# NASSP Honours Electrodynamics Part 1

## Revision Example Set 1: Electrostatics and Magnetostatics : SOLUTIONS

Q.1. Potential $V(x,y,z) = x^2 + 2xy + yz^3$ and $\vec{E}$ is minus grad $V$ :

$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = -\,(\,2x + 2y\,,\; 2x + z^3,\; 3yz^2\,)$

Q.2. Gauss's Law: $\nabla \cdot \vec{E} = \rho/\varepsilon_0$ so $\rho = \varepsilon_0 \nabla \cdot \vec{E} = \varepsilon_0 \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$

Here $\vec{E} = (x^2+y^2)\,\hat{x} + z\,\hat{y} + (x^2+y^2+z^2)\,\hat{z}$

So $\rho = \varepsilon_0\,(2x + 0 + 2z) = 2\varepsilon_0(x + z)$

Q.3. $\nabla \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z},\; \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x},\; \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = 0$ for electrostatic field.

(i) $\vec{E} = (x^2 + y)\hat{x} + (y^2 + z)\hat{y} + (z^2 + x)\hat{z}$

$\nabla \times \vec{E} = (0 - 1\;,\; 0 - 1\;,\; 0 - 1) = -(1,1,1) \neq 0$ NO, not electrostatic

(ii) $\vec{E} = y\hat{x} + x\hat{y}$

$\nabla \times \vec{E} = (0 - 0\;,\; 0 - 0\;,\; 1 - 1) = (0,0,0) = 0$ YES, could be electrostatic

Q.4. In a region containing no charge $V$ must satisfy Laplace's equation: $\nabla^2 V = 0$

For $V = A(x^2+y^2+z^2)$ [ $\partial V/\partial x = 2x$ , $\partial V/\partial y = 2y$ , $\partial V/\partial z = 2z$ ]

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = A(2 + 2 + 2) = 6A \neq 0$$

Hence for this $V$ the region must contain charge.

Q.5. $\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$ for a magnetostatic field.

(i) For $\vec{B} = (x^2+y^2)\,\hat{x} + x\,\hat{y} - 2xz\,\hat{z}$

$\nabla \cdot \vec{B} = 2x + 0 - 2x = 0$ YES

(ii) For $\vec{B} = (x^2+y^2z^2)\hat{y} + (y^2-z^2)\hat{z}$

$\nabla \cdot \vec{B} = 0 + 2yz^2 - 2z \neq 0$ NO

Q.6. $\vec{A} = 3(y^2\hat{x} + x^2\hat{y}\,)$ and $\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y^2 & 3x^2 & 0 \end{vmatrix}$

$$= \left(\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}3x^2\right)\hat{x} + \left(\frac{\partial}{\partial z}3x^2 - \frac{\partial}{\partial x}0\right)\hat{y} + \left(\frac{\partial}{\partial x}3x^2 - \frac{\partial}{\partial y}3y^2\right)\hat{z}$$

$$= (0 - 0)\,\hat{x} + (0 - 0)\,\hat{y} + (6x - 6y)\,\hat{z} = 6(x - y)\,\hat{z}$$

Q.7. Ampere's law: $\nabla \times \vec{B} = \mu_0 \vec{J}$ so $\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy + z^2 & 0 & x^2 + y^2 \end{vmatrix}$

$$= \left(\frac{\partial}{\partial y}(x^2+y^2) - \frac{\partial}{\partial z}0\right)\hat{x} + \left(\frac{\partial}{\partial z}(xy+z^2) - \frac{\partial}{\partial x}(x^2+y^2)\right)\hat{y} + \left(\frac{\partial}{\partial x}0 - \frac{\partial}{\partial y}(xy+z^2)\right)\hat{z}$$

$$= (2y - 0)\,\hat{x} + (2z - 2x)\,\hat{y} + (0 - x)\,\hat{z} = 2y\,\hat{x} + 2(z - x)\,\hat{y} - x\,\hat{z}$$

Q.8. We have to solve the 1D Poisson's eqn. $\nabla^2 V = \frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0}$ with $\rho$ constant.

Integrate: $\frac{dV}{dx} = -\frac{\rho}{\varepsilon_0}x + A$ ($A$ = constant of integration)

Integrate: $V = -\frac{\rho}{\varepsilon_0}\frac{x^2}{2} + Ax + B$ ($B$ = constant of integration)

Boundary conditions: (1) $V = 0$ at $x = 0$. Substitute $\Longrightarrow B = 0$

(2) $V = V_0$ at $x = a$. Substitute $\Longrightarrow V_0 = -\frac{\rho}{\varepsilon_0}\frac{a^2}{2} + Aa \Longrightarrow A = \frac{V_0}{a} + \frac{\rho a}{2\varepsilon_0}$

Thus final solution is $V = -\frac{\rho}{\varepsilon_0}\frac{x^2}{2} + \left(\frac{V_0}{a} + \frac{\rho a}{2\varepsilon_0}\right)x$ or $V = \frac{\rho x}{2\varepsilon_0}(a - x) + \frac{V_0 x}{a}$ or...

Now $E_x = -\frac{dV}{dx} = \frac{\rho}{\varepsilon_0}x - A$ from the first integration,

i.e. $E_x = \frac{\rho}{\varepsilon_0}x - \frac{V_0}{a} - \frac{\rho a}{2\varepsilon_0}$ or $E_x = \frac{\rho}{\varepsilon_0}(x - \frac{a}{2}) - \frac{V_0}{a}$ (several ways to write both)