# A statistical basis for estimating the proportion of South Coast spawning biomass that contributes to West Coast recruitment and of West Coast spawning biomass that contributes to South Coast recruitment

by

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### Background

FISHERIES/2016/OCT/SWG-PEL/47rev proposes that the proportional contribution of spawning biomass found on the SC to the number of recruits appearing on the WC could be estimated by treating the proportion contribution as a free parameter in a model fit of WC recruits versus the effective WC spawning biomass, the latter comprising a proportion of the spawning biomass on the SC. In FISHERIES/2016/OCT/SWG-PEL/47rev, explicit forms for the S/R relationship are considered. These are the Beverton-Holt, Ricker and Hockey-Stick forms. This document makes a contribution to this general approach, but effectively for the power form R=S<sup>γ</sup>.

#### Method

The method reported upon here considers that a proportion of the spawning biomass on the SC contributes to WC recruitment, and a proportion of the spawning biomass on the WC contributes to the SC recruitment. The spawning biomass on the WC augmented by spawning biomass from the SC is referred to as the effective WC spawning biomass, and the spawning biomass on the SC augmented by spawning biomass from the WC is referred to as the effective SC spawning biomass. It is proposed that these proportions should satisfy one or both of the following criteria:

- 1. Maximum correlation between the logarithm of recruitment and the logarithm of spawning biomass, where the value maximised is the sum of the squares of the WC and SC correlations.
- 2. Equality of the ratio between recruitment and effective spawning biomass for the WC and SC.

#### Results

The results obtained using the data produced for the paper FISHERIES/2016/NOV/SWG-PEL/57 (we thank C. de Moor for providing us with these estimates) were:

Table 1. Results obtained by relating spawning biomass in year y to recruitment in year y as supplied to OLRAC SPS by C. de Moor. See Figure 1 for relevant figures. In these results the yearly recruitment and spawning biomass were adjusted by 10% of their mean value across all the years when taking logarithms.

	%WC cont to WC	%SC cont to WC	%WC cont to SC	%SC cont to SC	Ratio (WC/SC)	Median Ratio (WC/SC)	$R^2_{wc}+R^2_{sc}$
1	98%	84%	2%	16%	0.817	0.584	0.534
2	78%	97%	22%	3%	0.814	1.000	0.349
3	80%	94%	20%	6%	0.884	1.000	0.406
4	84%	90%	16%	10%	1.000	1.000	0.468
5	89%	84%	11%	16%	1.158	1.000	0.513
6	95%	76%	5%	24%	1.354	1.000	0.532
7	98%	72%	2%	28%	1.465	1.000	0.533

In the above table,

1. The quantities "% WC cont to WC", "% SC cont to WC", "% WC cont to SC" and "% SC cont to SC" are the various contributions of spawning biomass to the effective spawning biomass on each coast, where each are constrained to lie between 0% and 98%, and by definition:

"% WC cont to WC" + "% WC cont to SC" = 100%

"% SC cont to WC" + "% SC cont to SC" = 100%.

- The quantity "Ratio (WC/SC)" is the ratio of "the ratio of mean recruitment to effective spawning biomass (calculated from mean spawning biomasses) for the WC" divided by "the ratio of mean recruitment to effective spawning biomass (calculated from mean spawning biomasses) for the SC". Means are calculated over all available years.
- 3. The quantity "Median Ratio (WC/SC)" is the ratio of "the ratio of median recruitment to effective spawning biomass (calculated from median spawning biomass) for the WC" divided by "the ratio of median recruitment to effective spawning biomass (calculated from median spawning biomass) for the SC". Medians are calculated over all available years.
- 4. R<sup>2</sup><sub>wc</sub>+R<sup>2</sup><sub>sc</sub> is the sum of the squares of the correlations between the logarithm of WC recruitment and the logarithm of WC effective spawning biomass and between the logarithm of SC recruitment and the logarithm of SC effective spawning biomass.

Row 1 of the table shows the results that are obtained when the percentage contributions are selected to maximise  $R^2_{wc}+R^2_{sc}$ , where it is emphasised that 98% was an upper bound imposed for numerical efficiency reasons.

Rows 2 – 7 of the table are a series of results produced subject to the constraint "Median Ratio (WC/SC)" = 1, and across a range of values of "% SC cont to WC" from the maximum possible to the minimum possible that still satisfies "Median Ratio (WC/SC)" = 1. This constraint cannot be satisfied for "% SC cont to WC" < 72%.

#### Conclusions

- When R<sup>2</sup><sub>wc</sub>+R<sup>2</sup><sub>sc</sub> is maximised then the estimates are "% SC cont to WC" = 84% and "% WC cont to SC" = 2%.
- 2. When  $R_{wc}^2+R_{sc}^2$  is maximised subject to the constraint "Median Ratio (WC/SC)" = 1, then the estimates are **"% SC cont to WC" = 72%** and "% WC cont to SC" = 2%.
- 3. The full estimation distribution was not produced here, but we note that for the second method, "% SC cont to WC" < 72% is excluded.

4. Table 2 and Figure 2 are results produced where the logged values of the yearly recruitment and spawning biomass were not adjusted by 10% of their mean value across all the years, and it was necessary to omit the values on the SC for 1983, 1984 and 1985.



Figure 1: Scatterplots where the logged values of the yearly recruitment and spawning biomass were adjusted by 10% of their mean value across all the years.



Figure 2: Scatterplots where the logged value of the recruitment and spawning biomass were unadjusted, and as a result the values for 1983, 1984 and 1985 for SC.

	%WC cont to WC	%SC cont to WC	%WC cont to SC	%SC cont to SC	Ratio (WC/SC)	Median Ratio (WC/SC)	$R^2_{wc} + R^2_{sc}$
1	98.0%	84.3%	2.0%	15.7%	0.793	0.568	0.345
2	78.2%	97.0%	21.8%	3.0%	0.814	1.000	0.321
3	80.5%	94.2%	19.5%	5.8%	0.884	1.000	0.314
4	84.1%	89.6%	15.9%	10.4%	1.000	1.000	0.319
5	89.0%	83.5%	11.0%	16.5%	1.158	1.000	0.331
6	94.8%	76.3%	5.2%	23.7%	1.354	1.000	0.342
7	97.9%	72.4%	2.1%	27.6%	1.465	1.000	0.344

Table 2: West & south coast spawning biomass contributions to the effective biomass for each coast and other statistics.