

Table 4: Hessian based CVs associated with the estimable parameters for S4a (include autumn and spring new gear survey indices in the model fit) and S4b (include the autumn new gear survey index in the model fit, but not the spring new gear index). The Base Case CVs are also reported.

Parameter	Base Case	S4a	S4b
ℓnX	0.04	0.04	0.04
h	0.07	0.06	0.07
η	1.82	1.67	1.88
g	0.08	0.08	0.08
σ_R(estimated)	0.10	0.10	0.10

Table 5: Risk, in the form of the 5th percentile associated with $\frac{B^*_{lowest}}{K}$, for i) the Base Case (shaded rows) with different levels of jig effort and ii) scenarios where constant catch allocations are made to small scale fishers while holding jig effort (expressed in man-days) constant. The jig catch (tons) associated with each scenario and consequent loss as a result of allocations made to the small scale fishers is also reported, as is the average annual variation in catches.

	Effort	5th %-ile	median jig catch	small scale catch	Total catch	% jig catch lost	median AAV
BC (no small scale catch)	300000	0.192	8181.2	0.0	8181.2		0.29
10%		0.165	7905.0	963.8	8868.8	3.4	0.29
15%		0.150	7761.0	1445.7	9206.7	5.1	0.29
20%		0.133	7613.3	1927.6	9540.9	6.9	0.29
50%		0.000	6658.4	4819.0	11477.4	18.6	0.31
BC (no small scale catch)	400000	0.136	9336.3	0.0	9336.3		0.30
10%		0.106	8960.5	963.8	9924.3	4.0	0.30
15%		0.088	8775.6	1445.7	10221.3	6.0	0.30
20%		0.065	8585.1	1927.6	10512.7	8.0	0.30
50%		0.000	7220.2	4819.0	12039.2	22.7	0.33
BC (no small scale catch)	500000	0.089	10008.0	0.0	10008.0		0.32
10%		0.054	9540.2	963.8	10504.0	4.7	0.32
15%		0.020	9294.5	1445.7	10740.2	7.1	0.32
20%		0.000	9044.3	1927.6	10971.9	9.6	0.33
50%		0.000	7157.6	4819.0	11976.6	28.5	0.39
BC (no small scale catch)	600000	0.053	10286.7	0.0	10286.7		0.34
10%		0.001	9685.1	963.8	10648.9	5.8	0.35
15%		0.000	9371.2	1445.7	10816.9	8.9	0.36
20%		0.000	9049.8	1927.6	10977.4	12.0	0.37
50%		0.000	6421.9	4819.0	11240.9	37.6	0.47

Table 6: Risk, in the form of the 5th percentile associated with $\frac{B^*_{lowest}}{K}$, for i) the Base Case (shaded rows) with different levels of jig effort and ii) scenarios where constant catch allocations are made to small scale fishers while holding risk constant. Given the risk associated with the Base Case, the jig effort (man-days) for the small scale catch scenarios which produce a similar level of risk is reported. The jig catch (tons) associated with each scenario and consequent loss as a result of allocations made to the small scale fishers is also reported.

	Effort	5th %-ile	median jig catch	small scale catch	Total catch	% jig catch lost	median AAV
BC (no small scale catch)	300000	0.192	8181.2	0.0	8181.2		0.29
10%	260000	0.190	7305.0	963.8	8268.8	10.7	0.30
15%	230000	0.196	6668.9	1445.7	8114.6	18.5	0.30
20%	210000	0.194	6199.4	1927.6	8127.0	24.2	0.31
50%	80000	0.196	2704.3	4819.0	7523.3	66.9	0.35
BC (no small scale catch)	400000	0.136	9336.3	0.0	9336.3		0.30
10%	340000	0.140	8397.8	963.8	9361.6	10.1	0.29
15%	320000	0.136	8006.6	1445.7	9452.3	14.2	0.29
20%	290000	0.140	7489.5	1927.6	9417.1	19.8	0.29
50%	150000	0.142	4432.1	4819.0	9251.1	52.5	0.33
BC (no small scale catch)	500000	0.089	10008.0	0.0	10008.0		0.32
10%	430000	0.089	9182.5	963.8	10146.3	8.2	0.30
15%	390000	0.094	8695.9	1445.7	10141.6	13.1	0.30
20%	360000	0.093	8261.4	1927.6	10189.0	17.5	0.30
50%	200000	0.091	5380.9	4819.0	10199.9	46.2	0.31
BC (no small scale catch)	600000	0.053	10286.7	0.0	10286.7		0.34
10%	500000	0.054	9540.2	963.8	10504.0	7.3	0.32
15%	450000	0.057	9095.3	1445.7	10541.0	11.6	0.31
20%	410000	0.057	8645.9	1927.6	10573.5	16.0	0.30
50%	220000	0.060	5693.0	4819.0	10512.0	44.7	0.31

Figure 1: Median average annual jig catch for the Base Case and sensitivity tests S1-S4 for various fixed future effort level scenarios. The 90% probability intervals are also shown. The joint posterior mode vector was used to provide the starting point for projections over 2012-2021.

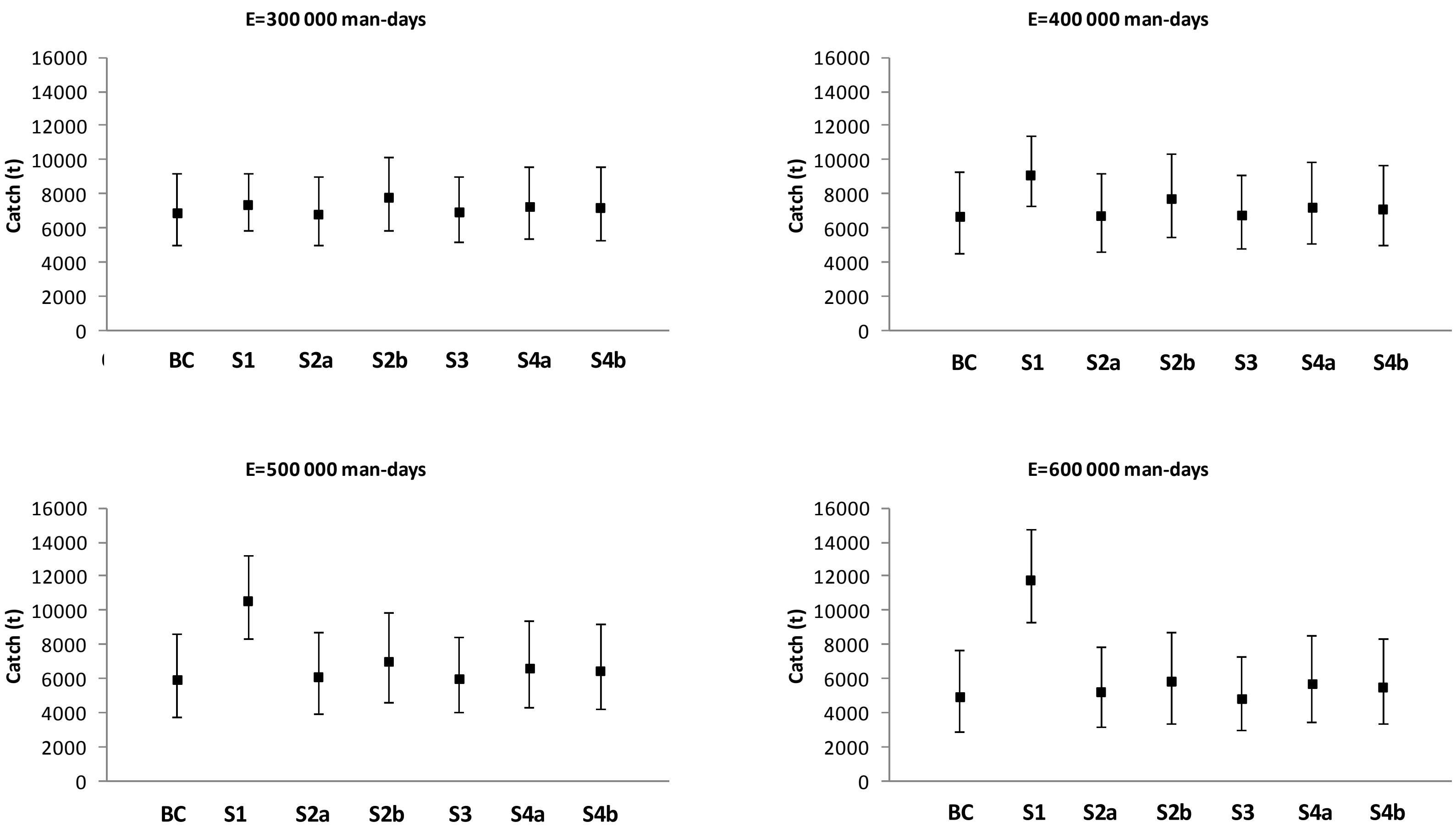


Figure 2: Median average annual variability in jig catch for the Base Case and sensitivity tests S1-S4 for various fixed future effort level scenarios. The 90% probability intervals are also shown. The joint posterior mode vector was used to provide the starting point for projections over 2012-2021.

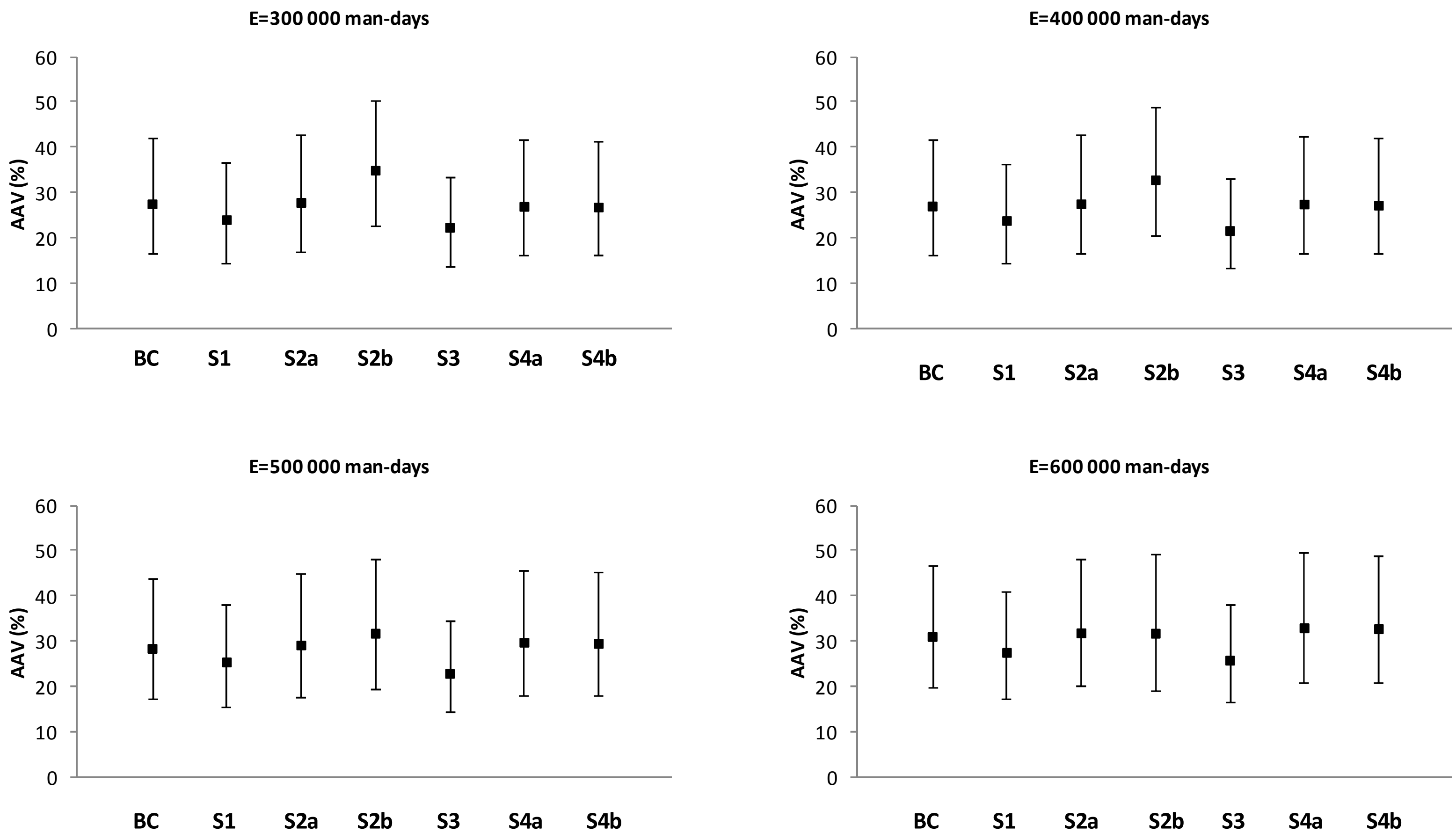


Figure 3: Median depletion for the Base Case and sensitivity tests S1-S4 for various fixed effort levels. The closed squares show depletion in 2022, while the open diamonds show the lowest level of depletion in the projection period. The 90% probability intervals are also shown. The joint posterior mode vector was used to provide the starting point for projections over 2012-2021.

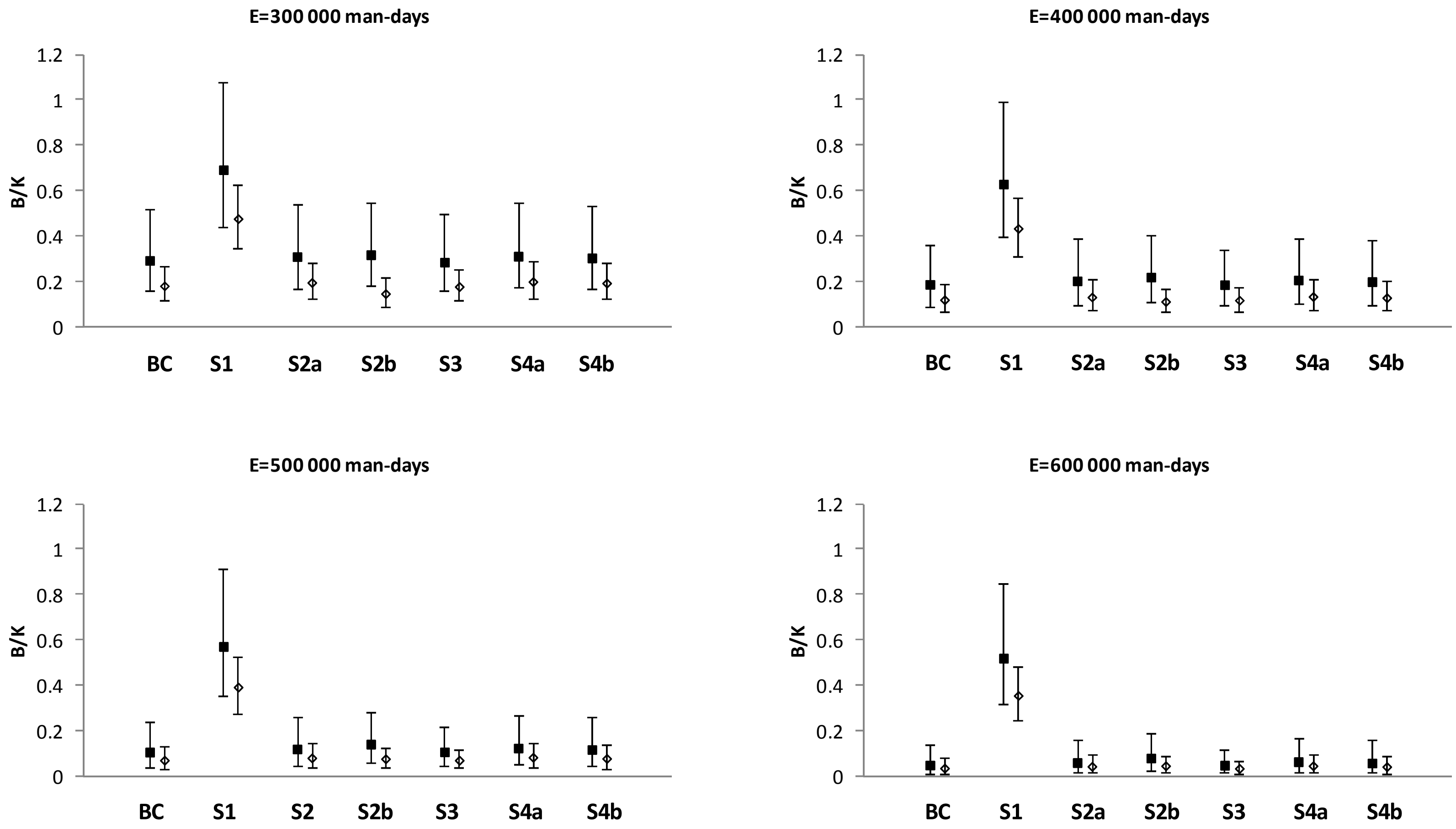


Figure 4: Median average annual catches for the Base Case and sensitivity test S5 where alternative scenarios for fixed catch allocations to small scale fishers are tested in addition to jig effort being maintained at the levels considered previously. Results are shown for various fixed future jig effort level scenarios. The 90% probability intervals are also shown. 5000 randomly selected samples, with replacement, from the MCMC chain were used to project forward. Closed squares show jig catches and open squares show jig+small scale catches.

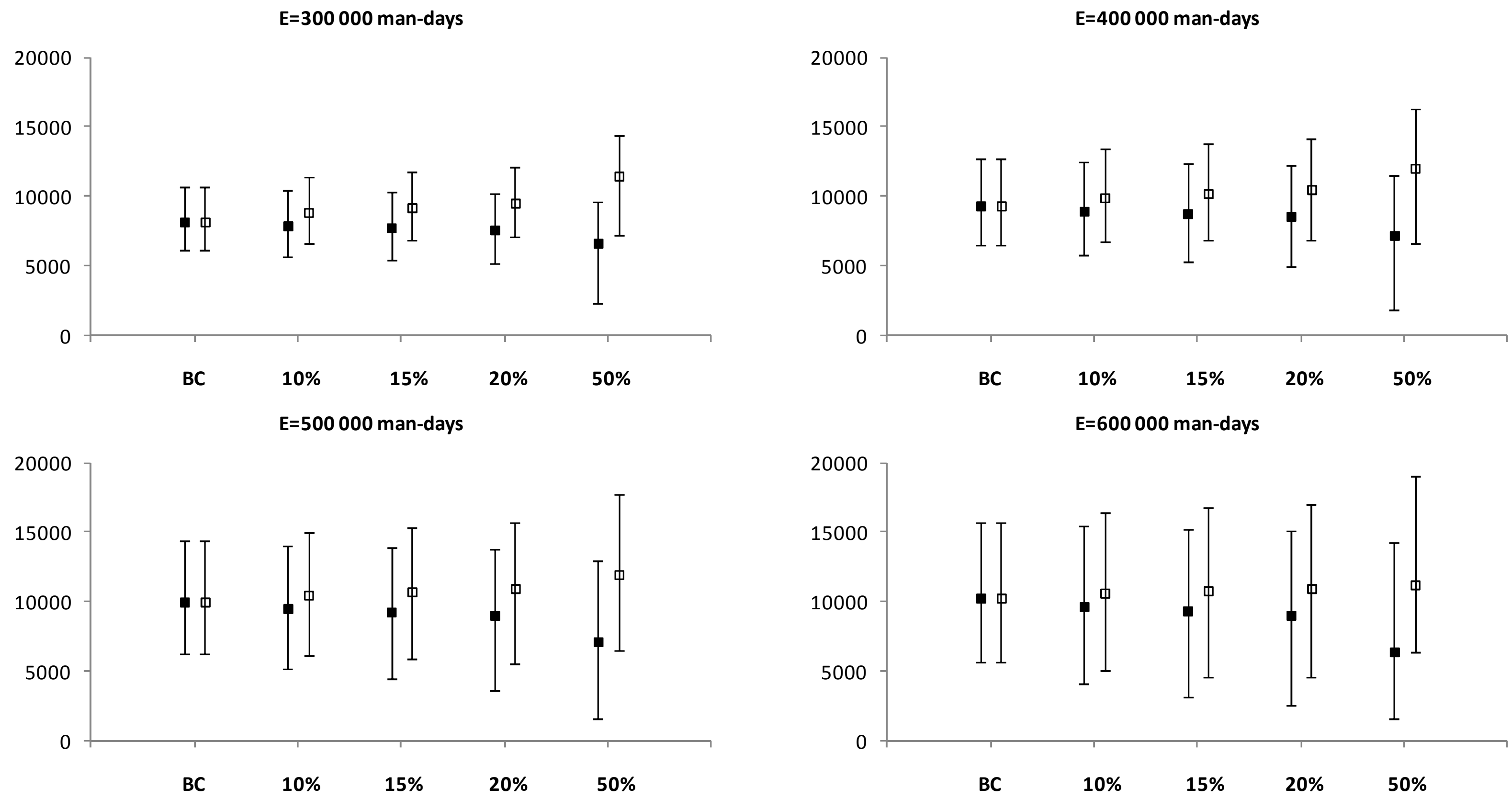


Figure 5: Median average annual variability (2012-2021) in jig catches for the Base Case and sensitivity test S5 where alternative scenarios for fixed catch allocations to small scale fishers are tested. Results are shown for various fixed future jig effort level scenarios. The 90% probability intervals are also shown. 5000 randomly selected samples, with replacement, from the MCMC chain were used to project forward.

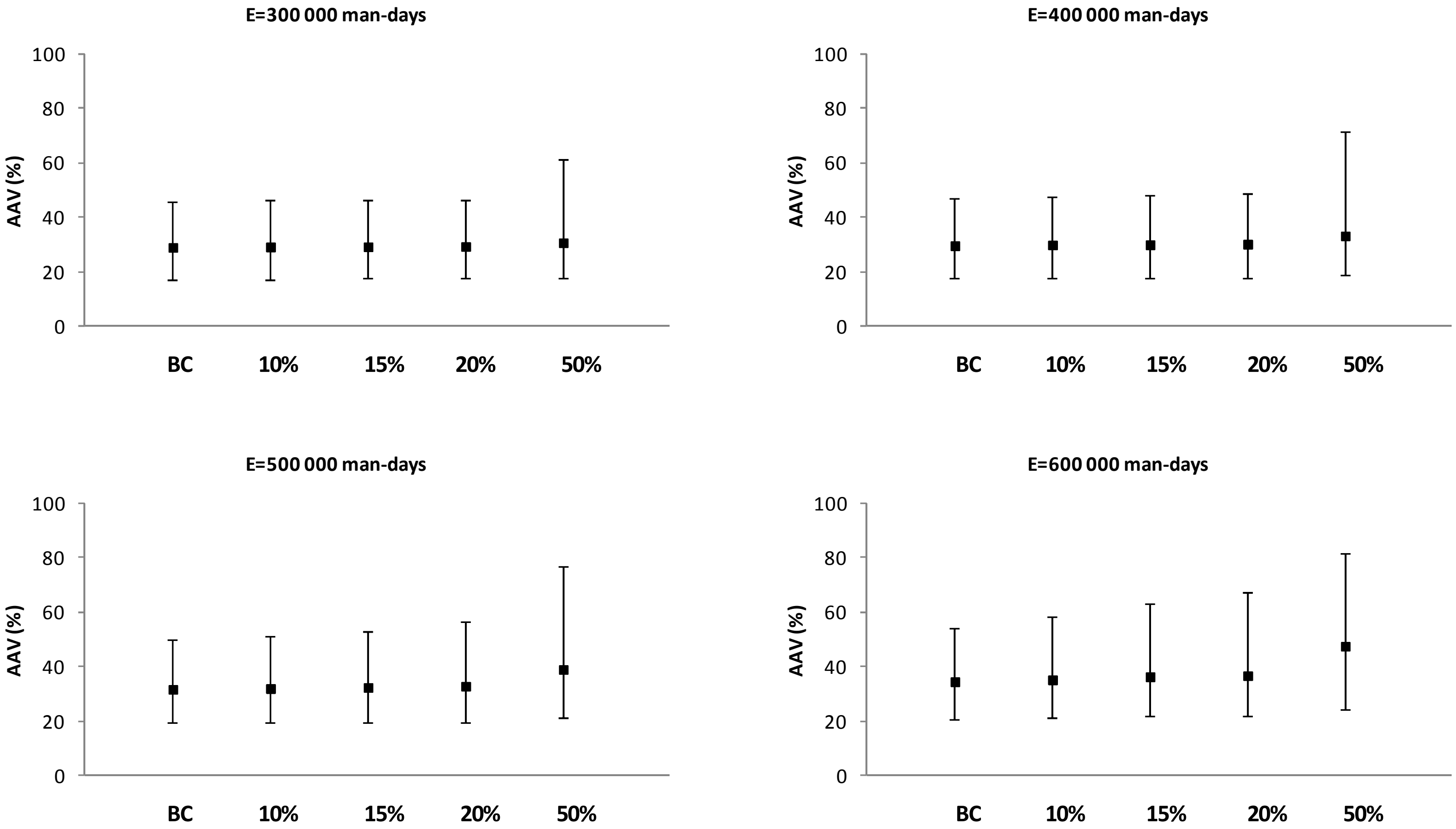


Figure 6: Median depletion for the Base Case and sensitivity test S5 where alternative scenarios for catch allocations to small scale fishers are tested. Results are shown for various fixed future jig effort level scenarios. The closed squares show depletion in 2022, while the open diamonds show the lowest level of depletion in the projection period. The 90% probability intervals are also shown. 5000 randomly selected samples, with replacement, from the MCMC chain were used to project forward.

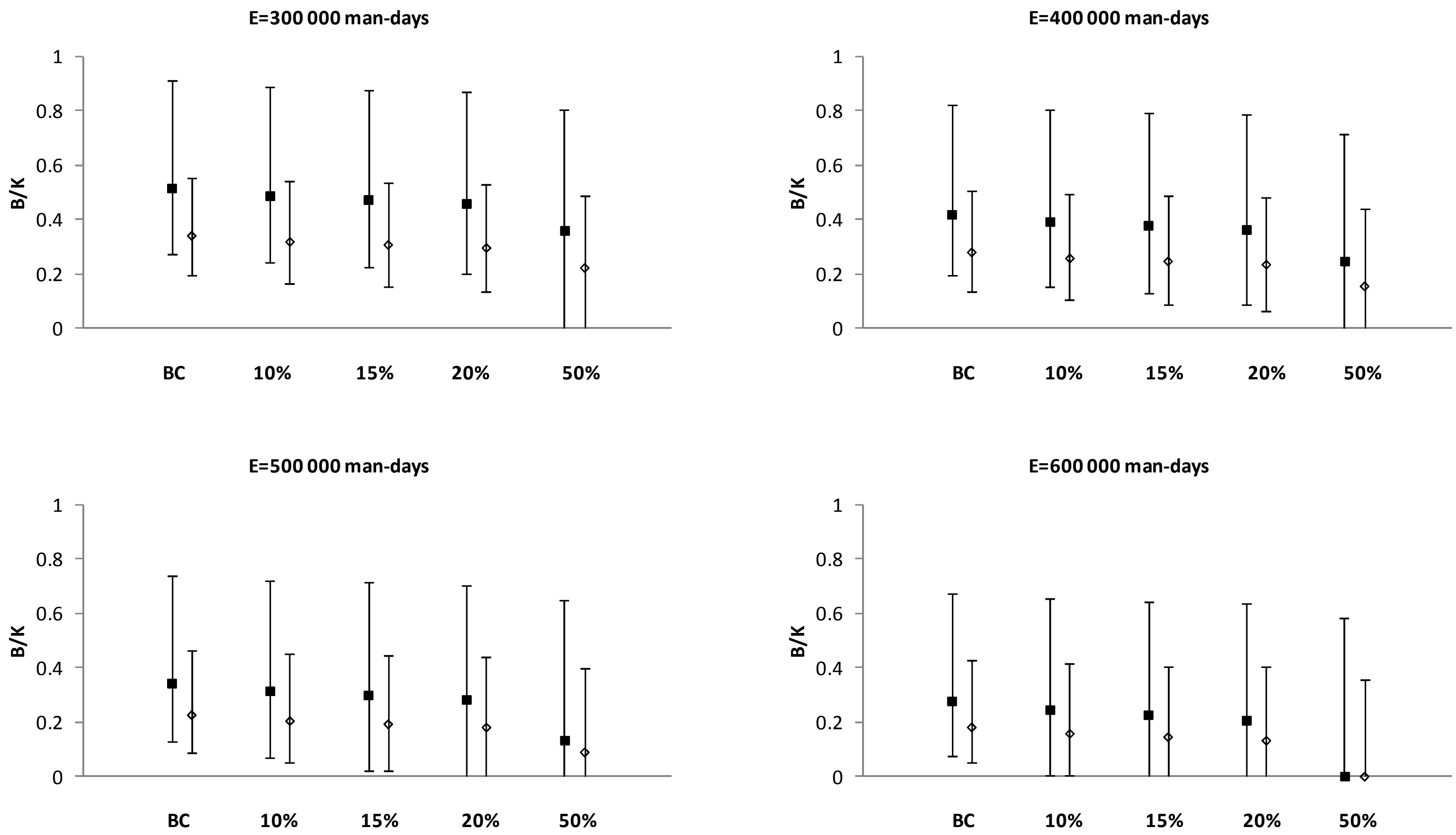


Figure 7: Median average annual catches for the Base Case and sensitivity test S5 where alternative scenarios for fixed catch allocations to small scale fishers are tested with jig effort being reduced to keep risk in terms of $\frac{B^{*}_{lowest}}{K}$ at the same level as that of the Base Case. Results are shown for various fixed future effort level scenarios. The 90% probability intervals are also shown. 5000 randomly selected samples, with replacement, from the MCMC chain were used to project forward. Closed squares show jig catches and open squares show jig+small scale catches.

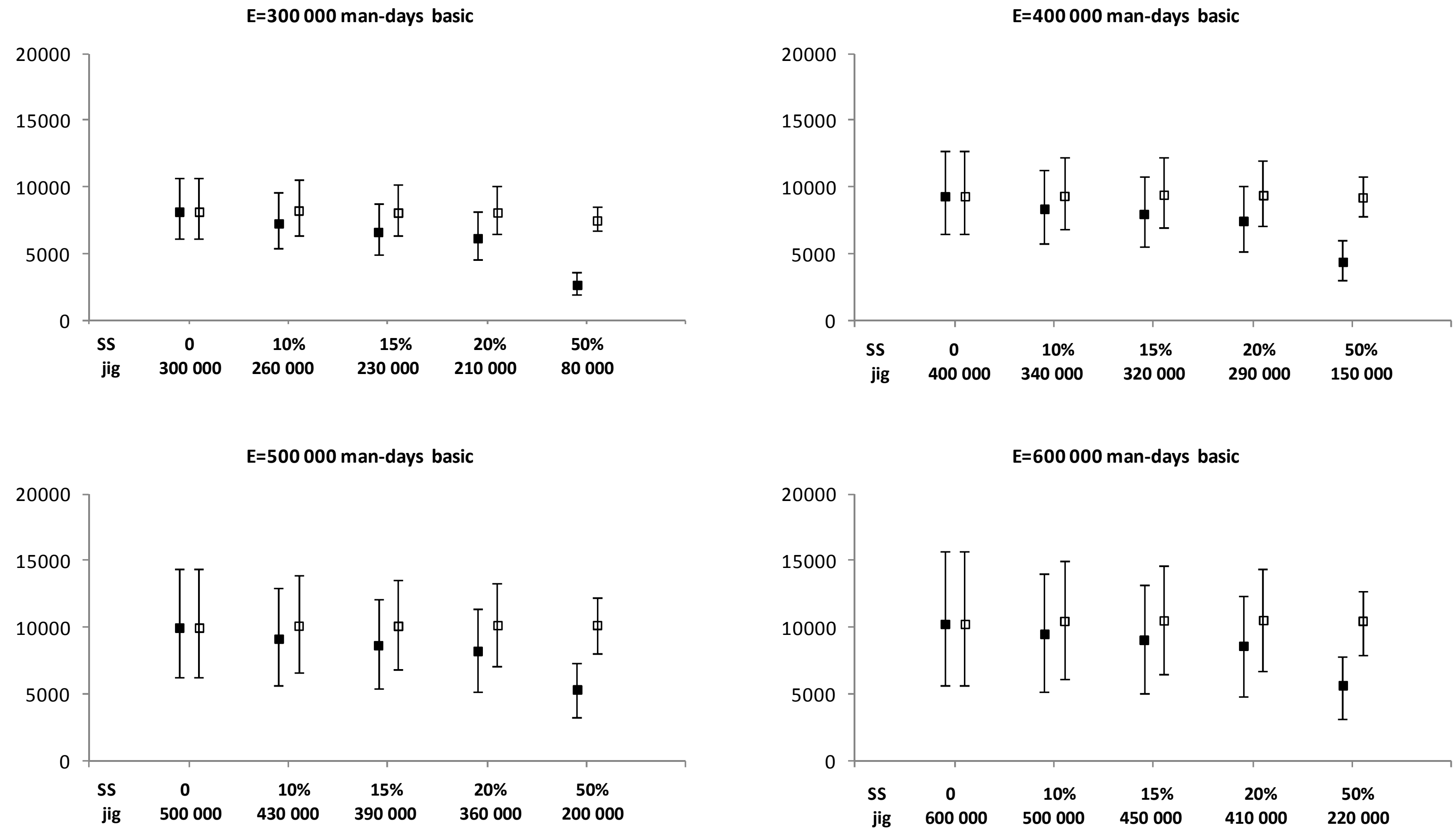


Figure 8: Median average annual variability (2012-2021) in jig catches for the Base Case and sensitivity test S5 where alternative scenarios for fixed catch allocations to small scale fishers are tested. Results are shown for effort level scenarios that would keep risk in terms of ($\frac{B^*_{lowest}}{K}$) at the same level as that for the Base Case. The 90% probability intervals are also shown. 5000 randomly selected samples, with replacement, from the MCMC chain were used to project forward.

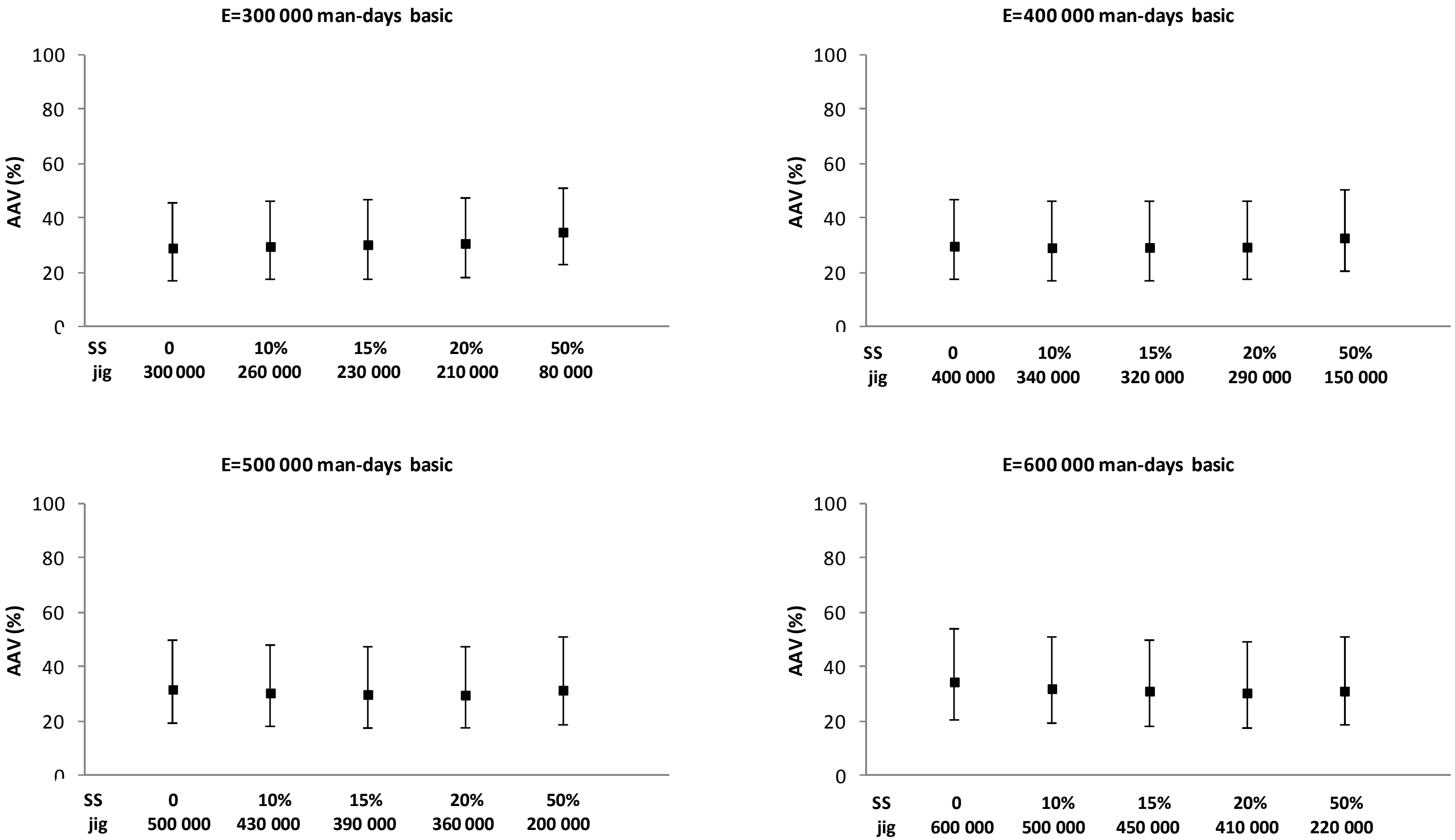
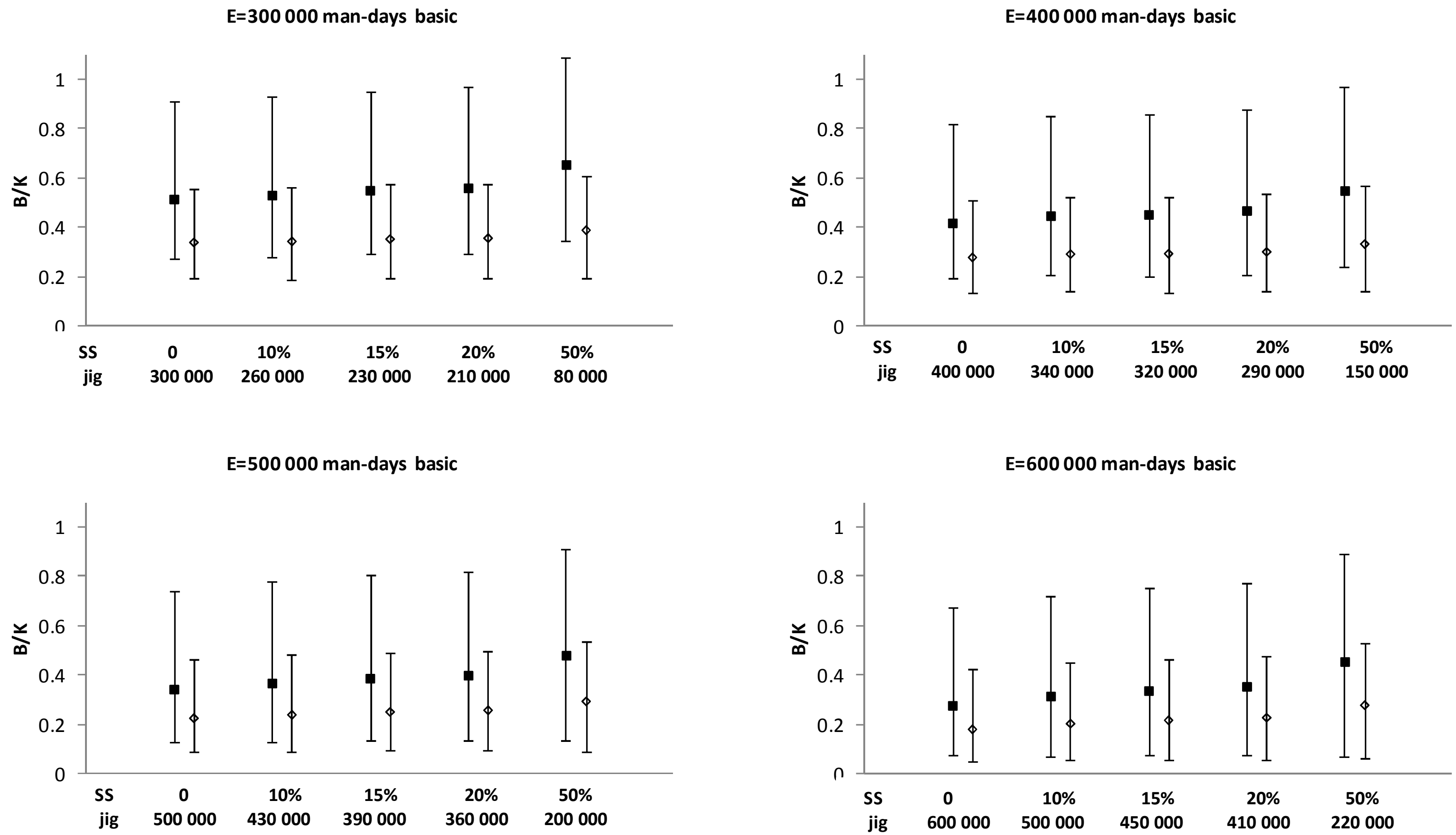


Figure 9: Median depletion for the Base Case and sensitivity test S5 where alternative scenarios for catch allocations to small scale fishers are tested. Results are shown for effort level scenarios that would keep risk in terms of ($\frac{B^*_{lowest}}{K}$) at the same level as that for the Base Case. The closed squares show depletion in 2022, while the open diamonds show the lowest level of depletion in the projection period. The 90% probability intervals are also shown. 5000 randomly selected samples, with replacement, from the MCMC chain were used to project forward.



APPENDIX A: The biomass dynamics model specifications and projection-related catch equations and rules

The population model splits a year into two time periods, January-March and April-December, to better reflect the dynamics of the stock and the two fisheries (jig and trawl) that exploit it. Hardly any recruitment takes place in the January – March period, and jig and trawl catches are disproportionately divided between this and the April-December period (Roel and Butterworth, 2000). The biomass time series is estimated by projecting the assumed pristine biomass at the start of the period B_0^* (= B_{1971}^* = *K*) forward given the historic annual catches.

The biomass dynamics for the two periods are given by:

$$B_y = B_y^* e^{-g/4} - C_y^{jig\ J-M} - C_y^{trawl\ J-M} \tag{A.1}$$

$$B_{y+1}^* = B_y e^{-3g/4} + R_y - C_y^{jig\ A-D} - C_y^{trawl\ A-D} \tag{A.2}$$

where B_y^* is the biomass in year *y* at the start of January,

B_y is the biomass in year *y* at the start of April,

$C_y^{jig\ J-M}$ is the jig catch taken in year *y* between January and March,

$C_y^{jig\ A-D}$ is the jig catch taken in year *y* between April and December,

$C_y^{trawl\ J-M}$ is the trawl catch taken in year *y* between January and March,

$C_y^{trawl\ A-D}$ is the trawl catch taken in year *y* between April and December, and

g is a composite parameter that accounts for natural mortality, emigration and growth.

R_y is the recruitment in year *y*:

$$R_y = \frac{\alpha\beta_y^*(1-\eta F_{y-1}^{jig})}{\beta+B_y^*} e^{(\xi_y - \frac{\sigma_R^2}{2})} \tag{A.3}$$

where:

$$F_y^{jig} = \frac{C_y^{jig\ A-D}}{B_y e^{-3g/4} + R_y} \tag{A.4}$$

η is an estimable parameter and controls the extent to which recruitment is affected by jig fishing mortality. ξ_y is the process error reflecting fluctuation about the expected recruitment for year *y*, drawn from $N(0, \sigma_R^2)$. These residuals are treated as estimable parameters in the model fitting process (σ_R is assumed to be 0.3 on input). The estimated residuals may be used to calculate an estimated $\hat{\sigma}_R = \sqrt{\frac{1}{n}\sum_y \xi_y^2}$ on output. The $\frac{\sigma_R^2}{2}$ term is to correct for bias given the skewness of the log-normal distribution.

α and β are stock-recruit relationship parameters. In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterized in terms of pre-exploitation equilibrium biomass, *K*, and the "steepness", *h*, of the stock-recruitment relationship ("steepness" being the fraction of pristine recruitment that results when biomass drops to 20% of its pristine level):

$$hR_0 = R(0.2K) \qquad \text{A.5}$$

from which it follows that:

$$h = \frac{0.2(\beta + K)}{\beta + 0.2K} \qquad \text{A.6}$$

and hence:

$$\alpha = \frac{4hR_0}{5h - 1} \qquad \text{A.7}$$

and

$$\beta = \frac{K(1-h)}{5h-1} \qquad \text{A.8}$$

The likelihood is calculated assuming that the abundance indices are log-normally distributed about their expected values:

$$I_y^i = \hat{I}_y^i e^{\varepsilon_y^i} \quad \text{or} \quad \varepsilon_y^i = \ell n(I_y^i) - \ell n(\hat{I}_y^i) \qquad \text{A.9}$$

where

I_y^i is the abundance index for year *y* and series *i*, $\hat{I}_y^i = \hat{q}^i \overline{B}_y$ is the corresponding model estimate ($\hat{q}^i$ being the catchability coefficient corresponding to series *i* and $\overline{B}_y$ the average biomass during a given period in year *y*), and ε_y^i is the observation error corresponding to series *i* in year *y*.

For the January-March trawl index,

$$\overline{B}_y = \frac{B_y^* + B_y^* e^{-g/4} - C_y^{jig\ J-M} - C_y^{trawl\ J-M}}{2} \qquad \text{A.10}$$

For the April-December jig and trawl indices,

$$\overline{B}_y = \frac{B_y + R_y + B_{y+1}^*}{2} \qquad \text{A.11}$$

For the autumn survey biomass index,

$$\overline{B}_y = B_y + 0.5R_y \qquad \text{A.12}$$

For the spring survey biomass index

$$\overline{B}_y = B_y + R_y \qquad \text{A.13}$$

The contribution of each abundance index to the negative log-likelihood function (after the removal of constants) is given by:

$$-\ell n L_i = n\ \ell n \sigma^{*i} + \frac{1}{2(\sigma^{*i})^2} \sum_{y=1}^{n_i} (\varepsilon_y^i)^2 \qquad \text{A.14}$$

where $\hat{\sigma}^{*i} = \sqrt{(\hat{\sigma}^i)^2 + C^2}$ A.15

$$\hat{\sigma}^i = \sqrt{\frac{1}{n_i} \sum_y (\varepsilon_y^i)^2} \qquad \text{A.16}$$

and *C*=0.2. The introduction of the *C* factor is to ensure that no abundance index receives unrealistically high weight in the fitting process.

The contribution of the stock-recruitment residuals to the negative log-likelihood function is given by:

$$-\ell nL = \sum_{y} [\ell n\sigma_R + \frac{1}{2\sigma_R^2}\xi_y^2] \qquad \text{A.17}$$

This is a penalty term, being the equivalent in a frequentist framework of what would reflect a normal prior in a Bayesian context.

The derivation of future catches given variability about the catch-effort relationship

The catch-effort relationship $(\frac{C}{E}) = q\overline{B}e^{\varepsilon}$, may be re-arranged to yield $C = qE\overline{B}e^{\varepsilon}$. Substituting equation A.10 for $\overline{B}$ will yield the future catches made in the January-March period for the trawl and jig fisheries respectively. Ignoring the *y* subscripts, these are thus:

$$C^{trawl,J-M} = \frac{q_{trawl,J-M}E_{trawl,J-M}e^{\xi^{trawl,J-M}}B^*(1+e^{\frac{-g}{4}})}{(2+q_{jig,J-M}E_{jig,J-M}e^{\xi^{jig,J-M}}+q_{trawl,J-M}E_{trawl,J-M}e^{\xi^{trawl,J-M}})} \qquad \text{A.18}$$

$$C^{jig,J-M} = \frac{q_{jig,J-M}E_{jig,J-M}e^{\xi^{jig,J-M}}B^*(1+e^{\frac{-g}{4}})}{(2+q_{jig,J-M}E_{jig,J-M}e^{\xi^{jig,J-M}}+q_{trawl,J-M}E_{trawl,J-M}e^{\xi^{trawl,J-M}})} \qquad \text{A.19}$$

Similarly, for the second period (April-December), substituting equation A.11 for $\overline{B}$ will yield the future catches made in the trawl and jig fisheries respectively:

$$C^{trawl,A-D} = \frac{q_{trawl,A-D}E_{trawl,A-D}e^{\varepsilon_{trawl,A-D}}\{B(1+e^{\frac{-3g}{4}})+2R\}}{(2+q_{jig,A-D}E_{jig,A-D}e^{\varepsilon_{jig,A-Dy}}+q_{trawl,A-D}E_{trawl,A-D}e^{\varepsilon_{trawl,A-D}})} \qquad \text{A.20}$$

$$C^{jig,A-D} = \frac{q_{jig,A-D}E_{jig,A-D}e^{\varepsilon_{jig,A-D}}\{B(1+e^{\frac{-3g}{4}})+2R\}}{(2+q_{jig,A-D}E_{jig,A-D}e^{\varepsilon_{jig,A-Dy}}+q_{trawl,A-D}E_{trawl,A-D}e^{\varepsilon_{trawl,A-D}})} \qquad \text{A.21}$$

$\varepsilon_i \sim N(0,(\hat{\sigma}*^i)^2)$, *i* denoting each index of abundance.

Rules for projections

If the estimated biomass in the second period was less than $0.05(B^* \times e^{\frac{-g}{4}})$ then the first period catches were set to $0.95p(B^* \times e^{\frac{-g}{4}})$ and the second period biomass to $0.05(B^* \times e^{\frac{-g}{4}})$. Similarly, if the estimated biomass in the first period of the following year was less than $0.05(B \times e^{\frac{-3g}{4}} + R)$ then the second period catches from the previous year were set to $0.95p(B \times e^{\frac{-3g}{4}} + R)$ and the first period biomass to $0.05(B \times e^{\frac{-3g}{4}} + R)$. *p* apportions the catches in the correct ratio for each period and each fishing type.