

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad c = 3.00 \times 10^8 \text{ m/s} \quad e = 1.60 \times 10^{-19} \text{ C}$$

Maxwell's Equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu \mathbf{J} + \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

Vector Identities:

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$$

$$\nabla (fg) = f \nabla g + g \nabla f$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{C}$$

$$\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) - \mathbf{A} \cdot (\nabla f)$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) + (\nabla f) \times \mathbf{A}$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\nabla \cdot \mathbf{B}) \mathbf{A} - (\nabla \cdot \mathbf{A}) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$$

$$\nabla^2 f = \nabla \cdot \nabla f$$

$$\nabla \times \nabla f = 0$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \cdot \mathbf{r} = 3$$

$$\nabla \times \mathbf{r} = 0$$

$$\nabla r = \frac{\mathbf{r}}{r}$$

$$\nabla \frac{1}{r} = -\frac{\mathbf{r}}{r^3} \quad (\mathbf{S} \cdot \nabla) \mathbf{r} = \mathbf{S}$$

Integral Relations:

Divergence Theorem

$$\int_V (\nabla \cdot \mathbf{A}) dV = \oint_S \mathbf{A} \cdot d\mathbf{a}$$

$$\oint_S f d\mathbf{a} = \int_V (\nabla f) dV$$

Stokes Theorem

$$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint_C \mathbf{A} \cdot d\vec{\ell}$$

$$\oint_C f d\vec{\ell} = \int_S d\mathbf{a} \times \nabla f$$

$$\int_V (\nabla \times \mathbf{A}) dV = \oint_S (\hat{\mathbf{n}} \times \mathbf{A}) da$$

$$\text{Poisson's Equation } \nabla^2 \varphi(\mathbf{r}) = f(\mathbf{r}')$$

Solution

$$\varphi(\mathbf{r}) = -\frac{1}{4\pi} \int_{V'} \frac{f(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' \quad [f(\infty) \rightarrow 0]$$

Cartesian Coordinates (x, y, z) :

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

Cylindrical Coordinates (r, ϕ, z) :

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r} \frac{\partial f}{\partial \phi}, \frac{\partial f}{\partial z} \right)$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r}(rA_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla \times \mathbf{A} = \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}, \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}, \frac{1}{r} \frac{\partial}{\partial r}(rA_r) - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right)$$

Spherical Coordinates (r, θ, ϕ) :

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r} \frac{\partial f}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \right)$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \left(\frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta}(\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right], \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi) \right], \frac{1}{r} \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta} \right] \right)$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$