Antennas & Radiometry

Definition of antenna:

A device for converting electromagnetic radiation in space into electrical currents in conductors or vice-versa.

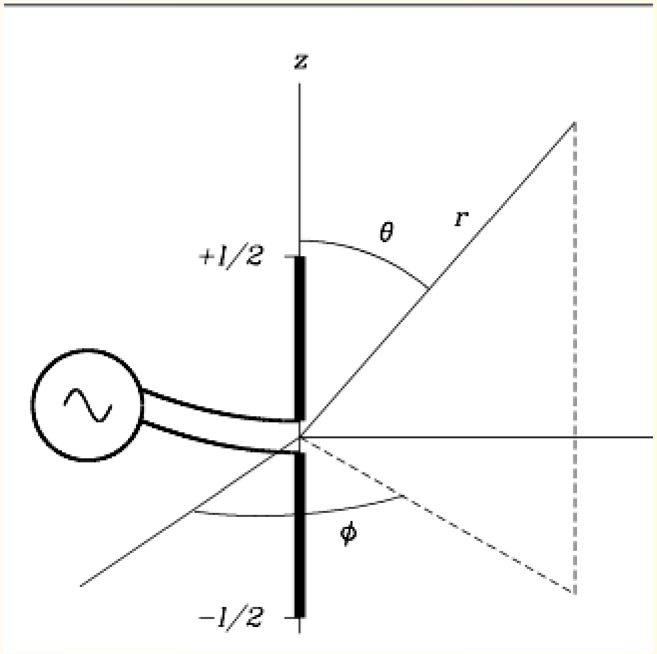
- Radio telescopes are antennas
- **Reciprocity** says we can treat them as transmitters as well as receivers
- Some things are easier to calculate as transmitters

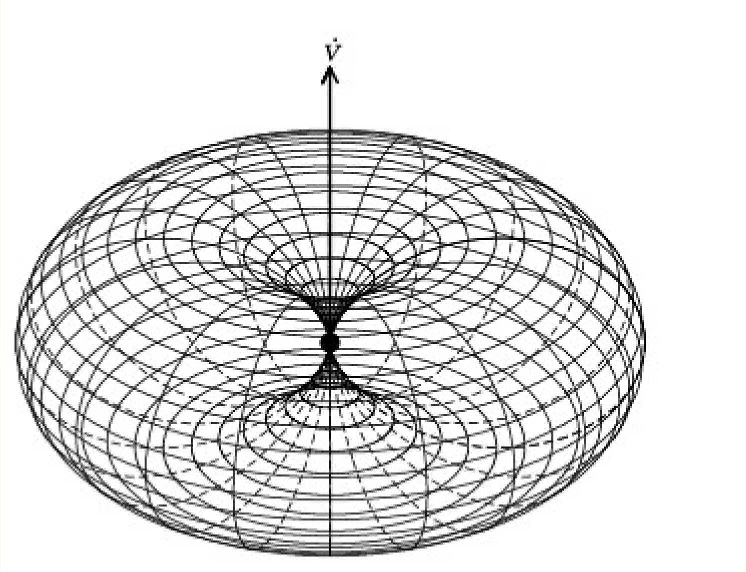
Short dipole antenna

- if the antenna is **much** shorter than the wavelength (λ) we can calculate with the Larmor formula
- The electrons are not relativistic
- Power radiated proportional to current^2 and length^2
- A monopole with a ground plane (reflection) looks like a dipole
- If the dipole is $\lambda/2$ we have resonance (but that case isn't 'short')

Dipole and radiation pattern

- Linearly polarized in direction of dipole

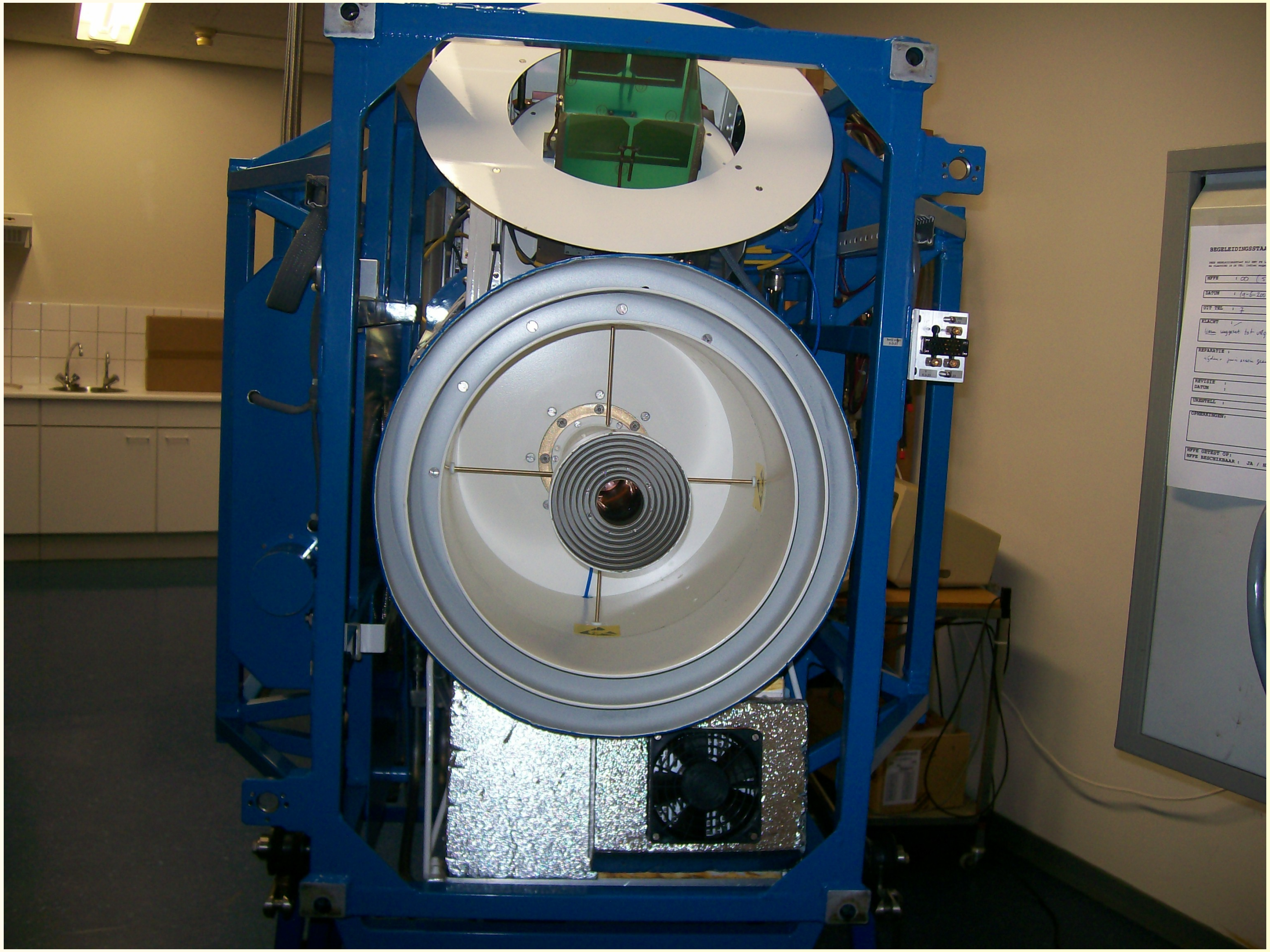




Example: LOFAR

a bent dipole

Example: Westerbork



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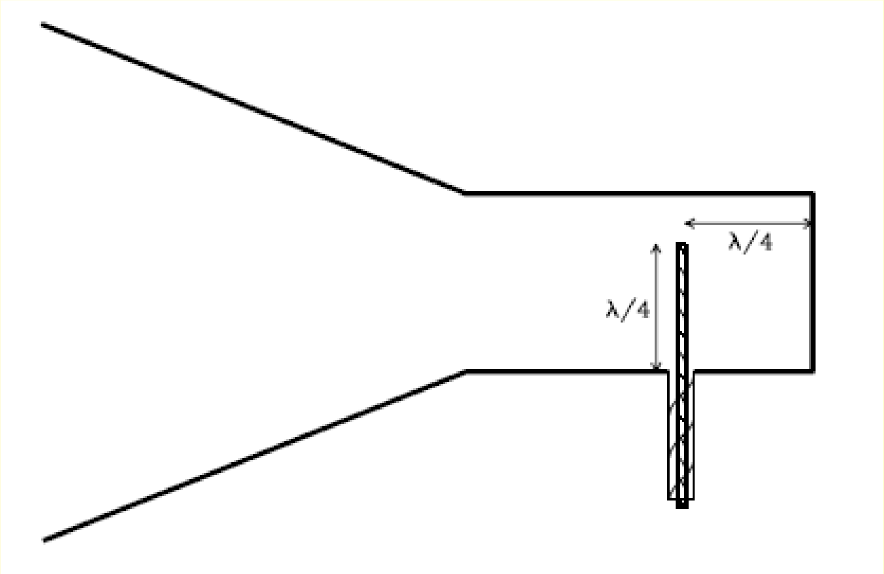
What about circular polarization?

- Either combine linear polarizations with a 90° phase shift
- or a helix

Feeds

- Parabolic reflectors focus incoming radio waves onto a *feed* which (usually) looks like either a dipole or monople with relector in a wavegiude horn
- The Allen Telescope Array is an exception



Gain and Power

If the total spectral P_ν power is fed into a lossless **isotropic** antenna, this would transmit power P units per solid angle per Hertz. Then the total radiated power at frequency ν is $4\pi P_\nu$.

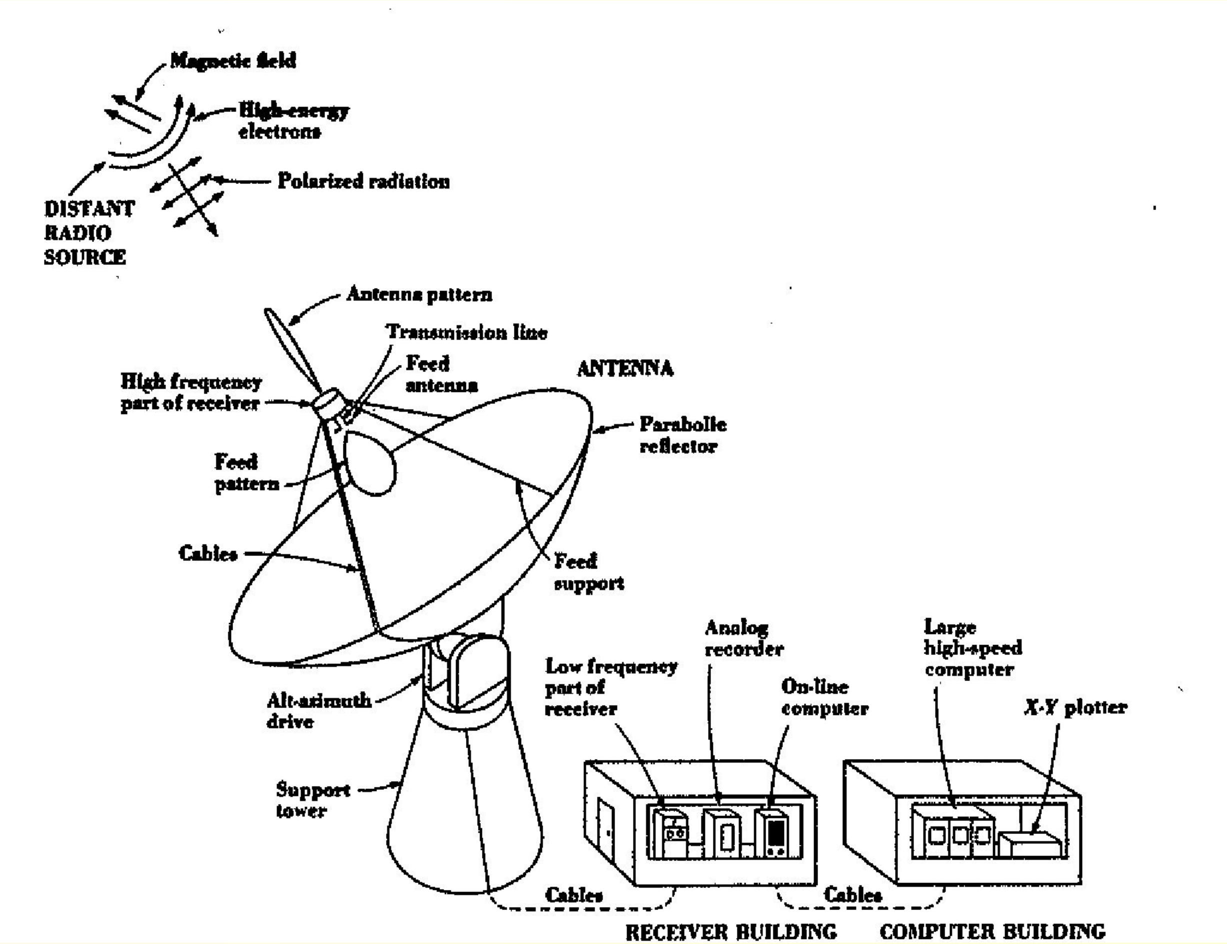
In a real antenna a power $P(\theta,\varphi)$ per unit solid angle is radiated in the direction (θ,φ)

We define the directive gain $P(\theta,\varphi)$

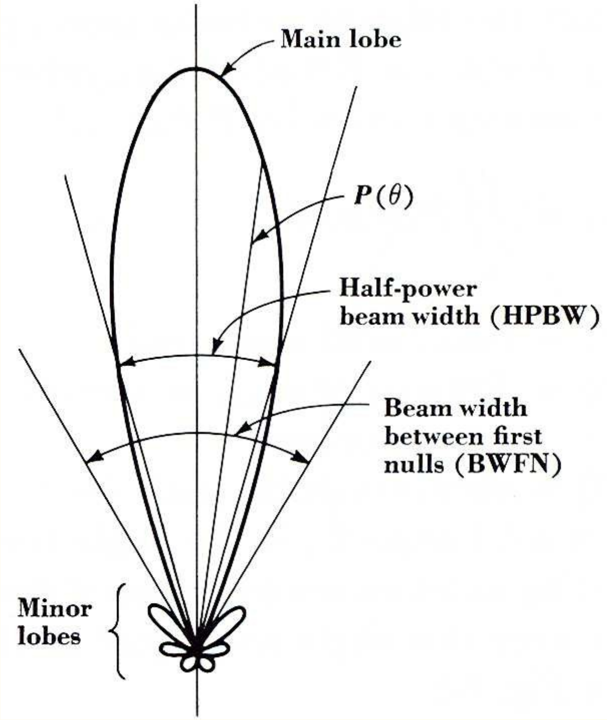
$$P(\theta,\phi)=PG(\theta,\phi)$$

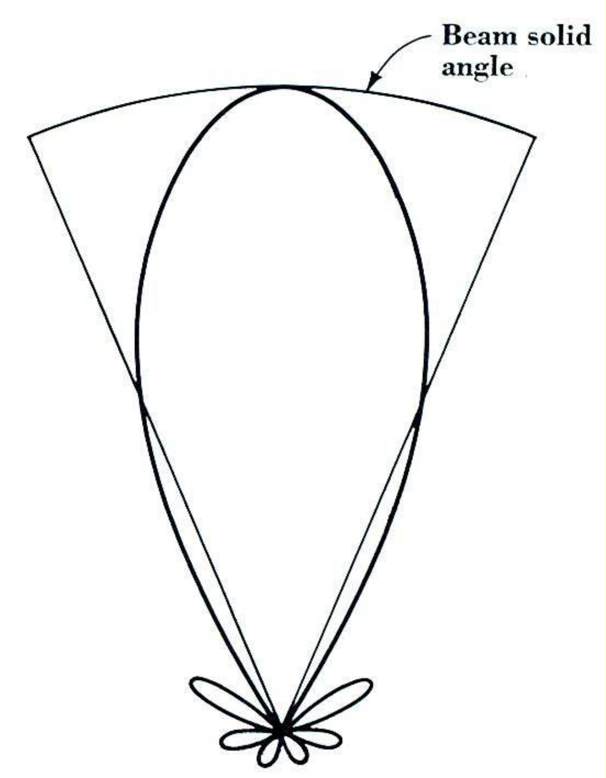
Simple radio telescope

no digitization, single dish, plane parallel waves



Beam patterns





Antenna Gain

- This comes down to

$$G(\theta,\phi)=\frac{4\pi P(\theta,\phi)}{\iint P(\theta,\phi)\,d\Omega}$$

- Gain is often measured (by engineers) in dB

$$G(dB)\equiv 10\times\log_{10}(G)$$

Lossless case

- If there is no loss then the power in gets radiated - in *some* direction

$$\langle G \rangle \equiv \frac{\int_{sphere} G \, d\Omega}{\int_{sphere} d\Omega} = \frac{4\pi}{4\pi} = 1$$

- So

$$\int_{sphere} G \, d\Omega = \int_{sphere} 1 \, d\Omega = 4\pi$$

So if the bulk is in one direction

- In general an antenna having peak gain G_{max} into a beam solid angle $\Delta\Omega$

$$\Delta\Omega \approx \frac{4\pi}{G_{max}}$$

- So a high gain means a narrow beam (and vice versa)

Matching

- No net power transfer

$$P_\nu = A_e S_{(matched)} = \frac{S}{2} = \int_{4\pi} A_e(\theta,\phi)\frac{B_\nu}{2}\,d\Omega$$

$$P_\nu = \frac{2\mathrm{kT}}{2\lambda^2}\int_{4\pi} A_e\,d\Omega = kT$$

$$\langle A_e\rangle \equiv \frac{\int_{4\pi} A_e\,d\Omega}{\int_{4\pi} d\Omega} = \frac{\lambda^2}{4\pi}$$

General case

- for the general case in a direction (Θ,ϕ)

$$A_e(\theta,\phi)=\frac{\lambda^2 G(\theta,\phi)}{4\pi}$$

Beam solid angle

- Beam solid angle for lossless antenna (P_n is normalised power)

$$\Omega_A \equiv \int_{4\pi} P_n(\theta, \phi)\, d\Omega$$

$$P_n(\theta, \phi) = \frac{G(\theta, \phi)}{G_{max}}$$

$$\Omega_A = \frac{1}{G_{max}} \int_{4\pi} G\, d\Omega = \frac{4\pi}{G_{max}}$$

Aperture efficiency/Effective area

- The effective collecting area of an antenna is the fraction of the power received compared to the total incident flux density.

$$A_e \equiv \frac{P_\nu}{\boldsymbol{S}_{matched}}$$

- We compare this to a geometrical area A_g and define efficiency η_A using

$$A_e = \eta_A A_g$$

Polarization matching

- We can decompose any EM wave into 2 orthogonal components (either linear or circular) and for a typical (low polarization) natural source

$$S_{matched} = \frac{S}{2}$$

- Blackbody and free-free are unpolarized; cyclotron and synchrotron **are** polarized, but in no particular direction with respect to our antenna

Antenna temperature

- Consider a receiving antenna with a normalised power pattern $P_n(\theta, \phi)$
- We point it at a sky brightness distribution $B_\nu(\theta, \phi)$
- The power in one polarization received per unit bandwidth P_ν is

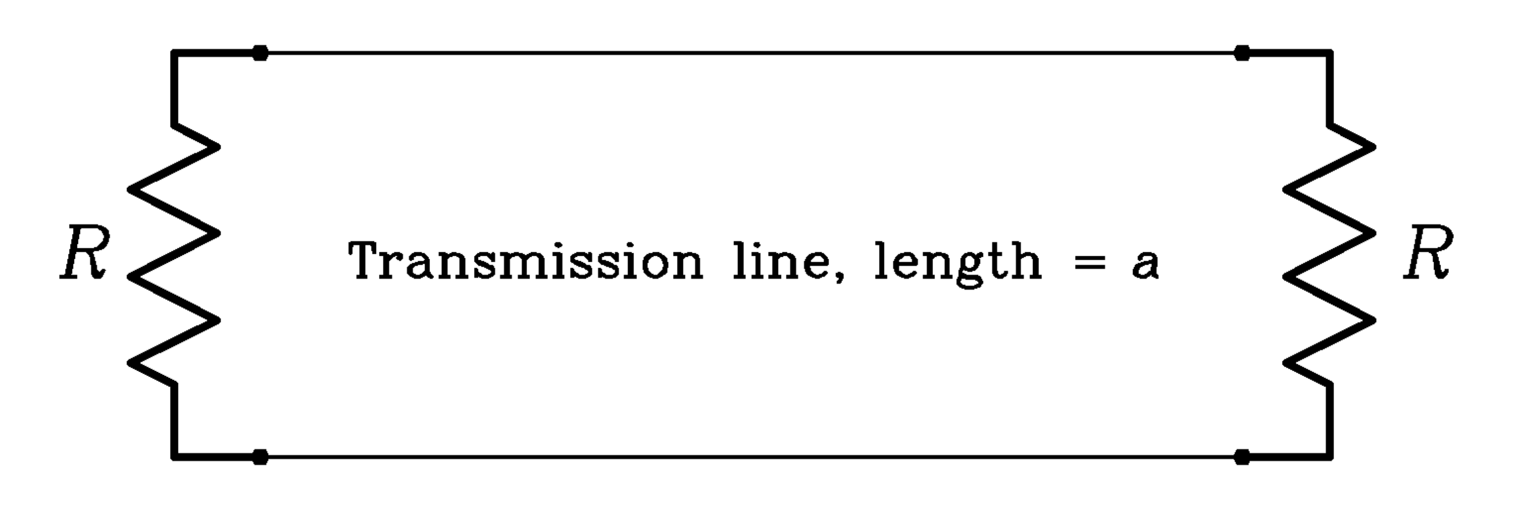
$$\frac{1}{2} A_e \iint B_\nu(\theta, \phi) P_n(\theta, \phi) \, d\Omega$$

- Astronomers give antenna temperature T_A of an equivalent resistor (Johnson-Nyquist noise theorem)

$$P_\nu = k T_A$$

Noise from resistors

- matched resistors on matched impedance line



- assume line length **a** much longer than wavelengths of interest

noise on line

- only wavelengths that fit can stand $a=\frac{n\lambda}{2} \quad n=1,2,3,\ldots$
- if velocity is **v** and frequency ν $\quad n=\frac{2a\nu}{v}$
- if a>> λ the number of modes per unit frequency is $\quad \frac{\Delta n}{\Delta \nu}=\frac{2a}{v}$
- In thermal equilibrium

$$E_\nu=\frac{\Delta n}{\Delta \nu}kT=\frac{2aKT}{v}$$

- Power = Energy per unit time $\quad \Delta t=\frac{a}{v}$

$$P_\nu=\frac{E_\nu}{\Delta t}=2kT$$

- From symmetry each resistor generates power P_ν = kT

Johnson-Nyquist resistor noise

- Power per unit bandwidth in a resisitor at temperature **T** is $P_\nu = kT$
- Uncorrelated powers add so

$$T_{sys} = T_{cmb} + \Delta T_{source} + T_{atm} + T_{spillover} + T_{rcvr} + ...$$

- But both our receiver noise and noise from a source are gaussian (random) and bandwidth limited

..actually

- A resistor acts like a 1-dimensional blackbody (ideal) radiator so for very high frequencies the quantum-mechanically correct version should be:

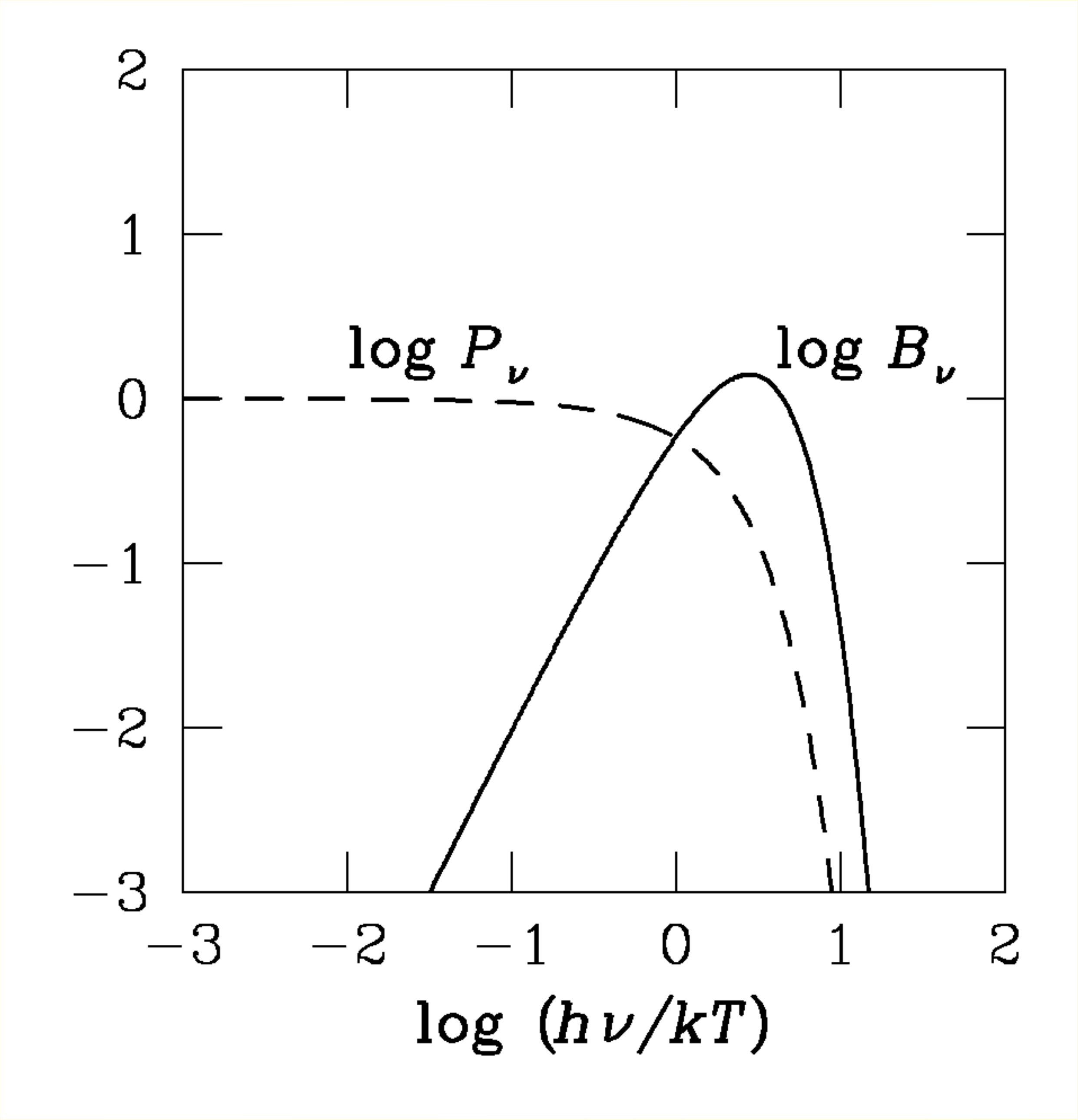
$$P_\nu(\nu,T)=\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

$$if \quad h\nu \ll kT \quad we\,can\,do\,a\,\mathrm{1st}\,order\,series\,expansion\,giving$$

$$P_\nu(\nu,T)=kT$$

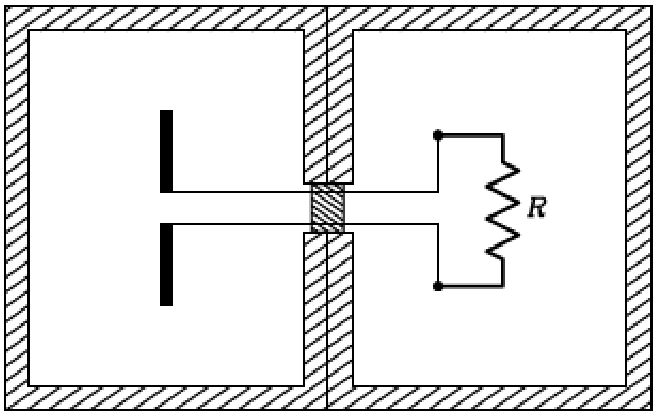
- We must use the QM correct version above about 400GHz for receivers at 20K

QM correction



Dipole example

- consider the equilibrium case of a dipole and noise source



A cavity in thermodynamic equilibrium at temperature T containing a resistor R is coupled to an antenna, also at temperature T, through a filter passing frequencies in the range ν to $\nu + d\nu$.

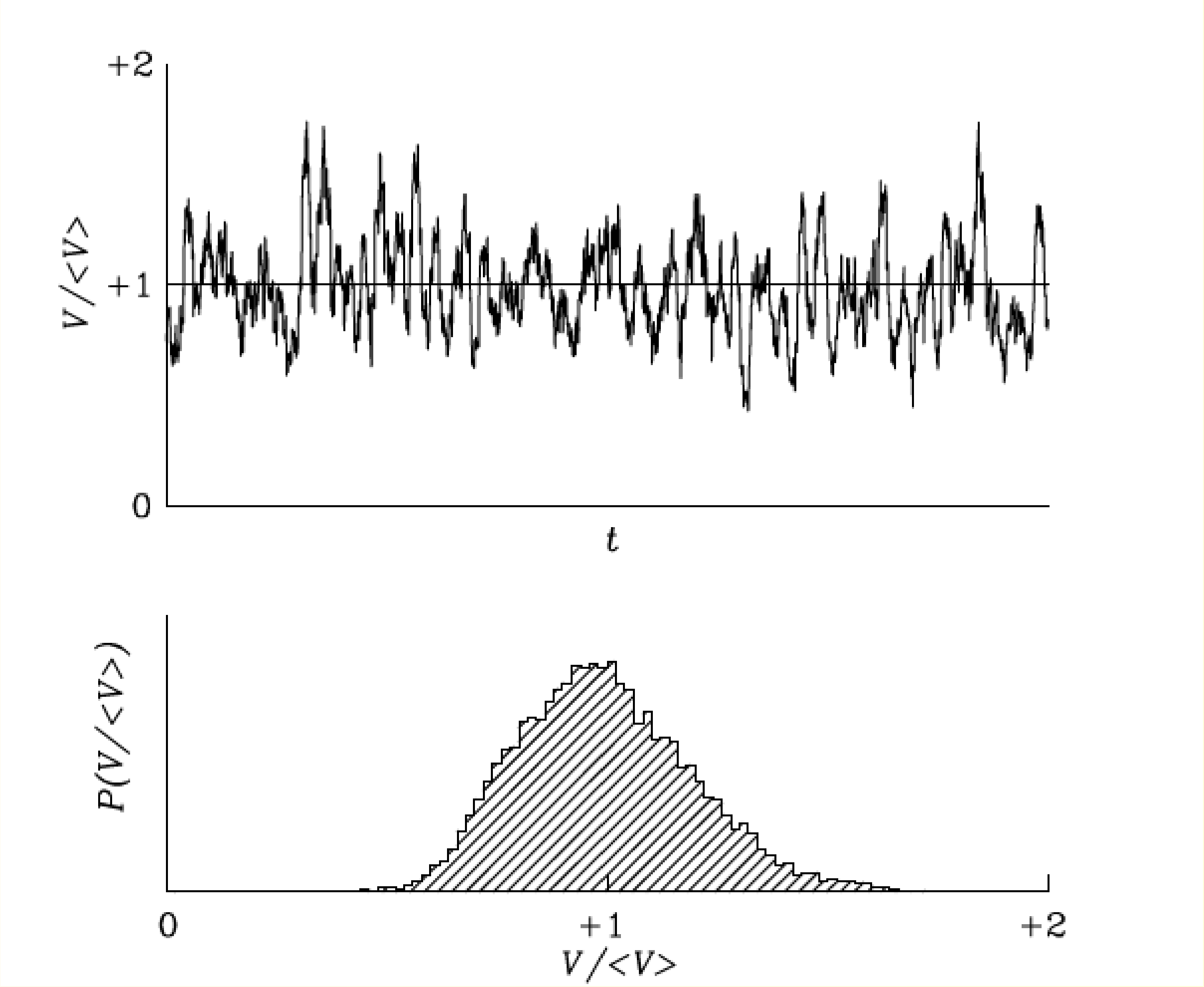
Band-limited noise

- Nyquist-Shannon sampling theory says that a signal of bandwidth $\Delta\nu$ and duration τ can be represented by N samples where $N = 2\tau\Delta\nu$
- If N is large we can use statistics of the central-limit theorem to sample the noise T_{sys}
- For each sample $\sigma_T = \sqrt{2}\, T_{sys}$
- So if we take N samples the r.m.s is reduced

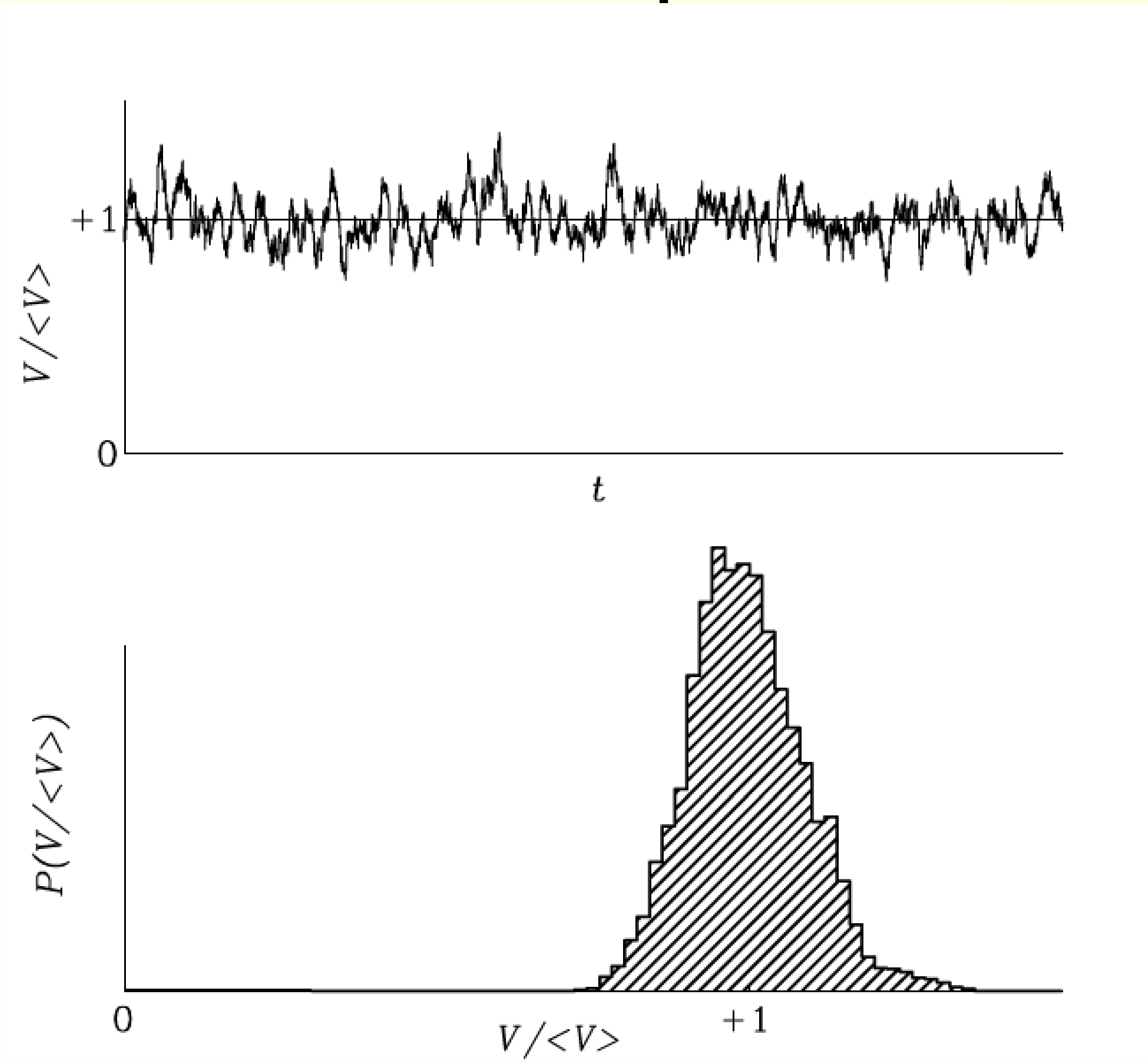
$$\sigma_T = \frac{\sqrt{2}\, T_{sys}}{\sqrt{N}}$$

$$\sigma_T \approx \frac{T_{sys}}{\sqrt{\Delta\nu\,\tau}}$$

Example: 50 samples



200 samples



Back to the result we had

- If we have a small source (much less than beam) and the rest of the sky is blank...

$$P_\nu \approx \frac{\lambda^2 G_{(max)} k T_B \Omega_s}{4\pi\lambda^2} = kT_B \frac{\Omega_s}{\Omega_A}$$

but $\quad T_A = \dfrac{P_\nu}{k}$

so $\quad T_A \approx T_B \dfrac{\Omega_s}{\Omega_A}$

Footnote on impedance & resistance

- If we use
$$Power = \frac{Voltage^2}{Resistance}$$
- Then free space (and a black hole) has a '*resistance*' of 377 Ω
- A half wave dipole in free space has a resistance of 73 Ω
- commercial coaxial cables usually are 30-80 Ω
  - the common choices are 50Ω and 75Ω

Impedance matching

- For any transfer we get the optimum *power transfer* if impedances match
    - if receiver has low impedance it looks like a short circuit and so voltage across it is low
    - if receiver has higher impedance it has low current
    - optimum is for a matched system
- the is an example of the **Maximum Power Theorem**