

# Assessment of the South African anchovy resource using data from 1984 – 2010: results at the posterior mode

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## Abstract

The operating model (OM) for the South African anchovy resource has been updated from that used to develop OMP-08 given four more years of data and a revised time series of commercial catch. A Hockey Stick stock recruitment relationship, and the same median juvenile and adult natural mortality rates as in previous assessments are used. When considering the Beverton Holt, Ricker and Hockey stick stock recruitment relationships,  $AIC_c$  model selection criterion do not show strong support for one relationship over another, yet the carrying capacity at the posterior mode differs considerably between the Hockey Stick and other relationships. Two base case hypotheses are chosen: one estimates random effects about adult natural mortality over time while the other assumes time-invariant annual adult natural mortality. There has been a decrease in recruitment residual standard deviation and in recruitment autocorrelation for this updated OM compared to that used in previous OMs. The impact of this on the appropriate choices of a risk definition and threshold for the new OMP to be developed needs to be considered. The resource abundance remains above the historic average, with a model-estimated 1+ biomass of 2.2-2.4 million tons in November 2010, having provided 8 years of above average recruitment in the past 11 years. The harvest proportion over the past 11 years has not exceeded 0.13.

## Introduction

The operating model of the South African anchovy resource has been updated from the last assessment (Cunningham and Butterworth 2007, with further updates) to take account of new data collected between 2007 and 2010. In addition there has been a change to the calculation of time series of commercial catch data. The monthly cut-off lengths for recruits now vary on an annual basis in accordance with the cut-off length estimated by the annual recruit survey (de Moor *et al.*, 2011). This operating model is to be used in developing and simulation testing OMP-12.

Initial results of the updated operating model (assessment) of the South African anchovy resource were presented by de Moor and Butterworth (2011a,b). This work led to the conclusion that the use of a random effects model, including autocorrelation, for adult natural mortality had resolved the former problem of perceived trends in the residuals from the model fit to May recruitment and the November proportion-at-age 1 data (de Moor and Butterworth, 2011a). It also resulted in the decision that juvenile natural mortality would best be treated as time-invariant, with model sensitivity to alternative values to be checked in robustness tests.

This document presents the updated base case operating models assuming a Hockey Stick stock recruitment relationship. One base case model estimates random effects about adult natural mortality over time while the other assumes constant (time-invariant) adult natural mortality. A number of robustness tests are also considered. There have been some changes to the data since the initial results reported in de Moor and Butterworth (2011b). Results are given at the posterior mode only. A separate document will show the full posterior distributions.

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## Population Dynamics Model

The operating model used for the South African anchovy resource is detailed in Appendix A. A glossary of all parameters used in this document is given in Appendix B. The data used in this assessment are listed in de Moor *et al.* (2011). The majority of prior distributions for the estimated parameters were chosen to be relatively uninformative.

### Stock recruitment relationship

The following alternative stock recruitment relationships have been considered (Table 1):

- A<sub>BH</sub> – Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity
- A<sub>2BH</sub> – two Beverton Holt stock-recruitment curves, with uniform priors on steepness and carrying capacity, one estimated using data from 1984 to 1999 and the other from 2000 to 2009
- A<sub>R</sub> – Ricker stock-recruitment curve, with uniform priors on steepness and carrying capacity
- A<sub>ModR</sub> – ‘Modified’ Ricker stock-recruitment curve, with uniform priors on steepness, carrying capacity and shape parameter.
- A<sub>HS</sub> – hockey stick stock-recruitment curve, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity
- A<sub>2HS</sub> – two hockey stick stock-recruitment curves, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity, one estimated using data from 1984 to 1999 and the other from 2000 to 2009
- A<sub>fixedHS</sub> – hockey stick stock-recruitment curve with a uniform prior on the log of the maximum recruitment, with the spawning biomass at the inflection point set equal to 20% of  $K$  (to correspond to the assumption made for the 2007 assessment)

### Natural mortality

A number of combinations of juvenile and median adult natural mortality values are tested, covering the range 0.6 to 2.1 year<sup>-1</sup>, and for the case where a Hockey Stick stock recruitment relationship is assumed. For realism, only combinations with  $\bar{M}_j^A \geq \bar{M}_{ad}^A$  are tested.

### Constant adult natural mortality

As projecting forward and simulation testing a new OMP using an operating model including random effects about adult natural mortality is novel, the previous approach of a constant natural mortality with time is also tested:

- A<sub>cstM</sub> – constant annual adult natural mortality, i.e. no random effects model

### Retrospective runs

A<sub>HS</sub> is run using data from 1984 to 1999, to 2003 and to 2007 to compare the base case model estimates to those which would have resulted from data corresponding to the years used as input to the operating models

used for testing OMP-02, OMP-04 and OMP-08. Note that the data used in  $A_{HS}$  and the retrospective runs do not compare directly with those used for the former OMs due to methodological updates over time as well as corrections to historic time series of data.

### Further robustness tests

The following robustness tests to  $A_{HS}$  are also considered:

$A_{10}$  – 10cm cut-off length for calculating the proportion of 1-year-olds in the November survey (Table 6 of de Moor *et al.* 2011)

$A_{10.5}$  – 10.5cm cut-off length for calculating the proportion of 1-year-olds in the November survey (Table 6 of de Moor *et al.* 2011)

$A_{11}$  – 11cm cut-off length for calculating the proportion of 1-year-olds in the November survey (Table 6 of de Moor *et al.* 2011)

$A_{kegg1}$  – negatively biased egg surveys, i.e.,  $k_g^A = 0.75$  (testing sensitivity to assumption 7 of Appendix A)

$A_{kegg2}$  – positively biased egg surveys, i.e.,  $k_g^A = 1.25$  (testing sensitivity to assumption 7 of Appendix A)

$A_{lam1}$  – fix the additional variance (over and above the survey sampling CV) associated with the recruit survey  $(\lambda_r^A)^2 = 0$

$A_{lam2}$  – fix the additional variance (over and above the survey sampling CV) associated with the November survey  $(\lambda_N^A)^2 = 0.02$

$A_{p1}$  – no autocorrelation, i.e.  $\rho = 0$  in the residuals of  $M_{ad,y}^A$  (de Moor and Butterworth 2011b)

## **Results**

### Prior on standard deviation in residuals about adult natural mortality

Figure 1 shows that the likelihood profile of the objective function for a fit assuming the Hockey Stick stock recruitment relationship is bi-modal over a range of fixed  $\sigma_{ad}$  values, where  $\sigma_{ad}$  is the standard deviation of the residuals about the central value of the log of adult natural mortality – see equation A.8. However, by considering the likelihood profiles of all the individual contributions to the objective function it becomes clear that once  $\sigma_{ad} < 0.15$ , the model “flips” into a space where it is mis-specified. Thus for  $\sigma_{ad} < 0.15$ , the fits to the May recruitment survey and to the November proportion-at-age 1 data become poor (Figure 1) with apparent trends in the residuals (see de Moor and Butterworth 2010); the small residuals about annual adult natural mortality cause the steep drop in the log prior contribution for these residuals evidenced in Figure 1. For alternative stock recruitment relationships, the same shape is evident for the likelihood profile, though the  $\sigma_{ad}$  value at which this ‘flip’ occurs differs slightly. For this reason a lower bound of 0.20 is used in the uniform prior on  $\sigma_{ad}$ , thus avoiding scenarios which reflect model mis-specification in fits to May recruitment survey and November proportion-at-age data.

### Natural mortality

Table 2 lists the various contributions to the objective function at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. The following criterion was used to distinguish “reasonable” from “unrealistic” combinations (unrealistic combinations are shaded in Table 2):

- the ratio  $k_r^A/k_N^A \in [0.5, 1.0]$ , as the November spawner biomass survey is expected to have a greater coverage of the full distribution of the resource than the May recruit survey so that the latter should reflect a smaller relative bias.

One further “reality check” was provided by the criterion that the multiplicative bias for the proportion-at-age 1 in the November survey,  $k_p^A$ , should not be markedly different from 1.

There is little change in the posterior distribution as  $\bar{M}_j^A$  is changed for a given  $\bar{M}_{ad}^A$  (about 2 likelihood points, improving as  $\bar{M}_j^A$  decreases). Given  $\bar{M}_j^A$ , the posterior distribution indicated an improved fit to the data for increasing  $\bar{M}_{ad}^A$ , with a slight deviation from this ‘rule’ for  $\bar{M}_{ad}^A = 1.8$  and  $\bar{M}_{ad}^A = 2.1$ . This latter feature may, however, be an artefact of the assessment methodology in that a higher natural mortality results in a higher loss of “memory” of cohorts, making the November survey data easier to fit. Considering  $k_p^A$  then, the following combinations were chosen for a set of robustness tests:

A<sub>HS</sub> -  $\bar{M}_j^A = 0.9$  and  $\bar{M}_{ad}^A = 0.9$  (base case)

A<sub>M1</sub> -  $\bar{M}_j^A = 0.9$  and  $\bar{M}_{ad}^A = 0.6$  (robustness test: alternative  $\bar{M}_{ad}^A$ , worse objective function value, but a high  $k_p^A$  value)

A<sub>M2</sub> -  $\bar{M}_j^A = 1.2$  and  $\bar{M}_{ad}^A = 0.9$  (robustness test: alternative  $\bar{M}_j^A$ , little difference from A<sub>HS</sub> in terms of value of objective function and  $k_p^A$ )

A<sub>M3</sub> -  $\bar{M}_j^A = 1.5$  and  $\bar{M}_{ad}^A = 0.9$  (robustness test: alternative  $\bar{M}_j^A$ , with the objective function value not substantially worse than that for A<sub>HS</sub> and little difference in  $k_p^A$  from A<sub>HS</sub>)

A<sub>M4</sub> -  $\bar{M}_j^A = 1.8$  and  $\bar{M}_{ad}^A = 0.9$  (robustness test: alternative  $\bar{M}_j^A$ , with the objective function value not substantially worse than that for A<sub>HS</sub> and little difference in  $k_p^A$  from A<sub>HS</sub>)

A<sub>M5</sub> -  $\bar{M}_j^A = 1.2$  and  $\bar{M}_{ad}^A = 1.2$  (robustness test: improved objective function compared to A<sub>HS</sub>, though  $k_p^A = 0.87$  is on the low side).

### Stock recruitment relationship

Table 3 lists the various contributions to the objective function at the posterior mode for the alternative stock-recruitment relationships considered. From a frequentist viewpoint, this is strictly a random effects model as

regards the annual variations in adult natural mortality and recruitment. However, the REML process to get unbiased estimates of the variances for these two effects has not been implemented as the key operating model(s) for use in developing OMP-12 will be Bayesian. Thus the use of AIC<sub>c</sub> to compare between alternative stock-recruitment relationships is approximate. AIC<sub>c</sub> suggests that the preferred stock-recruitment relationship is the Hockey stick, with the Beverton Holt and Ricker being close second choices. However, the estimated carrying capacity differs appreciably amongst these relationships. In particular, the carrying capacity for the four Beverton Holt and Ricker stock-recruitment alternatives is at the maximum defined by the prior distribution. Although this will likely have little effect on the simulation testing of OMP-12, the resultant projected biomass as a proportion of virgin abundance will differ substantially between assumed relationships! Sufficient data points are now available to estimate the inflection point of the hockey stick curve. Thus A<sub>HS</sub> is chosen as the base case operating model for OMP-12 development, with robustness being tested to A<sub>BH</sub> and A<sub>R</sub> (Figures 2 and 3). Models with different stock-recruitment relationships before and after the turn of the century as well as the Modified Ricker stock-recruitment relationship were not well supported by AIC<sub>c</sub>, primarily due to the greater number of estimable parameters required for these models. To enable comparison with the former assessment, the hockey stick curve with a fixed inflection point, A<sub>fixedHS</sub>, is also maintained as an alternative.

#### Base case (A<sub>HS</sub>) results at posterior mode

The estimated parameter values and key outputs for A<sub>HS</sub> are listed in Table 4. The population model fits to the time series of abundance estimates of November 1+ biomass, DEPM estimates of spawner biomass, May recruitment and proportion-at-age 1 in November are shown in Figures 4 to 7. The model projected posterior mode estimates of May recruitment in 2010 fall near the extremes of the 95% PI due to the model struggling to match a sharp decrease in the 1+ biomass estimate after a relatively good recruitment estimate. The annual adult natural mortality is plotted in Figure 8 together with the estimated residuals. Some autocorrelation between these residuals is estimated by the model ( $\rho = 0.43$ ). The historic annual harvest rates are plotted in Figure 9.

One new aspect of this operating model, compared to historic models, is that it has incorporated a random effects model for adult natural mortality. At the posterior mode of A<sub>HS</sub>, adult natural mortality is estimated to vary between 0.61 and 1.91. This variability is quite large, with 7 out of the past 10 years having above average adult natural mortality, where the historic average is 1.06year<sup>-1</sup>. The increase in natural mortality at the turn of the century implies that loss of anchovy to predation exceeded 6 million tons (Table 5). The autocorrelation in the residuals about adult natural mortality will affect future projections. As this is charting new territory, the cautious approach adopted is to consider two base case hypotheses with and without this random effects model when simulation testing OMP-12.

#### The alternative base case (A<sub>estM</sub>)

The fit of the model predictions from  $A_{\text{cstM}}$  to the data are also shown in Figures 4 to 7, with historic annual harvest rates plotted in Figure 9. The overall fit to the data is worse than for  $A_{\text{HS}}$  (Table 4) with trends in the residuals of the model fit to May recruitment and proportion-at-age 1, as discussed in de Moor and Butterworth (2011a). The difference between the largest and smallest annual losses to predation is 4.1 million tons compared to 7.3 million tons under  $A_{\text{HS}}$  (Table 5).

### Retrospective runs

There is little difference in the historic November 1+ biomass trajectory and key model parameters for the retrospective runs (Table 6, Figure 9).

### Further robustness tests

The model parameters, contributions to the objective function and key model outputs at the posterior mode for the robustness tests are given in Table 4. There were three cases which resulted in an overall improvement in the posterior at the mode. In the case of  $A_{\text{lam2}}$ , the larger additional variance on the November survey results in a significant improvement in the fit to the recruit survey ( $(\lambda_r^A)^2$  was estimated to be much smaller than in  $A_{\text{HS}}$ ) and at the expense of fitting to the November survey spawner biomass. Given the confidence scientists place in the November survey, and the lack of fit of  $A_{\text{lam2}}$  to the November survey, this case was not considered more plausible than the chosen base case  $A_{\text{HS}}$ . The improved fit for  $A_{\text{M5}}$  is coupled with a  $k_p^A$  value further removed from 1 than that for  $A_{\text{HS}}$ . The objective function for  $A_{\text{I1}}$  is worse primarily due to the larger estimated annual deviations about  $\bar{M}_{ad}^A$ , although the model is still able to fit the data well. However, as the cur-off length used to determine the proportions-at-age 1 decreases, the model is less able to fit the proportion-at-age data and for  $A_{\text{I0}}$  and  $A_{\text{I0.5}}$  the model is mis-specified resulting in an increasing trend in the residuals about  $\bar{M}_{ad}^A$ .

The risk threshold remains around 1.2 million tons for most robustness tests, except for  $A_{\text{Kegg1}}$  and  $A_{\text{Kegg2}}$ , which directly affect the scaling of the November acoustic surveys. In contrast, the estimated carrying capacity differs substantially between alternative robustness tests, indicating yet again this is not a reliable parameter upon which to base future depletion targets.

### **Discussion**

This document has detailed the updated assessment of the South African anchovy resource. Two base case hypotheses have been chosen, one assuming a random effects model for adult natural mortality,  $A_{\text{HS}}$ , and one assuming constant adult natural mortality,  $A_{\text{cstM}}$ . A Hockey stick stock recruitment relationship is assumed for the base case hypotheses. Results at the posterior mode have also been presented for a number of robustness tests to  $A_{\text{HS}}$ . The resource abundance remains above average, with a model-estimated 1+ biomass of 2.2-2.4 million tons in November 2010 under the base case hypotheses, having provided 8 years of above average recruitment in the past 11 years. The harvest proportion over the past 11 years has not exceeded 0.13 (Figure 11).

Although both base case hypotheses still lead to some questions (high variability in adult natural mortality vs trends in model fits to data), the use of both hypotheses when simulation testing OMP-12 should adequately cover likely possibilities.

### References

- Cunningham, C.L., and Butterworth, D.S. 2007. Assessment of the South African Anchovy Resource. Unpublished MCM Document MCM/2007/SEPT/SWG-PEL/05. 29pp.
- de Moor, C.L., and Butterworth, D.S. 2011a. Assessment of the South African anchovy resource using data from 1984 – 2010: initial results. Department of Agriculture, Forestry and Fisheries Document FISHERIES/2011/SWG-PEL/06. 11pp.
- de Moor, C.L., and Butterworth, D.S. 2011b. Assessment of the South African anchovy resource using data from 1984 – 2010: variability in natural mortality. DEAT: Branch Fisheries Document FISHERIES/2011/SWG-PEL/22. 14pp.
- de Moor, C.L., Coetzee, J., Durholtz D.,Merkle, D., and van der Westhuizen, J.J., 2011. A final record of the generation of data used in the 2011 sardine and anchovy assessments. Department of Agriculture, Forestry and Fisheries Document FISHEREIS/2011/SWG-PEL/51. 31pp.

**Table 1.** The alternative stock-recruitment relationships considered. The parameter  $h^A$  denotes the “steepness” of the stock-recruitment relationship, which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass  $K^A$  (shown in units of thousands of tons). For the hockey stick model,  $X = \sum_{a=1}^3 \bar{w}_a^A e^{-M_j^A - (a-1)\bar{M}_{ad}^A} + \bar{w}_{4+}^A e^{-M_j^A - 3\bar{M}_{ad}^A} \frac{1}{1 - e^{-\bar{M}_{ad}^A}}$ , where  $\bar{w}_a^A$  is the average of  $w_{y,a}^A$  as defined in Appendix A. For the hockey stick model,  $a^A$  denotes the maximum recruitment (in billions) and  $b^A$  denotes the spawner biomass below which the expectation for recruitment is reduced below the maximum.

Test	Stock recruitment relationship	$f(SSB_{y,N}^A) =$	Parameters
A <sub>BH</sub>	Beverton Holt	$\frac{\alpha^A SSB_y^A}{\beta^A + SSB_y^A}$	$h^A \sim U(0.2,1.5)$ $K^A \sim U(0,10)$ $\alpha^A = \frac{4h^A}{5h^A - 1} \frac{K^A}{X}$ $\beta^A = \frac{K^A(1 - h^A)}{5h^A - 1}$
A <sub>2BH</sub>	Beverton Holt (2 curves)	$\frac{\alpha_1^A SSB_y^A}{\beta_1^A + SSB_y^A}$ if $y < 2000$ $\frac{\alpha_2^A SSB_y^A}{\beta_2^A + SSB_y^A}$ if $y \geq 2000$	$h_{1/2}^A \sim U(0.2,1.5)$ $K_{1/2}^A \sim U(0,10)$ $\alpha_{1/2}^A = \frac{4h_{1/2}^A}{5h_{1/2}^A - 1} \frac{K_{1/2}^A}{X}$ $\beta_{1/2}^A = \frac{K_{1/2}^A(1 - h_{1/2}^A)}{5h_{1/2}^A - 1}$
A <sub>R</sub>	Ricker	$\alpha^A SSB_y^A e^{-\beta^A SSB_{y,N}^A}$	$h^A \sim U(0.2,1.5)$ $K^A \sim U(0,10)$ $\alpha^A = \frac{1}{X} \left( \frac{h^A}{0.2} \right)^{1/0.8}$ $\beta^A = \frac{\ln(h^A / 0.2)}{0.8K^A}$
A <sub>ModR</sub>	Modified Ricker	$\alpha^A SSB_y^A e^{-\beta^A (SSB_{y,N}^A)^c}$	$h^A \sim U(0.2,1.5)$ $K^A \sim U(0,10)$ $c \sim U(0,1)$ $\alpha^A = \frac{1}{X} \left( \frac{h^A}{0.2} \right)^{\frac{1}{1-0.2^c}}$ $\beta^A = \frac{\ln(h^A / 0.2)}{(K^A)^c [1 - 0.2^c]}$

Table 1 (continued).

Test	Stock recruitment relationship	$f(SSB_{y,N}^A) =$	Parameters
A <sub>HS</sub>	Hockey stick	$\begin{cases} a^A & , \text{if } SSB_y^A \geq b^A \\ \frac{a^A}{b^A} SSB_y^A & , \text{if } SSB_y^A < b^A \end{cases}$	$\ln(a^A) \sim U(0,7.2)^1$ $\frac{b^A}{K^A} \sim U(0,1)$ $K^A = a^A X^2$
A <sub>2HS</sub>	Hockey stick (2 curves)	<p>if <math>y &lt; 2000</math>:</p> $\begin{cases} a_1^A & , \text{if } SSB_y^A \geq b_1^A \\ \frac{a_1^A}{b_1^A} SSB_y^A & , \text{if } SSB_y^A < b_1^A \end{cases}$ <p>if <math>y \geq 2000</math>:</p> $\begin{cases} a_1^A & , \text{if } SSB_y^A \geq b_1^A \\ \frac{a_1^A}{b_1^A} SSB_y^A & , \text{if } SSB_y^A < b_1^A \end{cases}$	$\ln(a^A) \sim U(0,7.2)^1$ $\frac{b^A}{K^A} \sim U(0,1)$ $K^A = a^A X^2$
A <sub>fixedHS</sub>	Hockey stick	$\begin{cases} a^A & , \text{if } SSB_y^A \geq b^A \\ \frac{a^A}{b^A} SSB_y^A & , \text{if } SSB_y^A < b^A \end{cases}$	$\ln(a^A) \sim U(0,7.2)$ $b^A = 0.2K^A$ $K^A = a^A X$

<sup>1</sup> Given the lack of *a priori* information on the scale of  $a^A$ , a log-scale was used, with a maximum corresponding to about 10 million tons.

<sup>2</sup> For consistency,  $K$  relates throughout to corresponding MLEs. These will be less than the corresponding average pre-exploitation levels because of the lognormal distributions assumed for recruitment.

**Table 2.** The contributions to the objective function at the posterior mode for a range of combinations of juvenile,  $\bar{M}_j^A$ , and adult,  $\bar{M}_{ad}^A$ , natural mortality for models assuming the Hockey Stick stock recruitment relationship. The ratio of the multiplicative bias in the recruit survey to that in the November survey,  $k_r^A/k_N^A$ , and the multiplicative bias in the proportion-at-age 1 in the November survey,  $k_p^A$ , are given for diagnostic purposes. Shaded cells represent unrealistic choices in terms of the criteria applied.

$\bar{M}_j^A$	$\bar{M}_{ad}^A$	Poster-ior	-ln(Likelihood)				-ln(Prior)		$k_r^A$	$k_N^A$	$k_r^A/k_N^A$	$k_p^A$
			Nov	Egg	Rec	Prop	$\epsilon_y^A$	$\epsilon_y^{ad}$				
0.6	0.6	34.83	-6.48	6.98	5.21	-4.09	18.33	14.88	1.12	1.09	0.98	1.03
0.9	0.6	35.18	-6.42	7.04	5.14	-4.09	18.63	14.88	1.12	0.97	0.87	1.03
0.9	0.9	27.28	-6.49	7.17	4.92	-0.75	20.18	2.26	1.12	0.88	0.79	0.94
1.2	0.6	35.59	-6.38	7.08	5.15	-4.10	18.93	14.90	1.12	0.87	0.78	1.03
1.2	0.9	27.75	-6.32	7.22	4.86	-0.79	20.53	2.24	1.12	0.79	0.70	0.95
1.2	1.2	21.91	-8.29	6.67	5.01	4.07	22.08	-7.63	1.11	0.71	0.64	0.87
1.5	0.6	36.04	-6.36	7.12	5.24	-4.11	19.23	14.92	1.12	0.78	0.69	1.03
1.5	0.9	28.11	-6.39	7.25	5.09	-0.77	20.74	2.19	1.12	0.70	0.62	0.95
1.5	1.2	22.31	-8.25	6.71	5.13	4.05	22.33	-7.67	1.12	0.63	0.56	0.87
1.5	1.5	20.36	-11.63	6.01	4.65	9.74	23.27	-11.69	1.10	0.57	0.52	0.81
1.8	0.6	36.52	-6.36	7.14	5.42	-4.14	19.52	14.95	1.13	0.69	0.61	1.03
1.8	0.9	28.64	-6.26	7.28	5.21	-0.83	21.06	2.18	1.13	0.62	0.55	0.95
1.8	1.2	22.75	-8.23	6.73	5.33	4.03	22.57	-7.70	1.12	0.56	0.50	0.87
1.8	1.5	20.77	-11.63	6.04	4.85	9.76	23.48	-11.73	1.11	0.51	0.46	0.81
1.8	1.8	20.07	-14.00	5.61	5.08	13.95	23.79	-14.36	1.10	0.47	0.42	0.76
2.1	0.6	37.04	-6.38	7.15	5.66	-4.17	19.80	14.98	1.13	0.62	0.55	1.03
2.1	0.9	31.23	-13.08	5.96	7.51	18.97	24.08	-12.21	1.16	0.45	0.39	0.78
2.1	1.2	27.15	-14.01	5.72	7.33	18.64	24.60	-15.14	1.14	0.43	0.38	0.77
2.1	1.5	21.32	-11.74	5.96	5.17	10.22	23.59	-11.88	1.13	0.45	0.40	0.81
2.1	1.8	46.14	-14.48	5.44	2.52	14.96	23.33	13.26	1.13	0.41	0.37	0.75
2.1	2.1	60.73	-15.68	5.04	6.23	20.28	22.38	21.14	1.13	0.37	0.32	0.68

**Table 3.** The contributions to the objective function at the posterior mode for alternative stock recruitment relationships.

	$A_{BH}$	$A_{2BH}$	$A_R$	$A_{ModR}$	$A_{HS}$	$A_{2HS}$	$A_{fixedHS}$
Objective function	27.34	26.85	27.32	27.32	27.28	26.75	36.06
$-\ln(L_{Nov})$	-6.64	-6.92	-6.60	-6.62	-6.49	-6.59	-7.97
$-\ln(L_{Egg})$	7.12	6.89	7.13	7.12	7.17	6.99	6.33
$-\ln(L_{Rec})$	5.11	5.23	5.06	5.09	4.92	4.92	6.92
$-\ln(L_{Prop})$	-0.56	-0.12	-0.59	-0.57	-0.75	-0.30	0.16
$-\ln(\text{Prior rec residuals})$	20.37	19.98	20.33	20.35	20.18	19.58	29.88
$-\ln(\text{Prior Mad residuals})$	1.93	1.78	1.99	1.97	2.26	2.15	0.75
# parameters	67	70	67	68	67	69	66
Sample size (i.e. data points)	90	90	90	90	90	90	90
AIC	188.68	193.69	188.65	190.65	188.56	191.51	204.13
$AIC_c$	602.86	716.85	602.83	637.51	<b>602.74</b>	674.51	588.65
$h^A$	0.30	0.34	0.31	0.31			
$K^A$	10000	4145	10000	10000	6683	3030	2441
$c$				0.89			
$a^A$	2913	1005	0.212	0.217	834	378	305
$b^A$	13352	3913	0.00005	0.00016	4299	1957	488
$h_2^A$		0.32					
$K_2^A$		10000				6653	
$a_2^A$		2726				830	
$b_2^A$		11850				4298	

**Table 4.** Key parameter values estimated at the joint posterior mode together with key model outputs. All robustness tests are defined in the main text and all parameters are defined in the Appendix. Fixed values are given in **bold**. Numbers are reported in billions and biomass in thousands of tons.

	A <sub>HS</sub>	A <sub>cstM</sub>	A <sub>BH</sub>	A <sub>R</sub>	A <sub>fixedH</sub> <sub>S</sub>	A <sub>M1</sub>	A <sub>M2</sub>	A <sub>M3</sub>	A <sub>M4</sub>	A <sub>M5</sub>	A <sub>10</sub>	A <sub>10.5</sub>	A <sub>11</sub>	A <sub>kegg1</sub>	A <sub>kegg2</sub>	A <sub>lam1</sub>	A <sub>lam2</sub>	A <sub>p1</sub>
Objective function	27.28	66.75	27.34	27.32	36.06	35.18	27.75	28.11	28.64	21.91	40.34	37.99	36.96	28.49	26.32	30.29	27.03	30.32
-ln(L <sub>Nov</sub> )	-6.49	-3.52	-6.64	-6.60	-7.97	-6.42	-6.32	-6.39	-6.26	-8.29	-12.23	-12.21	-8.01	-6.36	-6.67	-0.69	4.62	-8.72
-ln(L <sub>Egg</sub> )	7.17	7.90	7.12	7.13	6.33	7.04	7.22	7.25	7.28	6.67	6.45	6.28	6.30	7.23	7.06	8.79	9.78	6.68
-ln(L <sub>Rec</sub> )	4.92	10.84	5.11	5.06	6.92	5.14	4.86	5.09	5.21	5.01	7.53	6.97	8.06	4.72	5.19	-2.50	-1.51	4.89
-ln(L <sub>Prop</sub> )	-0.75	28.65	-0.56	-0.59	0.16	-4.09	-0.78	-0.77	-0.83	4.07	27.02	25.13	-1.50	-0.89	-0.61	2.38	2.20	-1.69
-ln(Prior rec residuals)	20.18	22.88	20.37	20.33	29.88	18.63	20.53	20.74	21.06	22.08	26.42	25.78	23.00	20.79	19.67	19.35	18.17	20.34
-ln(Prior Mad residuals)	2.26	N/A	1.93	1.99	0.75	14.88	2.24	2.19	2.18	-7.63	-14.85	-13.95	9.11	3.00	1.67	2.96	-6.23	8.82
	Fixed/Estimated parameters																	
$\bar{M}_j^A$	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>1.2</b>	<b>1.5</b>	<b>1.8</b>	<b>1.2</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>
$\bar{M}_{ad}^A$	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.6</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>1.2</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>
$N_{1983,0}^A$	162	155	161	161	163	144	211	274	360	234	81	93	140	196	142	167	170	161
$N_{1983,1}^A$	138	141	137	137	140	105	138	137	138	178	217	215	149	179	113	142	154	150
$N_{1983,2}^A$	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
$N_{1983,3}^A$	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
$k_N^A$	1.12	1.18	1.13	1.13	1.15	1.12	1.12	1.12	1.13	1.11	1.18	1.17	1.15	0.84	1.40	1.04	1.01	1.11
$k_r^A$	0.88	1.06	0.89	0.89	0.90	0.97	0.79	0.70	0.62	0.71	0.90	0.86	0.94	0.69	1.05	0.82	0.84	0.89
$k_r^A/k_N^A$	0.79	0.90	0.79	0.79	0.78	0.87	0.70	0.62	0.55	0.64	0.77	0.73	0.82	0.83	0.75	0.78	0.83	0.81
$k_g^A$	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>0.75</b>	<b>1.25</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>
$k_p^A$	0.94	0.97	0.95	0.95	0.94	1.03	0.95	0.95	0.95	0.87	0.51	0.71	1.00	0.95	0.94	0.94	0.97	0.96
$(\lambda_N^A)^2$	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.020</b>	<b>0.000</b>

Table 4 (continued).

	A <sub>HS</sub>	A <sub>cstM</sub>	A <sub>BH</sub>	A <sub>R</sub>	A <sub>fixedHS</sub>	A <sub>M1</sub>	A <sub>M2</sub>	A <sub>M3</sub>	A <sub>M4</sub>	A <sub>M5</sub>	A <sub>10</sub>	A <sub>10.5</sub>	A <sub>11</sub>	A <sub>kegg1</sub>	A <sub>kegg2</sub>	A <sub>lam1</sub>	A <sub>lam2</sub>	A <sub>p1</sub>
$(\lambda_r^A)^2$	0.052	0.100	0.053	0.053	0.066	0.053	0.052	0.053	0.054	0.053	0.069	0.065	0.076	0.050	0.054	<b>0.000</b>	0.018	0.052
$(\sigma_p^A)^2$	0.09 <sup>3</sup>	0.49	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.43	0.38	0.09	0.09	0.09	0.09	0.09	0.09
$\sigma_{ad}$	0.26		0.26	0.26	0.25	0.42	0.26	0.26	0.26	0.20 <sup>4</sup>	0.20 <sup>5</sup>	0.20 <sup>5</sup>	0.34	0.27	0.26	0.27	0.20	0.34
$\rho$	0.43		0.43	0.43	0.45	0.46	0.43	0.43	0.43	0.43	0.93	0.94	0.38	0.42	0.44	0.43	0.48	<b>0.00</b>
$a^A$	834	402			305	775	1403	1469	2390	1190	587	837	831	1083	687	957	917	817
$b^A$	4299	2024			488	4381	5532	4342	5380	4220	2739	3944	4444	5945	3342	5005	4972	4294
$K^A$	6683	3223	10000	10000	2441	8865	8331	6462	7790	5667	4708	6709	6658	8684	5506	7669	7354	6549
$h^A$	0.31	0.32	0.30	0.31	1.00	0.40	0.30	0.30	0.29	0.27	0.34	0.34	0.30	0.29	0.33	0.31	0.30	0.31
$\sigma_r^A$	0.53	0.58	0.53	0.53	0.76	0.50	0.53	0.54	0.54	0.57	0.67	0.65	0.59	0.54	0.52	0.51	0.50	0.53
	Model Outputs																	
$B_{2010}^A$	2228	2368	2204	2209	2104	2171	2226	2218	2215	2243	2101	2128	2064	2981	1784	2491	2995	2218
$\bar{B}_{Nov}^A$ <sup>6</sup>	1189	1107	1180	1182	1175	1193	1190	1188	1189	1193	1132	1137	1172	1590	953	1249	1253	1197
$\eta_{2009}^A$	-0.76	-0.46	-0.59	-0.62	0.34	-0.78	-0.72	-0.71	-0.67	-0.74	-0.68	-0.76	-0.63	-0.71	-0.81	-0.67	-0.54	-0.78
$s_{cor}^A$	0.17	0.22	0.19	0.18	0.63	0.18	0.16	0.18	0.17	0.14	0.10	0.05	0.13	0.19	0.16	0.10	0.16	0.16

<sup>3</sup> On lower bound. Note that the exceptionally good fit to the proportion-at-age data allows this variance to be small.

<sup>4</sup> On lower bound.

<sup>5</sup> On lower bound. Note that for both A10 and A10.5 the objective function increased with increasing fixed  $\sigma_{ad}$  and did not appear to be bi-modal. Lower values of autocorrelation and better fits to the November 1+ biomass survey data were obtained at higher  $\sigma_{ad}$  values.

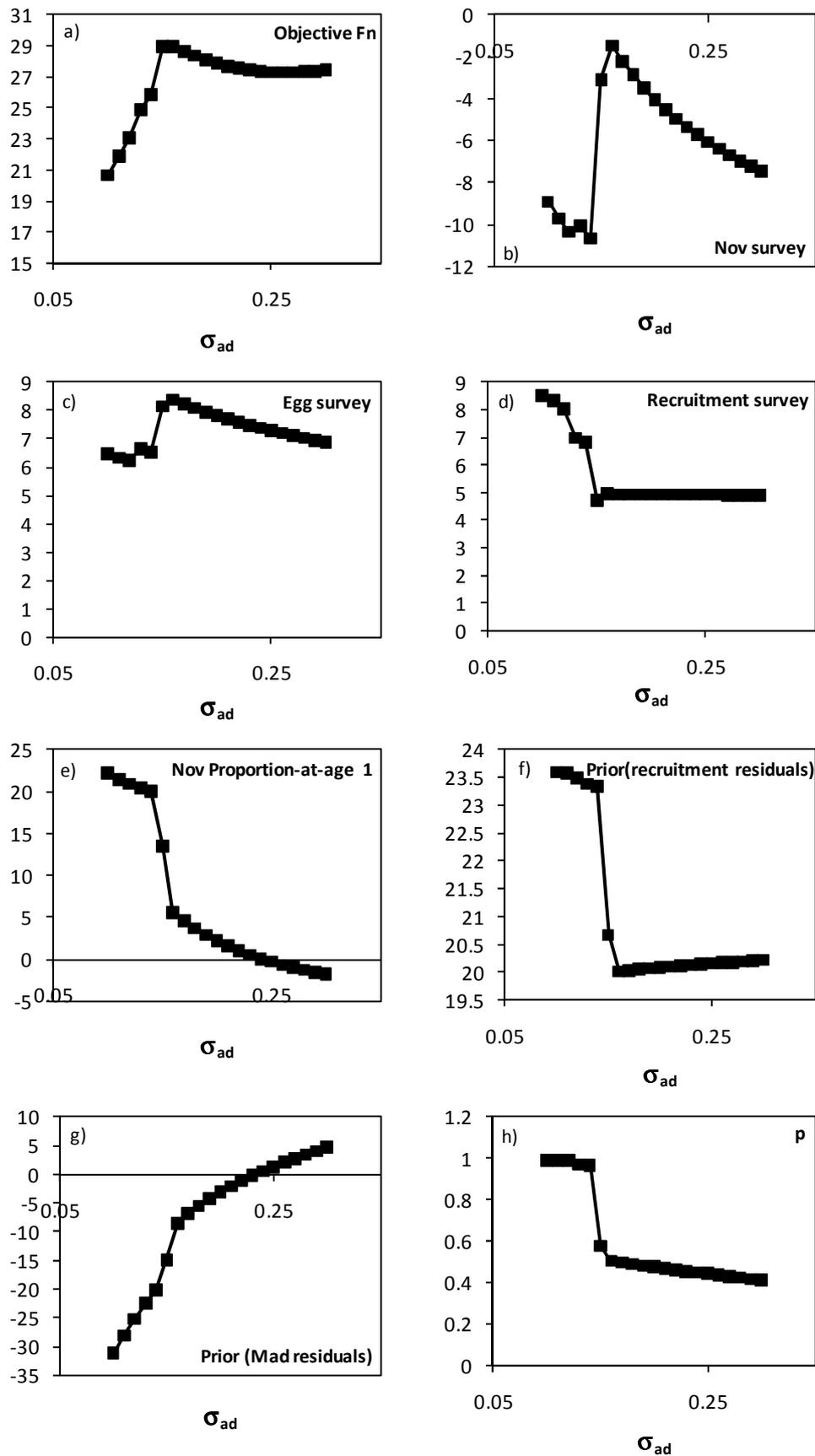
<sup>6</sup> OMP-04 and OMP-08 were developed using Risk defined as “the probability that adult anchovy biomass falls below 10% of the average adult anchovy biomass between November 1984 and November 1999 at least once during the projection period of 20 years”.

**Table 5.** The annual estimated anchovy loss to predation (in '000t),  $P_y^A$  in Appendix C, compared to the annual anchovy catch (in '000t).

Year	Catch	$A_{HS}$		$A_{estM}$	
		Loss to $M$	Catch: Loss to $M$	Loss to $M$	Catch: Loss to $M$
1984	265.7	1593.8	0.17	1651.5	0.16
1985	279.9	1301.7	0.22	1363.4	0.21
1986	299.6	1701.2	0.18	1809.0	0.17
1987	600.4	2015.9	0.30	1836.7	0.33
1988	569.7	1952.7	0.29	1563.9	0.36
1989	297.4	1197.8	0.25	957.1	0.31
1990	151.6	896.7	0.17	801.7	0.19
1991	151.0	1372.3	0.11	1473.5	0.10
1992	349.0	1924.7	0.18	1745.8	0.20
1993	235.8	1734.7	0.14	1228.6	0.19
1994	156.0	916.3	0.17	789.0	0.20
1995	176.8	688.7	0.26	613.5	0.29
1996	42.5	647.2	0.07	543.4	0.08
1997	60.4	768.7	0.08	850.7	0.07
1998	107.9	1095.5	0.10	1194.3	0.09
1999	178.9	1762.3	0.10	1783.0	0.10
2000	267.5	3550.5	0.08	3442.6	0.08
2001	285.4	6498.1	0.04	4620.0	0.06
2002	215.8	7980.7	0.03	4433.2	0.05
2003	255.8	5235.3	0.05	3672.0	0.07
2004	192.3	3765.3	0.05	2814.0	0.07
2005	282.2	2456.7	0.11	2585.7	0.11
2006	135.6	2287.5	0.06	2209.5	0.06
2007	251.3	2578.2	0.10	2332.3	0.11
2008	259.4	3892.8	0.07	2938.1	0.09
2009	181.2	4847.0	0.04	3204.6	0.06
2010	219.7	4067.6	0.05	2790.1	0.08

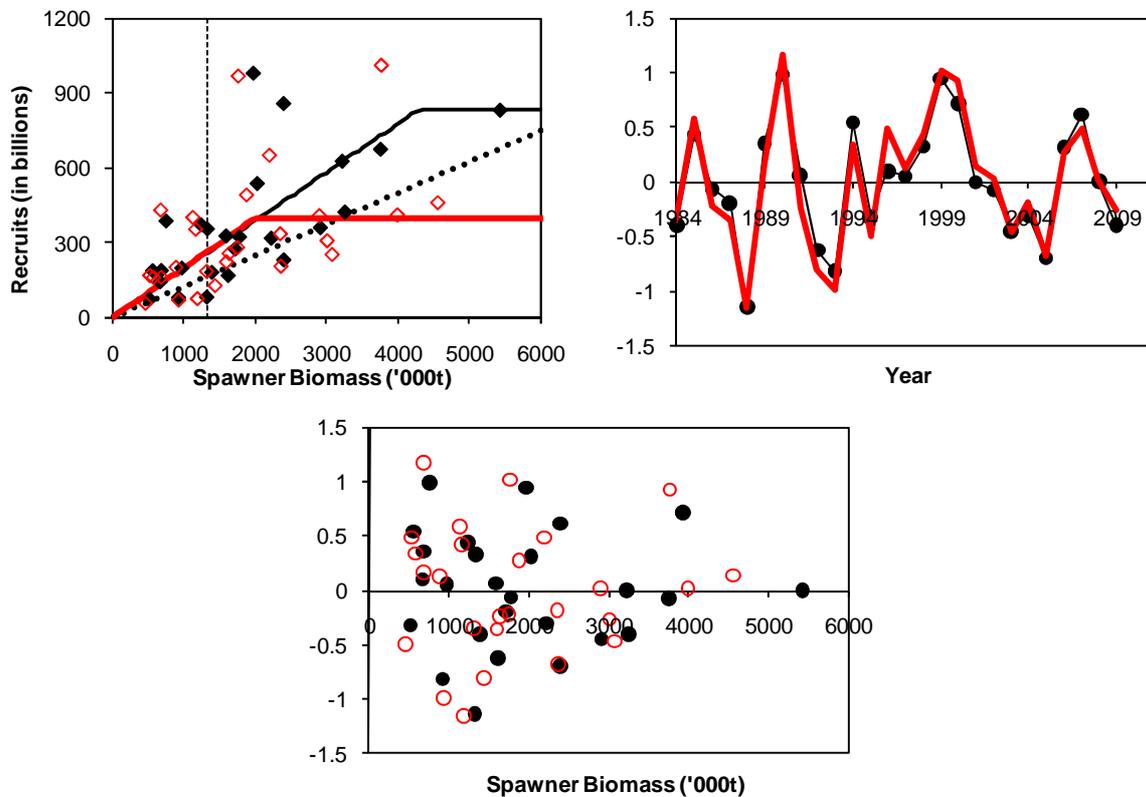
**Table 6.** Key parameter values estimated at the joint posterior mode for  $A_{HS}$  and the retrospective runs assuming a Hockey Stick stock recruitment relationship.  $A_{1999}$ ,  $A_{2003}$  and  $A_{2006}$  assume data available up to 1999, 2003 and 2006 only. Comparisons are also shown to the values at the posterior mode from former operating models used to develop OMP-02, OMP-04 and OMP-08. Note that the (non-peak) carrying capacity,  $K^A$ , is not directly comparable between  $A_{HS}$  and the retrospective runs on the one hand, and those from previous assessments on the other, as a bias correction factor was used for the latter. Numbers are reported in billions and biomass in thousands of tons.

	$A_{HS}$	$A_{2006}$	$A_{2003}$	$A_{1999}$	Previous assessments		
					OMP-02	OMP-04	OMP-08
$\bar{M}_j^A$	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>
$\bar{M}_{ad}^A$	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>	<b>0.9</b>
$k_N^A$	1.12	1.13	1.15	1.17	0.99	1.22	1.23
$k_r^A$	0.88	0.81	0.80	0.74	0.84	0.93	1.03
$a^A$	834	875	865	334	179	228	213
$b^A$	4299	4554	4127	1615	360	461	368
$K^A$	6683	7072	6904	2757	1802	2492	2925
$h^A$	0.31	0.31	0.33	0.34	1.00	1.00	1.00
$\sigma_r^A$	0.43	0.46	0.44	0.34	0.69	0.88	0.86
$\bar{B}_{Nov}^A$	1330	1190	1179	1154		1169	1103
$s_{cor}^A$	0.20	0.18	0.12	-0.05	0.32	0.47	0.43

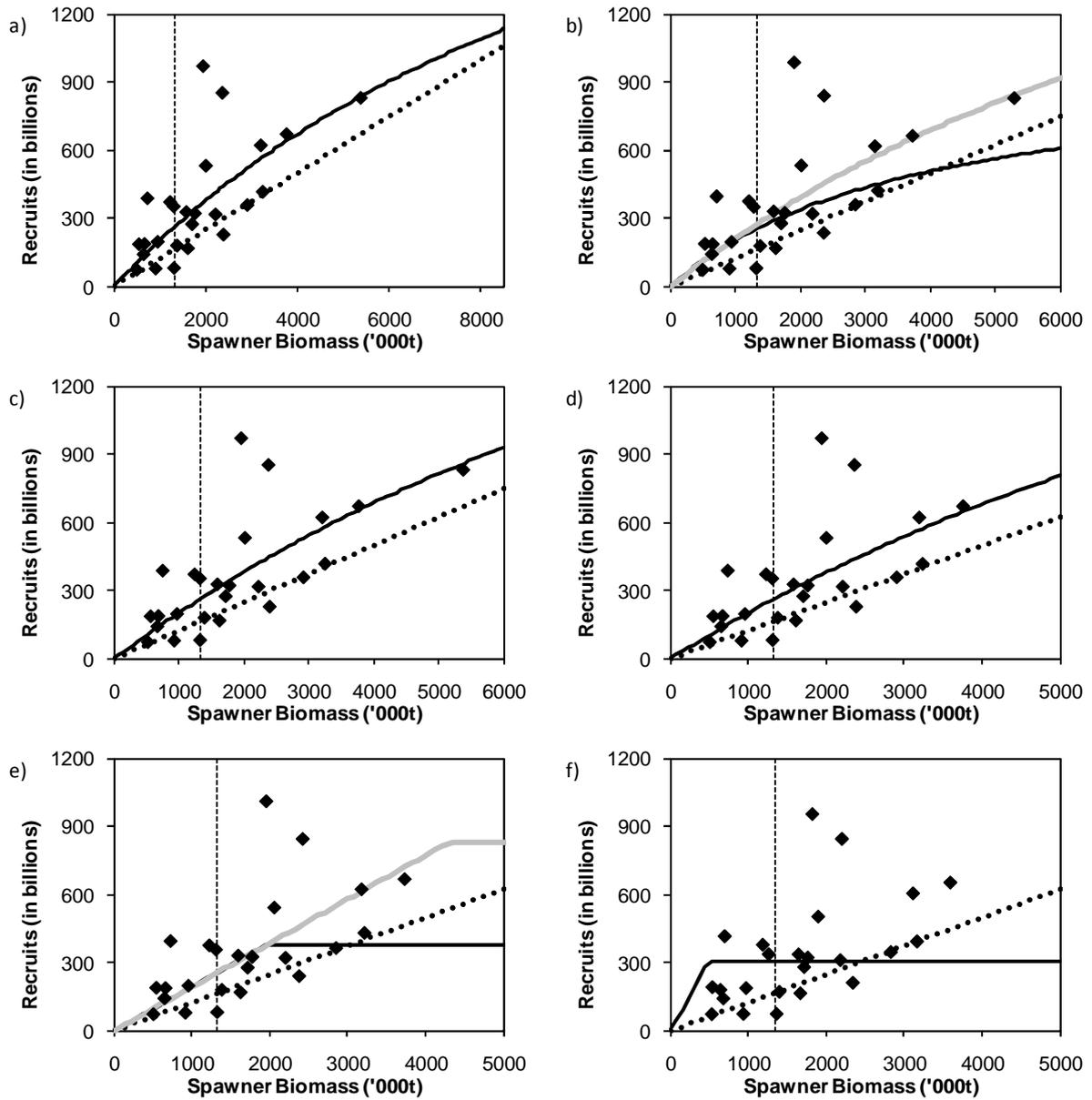


**Figure 1.** Likelihood profile, for models assuming the Hockey Stick stock-recruitment curve, for a) the objective function and b)-g) individual contributions to this objective function over a range of fixed values for

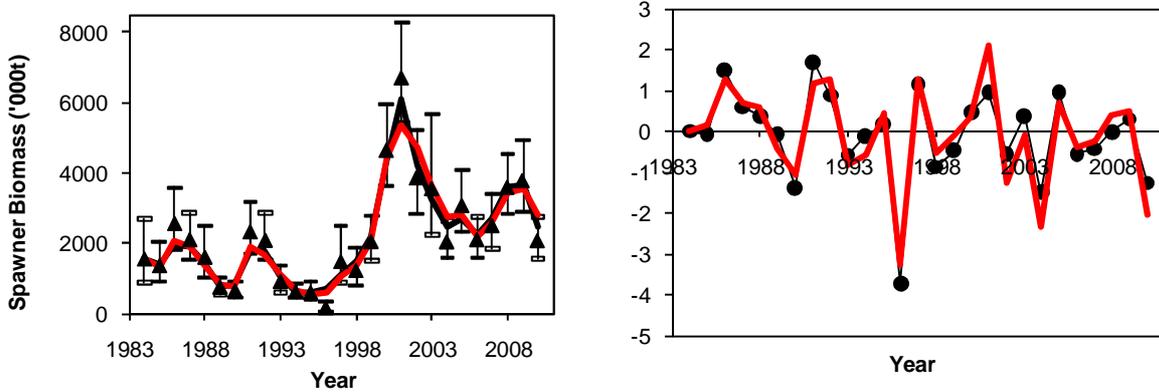
the standard deviation in residuals about adult natural mortality,  $\sigma_{ad}$ . The corresponding estimated values of autocorrelation in these residuals,  $\rho$ , are plotted in h).



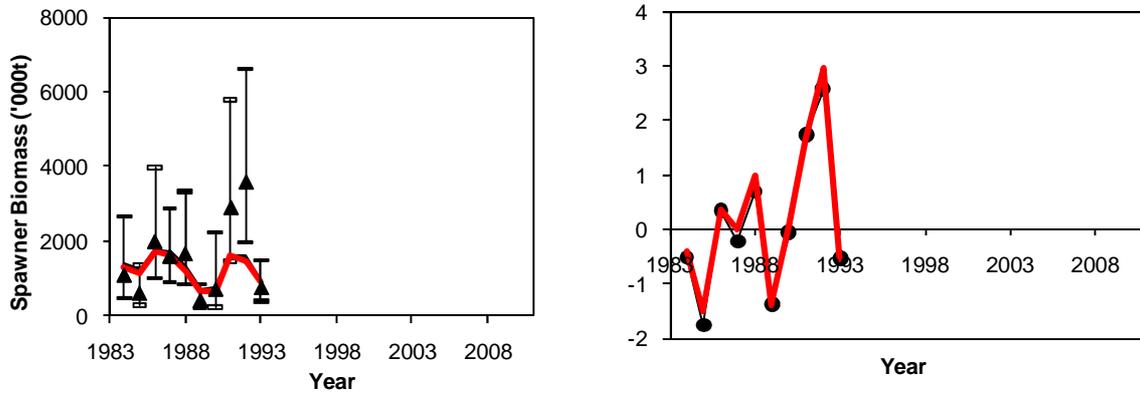
**Figure 2.** Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2009 for  $A_{HS}$  (black, filled symbols) and  $A_{ctM}$  (red, open symbols) with the Hockey stick stock recruitment relationship. The vertical thin dashed line indicates the average 1984 to 1999 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, against year and against spawner biomass.



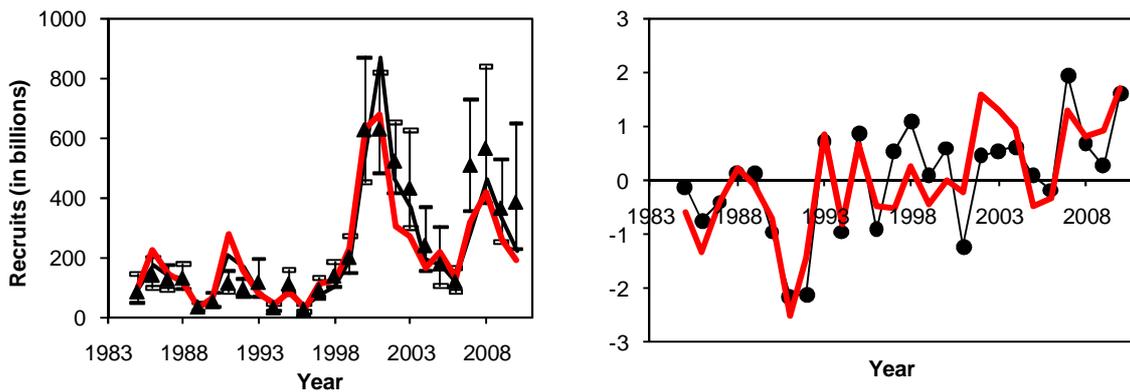
**Figure 3.** Stock-recruit relationships for a)  $A_{BH}$ , b)  $A_{2BH}$  (grey being 2000+ relationship), c)  $A_R$ , d)  $A_{ModR}$ , e)  $A_{2HS}$  (grey line showing the 2000+ relationship), and f)  $A_{fixedHS}$ .



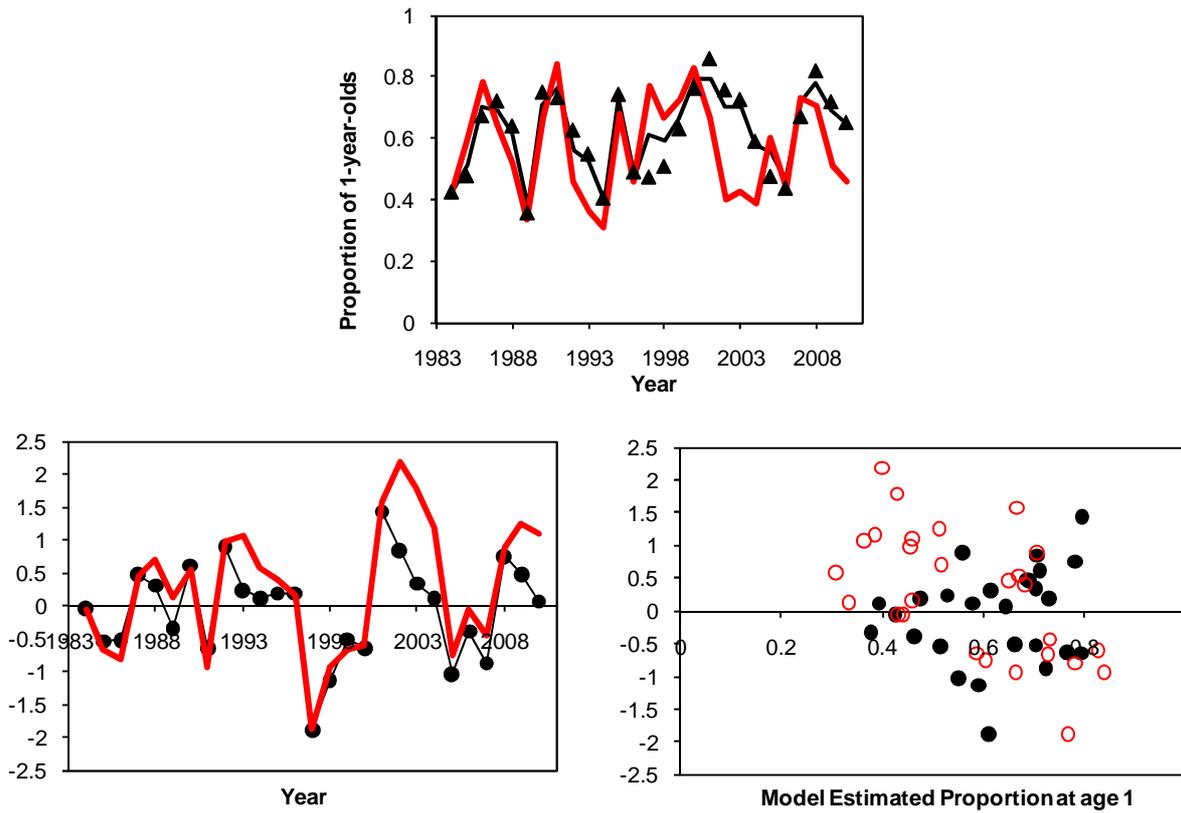
**Figure 4.** Acoustic survey results and model estimates for November anchovy spawner biomass from 1984 to 2010 for  $A_{HS}$  (black, connecting filled circles on the right side plot) and  $A_{cstM}$  (red). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.



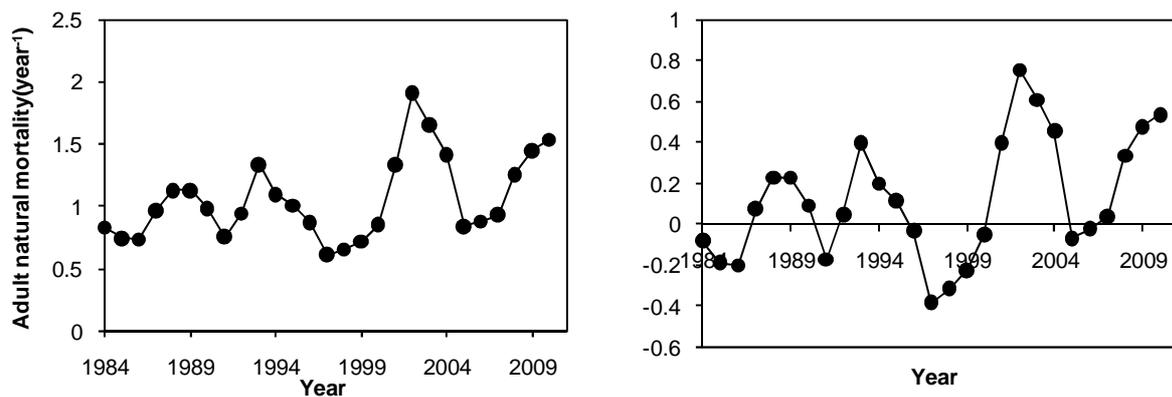
**Figure 5.** Egg survey results and model estimates for November anchovy spawner biomass from 1984 to 1991 for  $A_{HS}$  (black, connecting filled circles on the right side plot) and  $A_{cstM}$  (red). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.



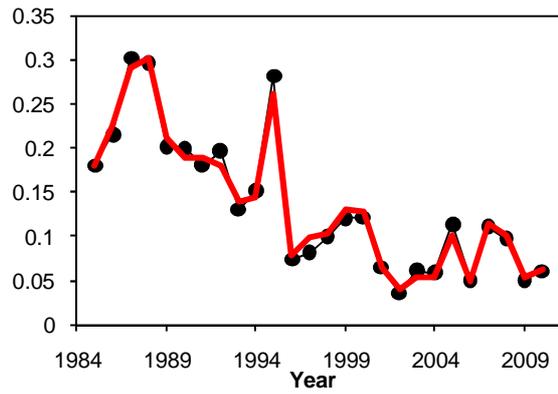
**Figure 6.** Acoustic survey results and model estimates for anchovy recruitment numbers from May 1985 to May 2010 for  $A_{HS}$  (black, connecting filled circles on the right side plot) and  $A_{cstM}$  (red). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.



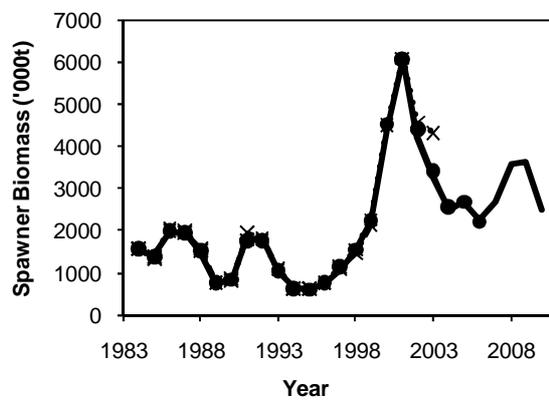
**Figure 7.** Acoustic survey results and model estimates for proportions of 1-year-olds in the November survey from 1984 to 2010 for  $A_{HS}$  (black, connecting filled circles in the bottom two plots) and  $A_{cstM}$  (red open circles). The standardised residuals from the fit are given in the lower plots, against year and against model estimates of proportions at age 1.



**Figure 8.** Model estimated annual adult natural mortality for  $A_{HS}$ . The random effects are plotted in the right hand panel.



**Figure 9.** The historic harvest proportion (catch by mass to 1+ biomass) for anchovy for  $A_{HS}$  (black, connecting filled circles) and  $A_{estM}$  (red).



**Figure 10.** The model predicted November anchovy spawner biomass for  $A_{HS}$  and the retrospective runs  $A_{2006}$  using data up to 2006,  $A_{2003}$  using data up to 2003 and  $A_{1999}$  using data up to 1999.

## Appendix A: Bayesian operating model for the South African anchovy resource

### Model Assumptions

- 1) All fish have a theoretical birthdate of 1 November.
- 2) Anchovy spawn for the first time (and are called adult anchovy) when they turn one year old.
- 3) A plus group of age 4 is used, thus assuming that natural mortality is the same for age 4 and older ages.
- 4) Two acoustic surveys are held each year: the first takes place in November and surveys the adult stock; the second is in May/June (known as the recruit survey) and surveys juvenile anchovy.
- 5) The November acoustic survey provides a relative index of abundance of unknown bias.
- 6) The recruit survey provides a relative index of abundance of unknown bias.
- 7) The egg survey observations (derived from data collected during the earlier November surveys) provide absolute indices of abundance.
- 8) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time.
- 9) Pulse fishing occurs five months after 1 November for 1-year-old anchovy; for 0-year-old anchovy this occurs 7½ months after 1 November prior to 1999, and 8½ months after 1 November from 1999 onwards; these two ages (0 and 1) are the only ages targeted by the fishery.
- 10) Catches are measured without error. (Selectivity of age 0 and age 1 anchovy varies from year to year. This would prove problematic were model predicted catch to be estimated and fitted to observed catch, but here the observed catches-at-age are directly incorporated into the dynamics.)
- 11) Natural mortality is year-invariant for juvenile fish, and age-invariant for adult fish.

### Population Dynamics

The basic dynamic equations for anchovy are as follows, where  $y_n = 2010$ .

#### *Numbers-at-age at 1 November*

$$\begin{aligned}
 N_{y,1}^A &= (N_{y-1,0}^A e^{-(7.5)M_{j,y}^A/12} - C_{y,0}^A) e^{-(4.5)M_{j,y}^A/12} & y = 1984, \dots, 1998 \\
 N_{y,1}^A &= (N_{y-1,0}^A e^{-(8.5)M_{j,y}^A/12} - C_{y,0}^A) e^{-(3.5)M_{j,y}^A/12} & y = 1999, \dots, y_n \\
 N_{y,2}^A &= (N_{y-1,1}^A e^{-5M_{ad,y}^A/12} - C_{y,1}^A) e^{-7M_{ad,y}^A/12} & y = 1984, \dots, y_n \\
 N_{y,3}^A &= N_{y-1,2}^A e^{-M_{ad,y}^A} & y = 1984, \dots, y_n \\
 N_{y,4+}^A &= N_{y-1,3}^A e^{-M_{ad,y}^A} & y = 1984 \\
 N_{y,4+}^A &= N_{y-1,3}^A e^{-M_{ad,y}^A} + N_{y-1,4+}^A e^{-M_{ad,y}^A} & y = 1985, \dots, y_n
 \end{aligned} \tag{A.1}$$

where

$N_{y,a}^A$  is the model predicted number (in billions) of anchovy of age  $a$  at the beginning of November in year  $y$ ;

$C_{y,a}^A$  is the model predicted number (in billions) of anchovy of age  $a$  caught from 1 November in year  $y-1$  to 31 October in year  $y$ ;

$M_{j,y}^A$  is the annual natural mortality (in year<sup>-1</sup>) of juvenile anchovy (i.e. fish of age 0) in year  $y$ ; and

$M_{ad,y}^A$  is the annual natural mortality (in year<sup>-1</sup>) of adult anchovy (i.e. fish of age 1+) in year  $y$ .

*Biomass associated with the November survey*

$$B_y^A = \sum_{a=1}^{4+} N_{y,a}^A w_{y,a}^A \quad y = 1984, \dots, y_n \quad (\text{A.2})$$

where:

$B_y^A$  is the model predicted biomass (in thousand tons) of adult anchovy at the beginning of November in year  $y$ , which are taken to be associated with the November survey; and

$w_{y,a}^A$  is the mean mass (in grams) of anchovy of age  $a$  sampled during the November survey of year  $y$ .

Anchovy are assumed to mature at age 1 and thus the spawning stock biomass is:

$$SSB_y^A = \sum_{a=1}^{4+} N_{y,a}^A w_{y,a}^A \quad y = 1984, \dots, y_n \quad (\text{A.3})$$

*Recruitment*

Recruitment at the beginning of November is assumed to fluctuate lognormally about a stock-recruitment curve (see Table 1):

$$N_{y,0}^A = f(SSB_y^A) e^{\varepsilon_y^A} \quad y = 1984, \dots, y_{n-1} \quad (\text{A.4})$$

where

$\varepsilon_y^A$  is the annual lognormal deviation of anchovy recruitment.

*Number of recruits at the time of the recruit survey*

The following equation projects  $N_{y,0}^A$  to the start of the recruit survey, taking natural and fishing mortality into account, and assuming pulse fishing of juveniles at 1 May (based on historic data).

$$N_{y,r}^A = (N_{y-1,0}^A e^{-0.5M_j^A} - C_{y,0bs}^A) e^{-t_y^A \times M_j^A / 12} \quad y = 1985, \dots, y_n \quad (\text{A.5})$$

where

$N_{y,r}^A$  is the model predicted number (in billions) of juvenile anchovy at the time of the recruit survey in year  $y$ ;

$C_{y,0bs}^A$  is the number (in billions) of juvenile anchovy caught between 1 November and the day before the start of the recruit survey in year  $y$ ;

$t_y^A$  is the time lapsed (in months) between 1 May and the start of the recruit survey that provided the estimate  $N_{y,rec}^A$  in year  $y$ .

*Proportions of 1-year-olds associated with November survey*

$$p_{y,1}^A = \frac{N_{y,1}^A}{\sum_{a=1}^{4+} N_{y,a}^A} \quad y = 1984, \dots, y_n \quad (\text{A.6})$$

where

$p_{y,1}^A$  is the model predicted proportion of 1-year-old anchovy at the beginning of November in year  $y$ , which is taken to be associated with the November survey.

### Fitting the Model to Observed Data (Likelihood)

The observations are assumed to be log-normally distributed, and sampling CVs (squared) of the untransformed survey observations are used to approximate the “sampling” component of the total variance of the corresponding log-distributions. The proportions of 1-year-olds are first logit-transformed before being used in the likelihood<sup>7</sup>. Thus we have:

$$\begin{aligned} -\ln L = & \frac{1}{2} \sum_{y=1984}^{yn} \left\{ \frac{(\ln \hat{B}_y^A - \ln(k_N^A B_y^A))^2}{(\sigma_{y,N}^A)^2 + (\lambda_N^A)^2} + \ln[2\pi((\sigma_{y,N}^A)^2 + (\lambda_N^A)^2)] \right\} \\ & + \frac{1}{2} \sum_{y=1984}^{1991} \left\{ \frac{(\ln \hat{B}_{y,egg}^A - \ln(k_g^A B_y^A))^2}{(\sigma_{y,egg}^A)^2} + \ln[2\pi(\sigma_{y,egg}^A)^2] \right\} \\ & + \frac{1}{2} \sum_{y=1985}^{yn} \left\{ \frac{(\ln \hat{N}_{y,r}^A - \ln(k_r^A N_{y,r}^A))^2}{(\sigma_{y,r}^A)^2 + (\lambda_r^A)^2} + \ln[2\pi((\sigma_{y,r}^A)^2 + (\lambda_r^A)^2)] \right\} \\ & + \frac{1}{2} \sum_{y=1984}^{yn} \left\{ \frac{(\ln(\hat{p}_{y,1}^A / (1 - \hat{p}_{y,1}^A)) - \ln(k_p^A p_{y,1}^A / (1 - k_p^A p_{y,1}^A)))^2}{(\sigma_p^A)^2} + \ln[2\pi(\sigma_p^A)^2] \right\} \end{aligned} \quad (\text{A.7})$$

where

$\hat{B}_y^A$  is the acoustic survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year  $y$ , with associated CV  $\sigma_{y,N}^A$  and constant of proportionality (multiplicative bias)  $k_N^A$ ;

$\hat{B}_{y,egg}^A$  is the egg survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year  $y$ , with associated CV  $\sigma_{y,egg}^A$  and constant of proportionality  $k_g^A$ ;

$\hat{N}_{y,r}^A$  is the acoustic survey estimate (in billions) of anchovy recruitment from the recruit survey in year  $y$ , with associated CV  $\sigma_{y,r}^A$  and constant of proportionality  $k_r^A$ ;

<sup>7</sup> This transformation proved adequate, resulting in no heteroscedasticity in the residuals of the logit transformed variable.

$\hat{p}_{y,1}^A$  is an estimate of the proportion (by number) of 1-year-old anchovy in the November survey of year  $y$ .

For the base case assessment an average Prosch age length key is used to derive these proportions;

$k_p^A$  is a multiplicative bias associated with the proportion of 1-year-olds in the November survey;

$(\lambda_{N/r}^A)^2$  is the additional variance (over and above the survey sampling CV  $\sigma_{y,N/r}^A$  that reflects survey inter-transect variance) associated with the November/recruit surveys;

$\sigma_p^A$  is the standard deviation associated with the proportion of 1-year-olds in the November survey;

### Fixed Parameters

Three parameters are fixed externally in this assessment (see main text for reasons and for variations for robustness tests):

$M_{j,y}^A = 0.9$  for all years,  $(\lambda_N^A)^2 = 0$ , and  $k_g^A = 1$ , as the egg survey estimates of abundance are assumed to be absolute.

Adult natural mortality varies around a median of  $\bar{M}_{ad}^A = 0.9$  as follows:

$$M_{ad,y}^A = \bar{M}_{ad}^A e^{\varepsilon_{ad,y}} \text{ with } \varepsilon_y^{ad} = p\varepsilon_{y-1}^{ad} + \sqrt{1-p^2}\eta_y^{ad} \quad (\text{A.8})$$

where  $\eta_y^{ad} \sim N(0, \sigma_{ad}^2)$  and

$\sigma_{ad}$  - is the standard deviation in the annual residuals about adult natural mortality; and

$p$  - is the annual autocorrelation coefficient.

### Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve:

$$\varepsilon_y^A \sim N\left(0, (\sigma_r^A)^2\right), \quad y = 1984, \dots, y_{n-1}$$

The remaining estimable parameters are defined as having the near non-informative prior distributions:

$$\ln(k_N^A) \sim U(-100, 0.7) \text{ (upper bound corresponding to } k_N^A = 2)$$

$$\ln(k_r^A) \sim U(-100, 0.7) \text{ (upper bound corresponding to } k_r^A = 2)$$

$$\ln(k_p^A) \sim U(-100, 0.7) \text{ (upper bound corresponding to } k_p^A = 2)$$

$$(\lambda_r^A)^2 \sim U(0, 100)$$

$$(\sigma_r^A)^2 \sim U(0.04, 10)$$

$$(\sigma_p^A)^2 \sim U(0.09, 10)$$

$$N_{1983,a}^A \sim U(0, 500), \quad a = 0, 1$$

$$N_{1983,a}^A \sim U(0, 0.01), \quad a = 2, 3$$

$$\sigma_{ad} \sim U(0.20, 0.5)$$

$$p \sim U(0,1)$$

### Further Outputs

Recruitment serial correlation:

$$s_{cor}^A = \frac{\sum_{y=1984}^{ym-2} \epsilon_y \epsilon_{y+1}}{\sqrt{\left( \sum_{y=1984}^{ym-2} \epsilon_y^2 \right) \left( \sum_{y=1984}^{ym-2} \epsilon_{y+1}^2 \right)}} \quad (\text{A.9})$$

and the standardised recruitment residual value for 2009:

$$\eta_{ym-1}^A = \frac{\epsilon_{ym-1}^A}{\sigma_r^A}. \quad (\text{A.10})$$

are also required as input into the OMP.

### Appendix B: Glossary of parameters used in this document

#### Annual numbers and biomass:

- $N_{y,a}^A$  - model predicted number (in billions) of anchovy of age  $a$  at the beginning of November in year  $y$
- $C_{y,a}^A$  - model predicted number (in billions) of anchovy of age  $a$  caught from 1 November in year  $y - 1$  to 31 October in year  $y$
- $B_{y,N}^A$  - model predicted biomass (in thousand tons) of adult anchovy at the beginning of November in year  $y$ , which are taken to be associated with the November survey
- $SSB_y^A$  - model predicted spawning stock biomass (in thousand tonnes) at the beginning of November in year  $y$
- $w_{y,a}^A$  - mean mass (in grams) of anchovy of age  $a$  sampled during the November survey of year  $y$
- $N_{y,r}^A$  - model predicted number (in billions) of juvenile anchovy at the time of the recruit survey in year  $y$
- $C_{y,obs}^A$  - number (in billions) of juvenile anchovy caught between 1 November and the day before the start of the recruit survey in year  $y$
- $t_y^A$  - time lapsed (in months) between 1 May and the start of the recruit survey in year  $y$ .

#### Natural mortality:

- $M_{j,y}^A$  - annual natural mortality (in year<sup>-1</sup>) of juvenile anchovy (i.e. fish of age 0) in year  $y$
- $M_{ad,y}^A$  - annual natural mortality (in year<sup>-1</sup>) of adult anchovy (i.e. fish of age 1+) in year  $y$
- $\overline{M}_{ad}^A$  - median adult rate of natural mortality (in year<sup>-1</sup>)
- $\varepsilon_y^{ad}$  - annual residuals about adult natural mortality
- $\eta_y^{ad}$  - normally distributed error used in calculating  $\varepsilon_y^{ad}$
- $\sigma_{ad}$  - standard deviation in the annual residuals about adult natural mortality
- $\rho$  - annual autocorrelation coefficient in annual residuals about adult natural mortality

#### Recruitment:

- $h^A$  - steepness associated with the stock-recruitment curve
- $K^A$  - carrying capacity
- $a^A$  - maximum median recruitment in the Hockey Stick stock-recruitment curve
- $b^A$  - biomass above which median recruitment is not impaired in the Hockey Stick stock-recruitment curve
- $\alpha^A$  - stock-recruitment curve parameter, linked to  $K^A$  and  $h^A$  (for Beverton Holt and Ricker curves)
- $\beta^A$  - stock-recruitment curve parameter, linked to  $K^A$  and  $h^A$  (for Beverton Holt and Ricker curves)
- $\varepsilon_y^A$  - annual lognormal deviation of anchovy recruitment
- $\sigma_r^A$  - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve

Proportions of 1-year-olds:

$p_{y,1}^A$  - model predicted proportion of 1-year-old anchovy at the beginning of November in year y

Likelihoods:

$\hat{B}_y^A$  - acoustic survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year y

$\sigma_{y,N}^A$  - survey sampling CV associated with  $\hat{B}_y^A$  that reflects survey inter-transect variance

$k_N^A$  - constant of proportionality (multiplicative bias) associated with  $\hat{B}_y^A$

$\hat{B}_{y,egg}^A$  - egg survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year y

$\sigma_{y,egg}^A$  - survey sampling CV associated with  $\hat{B}_{y,egg}^A$  that reflects survey inter-transect variance

$k_g^A$  - constant of proportionality (multiplicative bias) associated with  $\hat{B}_{y,egg}^A$

$\hat{N}_{y,r}^A$  - acoustic survey estimate (in billions) of anchovy recruitment from the recruit survey in year y

$\sigma_{y,r}^A$  - survey sampling CV associated with  $\hat{N}_{y,r}^A$  that reflects survey inter-transect variance

$k_r^A$  - constant of proportionality (multiplicative bias) associated with  $\hat{N}_{y,r}^A$

$\hat{p}_{y,1}^A$  - estimate of the proportion (by number) of 1-year-old anchovy in the November survey of year y

$k_p^A$  - multiplicative bias associated with the proportion of 1-year-olds in the November survey

$(\lambda_{N/r}^A)^2$  - additional variance (over and above  $\sigma_{y,N/r}^A$ ) associated with the November/recruit surveys

$\sigma_p^A$  - standard deviation associated with the proportion of 1-year-olds in the November survey

Other:

$s_{cor}^A$  - recruitment serial correlation

$\eta_{2009}^A$  - standardised recruitment residual value for 2009

$\bar{w}_a^A$  - mean mass (in grams) of anchovy of age  $a$  during each November survey

### Appendix C: Calculation of Loss to Predation for Anchovy

The assessment model assumes catch is taken in a single pulse during the year. The loss in numbers of age  $a$  in year  $y$  is calculated by:

$$P_{y,1}^A = N_{y-1,0}^A \left(1 - e^{-7.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-7.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-4.5M_{j,y}^A/12}\right) \quad y = 1984, \dots, 1998$$

$$P_{y,1}^A = N_{y-1,0}^A \left(1 - e^{-8.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-8.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-3.5M_{j,y}^A/12}\right) \quad y = 1999, \dots, y_n$$

$$P_{y,2}^A = N_{y-1,1}^A \left(1 - e^{-5M_{ad,y}^A/12}\right) + \left(N_{y-1,1}^A e^{-5M_{ad,y}^A/12} - C_{y,1}^A\right) \left(1 - e^{-7M_{ad,y}^A/12}\right) \quad y = 1984, \dots, y_n$$

$$P_{y,3}^A = N_{y-1,2}^A \left(1 - e^{-M_{ad,y}^A}\right), \quad y = 1984, \dots, y_n$$

$$P_{y,4+}^A = N_{y-1,3}^A \left(1 - e^{-M_{ad,y}^A}\right) + N_{y-1,4+}^A \left(1 - e^{-M_{ad,y}^A}\right) \quad y = 1985, \dots, y_n$$

The loss in biomass of fish of age  $a$  to predation in year  $y$  is therefore given by:

$$P_{y,1}^A = \left[ N_{y-1,0}^A \left(1 - e^{-7.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-7.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-4.5M_{j,y}^A/12}\right) \right] \frac{1}{2} (w_{y-1,0} + w_{y,1}) \quad y = 1984, \dots, 1998$$

$$P_{y,1}^A = \left[ N_{y-1,0}^A \left(1 - e^{-8.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-8.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-3.5M_{j,y}^A/12}\right) \right] \frac{1}{2} (w_{y-1,0} + w_{y,1})$$

$$P_{y,2}^A = \left[ N_{y-1,1}^A \left(1 - e^{-5M_{ad,y}^A/12}\right) + \left(N_{y-1,1}^A e^{-5M_{ad,y}^A/12} - C_{y,1}^A\right) \left(1 - e^{-7M_{ad,y}^A/12}\right) \right] \frac{1}{2} (w_{y-1,1} + w_{y,2}) \quad y = 1984, \dots, y_n$$

$$P_{y,3}^A = N_{y-1,2}^A \left(1 - e^{-M_{ad,y}^A}\right) \frac{1}{2} (w_{y-1,2} + w_{y,3}), \quad y = 1984, \dots, y_n$$

$$P_{y,4+}^A = N_{y-1,3}^A \left(1 - e^{-M_{ad,y}^A}\right) \frac{1}{2} (w_{y-1,3} + w_{y,4}) + N_{y-1,4+}^A \left(1 - e^{-M_{ad,y}^A}\right) \frac{1}{2} (w_{y-1,4} + w_{y,4}) \quad y = 1985, \dots, y_n$$

The assumption is made that  $w_{1983,a} = w_{1984,a}$ ,  $a = 1, \dots, 4+$ .

The total loss in anchovy biomass to predation in year  $y$  is then given by:

$$P_y^A = \sum_{a=1}^{4+} P_{y,a}^A.$$