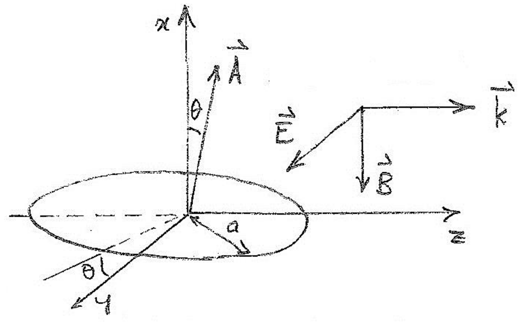


NASSP Honours Electrodynamics Part 1

Tutorial Problem Set 3: Electromagnetic Waves

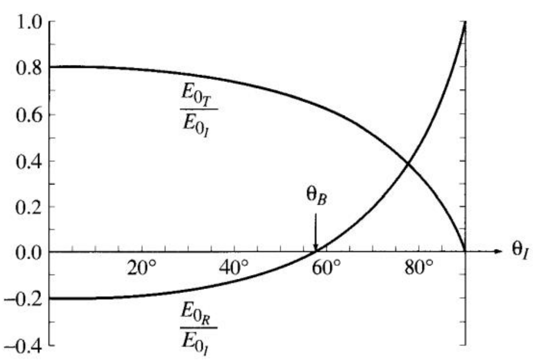
Q.1.* A plane electromagnetic wave travelling in air (treat as a vacuum) has $\vec{E} = E_0 e^{i(kz-\omega t)}\hat{y}$ where E_0 is real. A circular loop of radius a, with N turns, is located with its centre at the origin. The loop is orientated so that a diameter lies along the z axis and the plane of the loop makes an angle θ with the y axis (see sketch).
(a) Use Faraday's law to find the emf induced in the loop as a function of time.
You should assume $a \ll \lambda$, or $a \ll c/\omega$. (Why?)
(b) If this is an HF radio wave with frequency 6 MHz and an electric field amplitude of 50 mV/m, while the loop antenna has radius 0.5 m and 100 turns, and $\theta = 37°$, find the peak voltage induced.

Q.2. A particle with mass m and charge q is travelling with velocity $\vec{u}$ in the field of a plane electromagnetic wave in free space that is itself travelling in the z direction (take $\vec{E}$ to be in the x direction). (i) Find the force (vector!) on the particle. (ii) What does this become for the special case in which the particle is travelling in the same direction as the wave? (iii) What is the direction of the force? (iv) Under what conditions (if any) will the force vanish in this case?

Q.3.* A plane HF radio wave has a frequency of 5 MHz and the intensity of the wave is $20\ \mathrm{W.m^{-2}}$.
(a) What is the wavelength? (b) Find the amplitudes of $\vec{E}$ and $\vec{B}$ for this wave.

Q.4. Sea water has typical conductivity of 4.0 S/m and a relative permittivity of 81. Assume $\mu_r = 1$.
(a) Determine the skin depth in sea water for VLF radio waves with frequency 20 kHz.
(b) Determine the skin depth for VHF waves with frequency 200 MHz.
(c) Comment on the difference between the answers to (a) and (b) and what this means for radio communication with submarines.
[In case you didn't know, siemens (S) is the SI unit for conductance. 1 S = 1 Ω^{-1} (used to be "mho")]

Q. 5.* The refractive index of diamond is 2.42. Construct the graph analogous to the one on the right (which is for glass) for an air-diamond interface, for the case of the electric field in the plane of incidence. (Assume $\mu_1 = \mu_2 = \mu_0$.) In particular, calculate
(a) the amplitudes at normal incidence,
(b) Brewster's angle, and (c) the 'crossover' angle, at which the reflected and transmitted amplitudes are equal.
[Hint: Get $\alpha = (\cos\theta_t)/(\cos\theta_i)$ as a function of θ_i using Snell's law. Then you can evaluate r and t with the Fresnel equations for, e.g. $\theta_i = 0, 10°, 20°$, etc., in a spreadsheet.]



Q.6. Consider the electromagnetic wave whose electric field is given by
$$\vec{\boldsymbol{E}} = \hat{x}\, E_0 \cos(kz - \omega t) + \hat{y}\, E_0 \cos(kz - \omega t + \pi/2)\ .$$
(a) Describe completely the state of polarization of this wave.
(b) Write down the Jones vector which represents this state.
(c) Find a Jones vector orthogonal to the vector in (b)

Q.7. Mica is a birefringent material with $n_{\text{fast}} = 1.5940$ and $n_{\text{slow}} = 1.5990$. Determine the minimum ("zero order") thickness of a sheet of mica which would act as a quarter-wave plate for sodium yellow light ($\lambda_{\text{vac}} = 589$ nm).

* Solutions to Q.s 1, 3 & 5 to be handed in on Friday 14 March at 09:00.
Please attempt the other problems before the Friday Tutorial session.