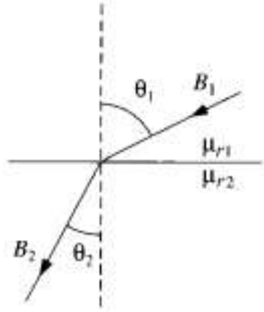
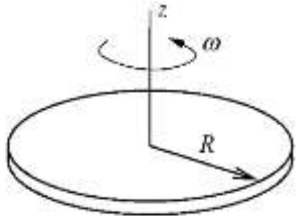
NASSP Honours Electrodynamics Part 1

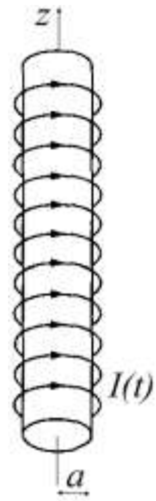
Tutorial Problem Set 2: Magnetic Materials, Time Varying Fields



Q.1. At the interface between one linear magnetic material and another (relative permeabilities μ_{r1} and μ_{r2}) the magnetic field lines bend (see Figure →).
Show that $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r1}}{\mu_{r2}}$, assuming there are no free currents at the interface.
Compare this with the result for the electric field (see ED-03, slide 11).



Q.2. The Zeeman Effect (spectral line splitting) observed in the spectra of sunspots reveals the existence of magnetic fields as large as 0.4 T.
In one model, this magnetic field is produced at the centre of a disc of electrons 10^7 m in radius rotating at an angular velocity $\omega = 0.03$ rad.s^{-1}.
The thickness of the disc is small compared to its radius.
(a) Show that the (assumed uniform) surface density of electrons required to achieve a magnetic field of 0.4 T in this model is about 10^{19} per square metre.
[Hint: Consider the current and resultant magnetic field due to an annulus (ring), width dr, of charge rotating with angular velocity ω and then integrate.
You can use the result $B = \mu_0 I/2r$ for the field at the centre of a circular loop.]
(b) Show that the current due to the whole rotating disc is about 3×10^{12} A.
(c) In what direction is the resulting magnetic field (with reference to the figure)?



Q.3. We assume the "quasistatic approximation" when we apply Faraday's law (with *changing* $\vec{B}$) but use the laws of magneto*statics* (e.g. Ampere's law) to determine $\vec{B}$, i.e. we assume the field does not vary too rapidly. (We can do this e.g. with solenoids in the lab, but *not* with EM waves.) As shown in the figure alongside, a long solenoid with radius a and n turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\phi}$ direction. Find the electric field (magnitude and direction) at a distance s from the axis, both inside ($s < a$) and outside ($s > a$) the solenoid, in the quasistatic approximation.
[You can use the result $B = \mu_0 n I$ (derived using Ampere's law!) for a long solenoid.]

Q.4. (a) An infinitely long cylinder carries a uniform magnetization parallel to its axis of $\vec{M} = k\hat{z}$, where k is a constant and $\hat{z}$ is the unit vector parallel to the cylinder axis.
Calculate the equivalent current densities $\vec{J}_e$ and $\vec{K}_e$. [Use cylindrical co-ordinates (s, ϕ, z) :
curl in cylindrical coords: $\nabla \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial\{s v_\phi\}}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$]
(b) Ignoring any applied field that caused the magnetization, find the magnetic field $\vec{B}$ due to $\vec{J}_e$ and $\vec{K}_e$ inside and outside the cylinder using Ampere's Law.
Comment on the direction and magnitude of $\vec{B}$ in relation to the magnetization.
(c) If the magnetization $\vec{M}$ was caused by an applied magnetic field intensity of $\vec{H} = 9k\,\hat{z}$, calculate the magnetic flux density $\vec{B}$ in the material, and relative permeability μ_r of the material (assume the material is linear and non-ferromagnetic).

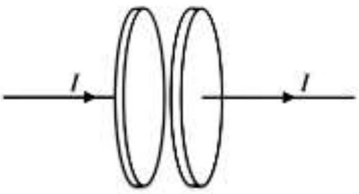
* Solutions to Q.5 (next page) to be handed in on Friday 7 March at 09:00 (please write your answers on the sheet). Please attempt the other problems before the Friday Tutorial session.

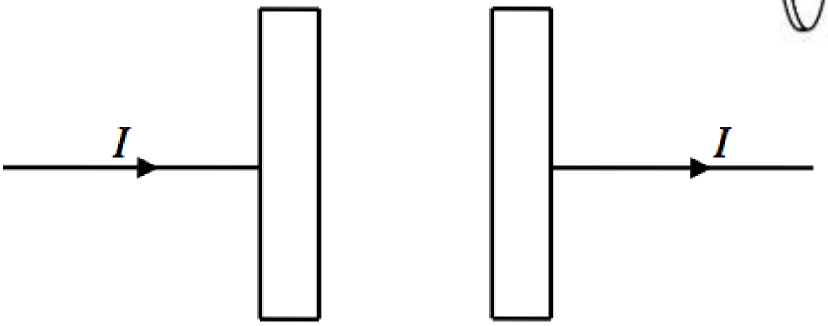
NAME: ____ TUT 2 HAND-IN

Q.5.* (a) The full Maxwell-Ampere Law in differential form is $\nabla \times \vec{B} = \mu_0 \vec{J}_f + \mu_0 \varepsilon_0 \, \partial \vec{E} / \partial t$.

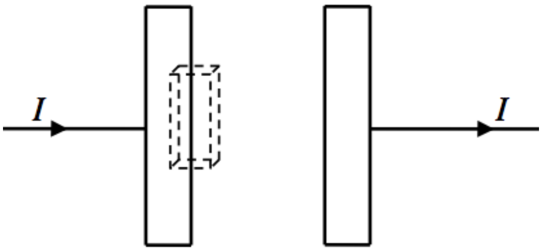
Rewrite this equation in integral form using Stokes' theorem. Show your steps.

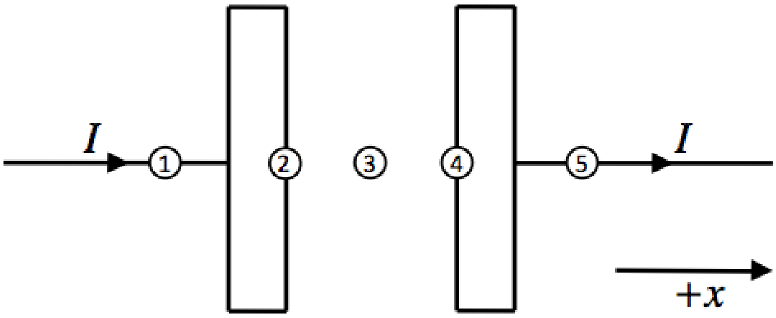
(b) Consider a capacitor in the process of charging up. The circular plates have radius R, area $A = \pi R^2$, and are so close together that fringe effects can be ignored. A current I is flowing in the long, straight wires (which are assumed to be very good conductors). Sketch the $\vec{E}$-field between the plates in the diagram below, which shows the plates edge-on. Is $\partial E / \partial t$ between the plates *positive*, *negative* or *zero*?





(c) Consider the surface of an imaginary volume (dashed lines, at right) that partly encloses the left capacitor plate. For this closed surface, is the *total* flux of the current density $\oint \vec{J} \cdot d\vec{a}$ *positive*, *negative* or *zero*? Briefly explain your answer.



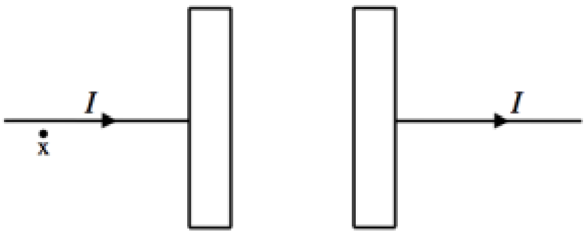


(d) For each of the five points in the diagram above (labeled 1-5), fill out the table below to indicate whether the quantity in each row is *positive* (+), *negative* (−) or *zero* (0) at that point. Be sure your answers are consistent with the continuity equation $\nabla \cdot \vec{J} + \partial\rho/\partial t = 0$.

	1	2	3	4	5
$\partial\rho/\partial t$					
$\nabla \cdot \vec{J}$					

Explain briefly how your entries in each column are consistent with charge conservation.

(e) Use the Maxwell-Ampere law to derive a formula for the magnetic field at the point "*x*" indicated in the diagram, a short distance r from the wire. [First year revision!]

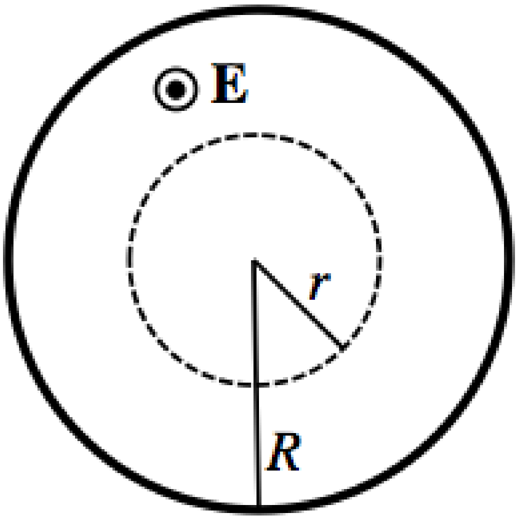


(f) Use Gauss' law in integral form to derive a formula for the electric field between the capacitor plates. Be specific about the Gaussian surface used, and write the answer in terms of σ, the charge per unit area on the plates. [First year revision!]

(g) Write down a relationship between the current I flowing in the wires and the rate of change of the charge per unit area on the plates $\partial\sigma/\partial t$.

(h) Use this information to find a relationship between the current I flowing in the wires and the rate of change of the electric field between the plates $\partial E/\partial t$.

(i) Use the diagram at right to indicate the direction of the magnetic field everywhere along the dashed loop, at a distance r from the center of the plates, shown here face-on.



(j) Use the Maxwell-Ampere Law to derive a formula for the magnetic field at a distance $r < R$ from the center of the plates. Express your final answer in terms of the current I flowing in the wires.

(k) Use the Maxwell-Ampere Law to derive a formula for the magnetic field at a distance $r > R$ from the center of the plates. Express your final answer in terms of the current I flowing in the wires.

(l) Consider the two line integrals of the magnetic field $\oint_{L1} \vec{B} \cdot d\vec{l}$ and $\oint_{L2} \vec{B} \cdot d\vec{l}$ for the loops $L1$ and $L2$ shown in the diagram below; both loops have the same radius r. How do the values of these two loop integrals compare? Is one larger than the other, or are they equal in magnitude? Explain your answer using the formulas you derived.