Fourier transforms

- From one domain to another
  - time and frequency (Wiener–Khintchine theorem)
  - angle and distance/λ (Van Cittert–Zernike theorem)
- Occurs naturally in all sorts of image and audio processing
- Heat diffusion
- Determining natural modes (vibration and QM)

Van Cittert–Zernike theorem

- Fourier transform of the mutual coherence function of a distant, incoherent source is equal to its complex visibility
- This is how radio astronomers make images of fields, since our correlators sample the mutual coherence function

Wiener–Khinchin theorem

- Power spectral density of a wide-sense-stationary random process is the Fourier transform of the corresponding autocorrelation function
- Einstein noticed this!
- Most radio spectrometers (and some spectrum analysers) work this way instead of using filters, as well as some infrared spectrometers

Properties

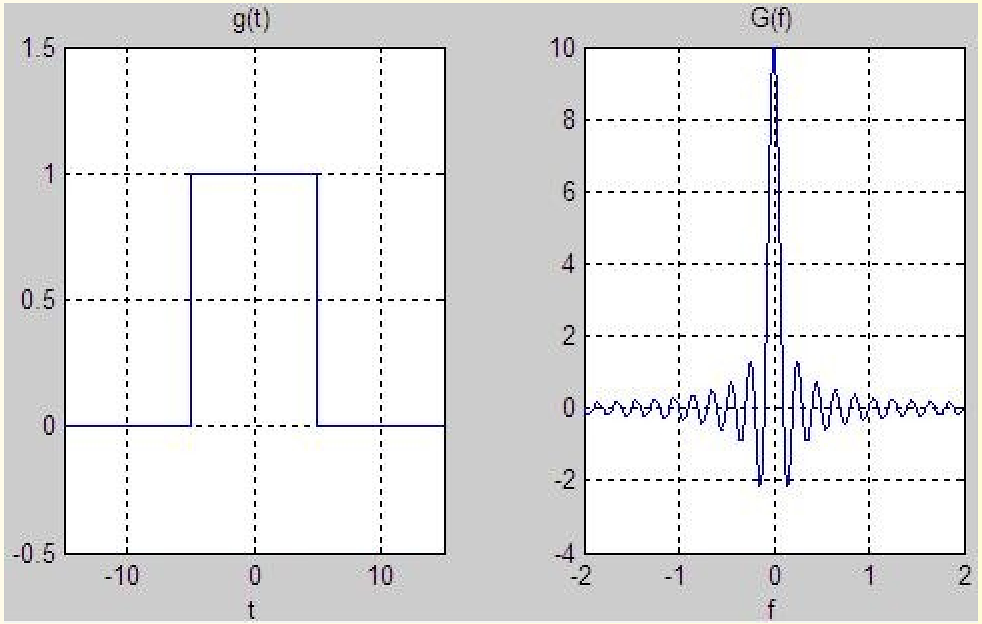
- If F(u) is a Fourier transform of f(x) and G is a Fourier transform of g

$$F(u)=\int_{-\infty}^{+\infty} f(x)\exp(-2\pi i x u)\,dx$$

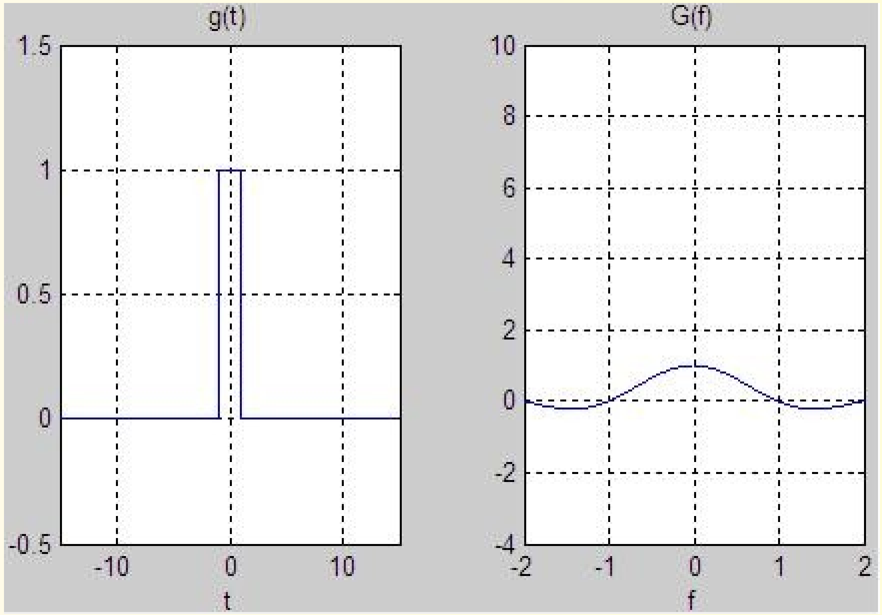
$$f(x)=\int_{-\infty}^{+\infty} F(x)\exp(2\pi i x u)\,dx$$

- Inverse FT just changes a sign (+i to -i)
- Phase is important it tells us where stuff is!
- Addition : FT of f(x)+g(x) is F(u)+G(u)
- Scale: FT of f(ax) is $\frac{1}{|a|}F(\frac{u}{a})$

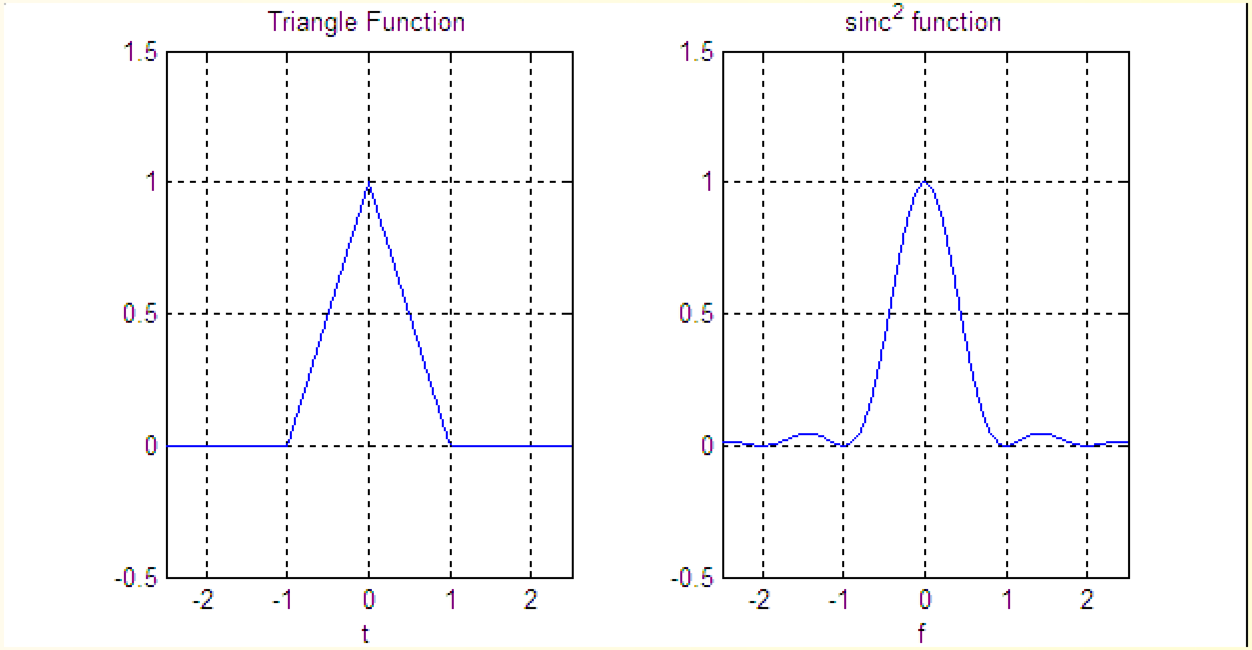
Example



Example 2



…



Triangle Function
1.5
1
0.5
0
-0.5
-2 -1 0 1 2
t
sinc2 function
1.5
1
0.5
0
-0.5
-2 -1 0 1 2
f

But

- FT of f.g(x) is F*G(u) , multiplication in one domain is convolution in the other-

$$\int_{-\infty}^{\infty} F(u)\, G(v-u)\, dv$$

or $$\int_{-\infty}^{\infty} F(v)\, G(v-u)\, dv$$

Convolution...

General properties of FT

- Sharp edges in one domain transform to ripples
- Wide in on domain is narrow in the other
- A shift in one domain is a phase slope in the other
- A cosine (or sine) in one domain is a point in the other (offset from zero)
- A constant value in one domain (alias cosine of infinite period) is a point at the origin
- A gaussian transforms to a gaussian
- An infinite series of spikes (Shah function) transforms to an infinite series of spikes

More

- If symmetrical about 0 it has only cosine components so it is *Real*
- If antisymmetrical about 0 it has only sine components so it is *Imaginary*
- All images can be decomposed into both a symmetrical and antisymmetrical part

$$f_{odd}(x,y)=\frac{1}{2}(f(x,y)-f(-x,-y))=-f_{odd}(-x,-y)$$

$$f_{even}(x,y)=\frac{1}{2}(f(x,y)+f(-x,-y))=f_{even}(-x,-y)$$

Phase & amplitude

- Amplitude info says how *strong* a signal is- and phase *where* it is:
  - FT of terminal & face; inverse with amplitude of *terminal* and phase information of *face*: