Cannibalism methods simplified.

by

OLRAC SPS
Silvermine House
Steenberg Office Park
Tokai 7945

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Basic dynamics and splitting natural mortality into a hake predation component and an "other component"

The basic dynamic equations use Pope's approximation:

$$N_{s,g,y+1,a+1} = \left(N_{s,g,y,a}e^{-\frac{M_{s,g,y,a}}{2}} - C_{s,g,y,a}\right)e^{-\frac{M_{s,g,y,a}}{2}} \qquad \text{(1rev)}$$

The total natural mortality, $M_{s,g,y,a}$, is modelled as the sum of a cannibalistic / interspecies predation component of natural mortality $M^{c}_{s,g,y,a}$and a base natural mortality coefficient $M^{base}_{s,g,a}$:

$$M_{s,g,y,a} = M^{c}_{s,g,y,a} + M^{base}_{s,g,a} \qquad \text{(2rev)}$$

The **total mortality for the pristine situation** is modelled by a logistic functional form plus an amount added so that the asymptote at large age is a non-zero positive amount rather than zero, i.e. the following 4 parameter model:

$$M_{s,g,y=p,a} = \frac{\phi_s}{\left(1+e^{\kappa_s(a-\theta_s)}\right)} + \rho_s \qquad \text{(3rev)}$$

(where y=p is the first year in the stock assessment run sequence at which point the population is assumed to be under pristine equilibrium conditions). The base natural mortality is calculated as

$$M^{base}_{s,g,a} = \frac{\phi_s}{\left(1+e^{\kappa_s(a-\theta_s)}\right)} + \rho_s - M^{c}_{s,g,y=p,a}.$$

The following notation is used later in this description:

$$n_{s,g,y,a,l} = N_{s,g,y,a}P_{s,g,a,l} \qquad \text{(4rev)}$$

- $n_{s,g,y,a,l}$ are the number of hake of species *s*, gender *g*, year *y*, age class *a* and length *l* at the beginning of the year.
- $N_{s,g,y,a}$ are the number of hake of species s, gender *g*, year *y* and age class *a* at the beginning of the year.
- $P_{s,g,a,l}$ are the proportion of hake at the beginning of the year of species *s*, gender *g*, age class *a* which are of length *l*.

Calculation sequence for a small time step, verbal description:

1. Calculate the mass of hake eaten by hake predators from ration and diet values.
2. Calculate the mass of hake eaten from preference functions X numbers of prey available X a scaling factor. Solve for the scaling factor between equations resulting from steps 1 and 2.
3. Calculate the number of hake prey eaten by hake predators using the scaling function and preference functions.
4. From step 3 calculate the total number of hake prey that are consumed by other hake, this is the total numbers of a given age/year class that are predated upon by other hake
5. Use the total number of hake predated by other hake as a way of calculating the portion of natural mortality due to hake cannibalism/predation (see algebra in next section) applicable at the beginning of the year. This relies on approximations that are valid for a small time step.
6. Adjusted approach to (4) and (5): In the calculation of the total predation by hake use mid year numbers rather than start of year numbers numbers, i.e. use $n_{s,g,y,a,l}e^{-\frac{(z_{s,g,y-1,a})}{2}}$ instead of $n_{s,g,y,a,l}$. This uses the value of Z from the year before to first adjust the numbers in the population to the middle of the year, before proceeding with the predation calculations leading to the calculation of the predatory component of total natural mortality, and the use of this in the application of Pope's approximation.

Calculation sequence for a small timestep, algebraic description:

Calculate mass of hake eaten from ration and diet considerations:

The mass of hake from species sp eaten by hake from species s, gender g, year y, age class a and length l in a time period Δt is

$$\boldsymbol{\Delta t HAKEwtEATEN_{s,g,y,a,l,sp} = \Delta t\ HAKESPP\%_{s,y,l,sp} 365.25/100\ Ration\%_{s,l} w_{s,g,l} n_{s,g,y,a,l}}$$

This is just the product of

the length of time involved in fractions of a year (Δt) X

the proportion of species sp hake in the diet of species s hake predators for hake predators of length l ($HAKESPP\%_{s,y,l,sp}$) X

the annual ration as a proportion of body weight ($365.25/100\ Ration\%_{s,l}$) X

the weight of $n_{s,g,y,a,l}$ hake in the population ($\boldsymbol{w_{s,g,l} n_{s,g,y,a,l}}$).

Calculate mass of hake eaten from preference function and prey number considerations:

$\boldsymbol{\Delta t} HAKEwtEATEN_{s,g,y,a,l,sp}$ **can be alternatively calculated** using preference functions and the number of available hake prey.

It is the product of

the length of time involved in fractions of a year (Δt) X a scaling factor ($\mathrm{K}_{s,g,y,a,l,sp}$) X the sum over all hake prey lengths lp of the preference of size l hake predators for size lp hake prey ($\sum_{lp} PREF_{s,l,sp,lp}$) X

the total weight of species sp hake in the population with length lp $\left[\sum_{gp}\sum_{ap} w_{sp,gp,lp} n_{sp,gp,y,ap,lp}\right]$.

$$\boldsymbol{\Delta t HAKEwtEATEN_{s,g,y,a,l,sp} = \Delta t \mathrm{K}_{s,g,y,a,l,sp} \sum_{lp} PREF_{s,l,sp,lp} \left[\sum_{gp}\sum_{ap} w_{sp,gp,lp} n_{sp,gp,y,ap,lp}\right]} \quad \textbf{(7rev)}$$

(Indexing system: hake as predator: *s, g, a, l*. hake as prey: *sp, gp, ap, lp*).

Calculation of the scaling factor:

The scaling factor $\mathrm{K}_{s,g,y,a,l,sp}$ is solved from equation (7rev):

$$\boldsymbol{\mathrm{K}_{s,g,y,a,l,sp} = \frac{HAKEwtEATEN_{s,g,y,a,l,sp}}{\sum_{lp} PREF_{s,l,sp,lp}\left[\sum_{gp}\sum_{ap} w_{sp,gp,lp} n_{sp,gp,y,ap,lp}\right]}} \quad (8rev)$$

Calculation of total numbers predated:

The **weight** of species, gender, age and size of hake eaten by hake of a given species, gender, age and size over a time period Δt, $\boldsymbol{\Delta t}v_{s,g,y,a,l,sp,gp,ap,lp}{}^{w}$, is given by a simplified version of equation (7rev):

$$\boldsymbol{\Delta t}v_{s,g,y,a,l,sp,gp,ap,lp}{}^{w} = \boldsymbol{\Delta t}\mathrm{K}_{s,g,y,a,l,sp} PREF_{s,l,sp,lp} \boldsymbol{w_{sp,gp,lp}} n_{sp,gp,y,ap,lp} \quad (9rev)$$

The **number** of species, gender, age and size of hake eaten by hake of a given species, gender, age and size over a time period Δt, $\boldsymbol{\Delta t}v_{s,g,y,a,l,sp,gp,ap,lp}$, is given by a simplified version of equation (9rev) which is numbers based rather than weight based:

$$\boldsymbol{\Delta t}v_{s,g,y,a,l,sp,gp,ap,lp} = \boldsymbol{\Delta t}\mathrm{K}_{s,g,y,a,l,sp} PREF_{s,l,sp,lp} n_{sp,gp,y,ap,lp} \quad (10rev)$$

Therefore the total number of sp, gp, y, ap hake which are consumed by other hake, $\boldsymbol{\Delta t}PRED_{sp,gp,y,ap}$, can be obtained by summing up $\boldsymbol{\Delta t}v_{s,g,y,a,l,sp,gp,ap,lp}$ across all predator indices s, g, a, l, and the prey index *lp* i.e.

$$\boldsymbol{\Delta t}PRED_{sp,gp,y,ap} = \sum_{lp}\sum_{s}\sum_{g}\sum_{a}\sum_{l} \boldsymbol{\Delta t}v_{s,g,y,a,l,sp,gp,ap,lp} \quad (11rev)$$

Calculating of a "predation" natural mortality coefficient from the total predation numbers

Updating the population numbers from y to y+Δt for small Δt. This can be expressed by subtracting the estimated numbers of hake predated, multiplied by survivorship from other sources of natural mortality:

$$N_{s,g,y+\Delta t,a} = \left(N_{s,g,y,a} - \boldsymbol{\Delta t}PRED_{sp,gp,y,ap}\right) e^{-\Delta t M_{s,g,a}^{base}}, \quad (13rev)$$

The above equation can be alternatively written in terms of $M_{s,g,y,a}^{c}$ and $M_{s,g,a}^{base}$:

$$N_{s,g,y+\Delta t,a} = N_{s,g,y,a} e^{-\Delta t\left(M_{s,g,y,a}^{c} + M_{s,g,a}^{base}\right)} \quad (14rev)$$

and from this,

$$\boldsymbol{\Delta t}PRED_{sp,gp,y,ap} = N_{s,g,y,a} \frac{M_{s,g,y,a}^{c}}{\left(M_{s,g,y,a}^{c} + M_{s,g,a}^{base}\right)}\left(1 - e^{-\Delta t\left(M_{s,g,y,a}^{c} + M_{s,g,a}^{base}\right)}\right) \quad (15rev)$$

In the limit for small Δt, equation (15rev) becomes

$PRED_{sp,gp,y,ap} = N_{s,g,y,a} M^{c}_{s,g,a}$, and

$$M^{c}_{s,g,y,a} = \frac{PRED_{sp,gp,y,ap}}{N_{s,g,y,a}} \qquad (16rev)$$

This would be the portion of total natural mortality that is due to predation by hake at the beginning of the year. This value is however unlikely to be invariant during the year given that the number of predators and prey are fluctuating through the year. Although as a first approximation this natural mortality value can be used in Pope's approximation to update population numbers:

$$N_{s,g,y+1,a+1} = \left(N_{s,g,y,a} e^{-\frac{(M^{c}_{s,g,y,a}+M^{base}_{s,g,a})}{2}} - C_{s,g,y,a} \right) e^{-\frac{(M^{c}_{s,g,y,a}+M^{base}_{s,g,a})}{2}} \qquad (17rev),$$

a better approximation was sought.

Improving the estimated "predation" natural mortality coefficient for use in the model

The approximation in equation (16rev) was felt to be too crude, and some improvement was sought. A possible improvement is to use as basis for the calculation of $M^{c}_{s,g,y,a} = \frac{PRED_{sp,gp,y,ap}}{N_{s,g,y,a}}$, the population numbers present in the middle of the year rather than at the start of the year. The calculation of the numbers in the middle of year y involves $M^{c}_{s,g,y,a}$, and so the approximation used instead is

$$n_{s,g,y,a,l} e^{-\frac{\left(Z_{s,g,y-1,a}\right)}{2}}$$

(rather than $n_{s,g,y,a,l}$). This uses the value of Z from the year before to first adjust the numbers in the population to the middle of the year, before proceeding with the predation calculations leading to the calculation of the predatory component of total natural mortality, $M^{c}_{s,g,y,a}$, and the use of this value in the application of Pope's approximation in equation (17rev).