

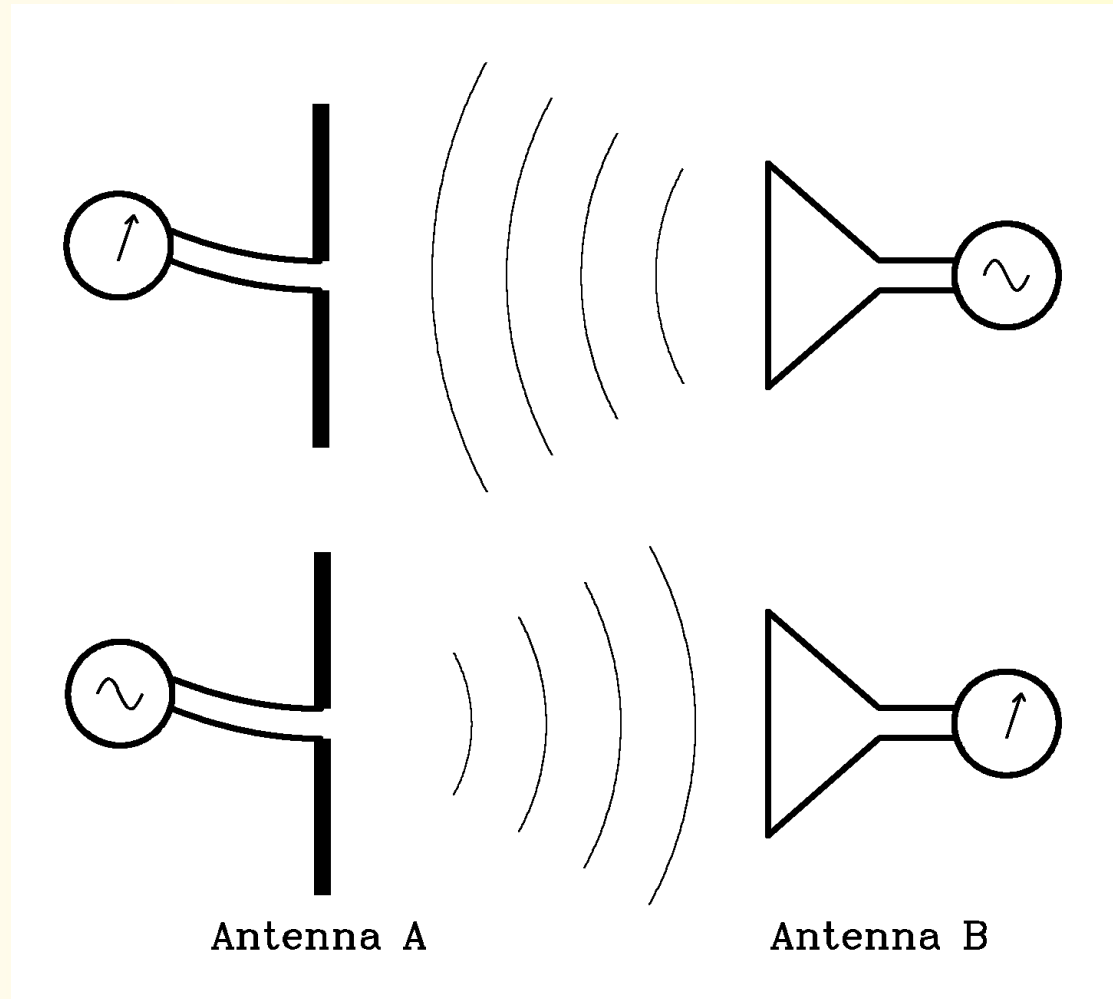
Antennas

Definition of antenna:

A device for converting electromagnetic radiation in space into electrical currents in conductors or vice-versa.

- Radio telescopes are antennas
- **Reciprocity** says we can treat them as transmitters as well as receivers
- Some things are easier to calculate as transmitters

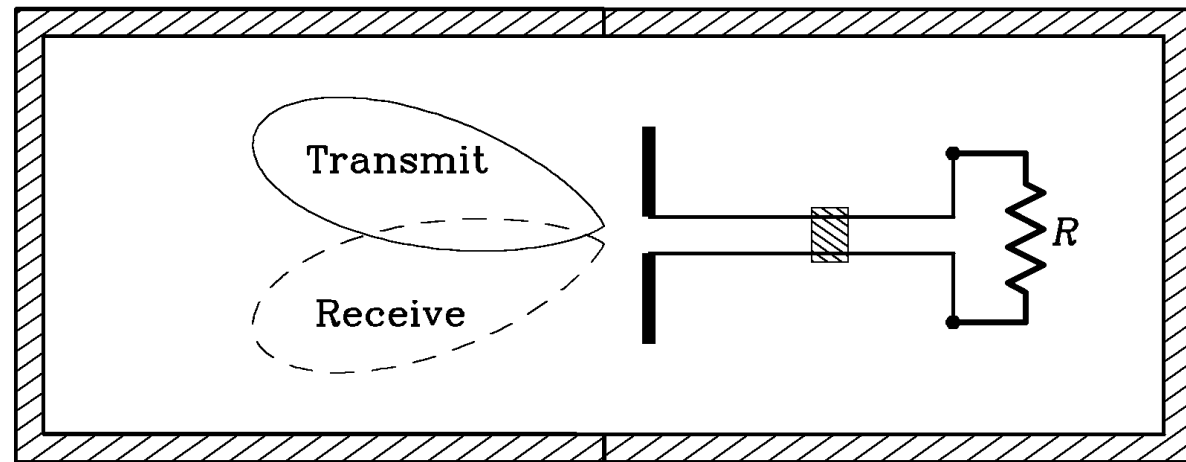
What does reciprocity mean



Why does reciprocity work

- Just consider Maxwell's equations – they are time reversible
- Also you can have a thought experiment of an isolated box, an antenna and a thermal noise source in equilibrium

(Weak) Reciprocity



Isolated box at some temperature

Radio noise from resistor at same temperature

Matching network

Antenna

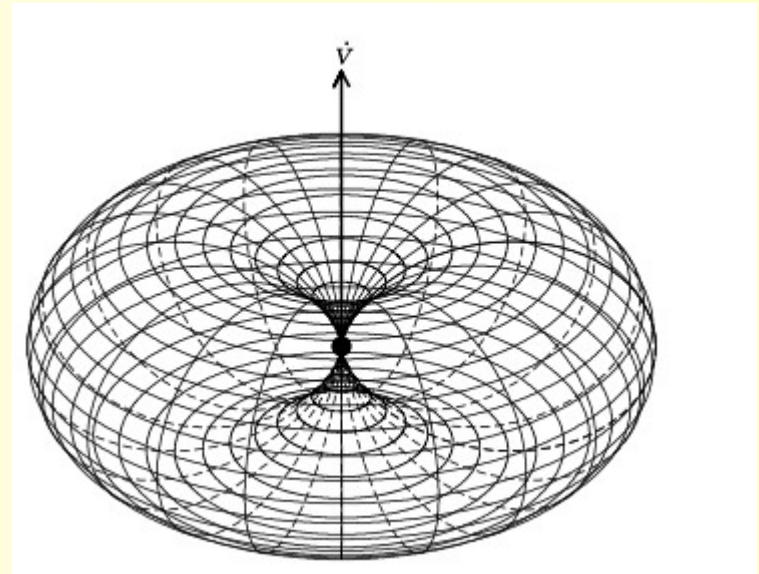
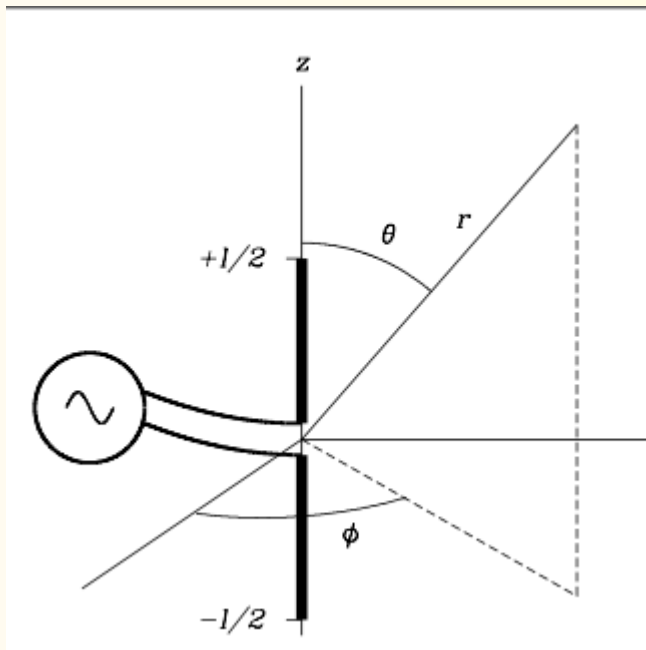
Equilibrium – same power in & out of antenna

Short dipole antenna

- if the antenna is **much** shorter than the wavelength (λ) we can calculate with the Larmor formula
- The electrons are not relativistic
- Power radiated proportional to current² and length²
- A monopole with a ground plane (reflection) looks like a dipole ... in the top half
- If the dipole is $\lambda/2$ we have resonance (but that case isn't 'short')

Dipole and radiation pattern

- Linearly polarized in direction of dipole

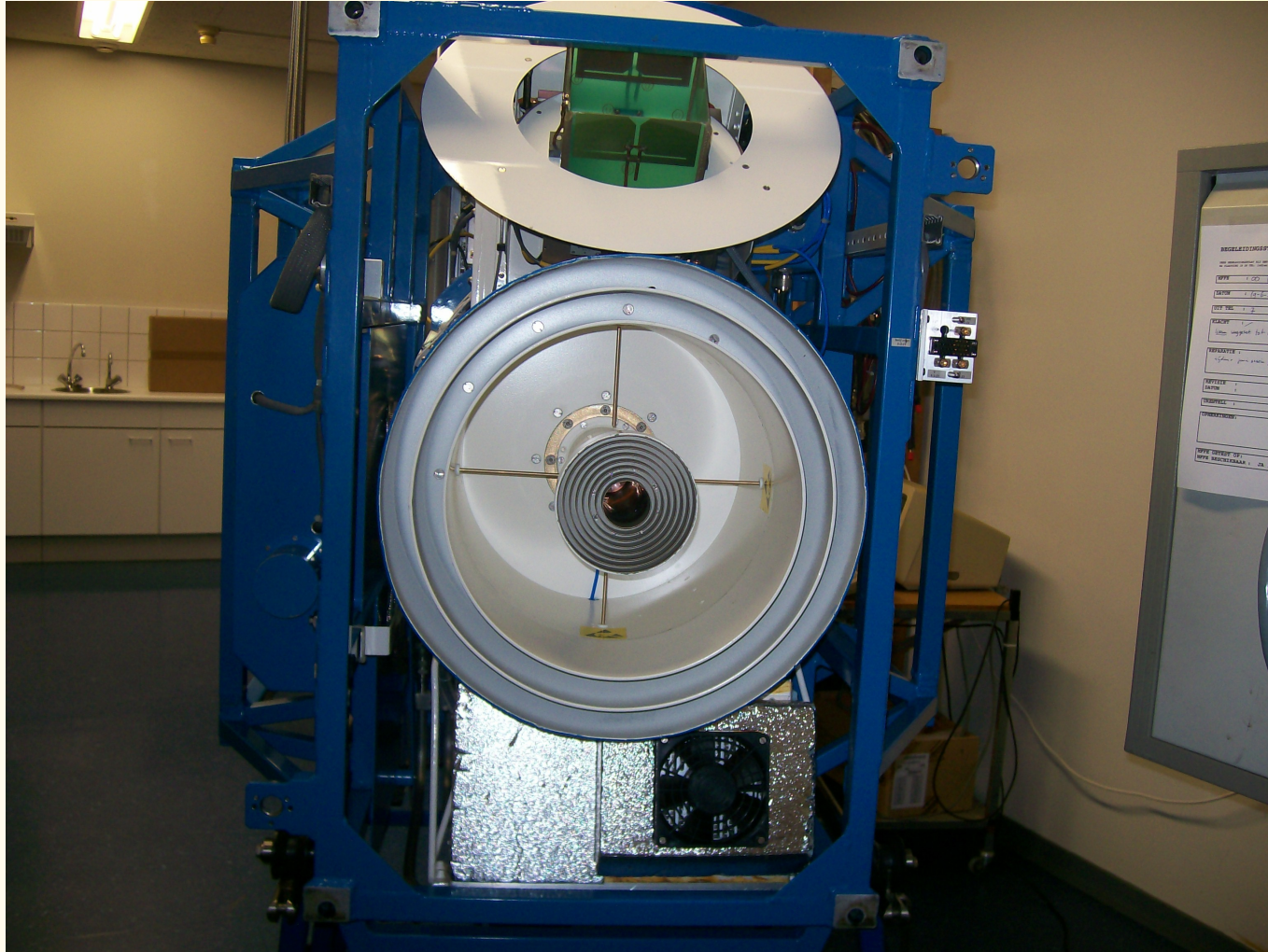


Example: LOFAR

a bent dipole



Example: Westerbork



anim



What about circular polarization?

- Either combine linear polarizations with a 90° phase shift
- or a helix (spiral)

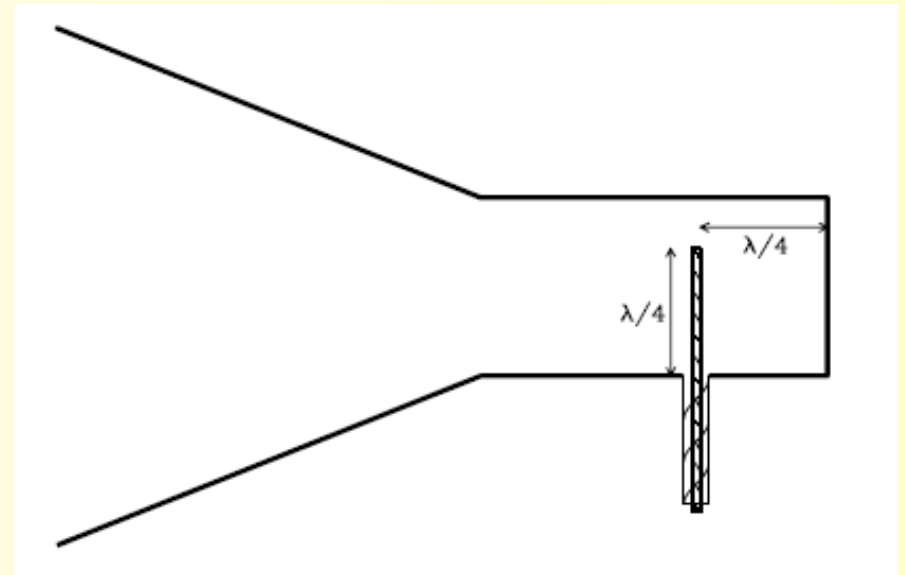


Other possible

- Boxing ring (folded dipole)
- Bow Tie
- Yagi
- Conical Spiral
- Bicone

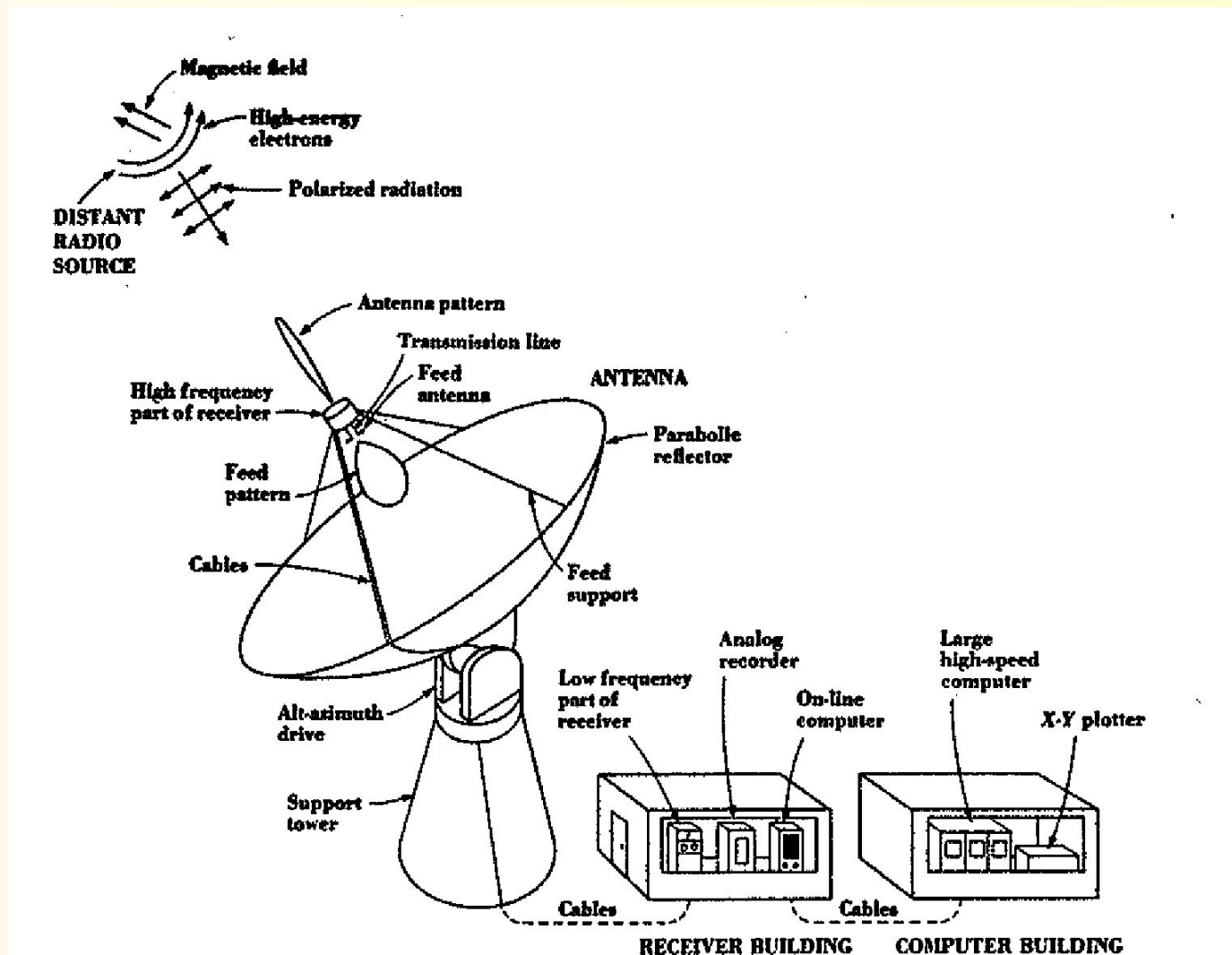
'Feeds'

- Parabolic reflectors focus incoming radio waves onto a *feed* which (usually) looks like either a dipole or monopole with relector in a waveguide horn
- The Allen Telescope Array is an exception

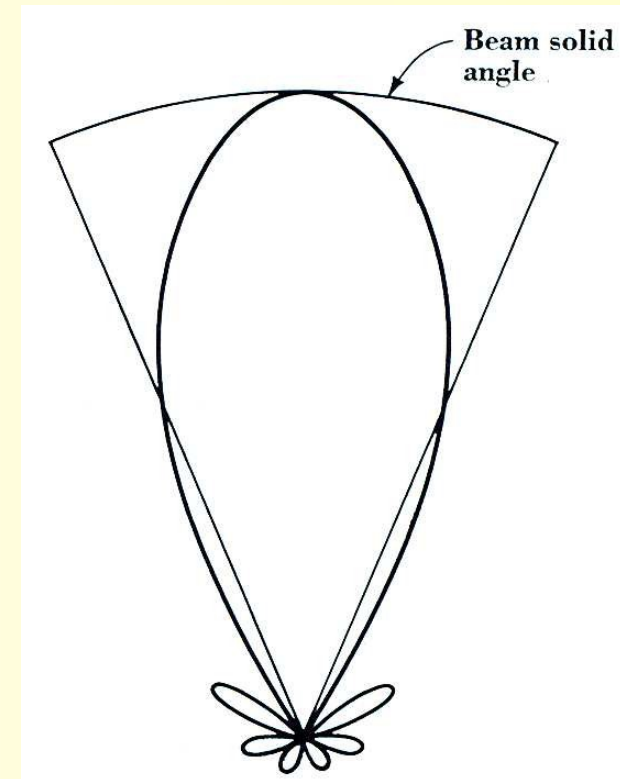
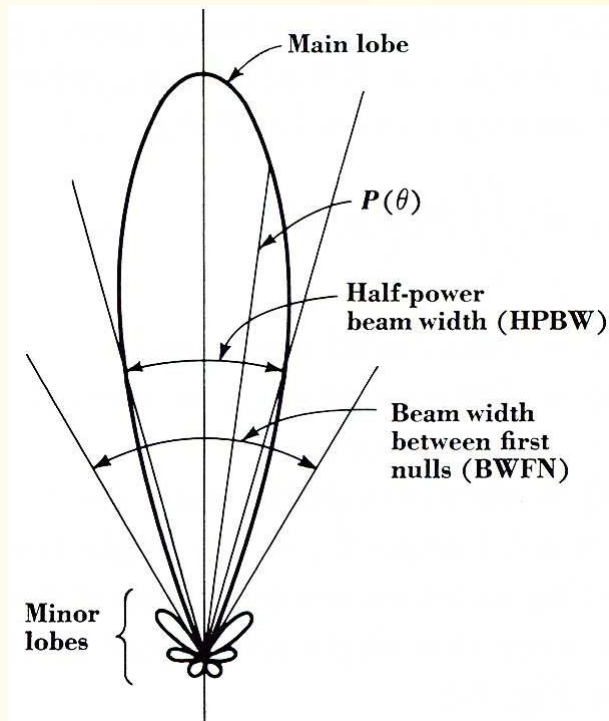


Simple radio telescope

no digitization, single dish, plane parallel waves



Beam patterns



Gain and Power

If the total spectral P_ν power is fed into a lossless **isotropic** antenna, this would transmit power $\mathbf{P}$ units per solid angle per Hertz. Then the total radiated power at frequency ν is $4\pi P_\nu$.

In a real antenna a power $P(\theta, \varphi)$ per unit solid angle is radiated in the direction (θ, φ)

We define the directive gain $G_\nu(\theta, \varphi)$ from

$$P_\nu(\theta, \varphi) = \mathbf{P}_\nu G_\nu(\theta, \varphi)$$

Antenna Gain

- This comes down to

$$G(\theta, \phi) = \frac{4\pi P(\theta, \phi)}{\iint P(\theta, \phi) d\Omega}$$

- Antenna Gain is often measured (by engineers) in dB

$$G(dB) \equiv 10 \times \log_{10}(G)$$

Lossless case

- If there is *no* loss then the power must be radiated
- in *some* direction

$$\langle G \rangle \equiv \frac{\int_{\text{sphere}} G d\Omega}{\int_{\text{sphere}} d\Omega} = \frac{4\pi}{4\pi} = 1$$

- So

$$\int_{\text{sphere}} G d\Omega = \int_{\text{sphere}} 1 d\Omega = 4\pi$$

- Where is the benefit then...

If the bulk is in one direction

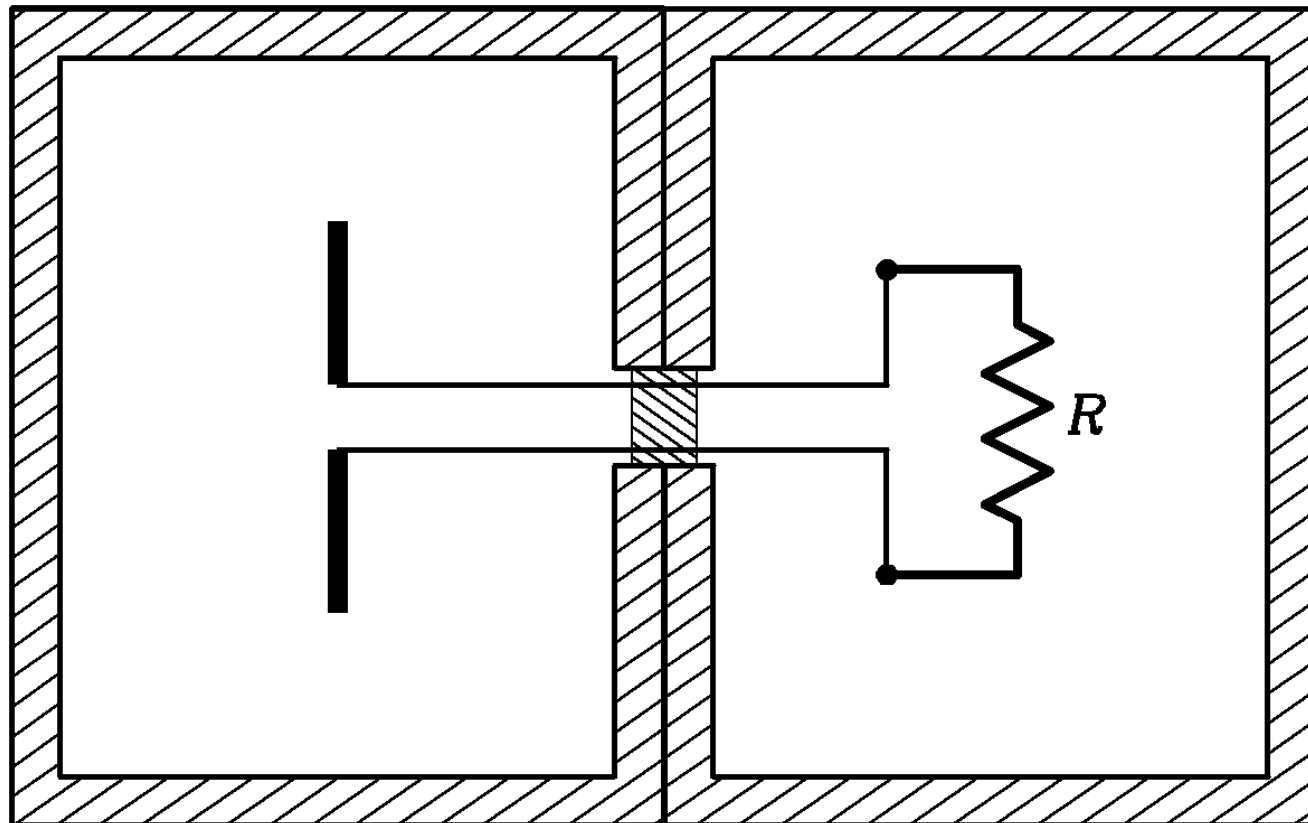
- In general an antenna having peak gain $G_{\max}$ into a beam solid angle $\Delta\Omega$

$$\Delta\Omega \approx \frac{4\pi}{G_{\max}}$$

- So a high antenna gain means a narrow antenna beam (and vice versa)
- If I don't want narrow beam I cannot use high gain antenna

Blackbody cavity

- Thermal equilibrium (T)
- Resistor R coupled to antenna with frequency filter



So

- We define an effective area A_e
 - Which is a function of angle (θ, φ)
- Flux density S (in $\text{W}/\text{m}^2/\text{Hz}$)
- Wavelength λ (wavelength ν)
- Temperature T
- Blackbody radiation

$$B_\nu = \frac{2kT}{\lambda^2}$$

Matching

- No net power transfer

$$P_v = A_e S_{(matched)} = \frac{S}{2} = \int_{4\pi} A_e(\theta, \varphi) \frac{B_v}{2} d\Omega$$

$$P_v = \frac{2kT}{2\lambda^2} \int_{4\pi} A_e d\Omega = kT$$

$$\langle A_e \rangle \equiv \frac{\int_{4\pi} A_e d\Omega}{\int_{4\pi} d\Omega} = \frac{\lambda^2}{4\pi}$$

General case

- for the general case in a direction (Θ, ϕ)

$$A_e(\theta, \phi) = \frac{\lambda^2 G(\theta, \phi)}{4\pi}$$

Beam solid angle

- Beam solid angle for lossless antenna (P_n is normalised power)

$$\Omega_A \equiv \int_{4\pi} P_n(\theta, \phi) d\Omega$$

$$P_n(\theta, \phi) = \frac{G(\theta, \phi)}{G_{max}}$$

$$\Omega_A = \frac{1}{G_{max}} \int_{4\pi} G d\Omega = \frac{4\pi}{G_{max}}$$

Aperture efficiency/Effective area

- The effective collecting area of an antenna is the fraction of the power received compared to the total incident flux density.

$$A_e \equiv \frac{P_v}{S_{matched}}$$

- We compare this to a geometrical area A_g and define efficiency η_A using

$$A_e = \eta_A A_g$$

- Typical efficiencies for a good antenna are 70%
 - Don't use full area... why
 - Blockage
 - Surface errors

Polarization matching

- We can decompose any EM wave into 2 orthogonal components (either linear or circular) and for a typical (low polarization) natural source

$$S_{\text{matched}} = \frac{S}{2}$$

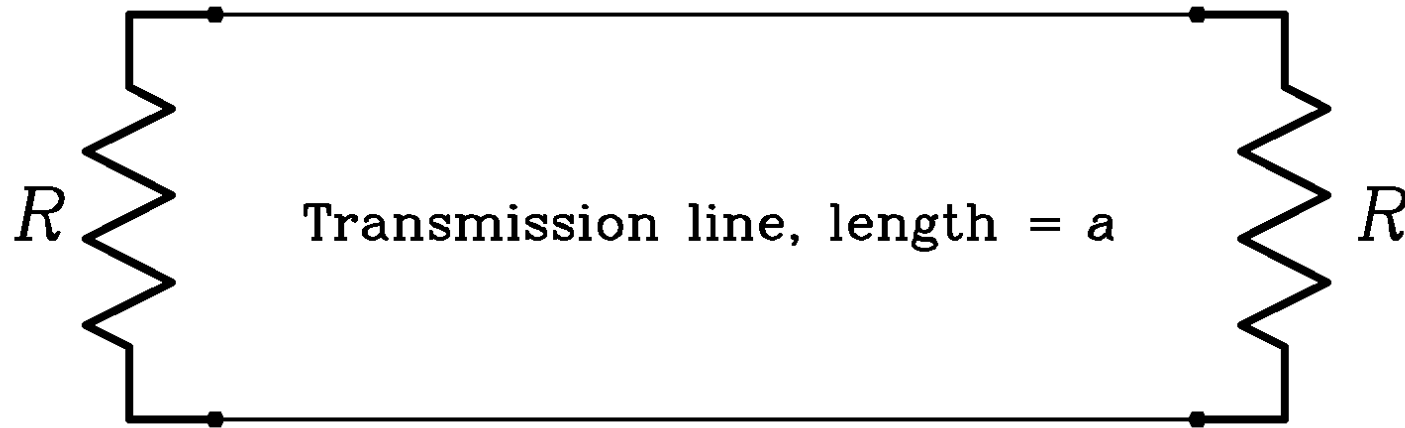
- Blackbody and free-free are unpolarized; cyclotron and synchrotron are polarized (but in no particular direction with respect to our antenna)

Antenna temperature

- Consider a receiving antenna with a normalised power pattern $P_n(\theta, \phi)$
- We point it at a sky brightness distribution $B_\nu(\theta, \phi)$
- The power in one polarization received per unit bandwidth P_ν is
$$\frac{1}{2} A_e \iint B_\nu(\theta, \phi) P_n(\theta, \phi) d\Omega$$
- Astronomers give antenna temperature T_A of an equivalent resistor (Johnson-Nyquist noise theorem)
$$P_\nu = kT_A$$

Noise from resistors

- matched resistors on matched impedance line



- assume line length a much longer than wavelengths of interest

noise on line

- only wavelengths that fit can stand $a = \frac{n\lambda}{2} \quad n=1,2,3,\dots$
- if velocity is v and frequency ν $n = \frac{2a\nu}{v}$
- if $a \gg \lambda$ the number of modes per unit frequency is $\frac{\Delta n}{\Delta \nu} = \frac{2a}{v}$
- In thermal equilibrium $E_\nu = \frac{\Delta n}{\Delta \nu} kT = \frac{2aKT}{v}$
- Power = Energy per unit time $\Delta t = \frac{a}{v}$
 $P_\nu = \frac{E_\nu}{\Delta t} = 2kT$
- From symmetry each resistor generates $P_\nu = kT$

Johnson-Nyquist resistor noise

- Power per unit bandwidth in a resistor at temperature T is $P_v = kT$

- Uncorrelated powers add so

$$T_{sys} = T_{cmb} + \Delta T_{source} + T_{atm} + T_{spillover} + T_{rcvr} + \dots$$

- But both our receiver noise and noise from a source are gaussian (random) and we bandwidth limited (as we limit it!)
- *Interference will not look gaussian*

..actually

- A resistor acts like a 1-dimensional blackbody (ideal) radiator so for very high frequencies the quantum-mechanically correct version should be:

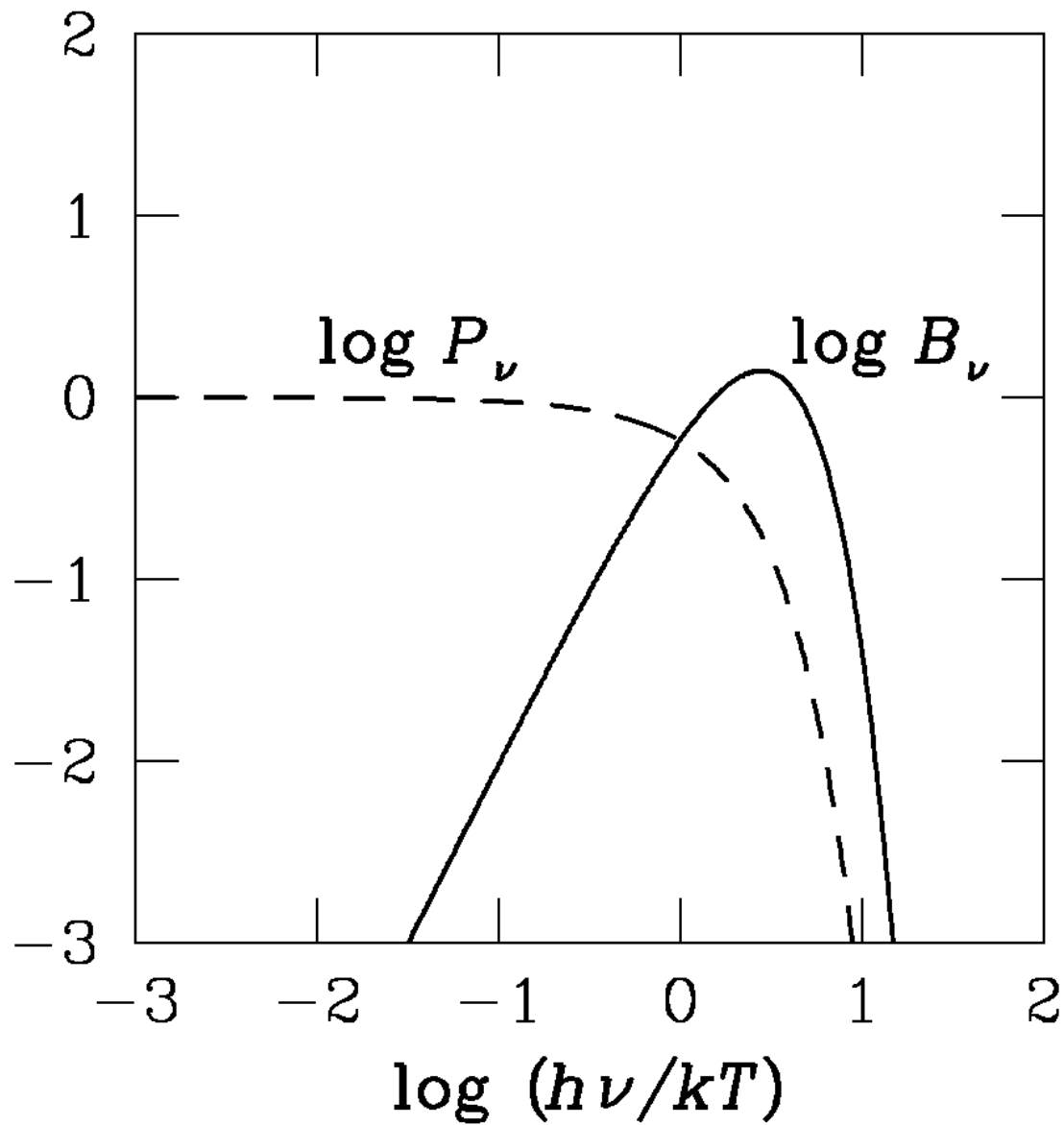
$$P_{\nu}(\nu, T) = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

if $h\nu \ll kT$ we can do a 1st order series expansion giving

$$P_{\nu}(\nu, T) = kT$$

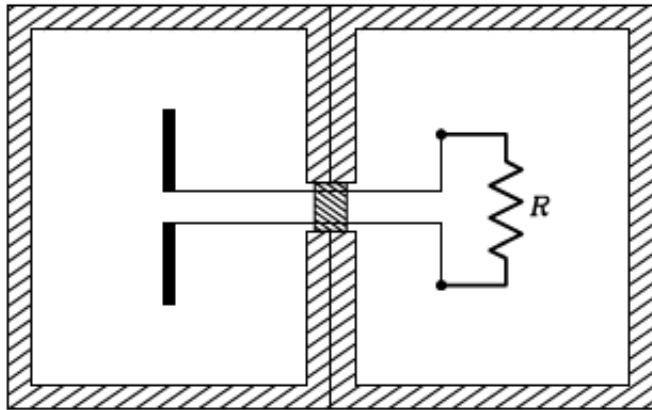
- We must use the QM correct version above about 400GHz for receivers at 20K

QM correction



Dipole example

- consider the equilibrium case of a dipole and noise source



A cavity in thermodynamic equilibrium at temperature T containing a resistor R is coupled to an antenna, also at temperature T , through a filter passing frequencies in the range ν to $\nu + d\nu$.

Back to the result we had

- If we have a small source (much less than beam) and the rest of the sky is blank...

$$P_v \approx \frac{\lambda^2 G_{(max)} k T_B \Omega_s}{4 \pi \lambda^2} = k T_B \frac{\Omega_s}{\Omega_A}$$

but antenna temperature $T_A = \frac{P_v}{k}$

so $T_A \approx T_B \frac{\Omega_s}{\Omega_A}$

Footnote on impedance (RF resistance)

$$Power = \frac{Voltage^2}{Impedance}$$

- Then free space (and a black hole) has a impedance of 377Ω
 $\sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$
- A half wave dipole in free space has a impedance of 73Ω
- commercial coaxial cables usually are 30-80 Ω
 - the common choices are 50Ω and 75Ω

Impedance matching

- For any transfer we get the optimum *power transfer* if impedances match
 - if receiver has low impedance it looks like a short circuit and so voltage across it is low
 - if receiver has higher impedance it has low current
 - optimum is for a matched system
- this is an example of the **Maximum Power Theorem**