NASSP Honours - Electrodynamics
First Semester 2014

Electrodynamics Part 1
12 Lectures

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Course Summary

- **Aim:** To provide a foundation in electrodynamics, primarily electromagnetic waves.
- **Content:** 12 Lectures and 3 Tutorials
  - Maxwell's equations: Electric fields, Gauss's law, Poisson's equation, continuity equation, magnetic fields, Ampere's law, dielectric and magnetic materials, Faraday's law, potentials, gauge transformations, Maxwell's equations
  - Electromagnetic waves – plane waves in vacuum and in media, reflection and transmission, polarization
  - Relativity and electromagnetism

Syllabus

1. Revision: Electrostatics
2. Revision: Currents and Magnetostatics
3. Dielectric and Magnetic Materials
4. Time Varying Systems; Maxwell's Equations
5. Plane Electromagnetic Waves
6. Waves in Media
7. Polarization
8. Relativity and Electrodynamics

Sources

- D.J. Griffiths, *Introduction to Electrodynamics,* 3rd ed., 1999 (or 4th edition, 2012)
- J.D. Jackson, *Classical Electrodynamics*, 3rd ed., 1999 (earlier editions non-SI)
- G.L. Pollack & D.R. Stump, *Electromagnetism*, 2002
- P. Lorrain & D.R. Corson, *Electromagnetic Fields and Waves*, 2nd ed., 1970 (out of print)
- R.K. Wangsness, *Electromagnetic Fields*, 2nd ed., 1986

First Year EM: (a) Electrostatics

1. Coulomb's law: force between charges

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q\,q\prime|}{r^2} \qquad \text{(magnitude)}$$

2. Define electric field $\vec{E}$ as force per unit charge
3. Gauss's Law $\oint \vec{E}\,.\,d\vec{A} = \Phi = Q_{enc}/\varepsilon_0$

   (derived from electric field of a point charge)
4. Define potential V as PE per unit charge

   p.d. $V_a - V_b = \int_a^b \vec{E}.\,d\vec{l}$, static $\vec{E}$ conservative
5. Potential gradient: $E_x = -{}^{dV}/_{dx}$ $\left(\vec{E} = -\nabla V\right)$

   \+ Capacitors etc…

(b) Magnetostatics

6. Define magnetic field $\vec{B}$ by force on moving charge:
   $\vec{F} = q\vec{v} \times \vec{B}$.
7. Magnetic flux: $\Phi = \int \vec{B} \cdot d\vec{A}$
8. $\vec{B}$ produced by moving charge: $\vec{B} = kq\frac{\vec{v}\times\hat{r}}{r^2}$
   Define $\mu_0\varepsilon_0 = {}^{1}/_{c^2}$
9. Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$ due to current element
10. $\vec{B}$ of long straight wire = circles around wire.
    => Net flux through closed surface $\oint \vec{B} . d\vec{A} = 0$
11. Ampere's Law: $\oint \vec{B} . d\vec{l} = \mu_0 I$
    (Biot-Savart and Ampere used to find $\vec{B}$ for given I.)

(c) Induced Electric Fields

12. emf $\mathcal{E} = \int \vec{E}_n \cdot d\vec{l}$ ("non-electrostatic E")
13. Faraday's Law (with Lenz's Law (– sign)) relating induced emf to flux: $\mathcal{E} = -{}^{d\Phi}/_{dt}$
14. Hence induced $\vec{E}$:

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{A}$$

This constitutes 'pre-Maxwell' electromagnetic theory, i.e. ca.1860.

Electric and Magnetic Fields – The Lorentz Force

- The electric field $\vec{E}$ is defined as the "force per unit charge" i.e. $\vec{F} = q\vec{E}$
- The magnetic field is defined in terms of the force on a moving charge: $\vec{F} = q\vec{v} \times \vec{B}$
- In general electric and magnetic fields, a (moving) charge experiences forces due to both fields, and

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

The Lorentz Force

In fact we can show that $\vec{B}$ arises from the application of the relativistic (Lorentz) transformations to the force between two charges in a moving frame. (Later!)

Gauss's Law in Integral Form

Consider charges as sources of $\vec{E}$.

The field is represented by "field lines" whose "density" represents the intensity of the field.

Electric flux through a surface $\Phi_E = \int_S \vec{E} \cdot d\vec{a}$ is the "total number of field lines" passing through the surface.

Gauss's Law relates the electric flux through a closed surface to the total charge enclosed:

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_{\mathcal{V}} \rho \, d\mathcal{V}$$

Faraday's Law in Integral Form

- The familiar form of Faraday's Law is written $\mathcal{E} = -\frac{d\Phi_m}{dt}$ (negative sign from Lenz's Law)
- Now the emf can be considered as the line integral of the induced electric field, i.e. $\mathcal{E} = \oint_c \vec{E} \cdot d\vec{l}$ (or $\vec{E}$ is the "*potential gradient*")
- Φ_m is of course the magnetic flux or the integral of $\vec{B}$ over the surface S bounded by C: $\int_s \vec{B} \cdot d\vec{a}$
- Thus in integral form

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{a}$$

Faraday's Law

Static Fields: Scalar Potential

$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{a}$ Faraday's Law

For **static fields** ($\frac{d}{dt} = 0$), Faraday's Law gives

$$\oint_c \vec{E} \cdot d\vec{l} = 0$$

"**The electrostatic field is conservative**"

This implies that the electrostatic field is described by a **scalar potential** V which **depends only on position**.

In 1-D $\vec{E} = -\frac{dV}{dx}$ or in 3-D $\vec{E} = -\nabla V$

since the curl of the gradient of any scalar is zero.

Solenoidal Magnetic Fields

- Fundamental Law: Unlike electric fields, **magnetic fields do not have a beginning or an end** – there are *no magnetic monopoles*.
- For any closed surface the number of magnetic field lines entering must equal the number leaving.
- i.e. the **net magnetic flux through any closed surface is zero**:

$$\oint \vec{B} \cdot d\vec{a} = 0$$

Sometimes called "Gauss's Law for magnetic fields" (✘).
It implies that $\vec{B}$ is given by a **vector potential** (see later).

Ampere's Law + Displacement Current

- Familiar form of Ampere's Law: $\oint \vec{B} \,.\, d\vec{l} = \mu_0 I$
- $I = \int \vec{J} \cdot d\vec{A}$ (current density $\vec{J} = nq\vec{v}$), is the current enclosed by the closed loop around which the line integral of $\vec{B}$ is taken
- This clearly shows current I (i.e. moving charges) as a source of $\vec{B}$.
- Maxwell: this eqn. is incomplete. $\vec{B}$ also generated by changing $\vec{E}$ (displacement current):

$$I_D = \varepsilon_0 \frac{d}{dt} \int_s \vec{E} \cdot d\vec{a}$$

Modified Ampere's Law

- The total current through any surface is then the conduction current + displacement current i.e. $I_{\text{total}} = I + I_D$
- Ampere's Law should read

$$\oint \vec{B} \,.\, d\vec{l} = \mu_0 I_{\text{total}} = \mu_0 (I + I_D) \qquad \text{or}$$

$$\oint \vec{B} \,.\, d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt} \int_s \vec{E} \cdot d\vec{a}$$

$$\oint \vec{B} \,.\, d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_s \vec{E} \cdot d\vec{a}$$

- In e.g. a wire (a good conductor), the displacement current is negligible and so the total current is just I,
- In e.g. a capacitor space, I is zero and the total current is just the displacement current due to the changing E field.

Maxwell's Equations in Integral Form

- The 4 fundamental equations of electromagnetism:

1. Gauss's Law: $\oint_s \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} \int_{\mathcal{V}} \rho \, d\mathcal{V}$
2. Faraday's Law: $\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{a}$
3. "No Monopoles": $\oint \vec{B} \cdot d\vec{a} = 0$
4. Ampere's Law with displacement current:
   $$\oint \vec{B} \, . \, d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_s \vec{E} \cdot d\vec{a}$$

- The fields which are the solutions to these equations are coupled through the $\frac{d}{dt}$ terms.

Maxwell's Equations continued

- In the static case (charges not moving, constant currents) Maxwell's equations are decoupled.
- This allows us to study electrostatics and magnetostatics separately, as in first year.
- The equations are then

1. Gauss's Law: $\oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0}$
2. Faraday's Law: $\oint_C \vec{E} \cdot d\vec{l} = 0$
3. "No Monopoles": $\oint \vec{B} \cdot d\vec{a} = 0$
4. Ampere's Law: $\oint \vec{B} \,.\, d\vec{l} = \mu_0 I = \int \vec{J} \cdot d\vec{a}$