Basics – II. Time, Magnitudes and Spectral types

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"We'd like to study the effects of gravity, loud music, and beer on astrophysicists."

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1 Time

Time is difficult; we don't really know what it is, and the more carefully we try to measure it, the more complex it gets. To give some idea of the complexity of the issue, the *Astronomical Almanac* lists the following time systems – TAI, UTC, TDT, TDB, UT0, UT1, UT2, GMST, GAST, LMST, LST – amongst others. We don't need to know everything about all of these, but the complexity of accurate time-keeping should be appreciated. **In astronomical measurements, it is often important to know what time system is being used for any particular application**. For more details on time (and all manner of subjects) see the *Explanatory Supplement to the Astronomical Almanac*.

There are two main ways of measuring time:

- using the rotation of the Earth and

- using the frequency of atomic oscillations.

The Earth's rotation is not uniform; the rate includes periodic and secular (long-term) changes of the order of a second per year. Atomic standards are uniform in the microseconds per year range. Since the 1950s, "atomic time" has taken over from Earth rotation times. Prior to that, the best accuracy was given by **Ephemeris Time** which was used until 1984 and took the best theory of the Earth's rotation to remove changes in the rotation rate.

Several time scales still follow the Earth's rotation (eg. civil and sidereal times) but these are now based on atomic clocks and actual measurements of rotation rate changes. See the *Explanatory Supplement to the Astronomical Almanac*.

1.1 Earth Rotation Times

1.1.1 Apparent Solar Time: The Equation of Time

It is convenient for many human pursuits to use the Earth's diurnal and annual motion (the day and year) as a basis for time-keeping. The rotation of the Earth on its axis is fundamental to us; our waking and sleeping cycles are determined by it. It is, however, not strictly constant. Perhaps the earliest time-keeping was based on the apparent diurnal motion of the Sun, and we can define a **Local Apparent Time** by calling the time that the Sun crosses the local meridian the local **noon** – or mid-day. The local time is then simply the **Local Hour Angle of the Sun + 12 hours** (so the day starts at midnight) – and a day is the interval between successive noons. This is the time displayed by a sundial, for example.

Using even moderately reliable clocks, it is clear that the day defined in this way is not constant, for two main (periodic) reasons:

- **The eccentricity of the Earth's orbit**. Because the Earth's orbit is slightly elliptical, the Earth moves slightly faster at **perihelion** (in January) than at **aphelion** (in July). So, in our (false) picture of a fixed Earth and moving heavens, the Sun appears to move slightly faster at **perigee** than at **apogee**. This introduces a variation from uniform motion which is a wave of period one year.

- **The obliquity of the ecliptic**. If the Earth's orbit were circular, the Sun would appear to have a constant velocity in celestial longitude (ecliptic co-ordinates). But, because the Earth's axis of rotation is tilted at an angle to the axis of the orbit, the ecliptic (along which

the Sun appears to move) is tilted at an angle to the celestial equator (this angle is called the obliquity of the ecliptic). Thus, when we measure the rate of the Sun's motion in RA (along the celestial equator) we see a varying rate due to the projection of the ecliptic on to the celestial equator. This introduces a variation from uniform motion which is a wave of period half a year.

The combination of these two effects leads to the **Equation of Time** (see figures). In effect, we define a **Mean Sun** which is an imaginary point which travels around the celestial equator at uniform speed. The equation of time is then the difference between the position of the mean Sun and that of the true Sun.
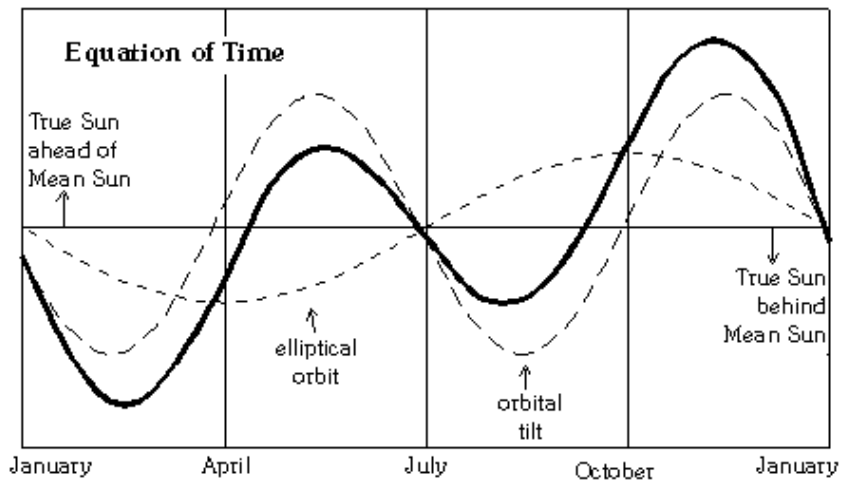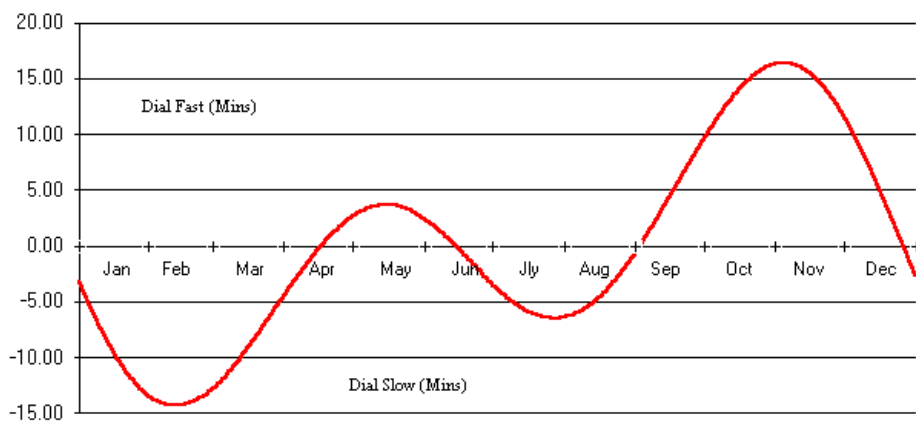

Figure 1:

Figure 2:

The position of the mean Sun thus defines **Local Mean Time** and the interval between successive transits of the meridian by the mean Sun is the mean solar day.

1.1.2 Mean Solar Time: Time Zones

Greenwich Mean Time (GMT) is defined by the location of the mean Sun relative to the Greenwich meridian, and other local mean times are defined by GMT with a correction for the observer's longitude.

On the one hand, it is not convenient to have every longitude with its own local time (imagine, for example, making railway timetables when each station has its own time frame), on the other hand, it is not generally desirable to have any locality *too* far from local mean time. This resulted in the setting up of the time "zone" system about 100 years ago.
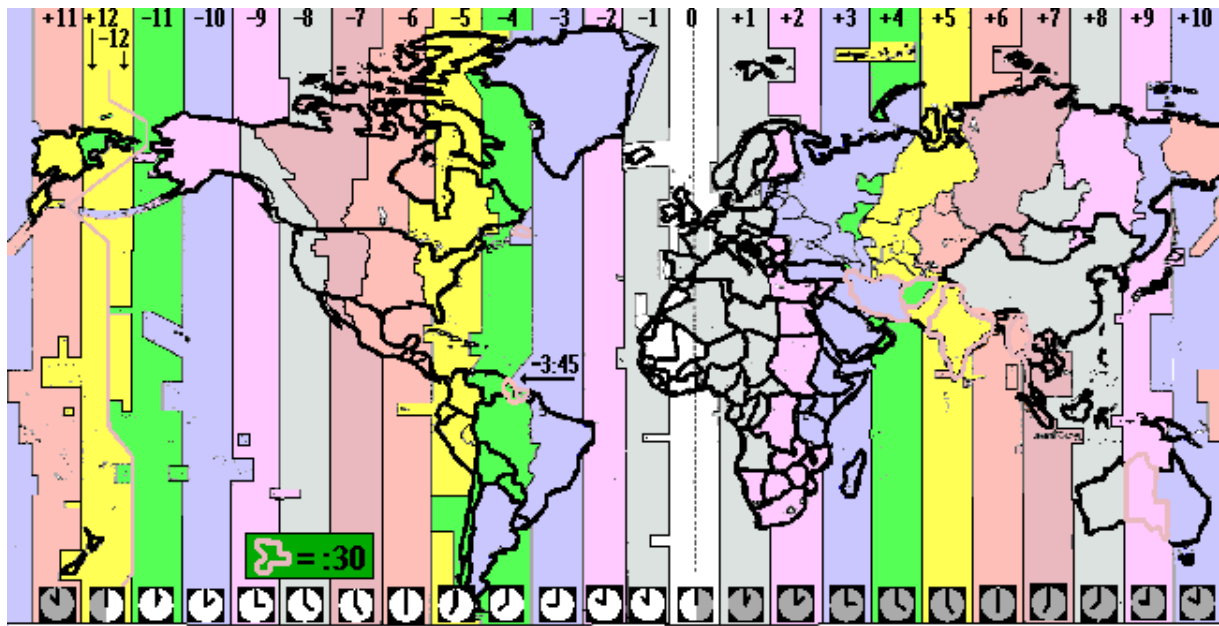
Figure 3:

1.1.3 Universal Time (UT1 or UT)

UT0 and UT2 are versions of **universal time** which are of decreasing use. UT1 (or just UT) is a measure of the actual rotation of the Earth, independent of location and is based on the mean solar day. It is essentially the same as the now discontinued GMT.

Since it is based on the not-completely-predictable rotation of the Earth, it drifts at about 1 second a year relative to TAI.

1.1.4 Sidereal Times

We have seen the basic idea of sidereal time – "time by the stars" – in the lecture on co-ordinate systems.

Greenwich Mean Sidereal Time (GMST) is the basic measure for sidereal time and is defined by the Greenwich meridian and the vernal equinox. GMST is the hour angle of the average position of the vernal equinox – neglecting the short-term effects of nutation. The International

Astronomical Union (IAU) conventions link GMST (in seconds at UT1=0) to UT1 by:

$$GMST \ = \ 24110.54841 \ + \ 8640184.812866 \ \times \ T \ + \ 0.093104 \ \times \ T^2 \ - \ 0.0000063 \ \times \ T^3$$

where:

$$T \ = \ d/36525 \quad and \quad d \ = \ JD \ - \ 2451545.0$$

(T is in **Julian centuries** from 2000 Jan 1, 12h UT; and JD is the **Julian date** – to be defined later)

Greenwich Apparent Sidereal Time (GAST) is GMST corrected for nutation. Precession is already allowed for in GMST. The RA component of nutation is called the **"equation of the equinox"** and so:

$$GAST \ = \ GMST \ + \ \text{the equation of the equinox}$$

Local Mean Sidereal Time (LMST) is GMST plus the observer's longitude measured positive *east* of the Greenwich meridian. This is the time usually displayed as LST in an observatory.

$$LMST \ = \ GMST \ + \ \text{observer's east longitude}$$

Local Sidereal Time (LST) we have seen defined as the local hour angle of the vernal equinox:

$$\text{Hour Angle} \ = \ \text{LST} \ + \ \text{RA}$$

1.2 Atomic Times

Perhaps the most accurate time-keeping is by so-called atomic clocks, which count atomic processes. These are currently operating at accuracies down to about 10^{-15}. If they reach an accuracy of 10^{-17}, they will be affected by relativistic effects such that raising or lowering the clock by 10cm will have a measurable effect (gravity variation) as will moving the clock at walking speed (velocity variation). As it is, the most accurate atomic clocks have to be corrected for the floor of the building on which they are used ! (see *Scientific American, September 2002*).

1.2.1 Atomic Time (TAI)

International Atomic Time (Temps Atomique International – TAI) is the basis of all modern time-keeping. The SI definition of a second is the duration of 9 192 631 770 cycles of the radiation corresponding to the transition between the two hyperfine levels of the ground state of Caesium–133. This definition was chosen to be as close as possible to the previous standard – the **ephemeris second**.

TAI is an earth-based time, since it is defined for a particular gravitational potential and inertial reference on the Earth. In practice, it is defined by a weighted average of ~ 200 atomic clocks, the Caesium clocks of the U.S. Naval Observatory in Washington being given considerable weight.

1.2.2 Co-ordinated Universal Time (UTC)

Co-ordinated Universal Time (UTC) is the time broadcast by the U.S. National Institute of Standards and Technology (NIST) and other national standards. UTC has the same *rate* as TAI, but has integer numbers of seconds added (about one per year) to keep solar noon at the

same UTC (i.e to keep UTC near UT1). The added seconds are called **"leap seconds"** and keep UTC within about 0.7 seconds of UT1. We have:

$$UTC = TAI - \Delta AT \text{ (number of leap seconds)}$$

and the current difference ΔAT is 32 seconds.

1.2.3 Terrestrial Dynamic Time (TDT or TT)

Before the widespread use of atomic clocks, **Ephemeris Time (ET)** was closest to a uniform time system. Since 1984, this has been replaced by **Terrestrial Dynamic Time**, which is related to TAI by a constant offset of 32.184 seconds:

$$TT = TAI + 32.184$$

The constant is applied to maintain continuity between ET and TT across the transition. The current difference between TT and UT ($\Delta T = TT - UT$) is about 64 seconds.

1.2.4 Barycentric Dynamic Time (TDB)

Barycentric Dynamic Time (TDB) is the same as TT except that relativistic corrections are applied to move the origin to the solar system barycentre, effectively removing terms due to the Earth's motion through the gravitational potential of the solar system. These periodic terms are smaller than about 1.6 milliseconds and so are not significant for much of what we do (they would be, for example, for accurate timing of millisecond pulsars).

1.3 Calendars

There are many different calenders in use, even today; the Islamic, Jewish and Christian, the Indian, Chinese and Japanese, for example. We know also of many "historical" calendars, such as the Egyptian, Mayan and Roman calendars.

All calendars suffer from the same problem; there are not an integer number of days in a year or month, nor months in a year. This means that they all tend to either slowly get out of phase with the year or need some jiggling to stay in phase.

1.3.1 The Year

Things are further complicated by the fact that there is more than one way of defining a year:

- The **tropical year** is the interval between two successive passages of the Mean Sun through the vernal equinox and is 365.2421988 mean solar days (i.e UT time). In a sense, this is the "natural" year as the seasons repeat on this period. (Note: it is decreasing at about 0.53 seconds per century).

- The **Sidereal year** is the interval between two successive passages of the Mean Sun at a (fixed) star and is 365.256366 days (UT). The sidereal day is longer than the tropical year because of the retrograde motion of the vernal equinox (due to precession).

- The **Anomalistic year** is the interval between two successive passages of the Earth through **perihelion** (or, equivalently, the Sun through **perigee**) and is 365.259636 days. Because of the precession of the **line of apsides** – the semi-major axis of the Earth's orbit – the anomalistic year is slightly longer than the sidereal year.

Don't you just love it ?

1.3.2 The Civil Year

For ordinary "civil" purposes, the year should:

- contain an integer number of days and

- stay in phase with the seasons

As we have seen, the tropical year marks the recurrence of the seasons, but is close to 365.25 days long. The **civil year** can contain either 365 (ordinary year) or 366 days (leap year) and by mixing these in approximately the ratio 3:1, the *average* year has a length close to that of the tropical year.

This concept was already known to the Egyptians and it was an Alexandrian scholar, Sosigenes, who advised Julius Caesar to introduce a similar calendar into the Roman empire in 46 B.C. The **Julian calendar** had every fourth year a leap year. (Incidentally, Julius Caesar ordered that 46 BC should have two extra months and be 445 days long, to bring the calendar back in line with the seasons).

In the short-term, this works well, but three years and one leap year give an average year of 365.25 days, different from the tropical year by about 0.0078 days, so in a thousand years, you're out by nearly 8 days.

In 1582, Pope Gregory XIII introduced calendar reform, producing the **Gregorian calendar** which we use today and which is a slightly modified Julian calendar, arranged so that 3 days in every 400 years are omitted. This is achieved by the following recipe:

- every year divisible by four is a leap year (like the Julian calendar),

- but every year which is a multiple of 100 is *not* a leap year,

- *unless* that year is also divisible by four, in which case, it *is* a leap year.

So, .., 2004, 2008, ... are leap years; but ..., 1700, 1800, 1900 are not leap years; but ..., 1200, 1600, 2000 are leap years.

This calendar will incur an error of just over a day in 4000 years.

In 1582, Pope Gregory XIII decreed that 10 days should be dropped from the calender to realign the seasons. This was adopted by Italy, Spain, Poland and Portugal immediately and in those countries, 4th October was followed by 15th October. Other Catholic countries followed soon after, but Protestant countries were reluctant to change and Greek Orthodox countries didn't change until the 20th century. Russia changed after the 1917 revolution and Turkey in 1927.

Britain and its dominions (including at that time North America) changed in 1752, when 2nd September was followed by 14th September. Interestingly, The Unix/Linux calender program, *cal* (cal 9 1752) produces:

```
    September 1752
 Su Mo Tu We Th Fr Sa
        1  2 14 15 16
 17 18 19 20 21 22 23
 24 25 26 27 28 29 30
```

1.4   The Julian Day (JD)

To avoid the complications of using calendars, we prefer to use, for many purposes, the **Julian Day Number** – or **Julian Day**.

This was devised by Josephus Scaliger (1540–1609) and probably named to honour his father, Julius Scaliger. The Julian Day count starts on 4713 BC January 1st at 12h GMT. This "zero point" is so far in the past that all historical dates are positive, but the apparently rather arbitrary start year was picked because three (then) important cycles (extrapolated into the past) had their start year in that year. These were the 19-year lunar Metonic cycle, the 15-year Indiction Cycle (a Roman taxation cycle) and the 28-year solar cycle (the period on which the Julian calendar repeated). All now largely of historic interest only.

So, 4713 January 1st, 12h GMT (or UT) is Julian day 0.0; 2008 January 1st 12h UT is JD 245 4466.00. Starting the JD at midday means, for European astronomers at least (and Africa) observations on any given night have the same JD.

Note that JD is based on GMT/UT so that for the most accurate work, correction might well need to be made to Terrestrial Dynamic Time, for example. And there are other, more significant corrections which we discuss briefly below.

The JD is a little unwieldy, so sometimes **Modified Julian Day** is used. This is just JD - 245 0000.5

1.5   Heliocentric Julian Day (HJD)

Imagine a star in the plane of the ecliptic (plane of the Earth's orbit) which is in the opposite direction to the Sun. Six months later, the same star will be very close to (or behind) the Sun. Neglecting the fact that the Sun is in the way, light from the star now reaching Earth has had to travel an extra distance equal to the diameter of the Earth's orbit – about 300 million kilometers. Since the velocity of light is near 300 000 km/sec, the light takes about 1000 seconds ($\sim$ 17 minutes) longer to reach the Earth when the Earth is on the opposite side of the Sun to the star than when it is on the same side.

If the star were at the ecliptic pole, then the position of the Earth relative to the Sun would have no effect on the **light travel time** (except for a small effect due to the eccentricity of the Earth's orbit).

Now, if we are measuring some periodic phenomenon – such as eclipses in a binary or pulsation of a variable star – we do not want errors of up to 17 minutes in the data. So, we usually correct all such observations to the centre of the Sun and get the **Heliocentric Julian Day (HJD)**.

Clearly, the correction will depend on the location of the star relative to the Earth – Sun radius vector. An approximate correction is:

$$HJD = JD + K R (cos\ L\ cos\ \alpha\ cos\ \delta + sin\ L\ (sin\ \epsilon\ sin\ \delta + cos\ \epsilon\ cos\ \delta\ sin\ \alpha))$$

where:
K = Sun–Earth light travel time ($\sim$ 0.000578 days)
R = Earth–Sun distance (in A.U.)   (1 AU - mean Earth–Sun distance)
L = true longitude of the sun (in degrees)
ϵ = the obliquity of the ecliptic ($23°26'21.448"$ for 2000.0)
α, δ = co-ordinates of the star in degrees

This is a simple, if tedious, calculation. Values for L and R are tabulated for each day in the *Astronomical Almanac*. Most reduction software will calculate the **heliocentric correction** automatically.

1.6 Barycentric Julian Day (BJD)

For the most accurate results, it is necessary to correct times to the centre of mass of the solar system – the **barycentre** of the solar system to get Barycentric Julian Day (BJD). Apart from the Sun, the solar system is dominated by the mass of Jupiter, so the difference between HJD and BJD is dominated by a cyclic variation of amplitude about 4 seconds and with a period of about 11 years – Jupiter's orbital period.

Note that since a day has 86400 seconds, an accuracy of 1 second in timing is about 0.00001 day. At this level it is necessary either to input TT (rather than UT) to the calculation of HJD and BJD, or to correct from UT to TT afterwards.

2 The magnitude system

2.1 Introduction

Stellar photometry is the measurement of the *apparent* brightness of stars, usually in more-or-less well-defined pass-bands. Ideally, we would like to be able to determine the distribution of radiation from a celestial body at all wavelengths and with "spectroscopic" resolution. In practice the technical difficulties are many (eg. that the atmosphere absorbs many spectral regions (X-ray, UV, etc) so we need very expensive satellite telescopes, dedicated to specific wavelength regimes), and the quantities of light we are dealing with are generally so small that resolution has to be sacrificed to get any kind of decent S/N (eg. high time resolution and high spectral resolution are only achievable with bright stars and big telescopes).

2.2 Apparent Magnitude – how bright do stars appear to be ?

Around 120 BC, Hipparchus divided the naked-eye stars into six groups which he called first magnitude (the brightest) down to sixth magnitude (the faintest) and we have been stuck with this upside-down scale ever since. During the 19th century, it was determined (Steinhel, Pogson) that the intensity of light received from a 6th magnitude star was about a hundred times less than a 1st magnitude star – and that the scale was logarithmic (like the decibel scale for sound/hearing) because the eye perceives equal *ratios* of intensity as equal *intervals* of brightness. The magnitude scale was then *defined* so that for two stars with measured brightnesses (light intensities) I_1 and I_2 different by a factor of a hundred, the magnitude difference was 5. So,

$$m_1 - m_2 = -2.5 \ log\frac{I_1}{I_2}$$

Where the minus sign gives the "inverted" magnitude scale, with brighter stars having smaller numbers (magnitudes). Alternatively, we can write for any star:

$$m = -2.5 \ log \ I + constant$$

which is sometimes called **Pogson's formula**, and where the constant is often called the **zero-point** of the magnitude scale. This has been established over decades (centuries, if you include Hipparchus), first visually, then photographically and photoelectrically, each method giving greater accuracy. The "zero" has remained approximately the same so that we can compare present observations to "archive" data – at least roughly.

Again alternatively, since 5 magnitudes is a factor of 100 in brightness, one magnitude is equal to $100^{\frac{1}{5}} = 2.512$, and we can write that:

$$\frac{I_2}{I_1} = 2.512^{m_1 - m_2}$$

Some examples:

The (apparently) brightest star in the sky, Sirius, has m = -1.4 (o yes - you can have negative magnitudes) and the faintest stars observed in things like the Hubble "deep" fields have apparent magnitudes near 29. So, the *difference* is about 30 magnitudes. This is then six steps of 5 magnitudes, so the *apparent* luminosity difference is a factor 100^6 or 10^{12}.

If star 1 has (apparent) $m_1 = 5.4$ and star 2 has $m_2 = 2.4$, then star 2 is clearly the brighter (smaller number) by

$$(2.512)^{5.4-2.4} = (2.512)^3 \approx 15.9$$

and you can see from this simple example that a star which is n magnitudes brighter than another (where n = 1,2,3,4,5,6, ...) has a luminosity which is approximately (2.5, 6.3, 16, 40 100, 250, 630, ...) times greater.

The two components of α Centauri (excluding Proxima) have apparent magnitudes of 0.33 (A) and 1.70 (B), what is their combined apparent magnitude ?

Since m = –2.5 log I (plus a constant), we have log I = –0.4m, so:

$$log\ I_A\ =\ -0.4\ (0.33)\ =\ -0.132\ =\ log\ 0.738$$

$$log\ I_B\ =\ -0.4\ (1.70)\ =\ -0.680\ =\ log\ 0.209$$

so

$$I_A\ +\ I_B\ =\ 0.945 \quad and \quad log(I_A\ +\ I_B)\ =\ -0.024$$

$$m_{A+B}\ =\ -2.5\ (-0.24)\ =\ 0.06$$

The fact that the magnitude scale is logarithmic enables the effective compression of large factors of brightness into a relatively small range of magnitude. This compact and easily understood form is really why the system has persisted – that and the fact that it annoys the bejasus out of physicists.

2.3 Absolute Magnitude – how bright are stars really ?

If we have a star at an unknown distance, d, with an apparent luminosity, ℓ, and corresponding apparent magnitude, m, and we define its **absolute magnitude**, M and corresponding luminosity, L, to be the apparent magnitude/luminosity when seen from a standard distance, D, then the inverse-square law for the propagation of light gives:

$$\frac{\ell}{L} = \frac{D^2}{d^2}$$

If we now set the standard distance to be 10pc and substitute

$$\frac{\ell}{L} = \frac{10^2}{d^2} \quad \text{into} \quad m_1 - m_2 = -2.5 \ log \ \frac{\ell_1}{\ell_2}$$

we get:

$$m \ - \ M = -2.5 \ log \ \frac{100}{d^2}$$

so

$$m \ - \ M = 5 \ log \ d - 5$$

or

$$m \ - \ M = -5 \ log \ \pi - 5$$

(because $\pi = 1/\text{d}$) and these equations relate the apparent magnitude (m), absolute magnitude (M) and distance (d) or parallax (π) of a star in a simple way, resulting from the definition:

The absolute magnitude of a star is the apparent magnitude the star would have if seen from a standard distance of 10 parsecs.

Important points:

- Obviously, m and M must be in the same passband and will often be subscripted to indicate this (eg, m_V–M_V, or V–M_V).

- The quantity $m - M$ is called the **distance modulus**.

- If we can measure the apparent magnitude of a star accurately (which we can) and we know the distance (which we generally don't), we can get the absolute magnitude (or luminosity) which then will allow us to determine important physical parameters for stars – radius, energy output, and so on.

- Alternatively, if we know the apparent magnitude and can determine the absolute magnitude is some devious way, we can get distance. Much of astronomy is thus spent being devious.

- The above equations assume that light travelling from a star to us is not blocked in any way. This is not true; interstellar dust and gas will scatter and absorb radiation, so that we typically write
$$m \ - \ M = 5 \ log \ d - 5 + A$$

  (but usually apply the "interstellar reddening" correction to the directly measured apparent magnitude, m). This will be discussed later.

- The absolute magnitude system must be calibrated. This can be done using:

  - The Sun, for which we know the fundamental parameters, including distance, very accurately.

  - Nearby stars for which we can determine trigonometric parallaxes accurately. The importance of the *Hipparcos* data was mentioned in the first lecture – this satellite substantially improved the accuracy of parallax/distance determination for nearby stars.

– Star clusters. If we can measure apparent magnitudes for a number of stars in a nearby cluster and find stars which are identical spectroscopically to the Sun (or some of the trig.parallax stars) we can use these to calibrate the cluster and get absolute magnitudes for hotter and more luminous stars. If we can thus calibrate the distance scale for (eg.) Cepheids – pulsating stars which have a well-established Period/Luminosity relationship – and which are very luminous – we can start to calibrate extra-galactic distances.

Some examples:

The apparent magnitude of the Sun is -26.78 (trust me, I'm an astronomer) and its distance is 1 AU (by definition). We know 1pc is 206265 AU and since

$$m_1 - m_2 = -2.5 \ log \ \frac{\ell_1}{\ell_2}$$

we can write

$$m - M = -2.5 log \ \frac{\ell}{L} = -2.5 \ log \ \frac{(2062650)^2}{1^2}$$

so

$$-26.78 - M = -5 \ log \ 2062650 = -31.57$$

thus

$$M = 4.79$$

and at 10pc, the Sun would appear to be an unimpressive star.

Sirius has an apparent magnitude of -1.44 and a parallax of 0.379 arcseconds. Its absolute magnitude is thus given by:

$$-1.44 - M = -5 \ log(0.379) - 5$$

so

$$M = +1.45$$

since we determined the absolute magnitude of the Sun to be 4.79, we can write:

$$log \ \frac{\ell_{Sirius}}{\ell_\odot} = 0.4(M_\odot - M_{Sirius}) = 0.4(4.79 - 1.45)$$

so

$$\ell_{Sirius} \approx 22 \ \ell_\odot$$

2.4   Bolometric Magnitude and effective temperature

The **Bolometric magnitude** of a star is simply the magnitude integrated over all wavelengths. As we have seen, this is not trivial to determine (because some regions of the spectrum are absorbed by the atmosphere (for example) and theoretical models are used to determine the **Bolometric correction** – the correction applied to a magnitude (apparent or absolute) to get the bolometric value. Usually:

$$M_{bol} = M_V + BC$$

where the subscript V refers to the "V" or "Visual" passband (a yellow-green filter) of the UBV or UBVRI photometric systems (of which, more later)
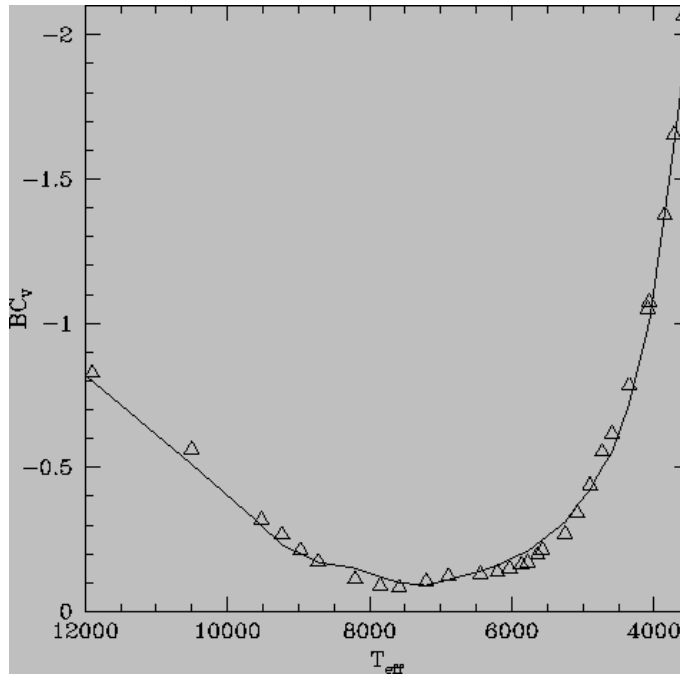

Figure 4: Bolometric correction vs temperature for "normal" dwarf stars

Fig 1.1 is a plot of the bolometric correction for "dwarf" stars (stars like the Sun – working on hydrogen–fusion). Note that the bolometric correction is small for stars near the solar temperature (6000 or 7000K) but rises rapidly for hotter or cooler stars. This is easily understood – stars near the solar temperature output a large fraction of their energy in the "visible" region - particularly in the yellow-green region covered by the "V" filter; very hot stars emit a lot of their light in the ultraviolet region and very cool stars emit mainly in the infrared.

As noted earlier, it is important when using equations such as $m - M = 5logd - 5$ to get the magnitudes m and M on the same system. It is important to remember that if you want the total luminosity of the star, you must use the bolometric absolute magnitude.

There are a number of ways of defining the temperature of a star and it is important to know which is being used.

We could use **Wien's law** which states that for a perfect radiator (a "black body") with a maximum in energy output at wavelength λ_{max} :

$$\lambda_{max} = \frac{2898}{T}\mu m$$

(in this case, the Sun, which has a surface temperature near 5800K would have $\lambda_{max} = 0.5\mu m$, whereas the Earth's atmosphere with a temperature of $\sim$ 300K would have λ_{max} near $10\mu m$.)

In practice, however, it is often very difficult to determine λ_{max} with any accuracy.

The **Planck formula** could be used to fit a Planck curve to a stellar energy distribution. The frequency form :

$$F_\nu \delta\nu = \frac{2\pi h}{c} \frac{\nu^3 \delta\nu}{e^{\frac{h\nu}{kT}} - 1}$$

usually has flux, F_ν, in Janskys or $Watts\ m^{-2}\ Hz^{-1}$ (where 1 Jansky $= 10^{-26}\ Watts\ m^{-2}\ Hz^{-1}$).

The wavelength version of the Planck law is :

$$F_\lambda \delta\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{\delta\lambda}{e^{\frac{hc}{\lambda kT}} - 1}$$

with units usually in $erg\ cm^{-2}\ \mathring{A}^{-1}$ or $erg\ cm^{-2}\ Hz^{-1}$ (cgs).

Stellar spectra, however are rarely black-body-like, so that fitting a Planck curve might not be simple.

Integration of the Planck formula results in the **Stefan–Boltzmann law** which gives the energy emitted per unit surface area as:

$$E = \sigma T^4$$

where σ is the Stefan-Boltzmann constant ($5.67 \times 10^{-8}\ Wm^{-2}T^{-4}$) and the total luminosity of a star is then:

$$L_* = 4\ \pi\ R_*^2\ \sigma\ T_{\text{eff}}^4$$

which leads to the definition of **effective temperature**, T_{eff}, of a star – which is the temperature of a black-body with the same total energy emission per unit surface area. And this is the definition of stellar temperature which is usually used.

Note that when $\lambda \gg \lambda_{max}$, $\frac{hc}{\lambda kT}$ is small and:

$$F_\lambda\ \alpha\ T\ \lambda^{-4}$$

which is the Rayleigh-Jeans approximation, and the region of the spectrum is known as the **Rayleigh-Jeans tail**.

The **solar constant** is the total radiation received from the Sun – **outside** the Earth's atmosphere – at the mean Earth–Sun distance. Currently, this is about 1367 W m^{-2}. Since the mean Earth–Sun distance is about 1.496×10^{11}m, then the total luminosity of the Sun is:

$$L_\odot\ =\ 4\ \pi\ (1.496 \times 10^{11})^2\ (1367)\ =\ 3.845 \times 10^{26}\ Watts$$

Star	M_V	BC	M_{bol}	T_{eff}
Sirius	1.4	-0.4	1.0	10400
Sun	4.8	-0.1	4.7	5800

Now, from the table, we can repeat an earlier rough calculation:

$$log \; \frac{L_{Sirius}}{L_\odot} \; = \; 0.4 \, (4.7 \; - \; 1.0) \; = \; 1.48$$

so

$$L_{Sirius} \; \simeq \; 30 \; L_\odot$$

and the difference between this result and the earlier one is because we have now used total luminosity – or bolometric magnitudes. The difference is large because Sirius is quite hot (compared to the Sun, anyway) and a lot more of its light is output in the blue/ultraviolet.

Sirius is a double star. The companion is about 10 magnitudes fainter – but is about the same colour. This means both stars are about the same temperature and therefore have about the same bolometric correction. Since they are also at the same distance (to well within the accuracy of this calculation), we can assume that they have the same difference in *absolute* magnitude as in apparent magnitude – and also in bolometric magnitude (since we are assuming the BCs are the same). Thus Sirius A and B are different in bolometric magnitude by 10 magnitudes or a factor of $(100)^2$ in luminosity.

Since:

$$L_* = 4 \; \pi \; R_*^2 \; \sigma \; T_{eff}^4$$

we can write:

$$\frac{L_A}{L_B} \; = \; \frac{R_A^2}{R_B^2} \; \frac{T_A^4}{T_B^4}$$

and, since the temperatures are assumed to be equal:

$$R_A^2 \; = \; (100)^2 \; R_B^2$$

so

$$R_A \; = \; 100 \; R_B$$

and since we have the radius of Sirius A as about 1.7 $R_\odot$, this makes the Sun about 60 times the size of Sirius B. Or, Sirius B is only about twice the radius of the Earth – it is in fact a degenerate **white dwarf** star.

3 Spectral Classification (in the MK system).

3.1 Spectral Types

With the earliest objective-prism spectra, over a hundred years ago, it was seen that there was a range of spectral appearance. To cut a long story short, these were classified A, B, C, etc., in order of decreasing strength of Hydrogen and other absorption lines.

It was soon realised that the spectra could be explained as a temperature sequence – also, that some of the assigned types were not stellar (gas clouds, for example) and the sequence which persists to today was established:

<div align="center">

O B A F G K M

</div>

This was not fine enough, so a decimal subdivision was added so the sequence looks like:

$$O3, O4, ...O9, B0, B1, B2, ...B9, A0, A1, ...A9, F0, F1, ...F9, G0, G1, ...$$

and so on, although note that not all decimal types exist. The sequence is NOT a linear sequence, and further subdivision has occurred in certain types, so that types like B1.5 exist.

Very recently, cooler stars than the latest M stars have been discovered; the so-called L and T stars which actually extend down into the "brown dwarf" region – objects which are not massive enough to sustain core nuclear reactions and therefore are not really stars.
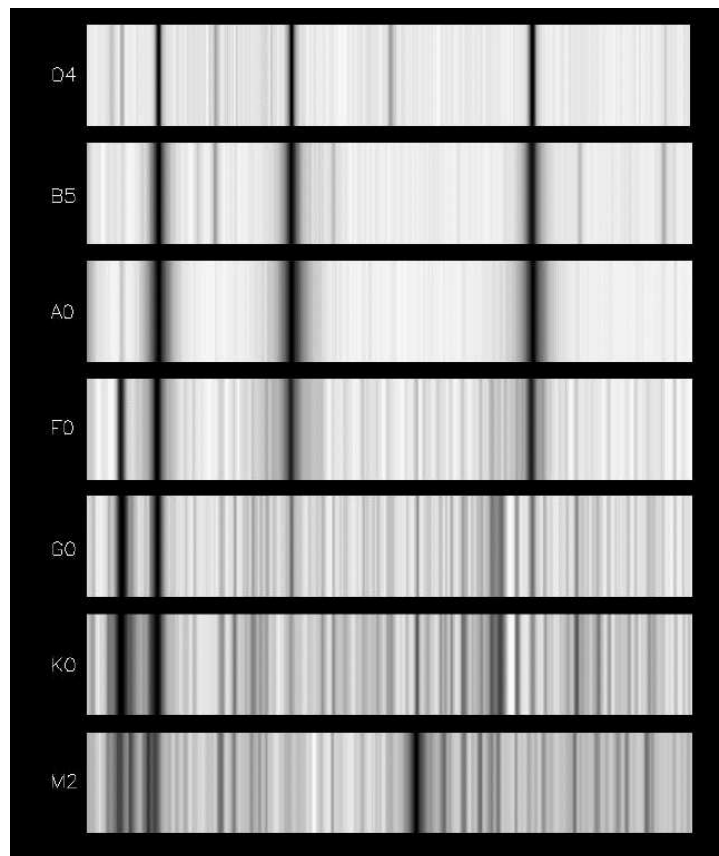
Figure 5: Sample spectral types.
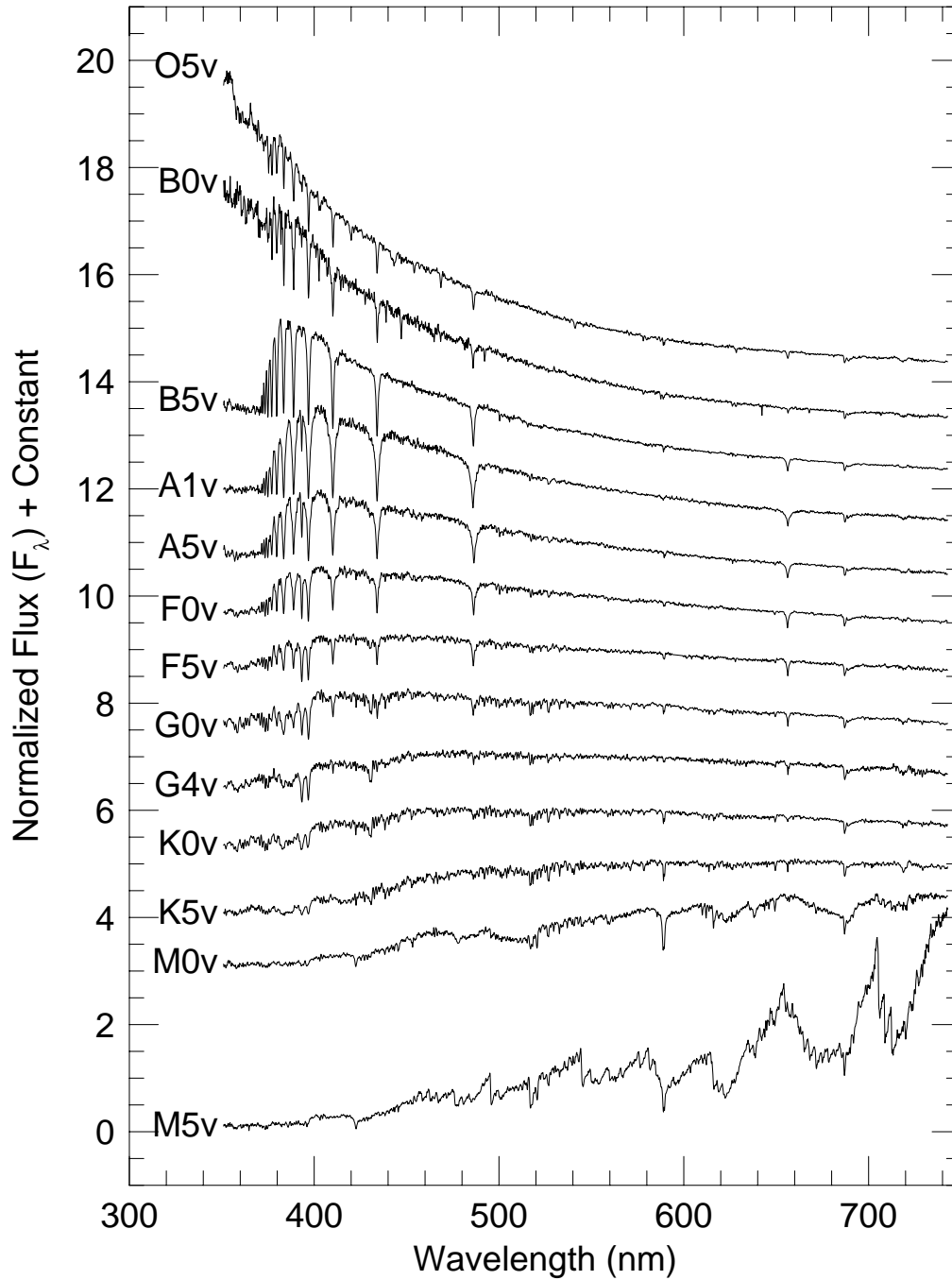
Figure 6: Sample spectral types in digital form, displayed in flux units.
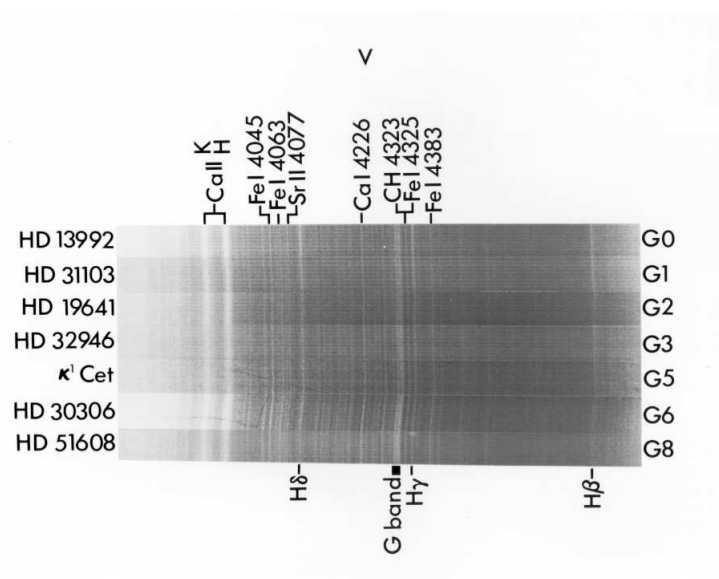
Figure 7: Details of G spectral type stars

Why the spectral sequence is a temperature sequence can be seen by considering the formation of the Balmer series of hydrogen – which is prominent in many stars (because nearly all stars are 80% hydrogen by mass).
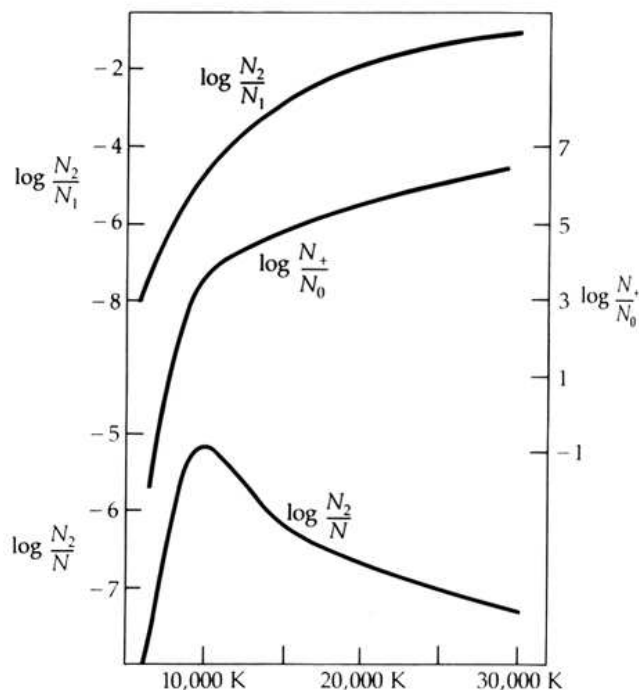

Figure 8: Variation of the Balmer series with temperature in stellar spectra. As temperature increases, the n=2 level, where the Balmer absorption originates, becomes more populated (top), but ionisation also increases (middle) and the combined effect shows a peak near 10 000 K (bottom) – around A0 in spectral type.

As indicated in the figure, as temperature increases in a sample of hydrogen atoms, the n=2 level becomes more populated at the expense of the n=1 level (ground state).

Additionally, as temperature increases, more hydrogen atoms are ionised – and clearly an ionised atom cannot produce Balmer series absorption. Ionisation increases rapidly towards 10 000 K and the combination of increased n=2 population with increased ionisation gives a peak in the Balmer series near 10 000 K – around spectral type A0.

Similar processes happen in other elements, as indicated in the figure, and the combination of all these gives the observed spectral sequence.
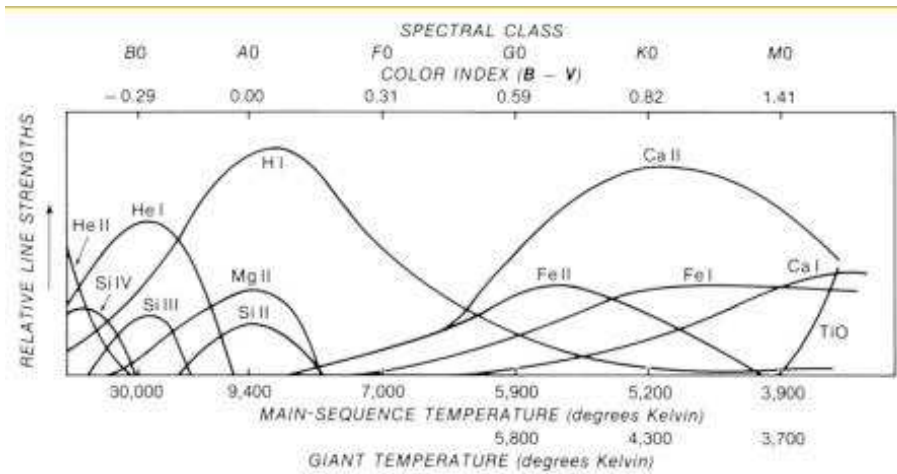
Figure 9: Variation of some spectral species with temperature/colour/ spectral type.

Type	Colour	Strongest lines (in visible)	Examples		
			Type	~B–V	~T$_{eff}$
O	Blue	HeI; HeII (H, NIII, SiIV)	O5	−0.32	38000°K
B	Blue-white	H; HeI (MgII, SiII, SiIII)	B0	−0.30	27000
A	White	H (very strong); CaII (H&K) MgII, SiII, FeII, weak neutral metals	B5 A5	−0.16 +0.15	16000 8200
F	Yellow-white	H; CaII; neutral & ionised metals (FeI, CrI, CaI, ..., FeII, SrII, ..)	A5 F5	+0.45 +0.45	7200 6700
G	Yellow	CaII (very strong); many metals are stronger than H; CH, CN band	G0 G5	+0.57 +0.70	6000 5500
K	Orange	H weak; neutral metals predominate	K0	+0.84	5100
M	Red	Neutral metals strong; TiO bands	M0	+1.39	3750

Figure 10: Summary of main spectral features.

We have then, that:
$$\text{Spectral Type} \; = \; f(\text{temperature})$$

3.2 Luminosity Types

We have already seen that Sirius (for example) is a binary with two stars of approximately the same colour (= temperature) but differing in luminosity by 10 magnitudes – a factor of 10 000. So, clearly, at least a two–dimensional classification is needed.

As we saw with the Sirius system, the difference must be a radius difference. If we assume that such stars are not *vastly* difference in mass, then the radius difference means that the more compact star must have a greater surface gravity. This has at least two effects that we can detect fairly easily (e.g. just by looking at the spectra):

- The increased gravity means a more compacted, denser atmosphere. This means that the absorbing atoms are subject to more electric fields from nearby charged particles and the quantum levels are broadened, so that a wider wavelength range can be absorbed. This is **pressure broadening** or **Stark effect** and is particularly strong for Hydrogen and Helium (Linear Stark effect).

- For elements which are partially ionised, the rate of ionisation is a function of gas temperature, whereas the rate of recombination is a function of gas density. Thus, where an element is about half ionised, the gas density can have a significant effect on the overall degree of ionisation. Comparison of the strengths of lines from ionised and neutral (or doubly-ionised and singly-ionised, etc) atoms of the same element can be luminosity criteria.
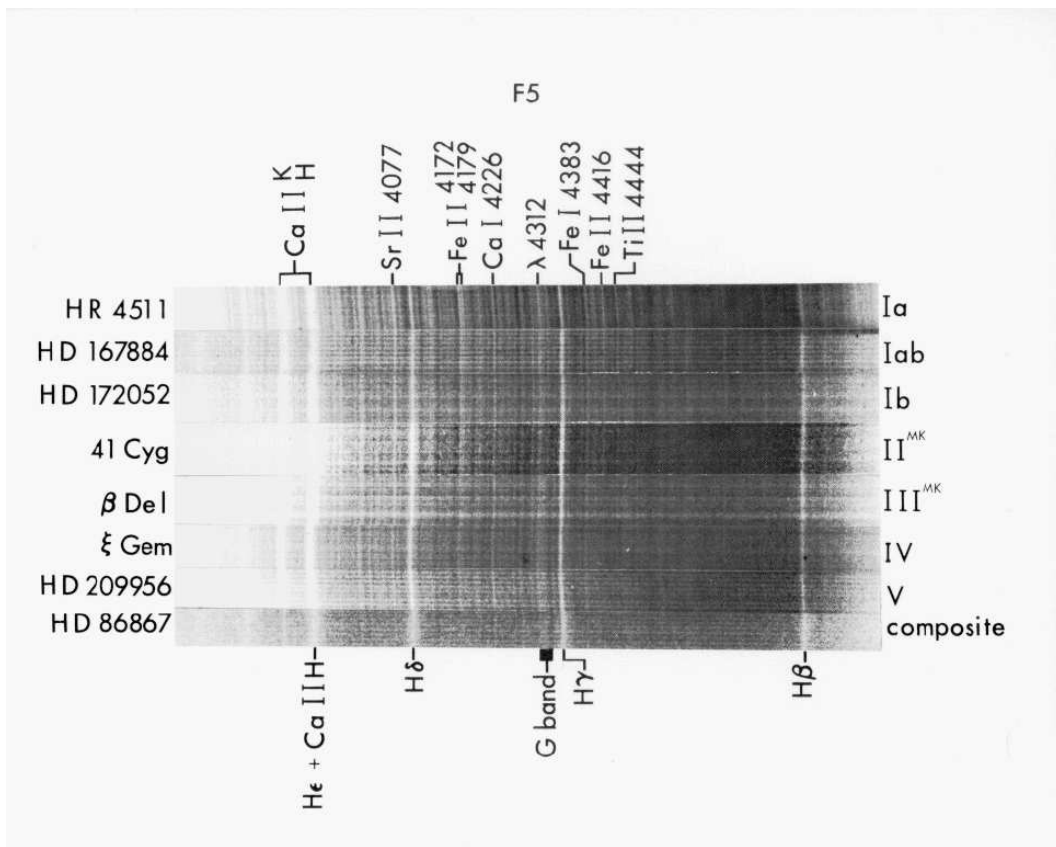

Figure 11: Objective-prism type spectra: luminosity effects at F5. Useful luminosity criteria are the lines SrII (4077Å) and FeII/TiII (4172-78Å), for example.
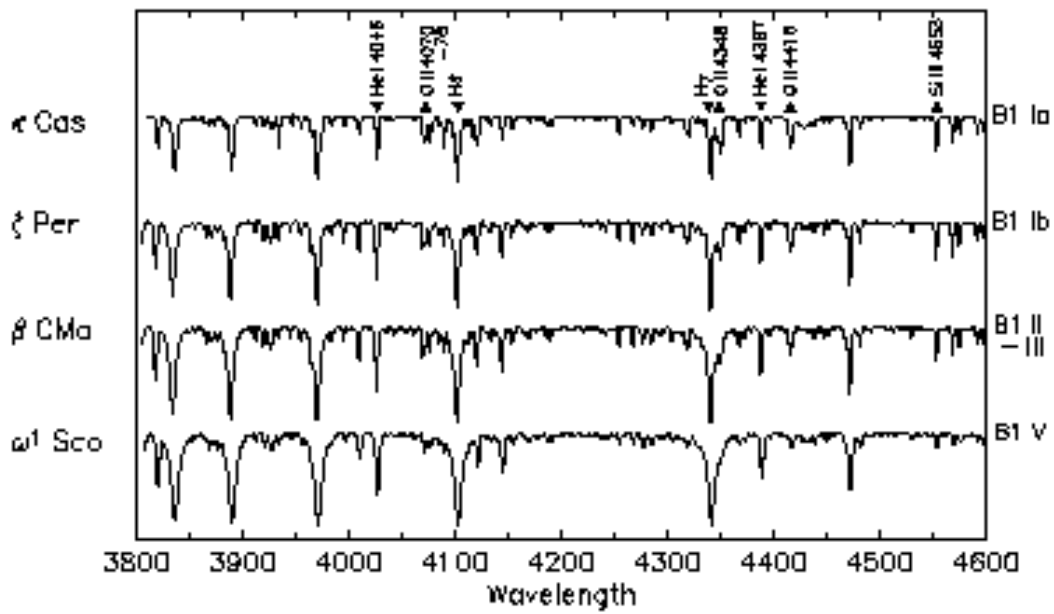
Figure 12: Digital spectra: luminosity effects at B1. Useful criteria are the OII lines 4070, 4348 and 4416Å, and the SiIII line at 4553Å.

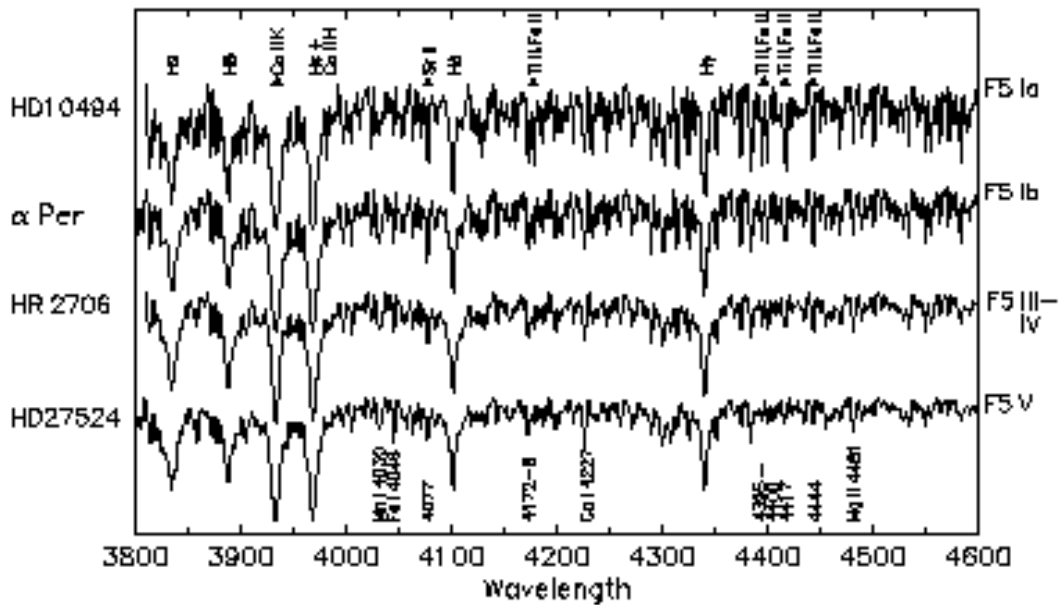Figure 13: Digital spectra: luminosity effects at F5.

We have then, that:
$$\text{Spectral Type} \ = \ f(\text{temperature})$$
$$\text{Luminosity Type} \ = \ f(\text{surface gravity})$$

These effects are not totally independent however, but nearly so.

The most commonly used classification system – the MK (Morgan–Keenan) system – uses the spectral types described in the previous section and luminosity classes:

- **I** – supergiants – now subdivided into Ia-0, Ia, Iab, Ib.

- **II** – bright giants

- **III** – giants - also sometines subdivided.

- **IV** –subgiants

- **V** – dwarfs or **main-sequence** stars

- **VI** – subdwarfs

- **VII** – white dwarfs (rarely used nowadays)

The MK types can be calibrated against temperature and luminosity (absolute magnitude) – using nearby stars, stars with trigonometric parallaxes and so on – giving us an indirect way of determining distances for more distant stars – provided we can classify them on the same system. This use of spectral/luminosity type to get absolute magnitude and hence distance is sometimes referred to as **spectroscopic parallax**.
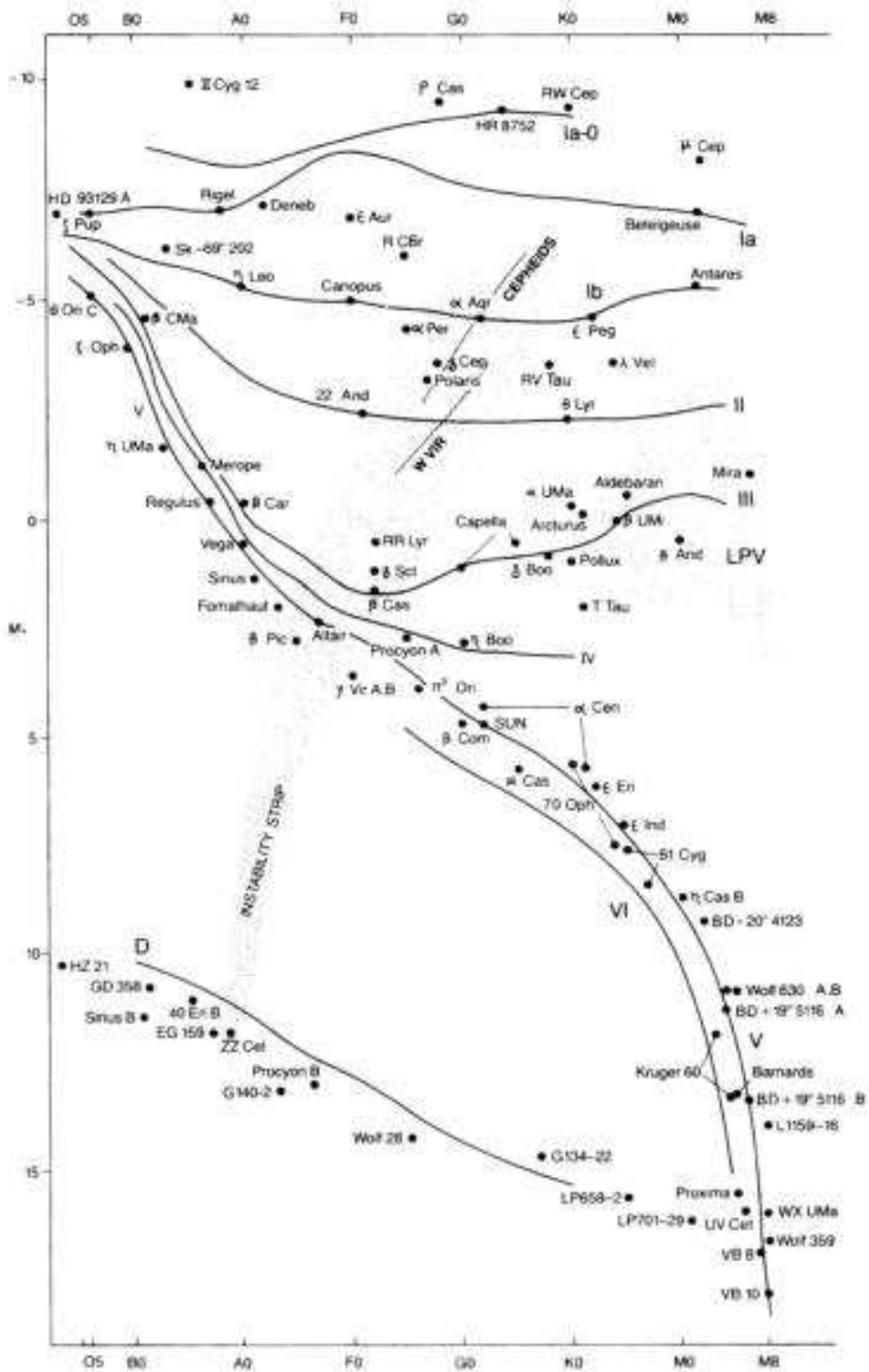
Figure 14: HR diagram with sample stars and luminosity calibrations.
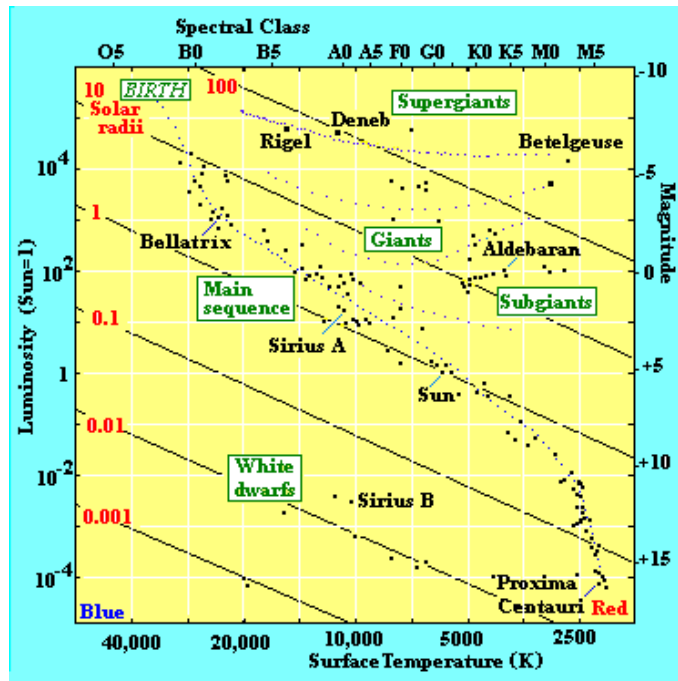
Figure 15: HR diagram with sample stars and indicating stellar radii.

It is worth remembering that many HR diagrams – such as the sketch above – are very biassed. If we look at the apparently brightest stars in the sky, we are sampling a *vastly* bigger volume for the supergiants than for the dwarfs. The sketch below shows a **volume limited sample** – note the complete absence of supergiants or even giants which are relatively rather rare.
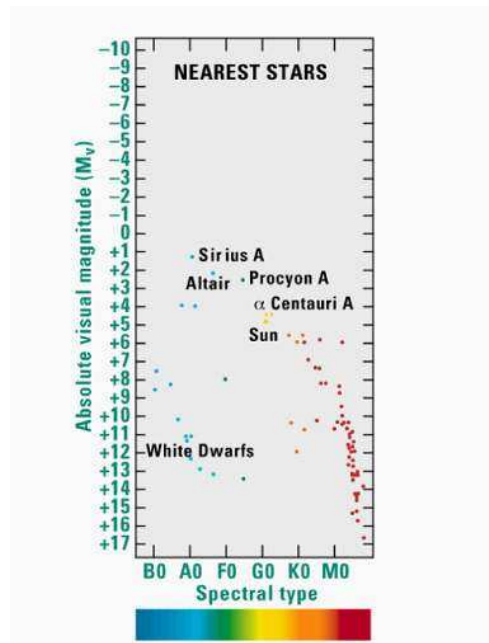

Figure 16: HR diagram for the nearest stars.

3.3 Chemical abundance

Although $\sim 99\%$ of classified stars fall into the **OBAFGKM** sequence (and are mostly dwarfs - or class V), there are many peculiarities.

- A number of suffices exist to further qualify spectra, for example:
  - **e** - to indicate the presence of emission (B2Ve)
  - **f** - to indicate NII and HeII emission in O stars (O5f)
  - **m** - enhanced metal features (Am)
  - **n, nn** - to indicate unusually "nebulous" or fuzzy lines (often due to rapid rotation) (B2nn)
  - **p** - to indicate abundance peculiarity (Ap)
  - **k** - to indicate the presence of CaII - often interstellar (B2Vek)

  and you can have hours (well, minutes) of fun making up your own odd types, such as B0p, B0nk, and so on.

- The so-called subdwarfs (class VI) can be divided into two unrelated groups
  - **F and G type** - stars with real metal-deficiency; these stars formed early in Galactic history (like the globular clusters) when metal abundances were lower.
  - **O and B type** - which are evolved (post red giant) stars.

- And there are many different classes of peculiar abundance stars, including:
  - **Wolf-Rayet stars** - very hot stars with extended atmospheres rich in Carbon (WC stars) or Nitrogen (WN)
  - **P Cyg stars** - hot stars in which the lines have an absorption component bluewards of an emission component, indicative of an expanding shell.
  - **Hydrogen-deficient stars** - a small number of hot ($\sim$ B type) stars with n_H from 1% down to undetectable, where most stars have $n_H \sim 95\%$
  - **Ap stars** - peculiar A stars with overabundances of Mn, Eu, Cr, Sr, etc., probably related to strong magnetic fields. This class includes the rapidly oscillating Ap (roAp) stars discovered at SAAO.
  - **Am and Fm stars** - "metallic" line A and F stars, which show enhanced metals relative to CaII.
  - **Carbon stars** - cool stars with overabundances of Carbon (C, R, and N stars).
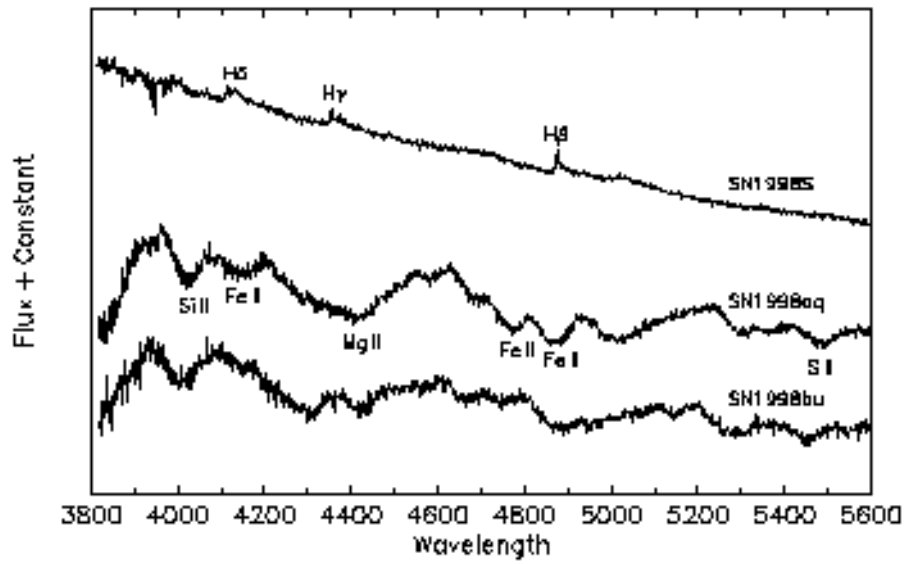 - **S type stars** - very cool stars with rare earth overabundances (ZrO, YO, LaO, etc)

Figure 17: Really "peculiar" spectra !

3.4 L and T dwarfs

The process of classification is simple but very useful. If we classify stars using a set of reference standards, we can characterise the stars in a number of ways (for example, by relating the MK type to luminosity/absolute magnitude and hence distance). If, at some stage, the calibration is improved, or we calibrate in terms of new parameters, this doesn't affect the original classification which is still valid.

Sometimes, though, we encounter "new" types of object and have to think about application – though the process remains similar. The L and T dwarf stars are an example. These are stars which are cooler than the latest dwarfs – they are the so-called "brown dwarfs" (of which it has been said "they are neither brown, nor dwarfs").

Examples of their spectra are shown below. Clearly, classification in the visible region is not such a good idea, because there's almost no flux there. A set of suitable classification features in the infrared will need to be developed ...
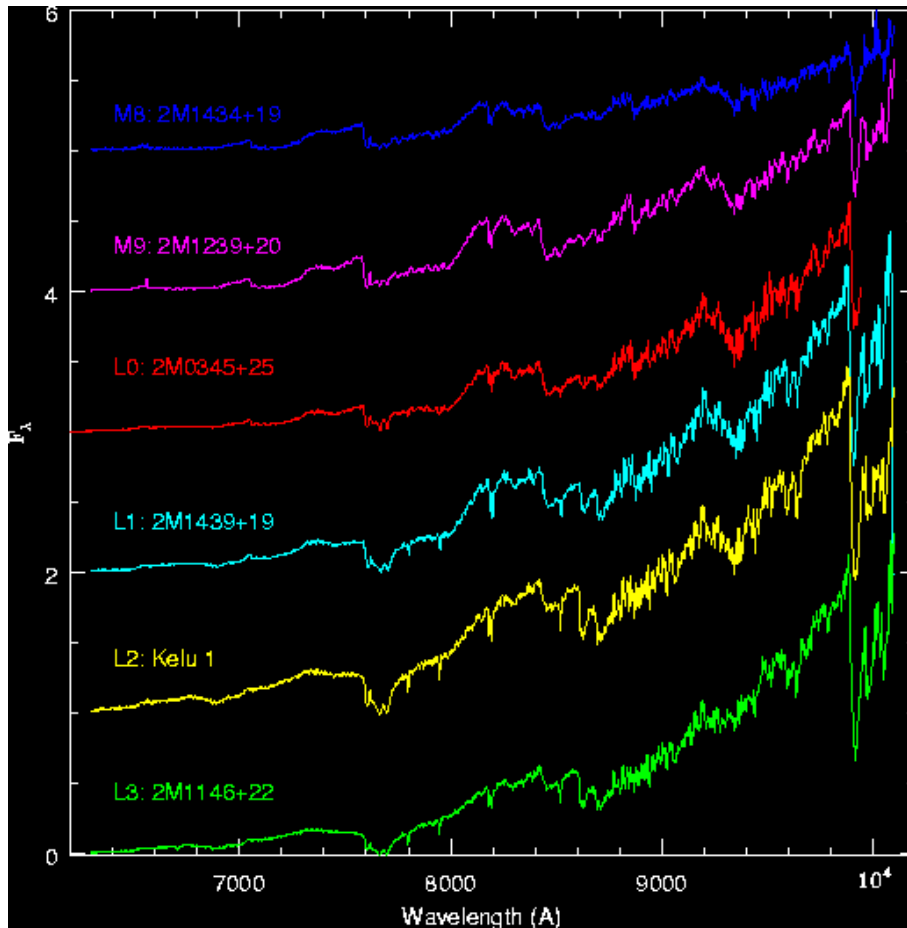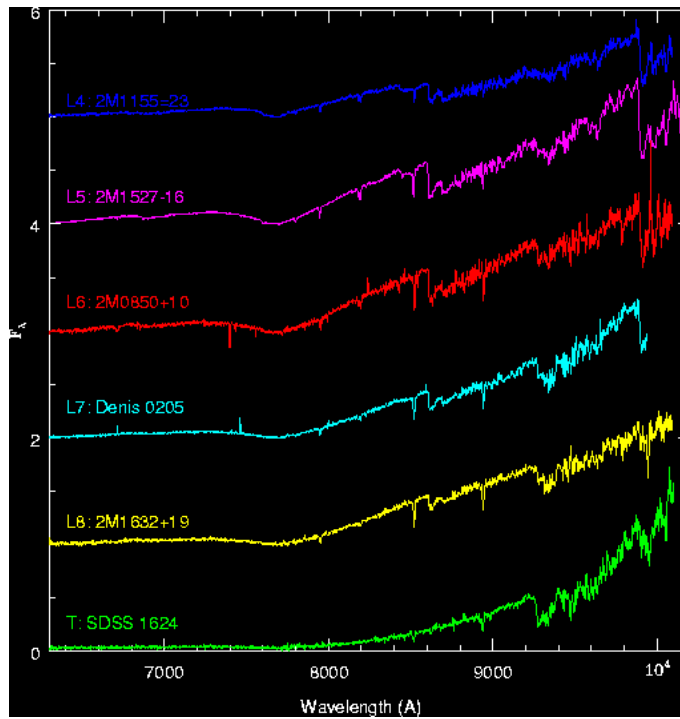

Figure 18: M and L dwarf stars in the near infra-red.

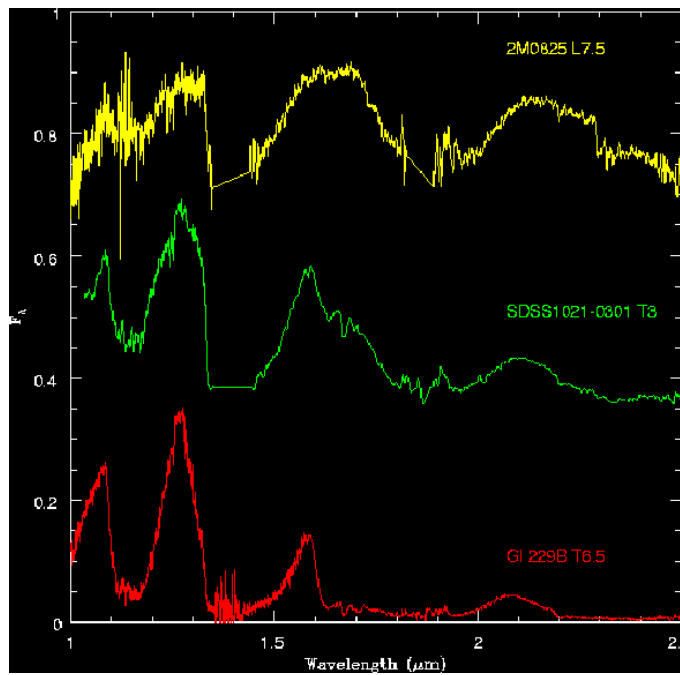Figure 19: L and T dwarf stars in the near infra-red.

Figure 20: L and T dwarf stars in the infra-red (1 – 2.5 μm.