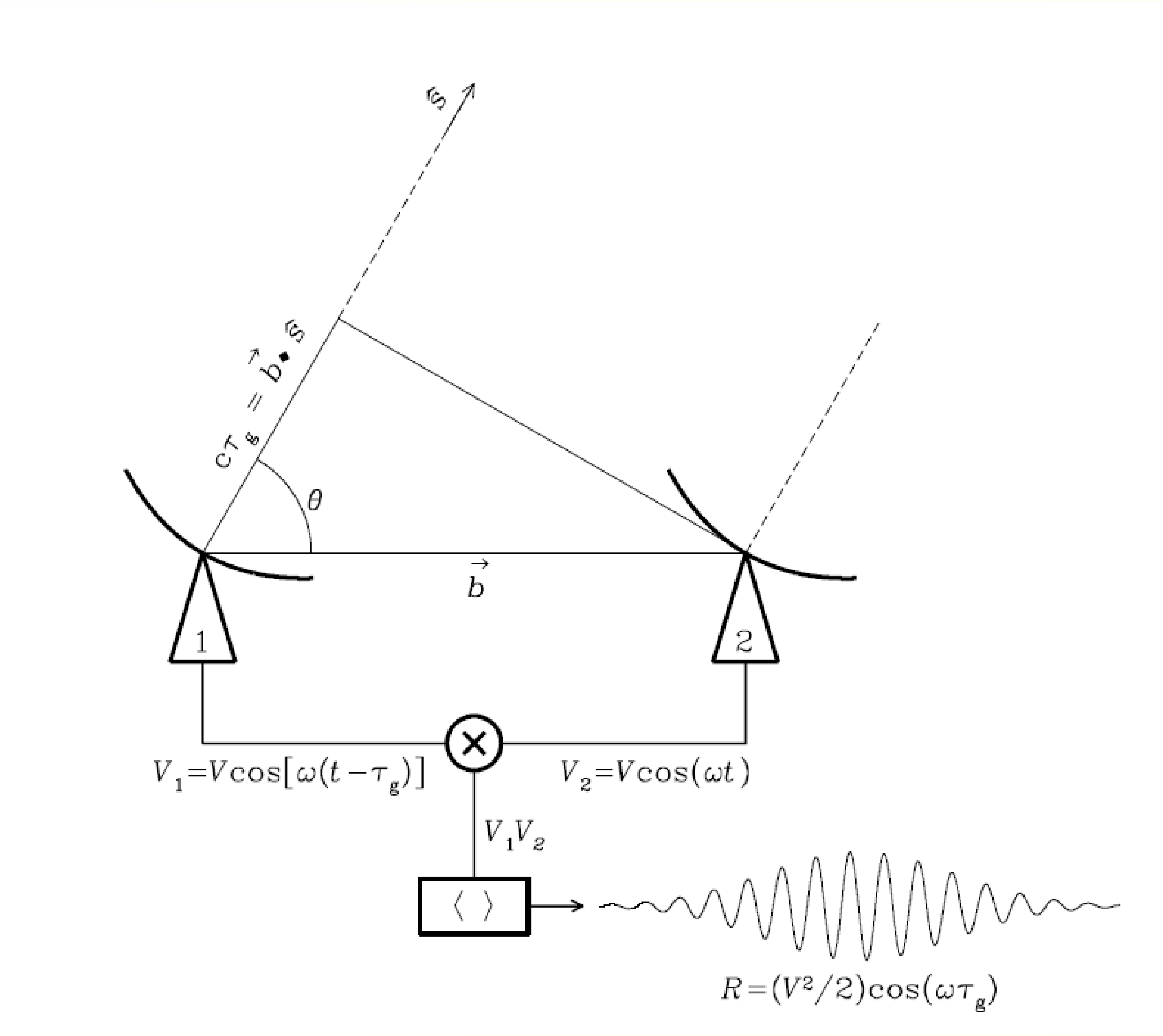
Interferometry

- Why bother!
  - Confusion at low spatial resolution and high sensitivity
  - We want high high resolution anyway and cannot build dishes >110m (want 10km or more)
  - Less vulnerable to some fluctuations (and some interference)

Technical issues

- Digital processing is cheap and easy
- Must preserve phase information (phase coherence)
  - Timing of signals is **CRITICAL**
- Modern interferometer arrays use correlator (multiply and average)

2 element interferometer



Interferometer analysis

- Quasi-monochromatic $\omega=2\pi\upsilon$
- Voltages V_1 and V_2 from the receivers, and one is delayed with respect to the other by geometry (pointing to a source)
- Averaging removes the high frequency component (2ω)
- As the source moves the amount of delay varies, giving a '**fringe**' with amplitude proportional to source flux density

more analysis...

delay $\tau = \dfrac{\vec{b}.\hat{s}}{c}$

$$V_1 = V\cos(\omega(t-\tau))$$
$$V_2 = V\cos(\omega t)$$

$$V_1 V_2 = V^2\cos(\omega(t-\tau))\cos(\omega t) = \frac{V^2}{2}[\cos(2\omega t)+\cos(\omega\tau)]$$

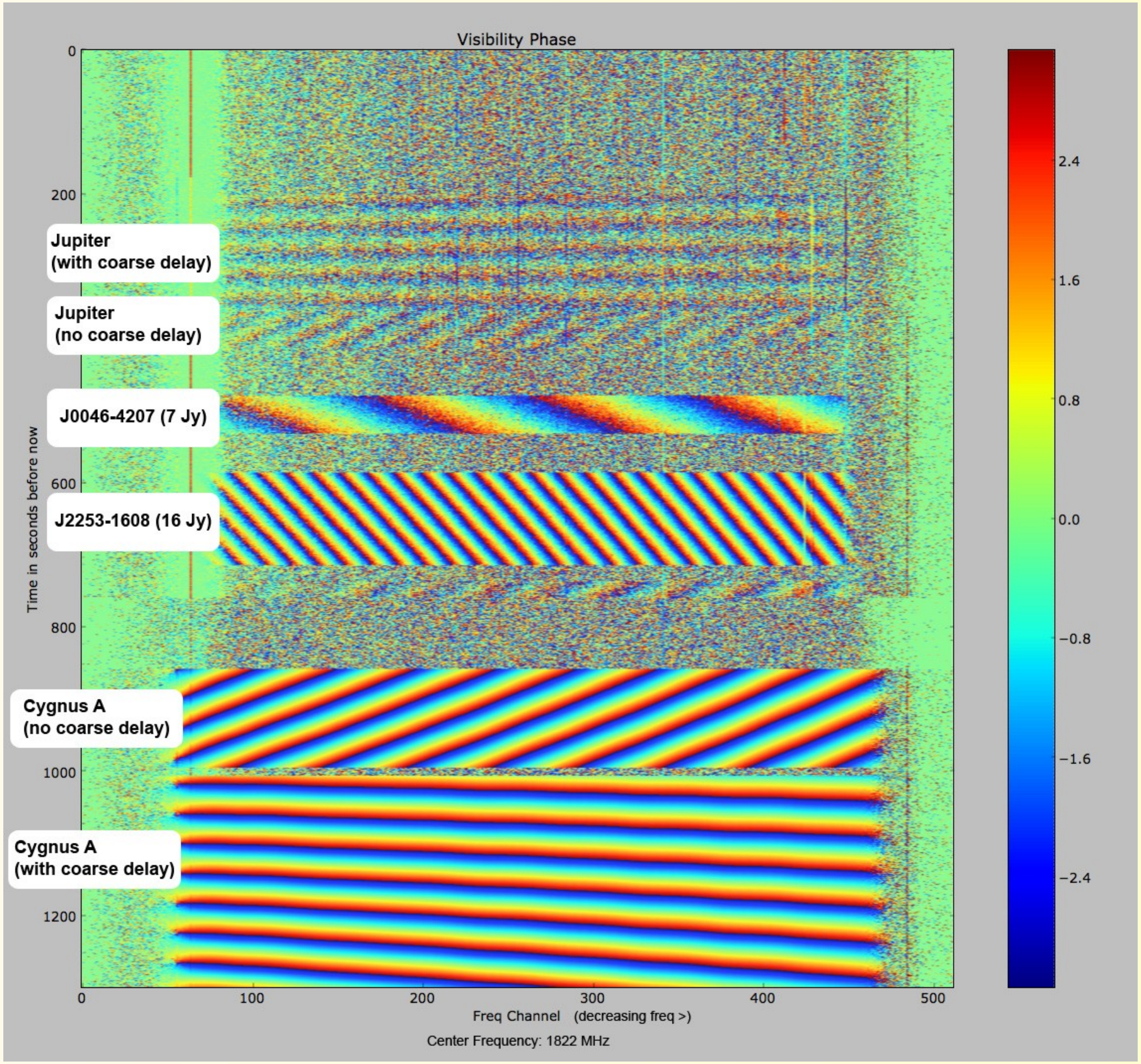
BUT ..... $\cos(2\omega t)$ averages to zero very fast

so we have a fringe phase $\phi = \omega\tau_g = \dfrac{\omega}{c} b\cos\theta$

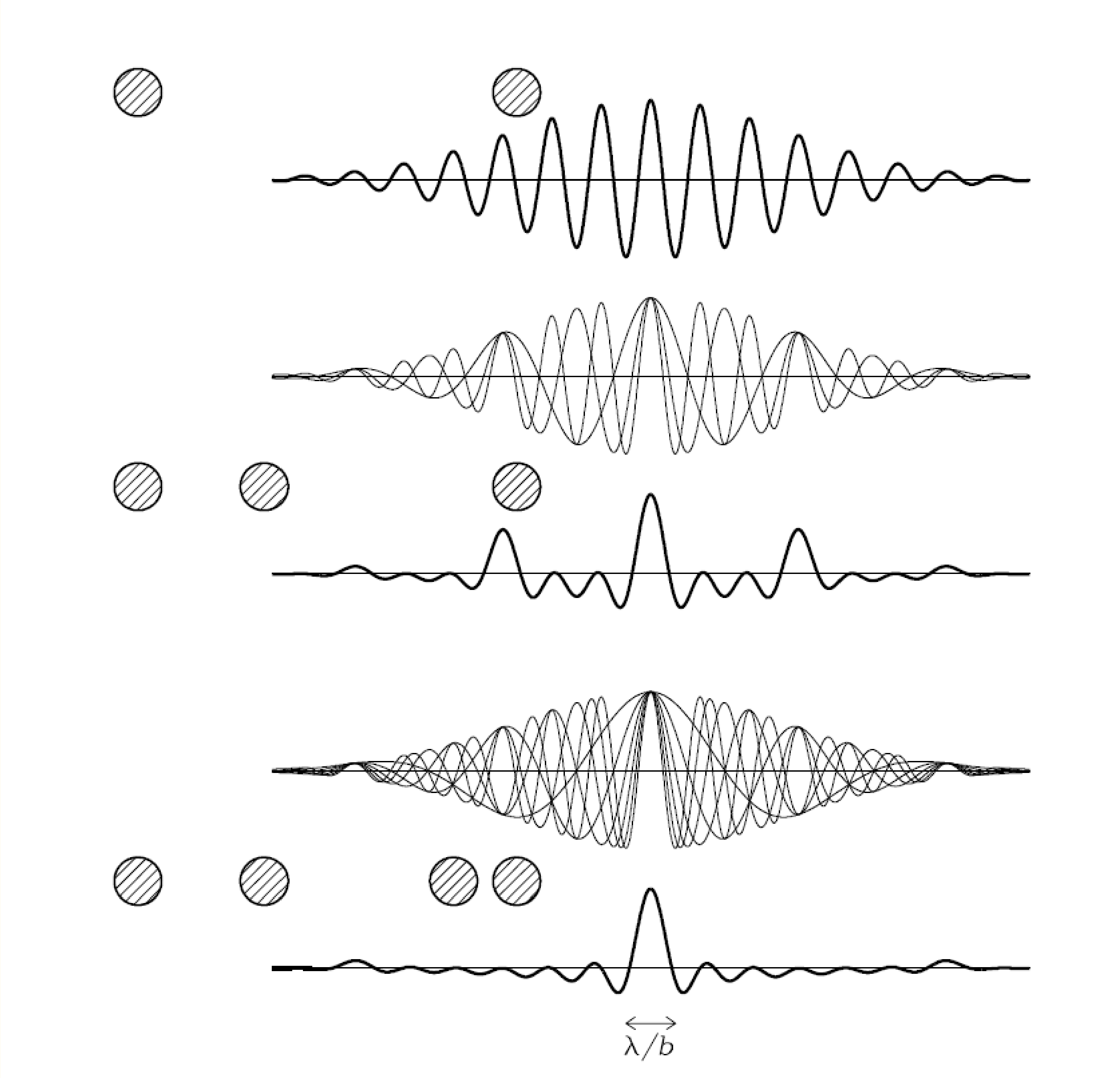
and the dependency on the angle to the source

$$\frac{d\phi}{d\theta} = \frac{\omega}{c} b\sin\theta = 2\pi\frac{(b\sin\theta)}{\lambda}$$

Fringes


Visibility Phase
Jupiter
(with coarse delay)
Jupiter
(no coarse delay)
J0046-4207 (7 Jy)
J2253-1608 (16 Jy)
Cygnus A
(no coarse delay)
Cygnus A
(with coarse delay)
Time in seconds before now
0
200
600
800
1000
1200
0
100
200
300
400
500
Freq Channel (decreasing freq >)
Center Frequency: 1822 MHz
2.4
1.6
0.8
0.0
−0.8
−1.6
−2.4

More baselines give more Fourier components



Extended sources and interferometers

We can treat a spatially incoherent extended source with sky brightness distribution $I_\nu(\hat{s})$ near frequency $\nu=\omega/2\pi$ the sum of independent point sources.

The response of the two-element interferometer with **cosine** correlator output to such a source is:

$$R_c = \int I_\nu(\hat{s}) \cos(2\pi\nu \vec{b}\cdot\hat{s}/c)\, d\Omega = \int I_\nu(\hat{s}) \cos(2\pi \vec{b}\cdot\hat{s}/\lambda)\, d\Omega$$

But this only corresponds to the symmetric part

Similarly

For the antisymmetric part (the sine correlator)

$$R_s = \int I_\nu(\hat{s}) \sin(2\pi \vec{b} \cdot \hat{s} / \lambda) d\Omega$$

We can combine cosine and sine part as a complex function

$$e^{i\phi} = \cos(\phi) + i \sin(\phi)$$

so if we define a complex visibility

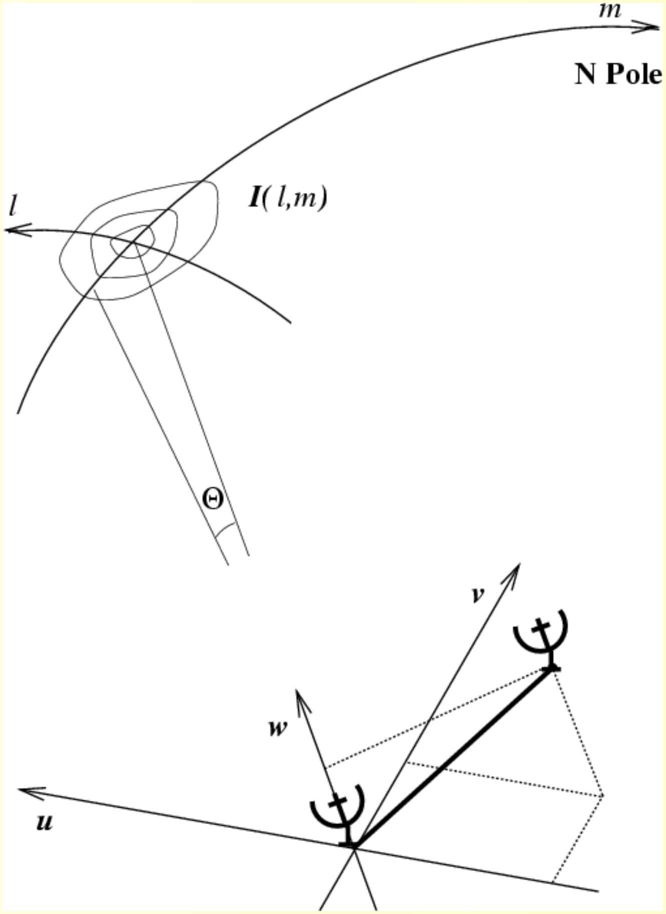
$$V \equiv R_c - i R_s \equiv A e^{-i\phi}$$

$$V_\nu = \int I_\nu(\hat{s}) \exp(-i 2\pi \vec{b} \cdot \hat{s} / \lambda) d\Omega$$

van Cittert- Zernike theorem

- For a small field of view, the complex visibility V(u,v) is the 2D fourier transform of the brightness of the sky
- u,v measured E-W and N-S
- in wavelengths
- on a tangent to the

  reference positions



Transforms

- Visibility is FT of sky

$$V(u,v)=\iint I(x,y)e^{2\pi i(ux+vy)}dxdy$$

- The 'w' coordinate (towards the source) is the geometrical delay
- BUT.. we cannot integrate to infinity

More realistically

- We need a bandwidth to get any sensitivity, so if we use bandwidth $\Delta\nu$ about a central frequency ν_c

$$V = \int \left[(\Delta \nu)^{-1} \int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta N/2} I_\nu(\hat{s}) \exp(-i 2\pi \vec{b} \cdot \hat{s}/\lambda) \, d\nu \right] d\Omega$$

$$V = \int \left[(\Delta \nu)^{-1} \int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta\nu/2} I_\nu(\hat{s}) \exp(-i 2\pi \nu \tau_g) \, d\nu \right] d\Omega$$

But this is a Fourier transform in frequency space

$$V = \int I_\nu(\hat{s}) \, sinc(\Delta \nu \tau_g) \exp(-i 2\pi \nu_c \tau_g) \, d\Omega$$

Delay effects

- For a finite bandwidth we reduce the amplitude by a factor sinc($\Delta\nu\ \tau$), so we introduce a compensating delay to keep this term close to zero at the field centre
- But the earth moves, so the direction of the source changes and we need to keep updating the delay (*delay tracking*) with tolerance

$$\left|\tau_0 - \tau_g\right| \ll (\Delta\nu)^{-1}$$

Delay Field size

- We can only compensate completely for one direction (the field centre) but if our field $\Delta\theta$ is not too big the loss is small

$$\Delta\nu(b\sin\theta)\Delta\theta/c \ll 1$$

Which corresponds to

$$\Delta\theta\Delta\nu \ll \theta_s\nu$$

- To map a larger field we are forced to split our observing band into many frequency channels

Fringe rate field size

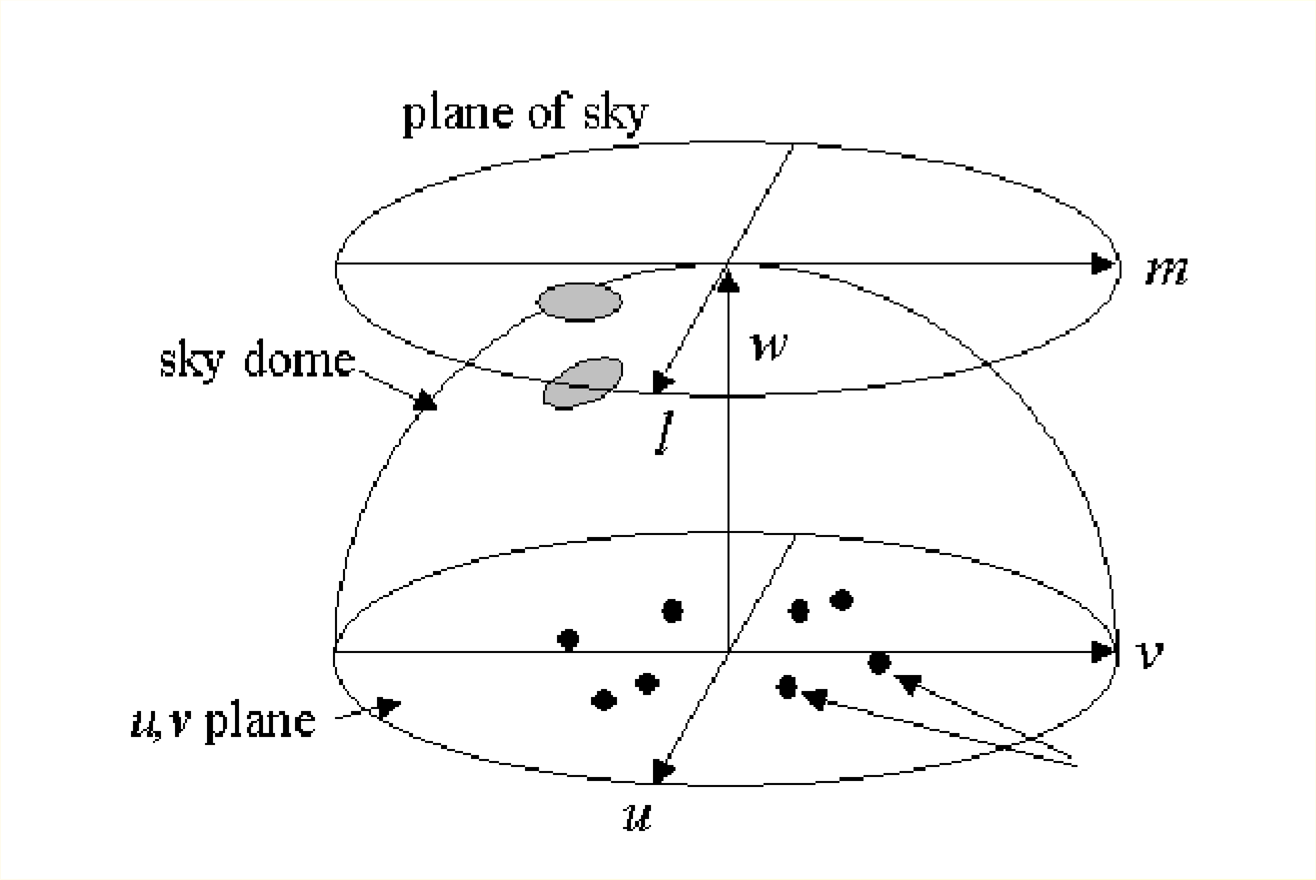
- We must also keep correlator integration periods short enough so that the source position does not change too much ($\Delta\theta$)
- Given the earth's period of rotation (23h56m04s) we are forced to have

$$\Delta\theta\,\Delta t \ll \frac{\theta_s P}{2\pi} \approx \theta_s \times 1.37 \times 10^4\ s$$

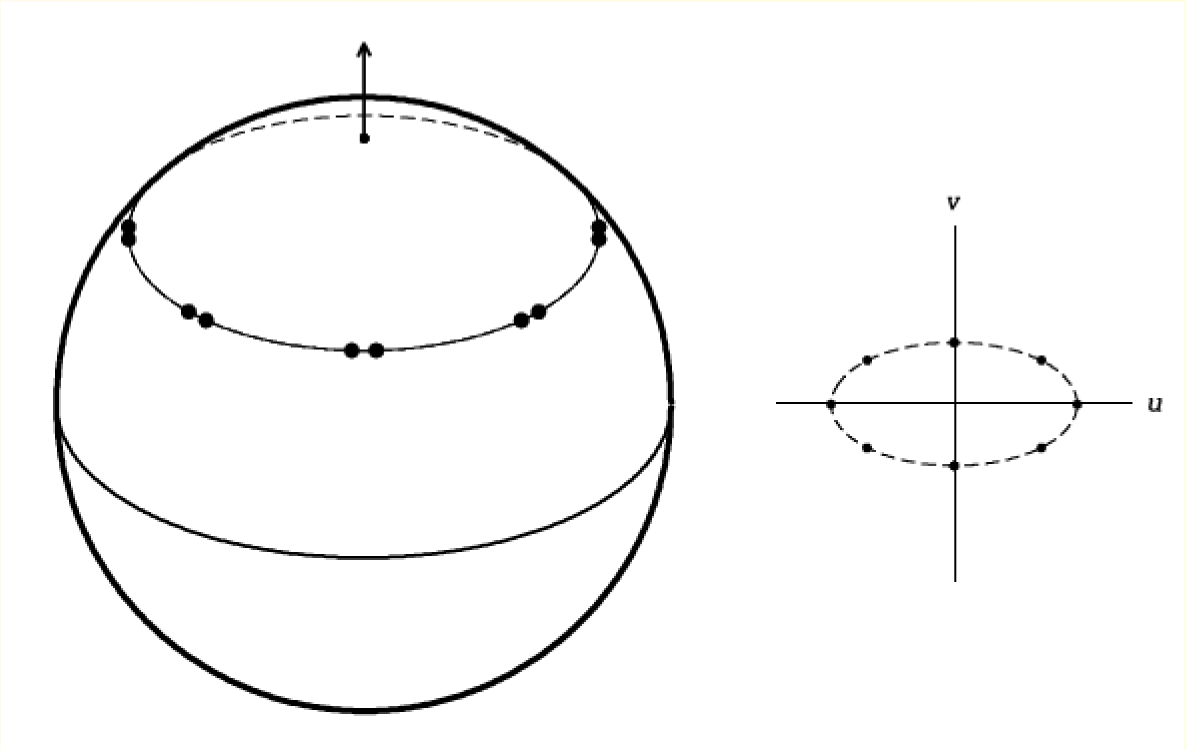
Earth rotation aperture synthesis

- We need a large number of baseline lengths and angles to cover the (u,v) plane to get a good image
  - We can build it this way
  - this needs lots of telescopes and is EXPENSIVE
- But the source is moving so we can use this fact to cover the (u,v) plane – provided that the field does not vary while we are observing
  - When might we be unable to use it???

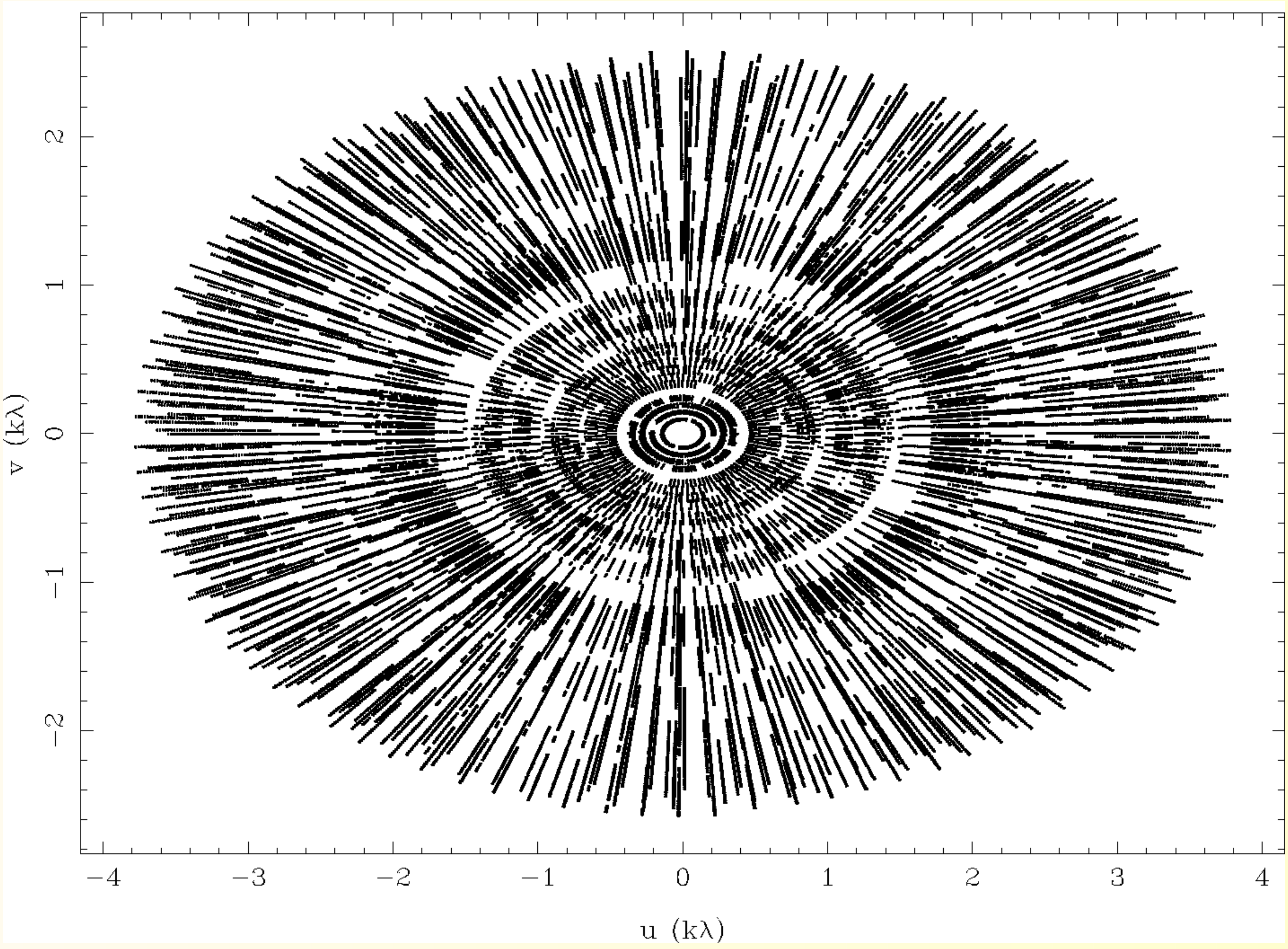
Aperture Synthesis



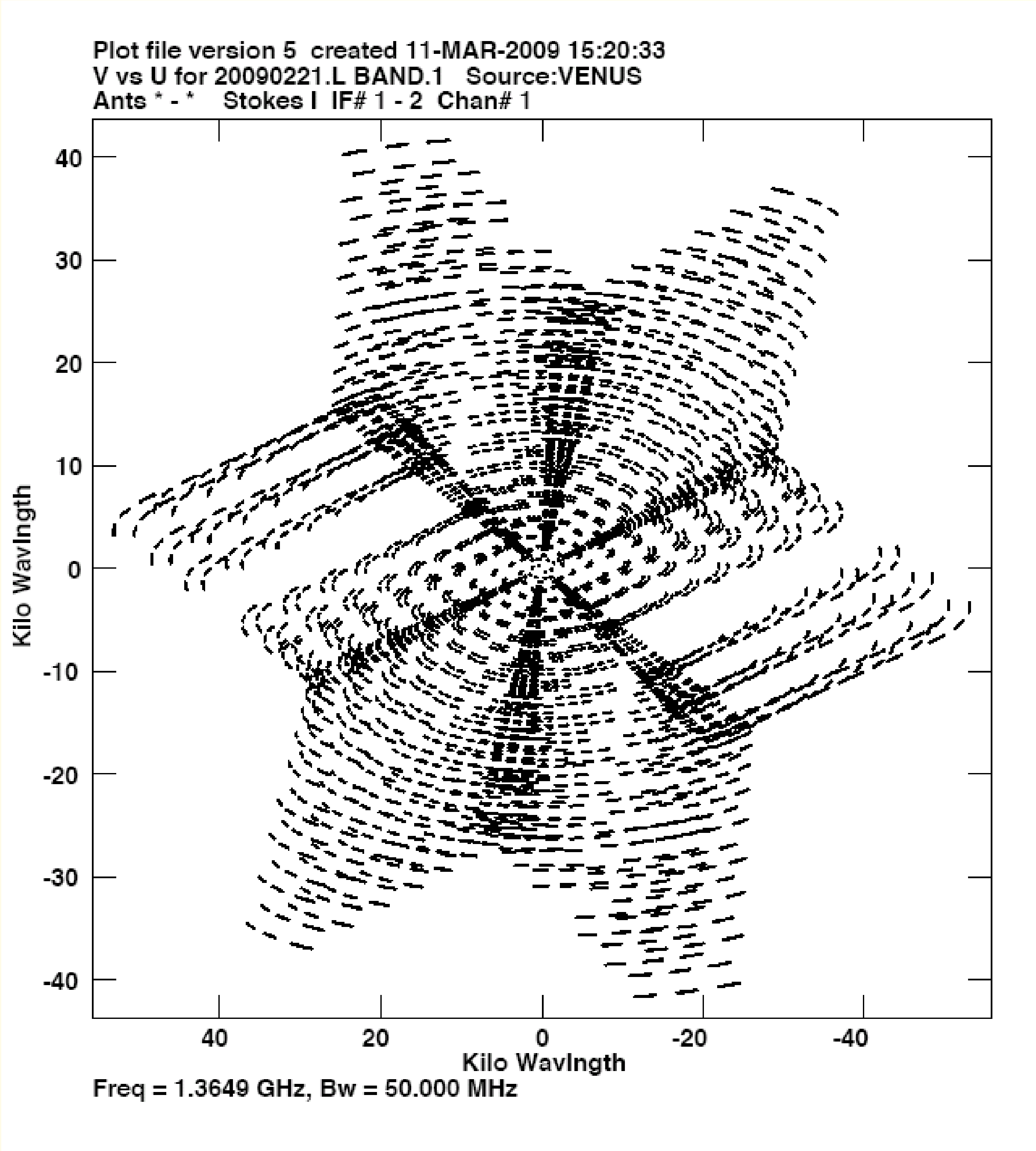
For an east-west baseline (ATNF,WSRT)



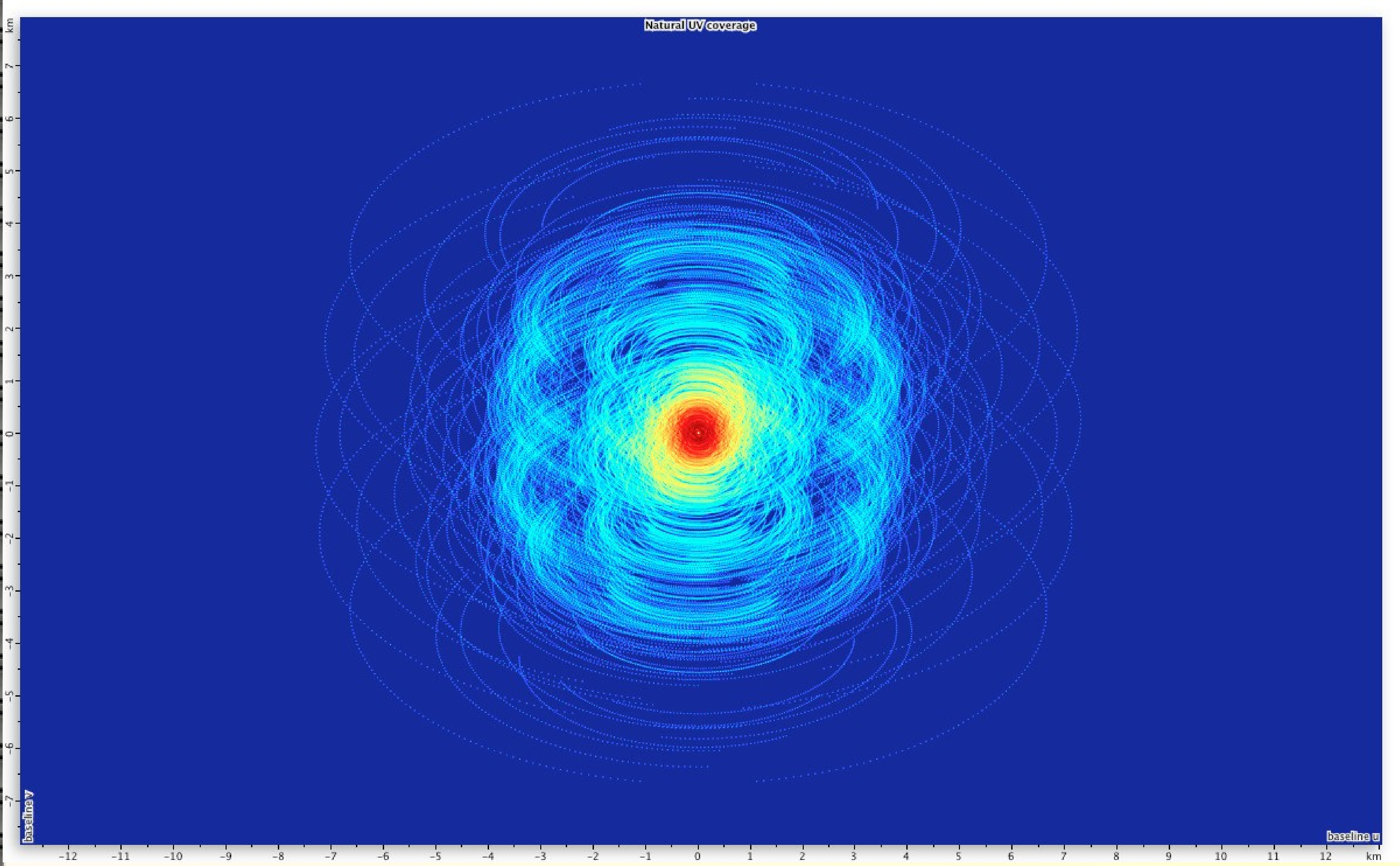
ATCA



VLA 'snapshot'



MeerKAT model 1



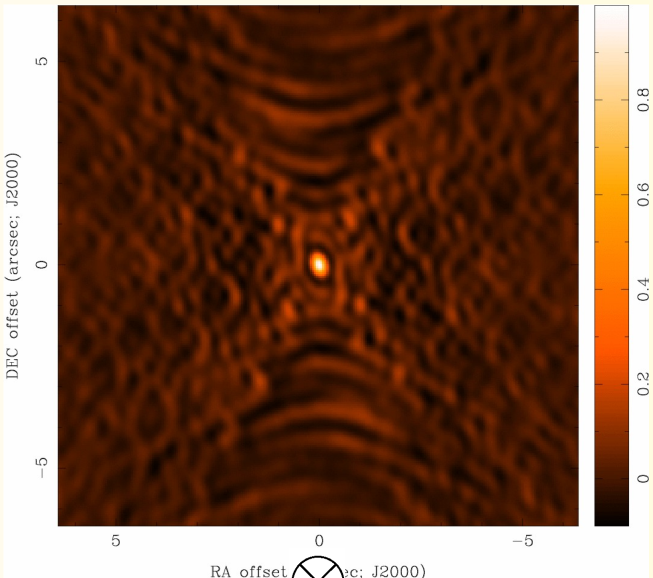
Dirty beam

- Our interferometer response to a point source is called the 'dirty beam'
- Any source on the sky we observe will be convolved with that
- So we want the 'dirt' to be as little as possible
- What would be ideal?

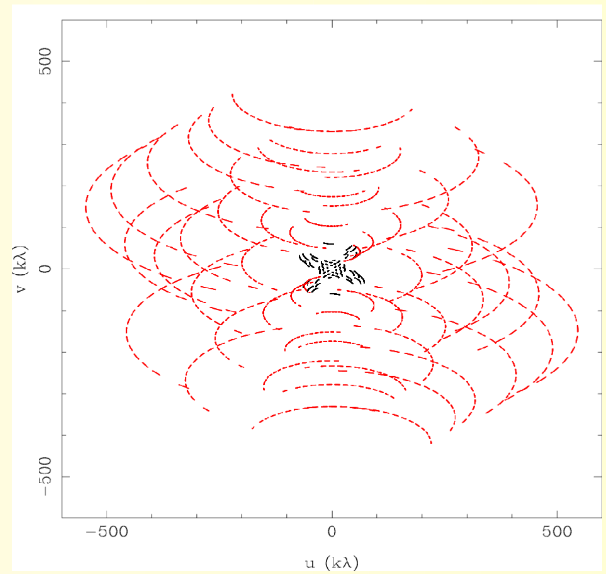
Examples of images and beams

Dirty Beam and Dirty Image

b(x,y)
(dirty beam)



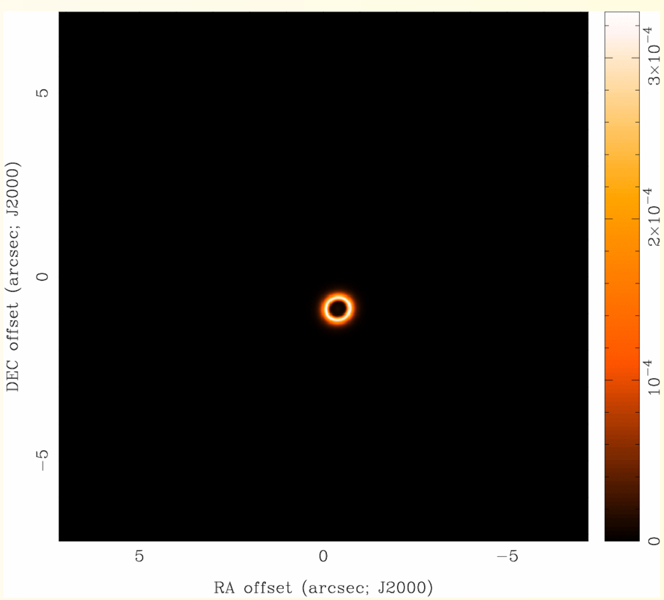
$\rightleftharpoons$



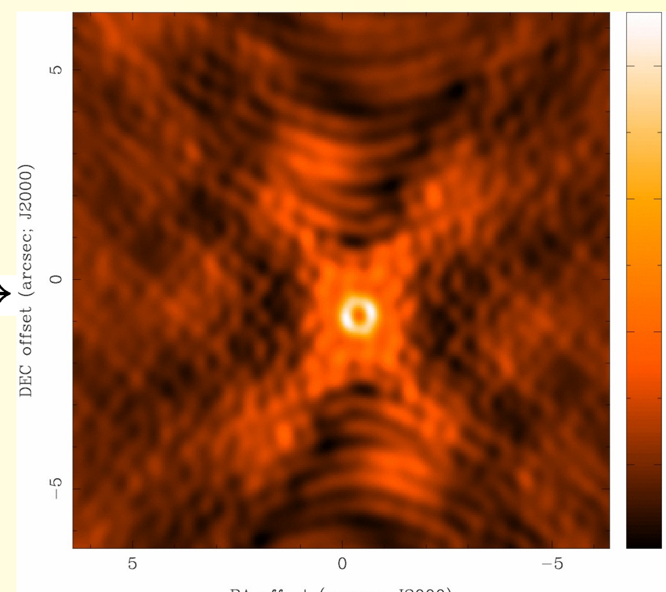
B(u,v)

$\otimes$

T(x,y)



$\rightarrow$

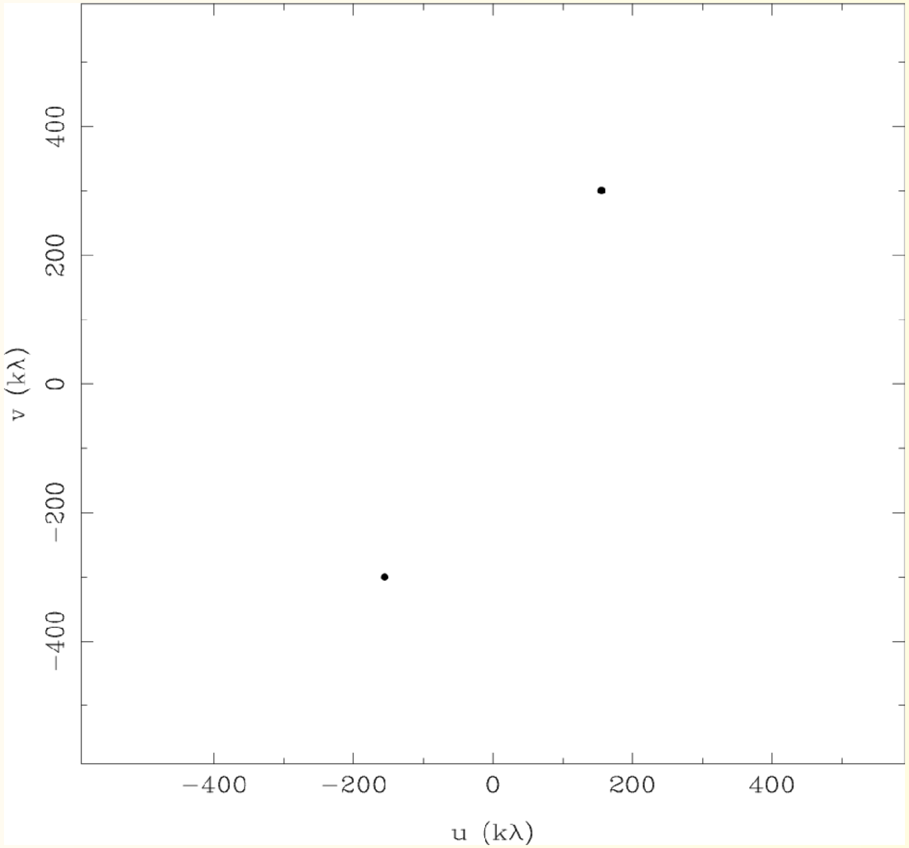


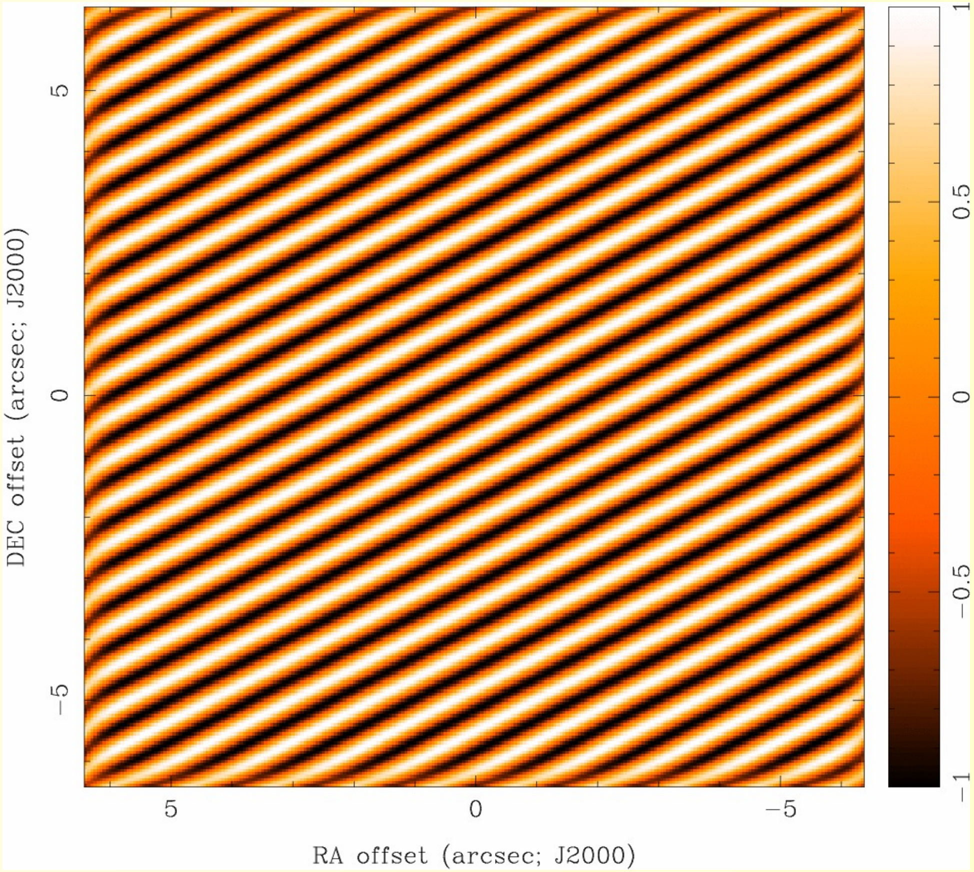
$T^D(x,y)$
(dirty image)

Dependency on numbers

Dirty Beam Shape and N Antennas

2 Antennas - 1 baseline -shown double

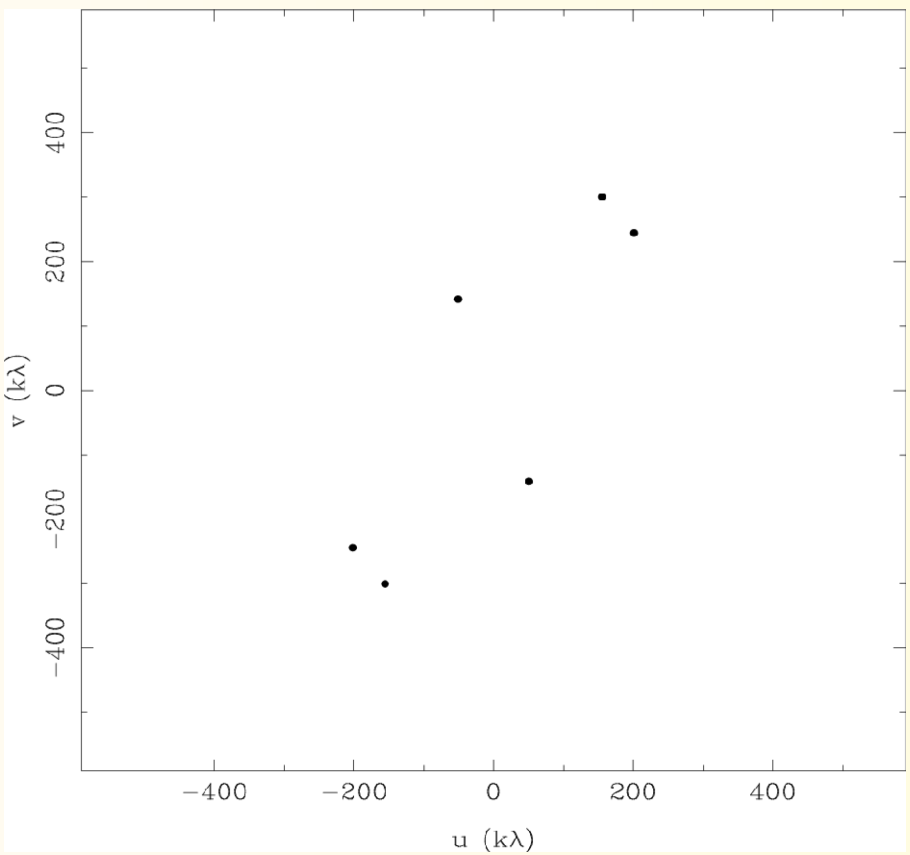


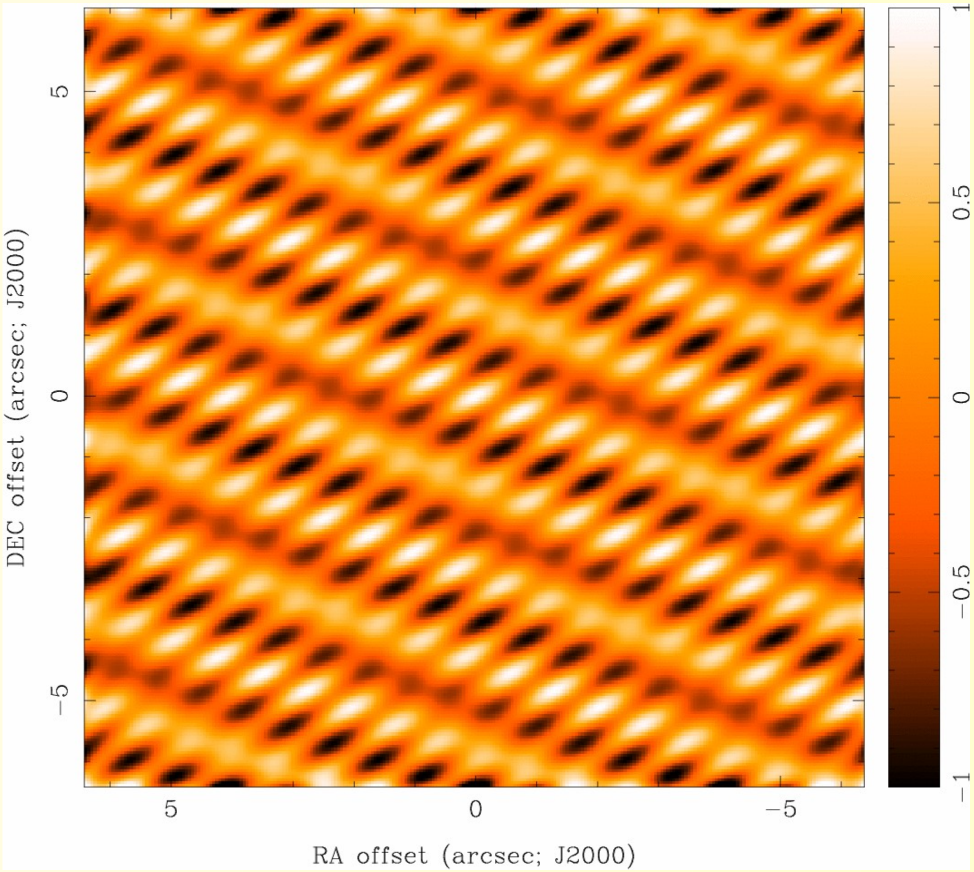


3 antennas , 3 baselines

Dirty Beam Shape and N Antennas

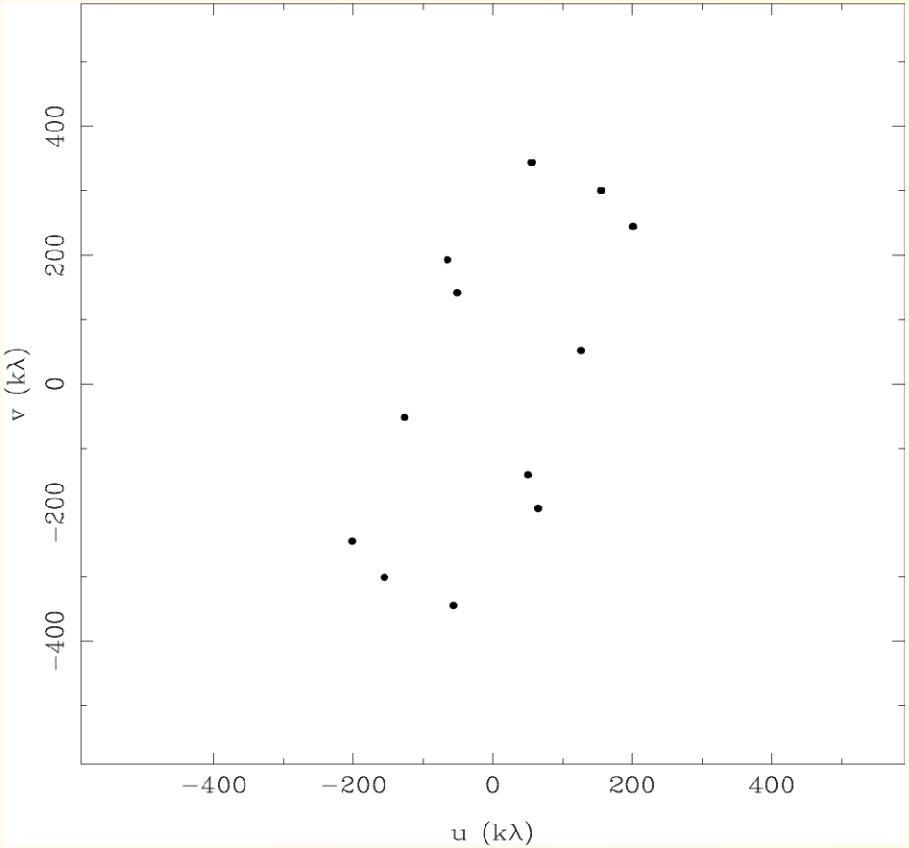
3 Antennas -shown double

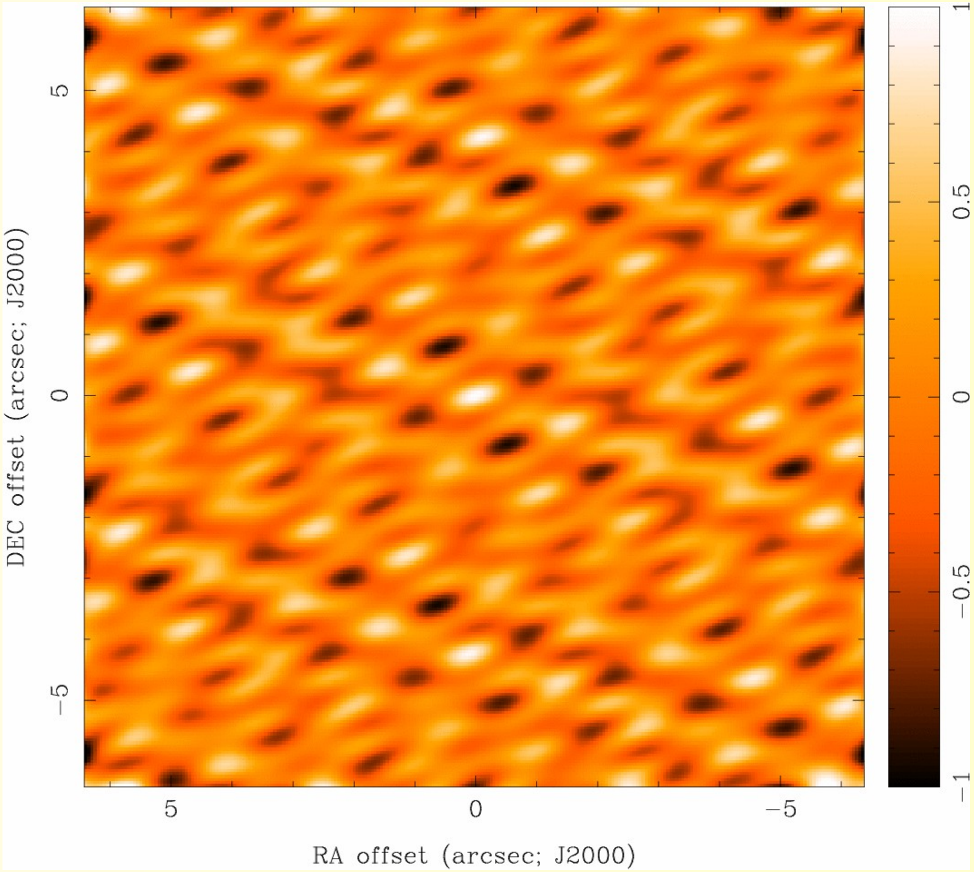




4 antennas, 6 baseliens
Dirty Beam Shape and N Antennas

4 Antennas

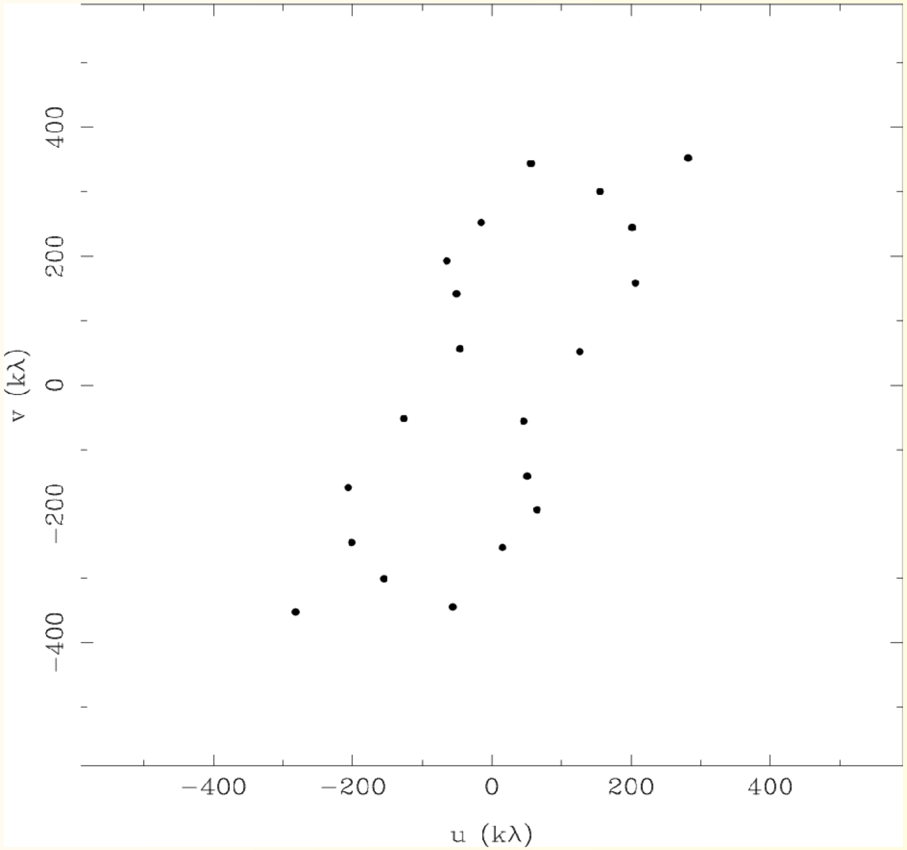


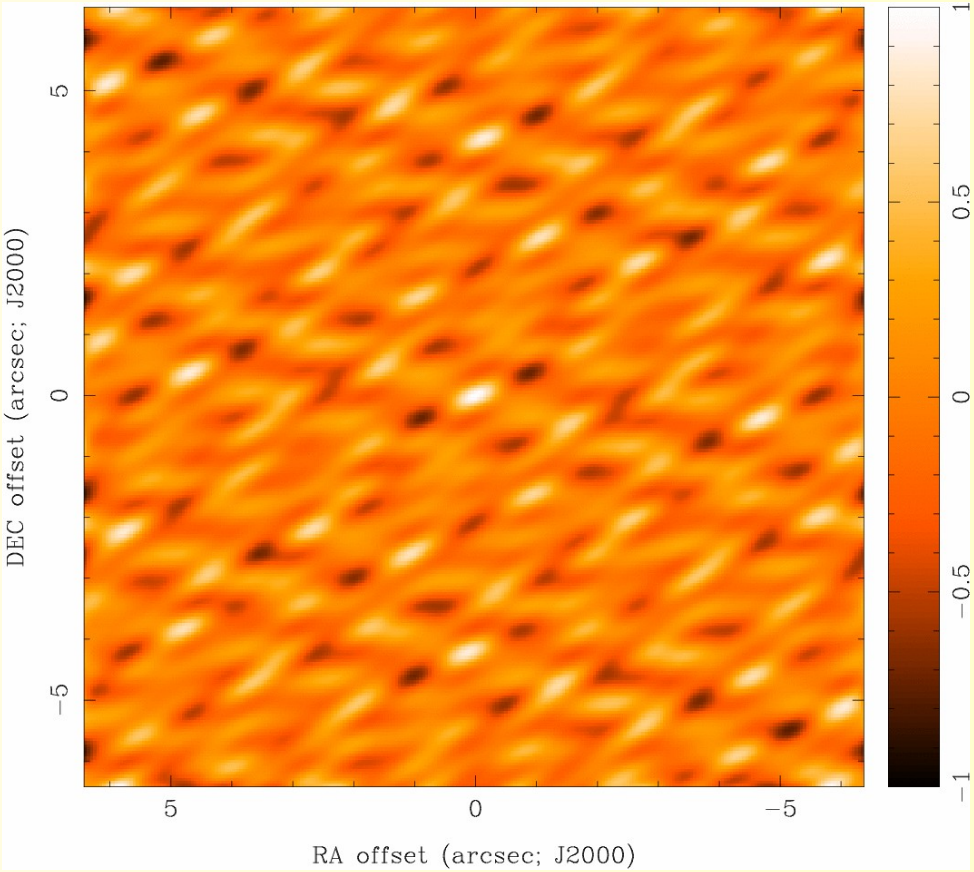


5 antennas, 10 baselines

Dirty Beam Shape and N Antennas

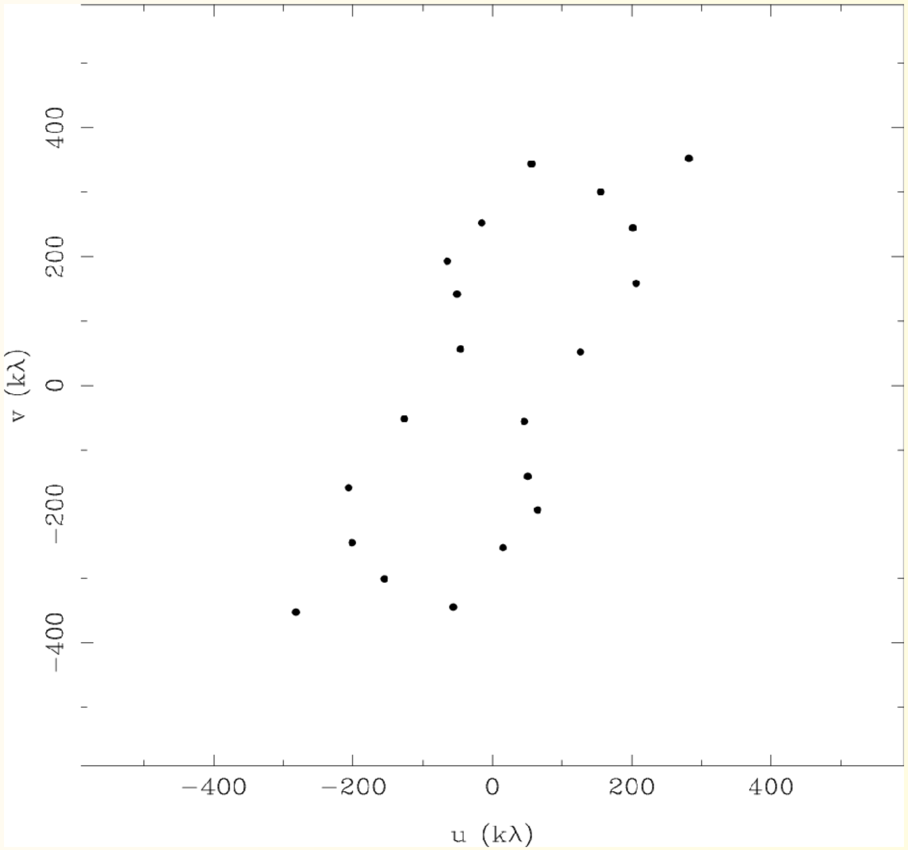
5 Antennas

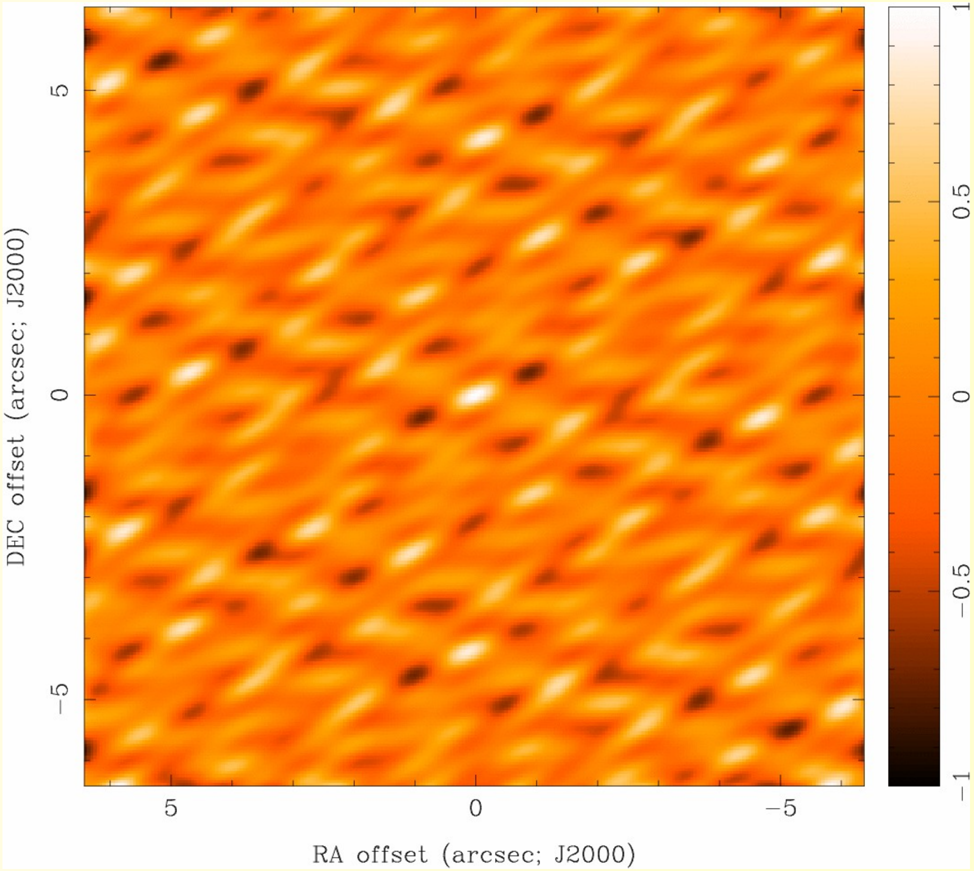




5
Dirty Beam Shape and N Antennas

5 Antennas

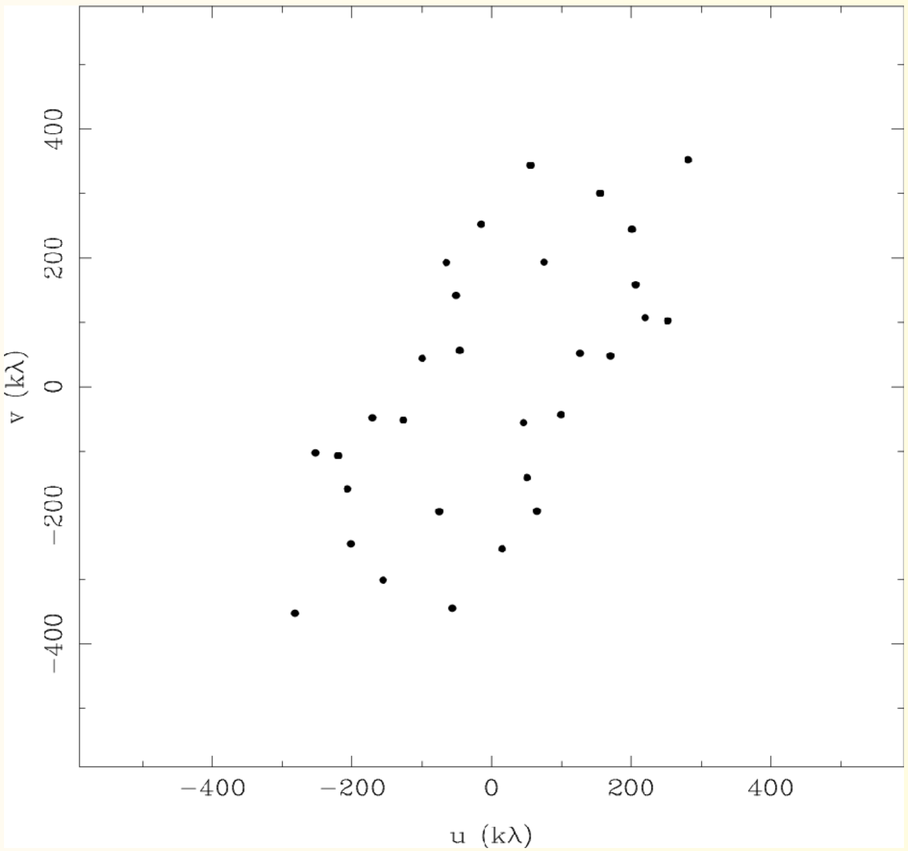


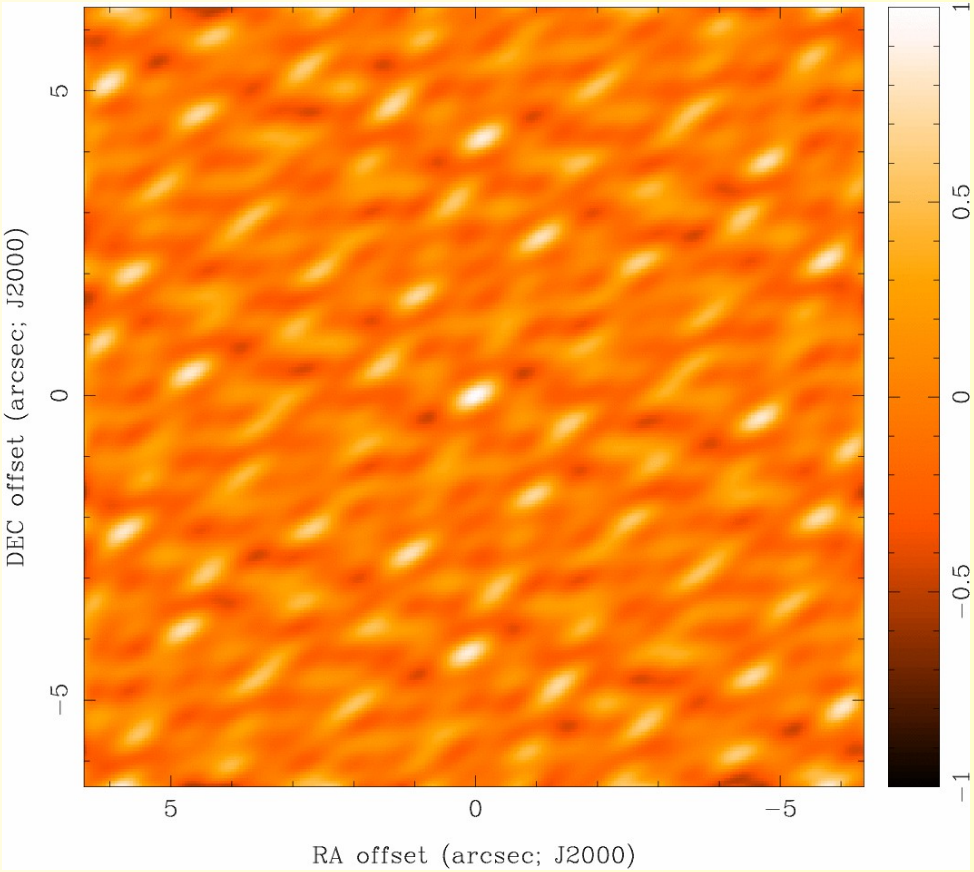


6 antennas, 15 baselines

Dirty Beam Shape and N Antennas

6 Antennas

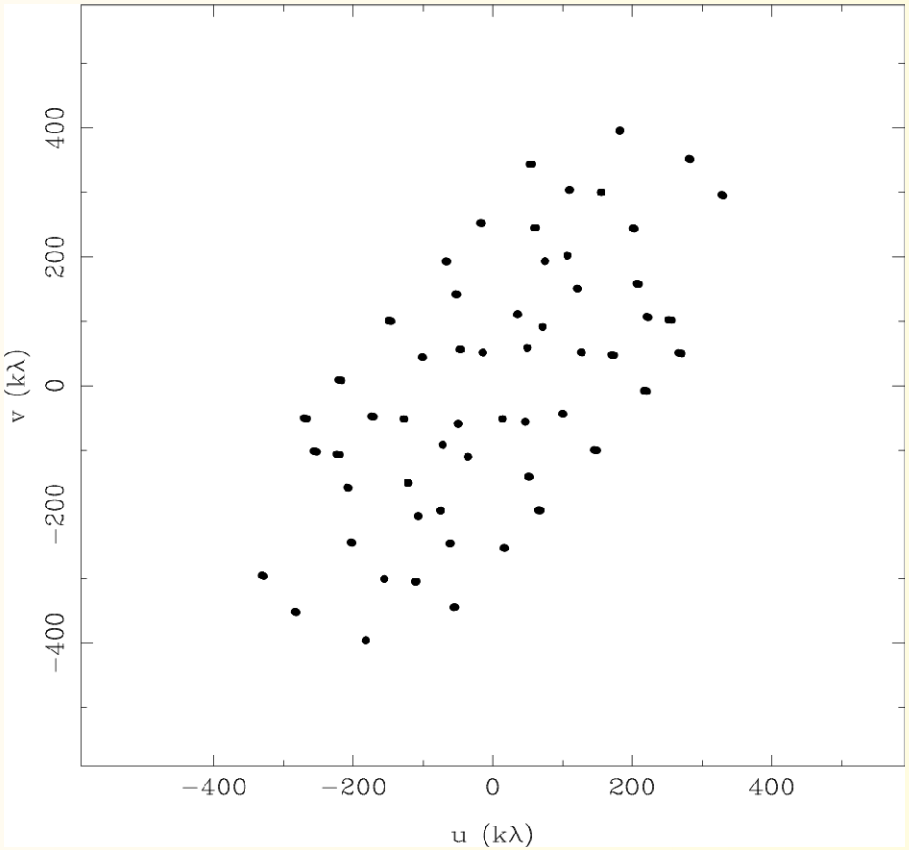


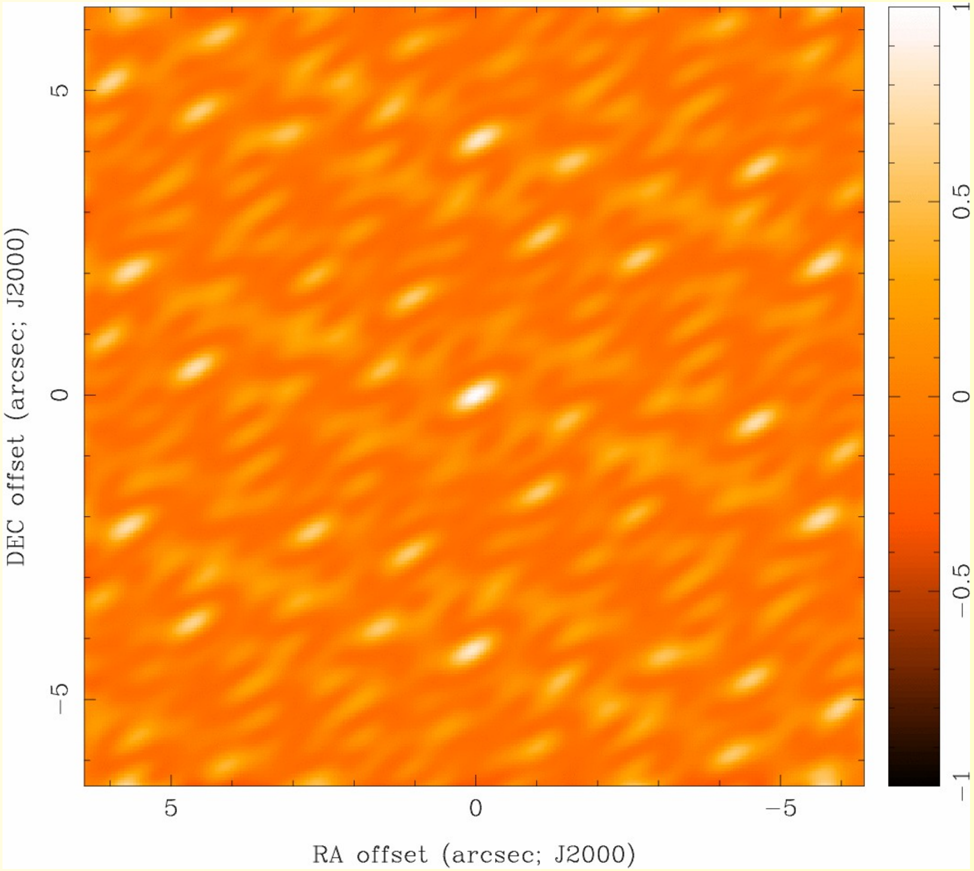


This is going slowly...

Dirty Beam Shape and N Antennas

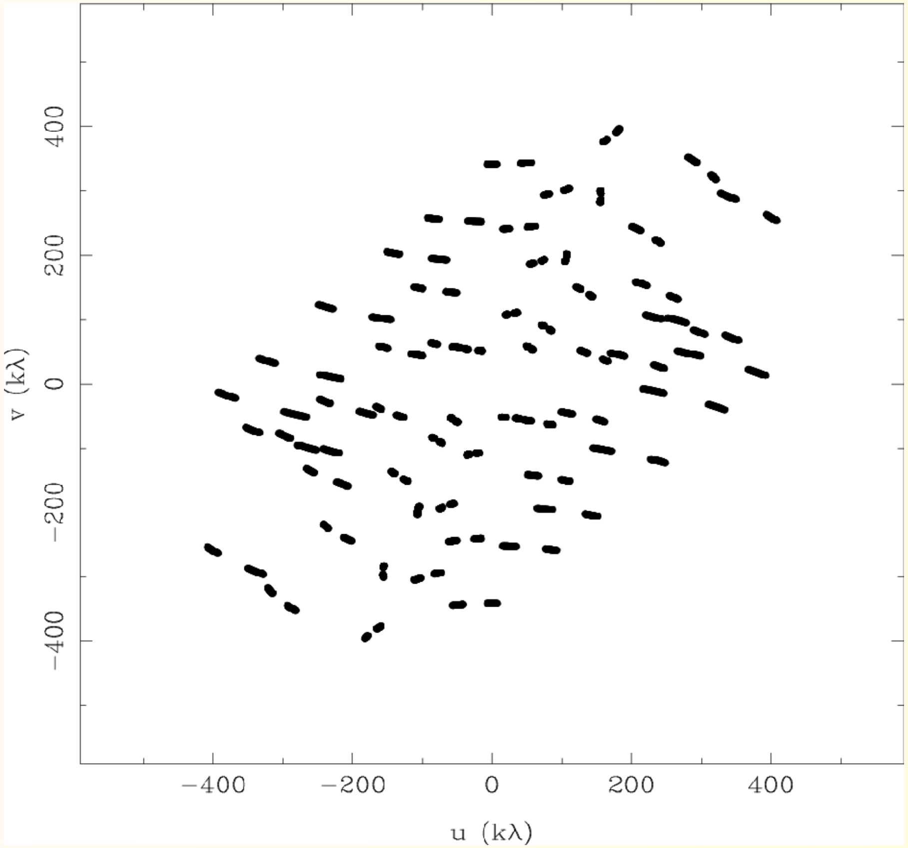
8 Antennas x 6 Samples

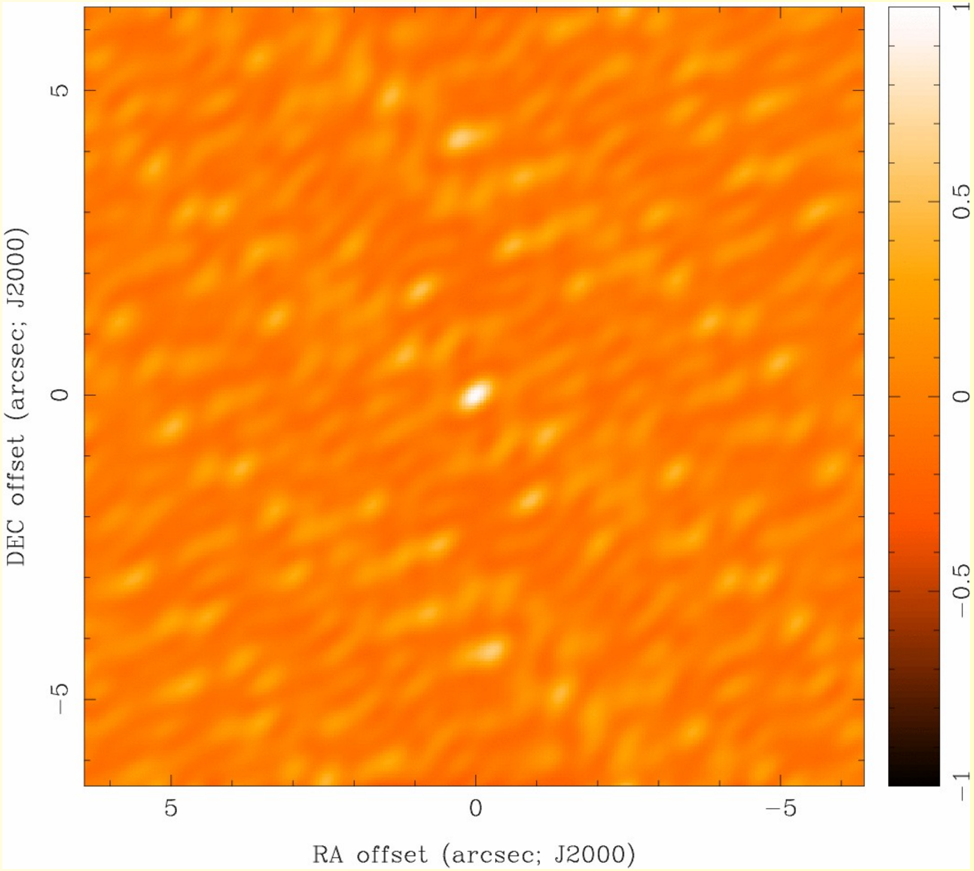




Dirty Beam Shape and N Antennas

8 Antennas x 60 Samples

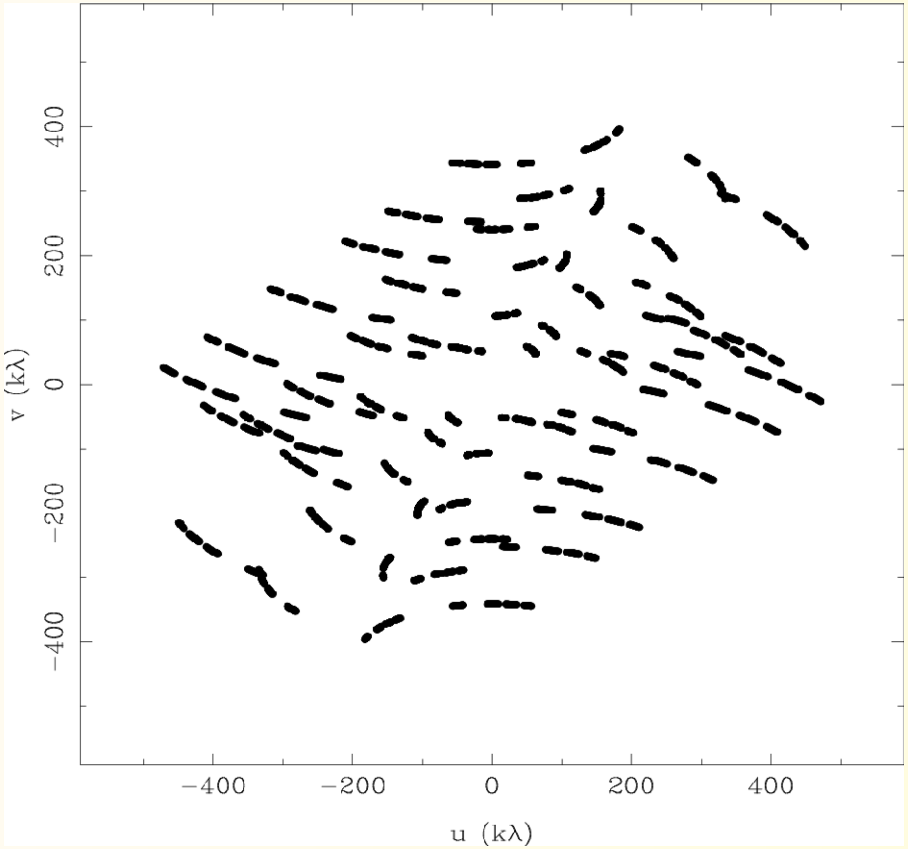


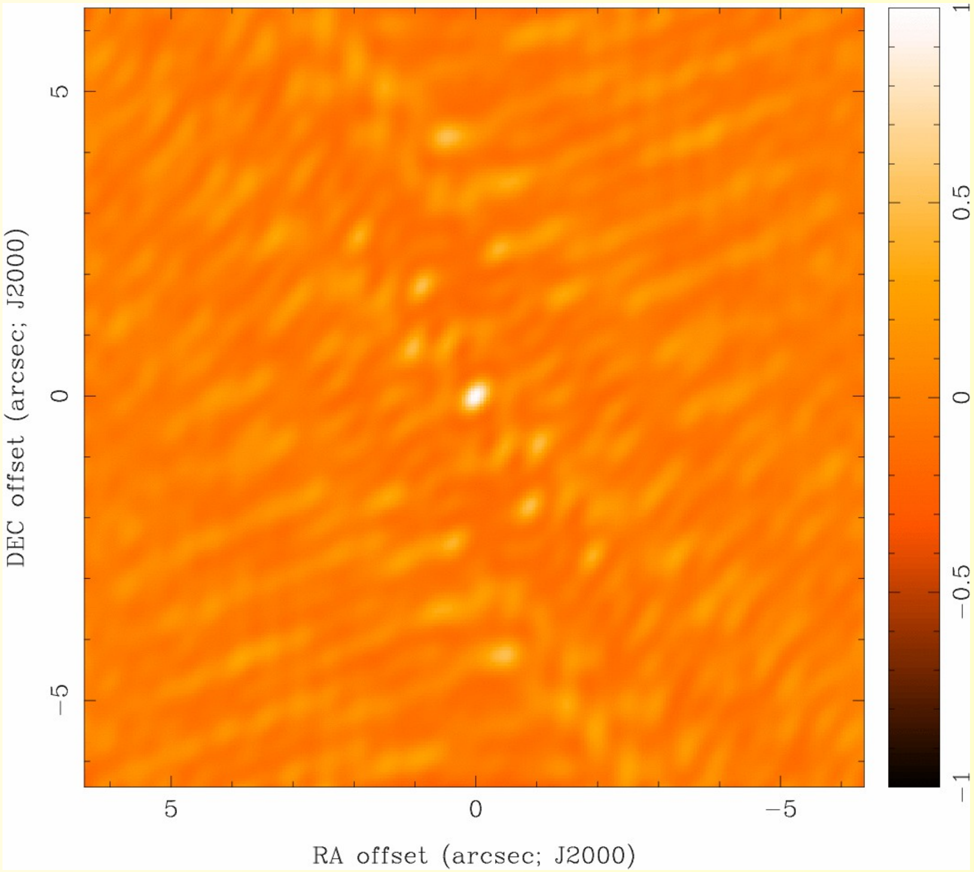


As we get better coverage

Better dirty beam

8 Antennas x 120 Samples





What happened to the sensitivity?

- If the effective collecting area of each (identical) dish is Ae
- Noise from different receivers is uncorrelated (GREAT!)
- If the contribution from the source is much less than the receiver temperature, then in our single baseline, the noise σ is:

$$\sigma = \frac{2^{1/2} k T_{sys}}{A_{eff} (\Delta \nu_{RF} \tau)^{1/2}}$$

And with more telescopes

- N dishes have N(N-1)/2 baselines, so the noise level σ_s for a point source is :

$$\sigma_s = \frac{2kT_{sys}}{A_{eff}\left[N(N-1)\Delta\nu_{RF}\tau\right]^{1/2}}$$

- In practice it is slightly noisier than that because we use a digitization, which introduces a little noise

Why aren't all radio telescopes like that?

- The point (unresolved) source sensitivity is great but the synthesizer beam of a single dish of the same area would be larger
- But the brightness sensitivity σ_T depends on beam area

$$\sigma_T = \left(\frac{\sigma_s}{\Omega_A}\right)\frac{\lambda^2}{2k}$$

- So low surface brightness (extended) regions can be missed. This is also called the 'zero spacing problem'
- Some observations need to fill this in