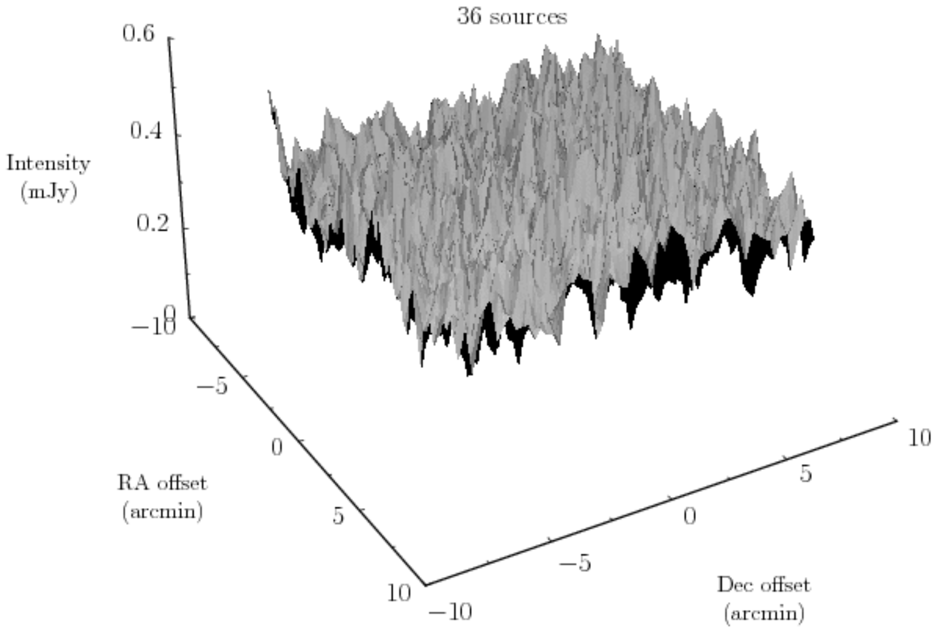
The Power of N



$\mathrm{Stackin}^{n^{n^{n^{n^{n^{n^{n}}}}}}}$

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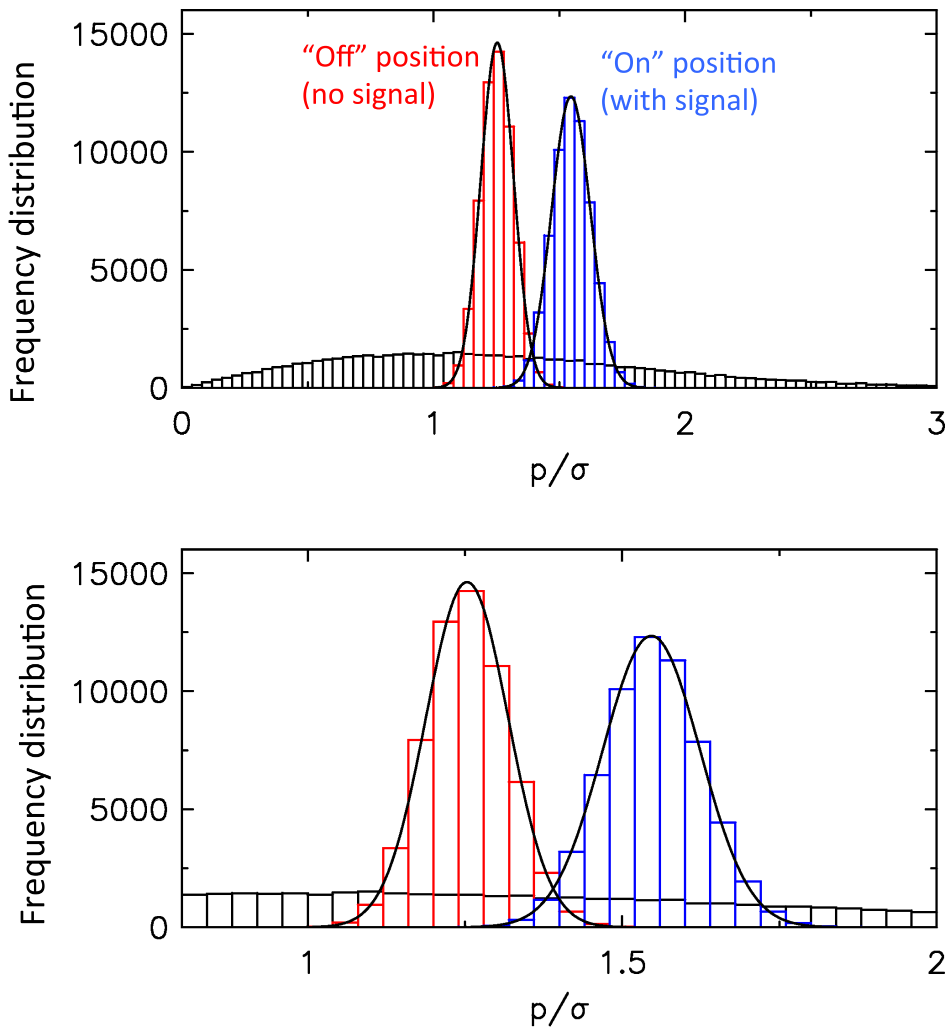
Thanks to the ATLAS team for providing the data

Introduction to Stacking



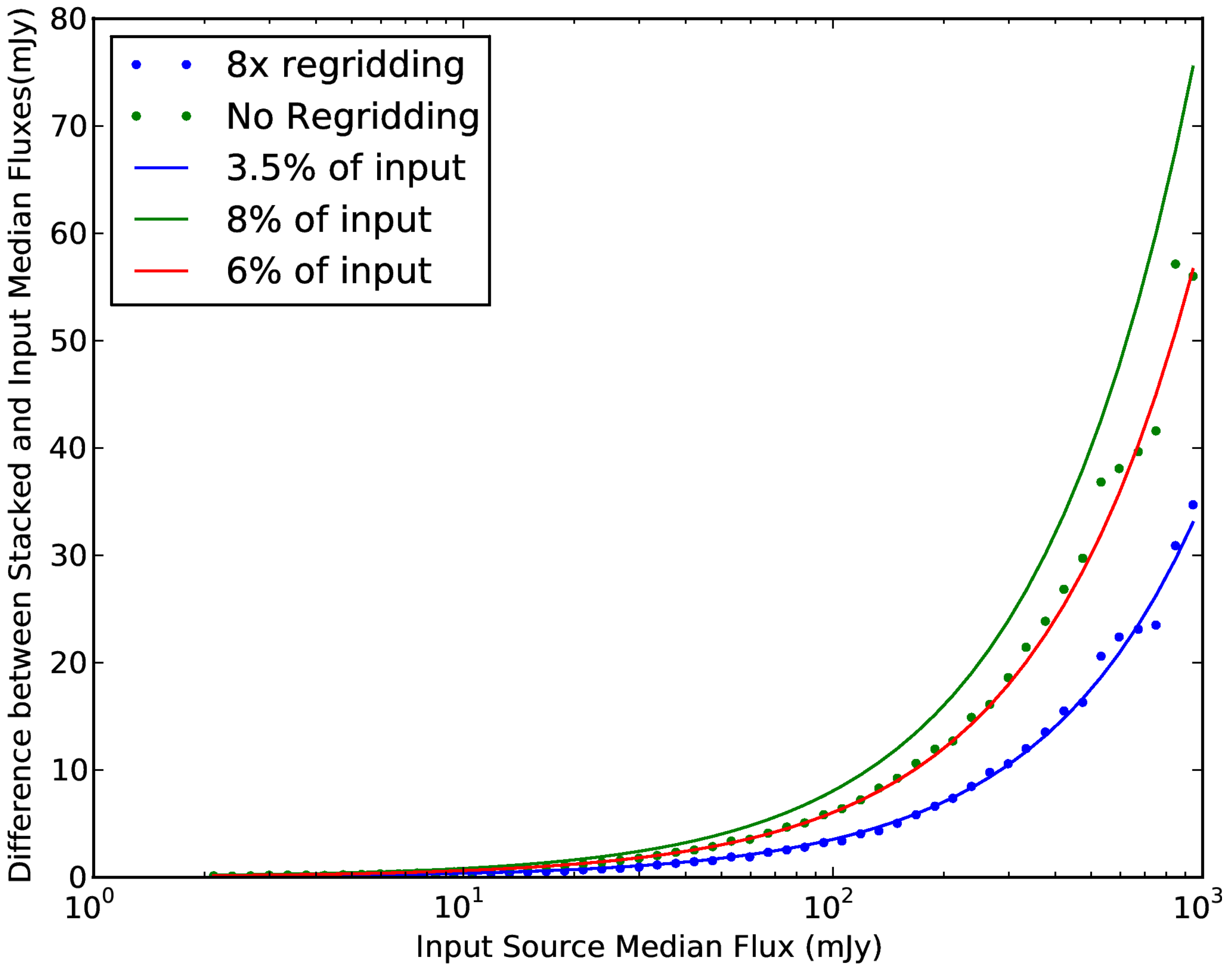
Sample mean, median, … for a large number of sources, each (or most, or many) below the formal detection threshold of a survey.

Target positions from another survey.

$\sqrt{N}$ improvement of the noise demonstrated for $N \lesssim 10^5$ in radio surveys.

Main challenge: keep systematic errors below the level of the formal statistical errors.

It Is Important to Over-sample The Beam For Target Position Alignment

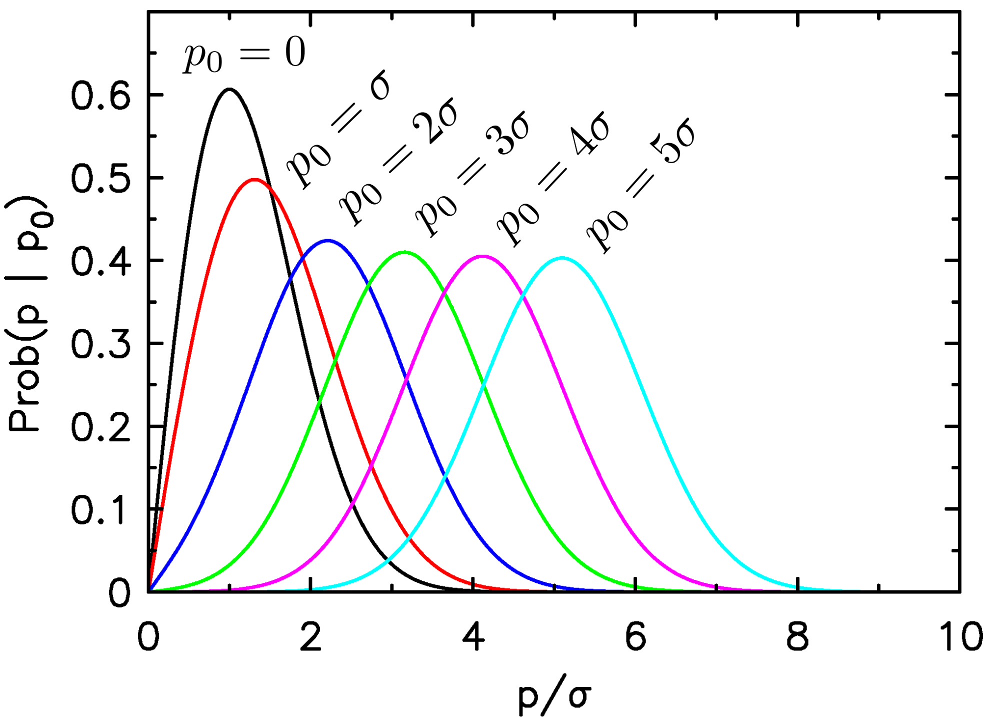


Alignment to nearest pixel introduces a downward systematic error depending on sampling.

Graph shows the **difference in median stacked intensity from true sample median** for a simulated survey images with Nyquist sampling of the beam.

Figure prepared by Ben Keller.

Statistics of Polarized Intensity



Rice distribution also applies to visibility amplitude!

Polarized intensity

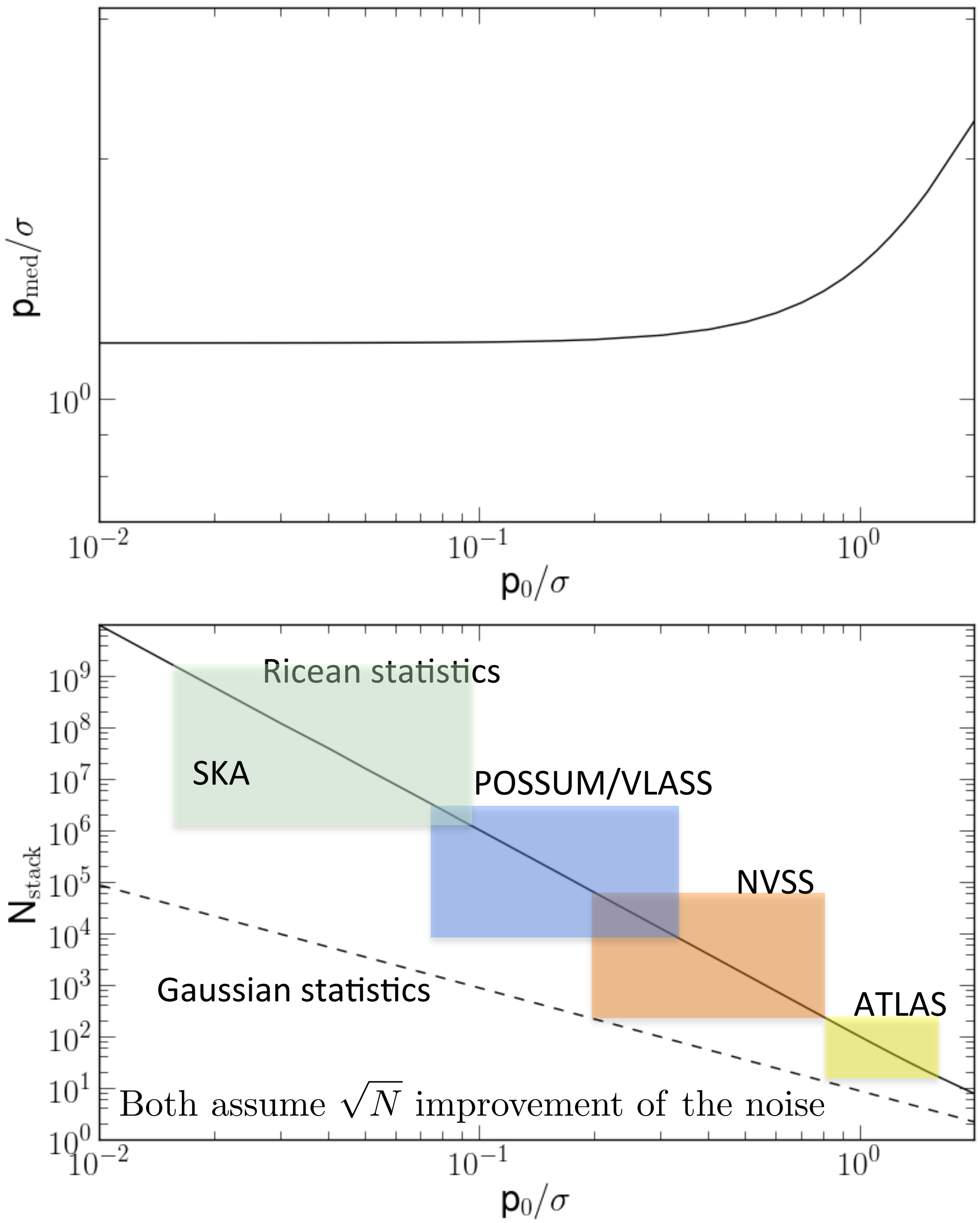
$$p = \sqrt{Q^2 + U^2}$$

If Q and U have independent Gaussian noise, then the statistics of p are given by the Rice (1945) distribution

$$F(\frac{p}{\sigma}|\frac{p_0}{\sigma}) = \frac{p}{2\pi\sigma} e^{-\frac{p^2+p_0^2}{2\sigma^2}} J_0(i\frac{pp_0}{\sigma^2})$$

Does <u>not</u> apply to polarized intensity derived from Faraday RM Synthesis (George et al. 2011, Macquart et al. 2012)

Challenge of Stacking Polarized Intensity



At small signal-to-noise ratio, the derivative

$$\frac{dp_{\text{med}}}{dp_0} \rightarrow 0$$

The number of sources that must be stacked to secure a 3-sigma detection in p_{med} as a function of true signal to noise ratio per source:

$$N = 100\left(\frac{p_0}{\sigma}\right)^{-4}$$

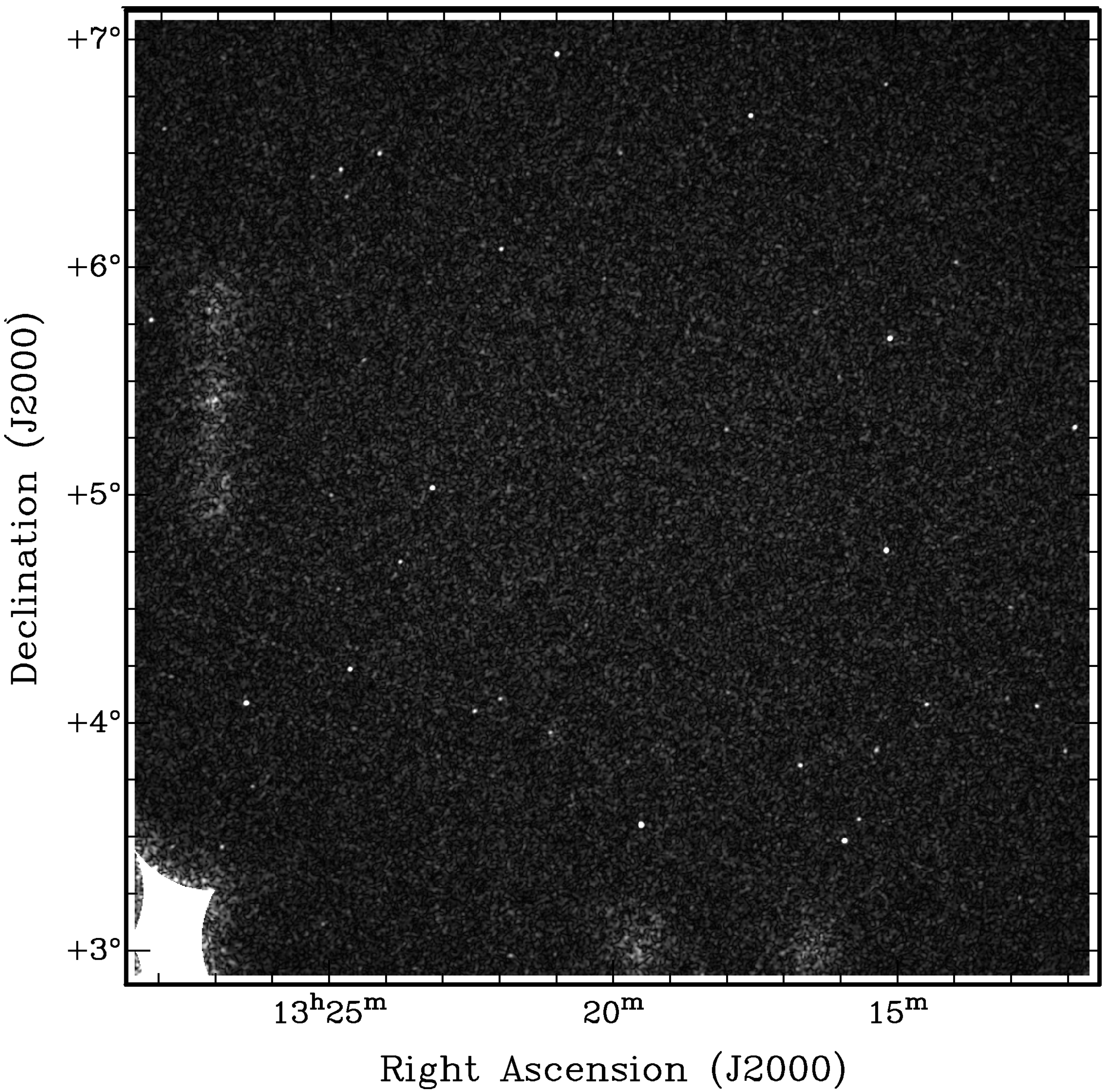
<u>If you need to stack to detect Stokes I, there is no sample today that is large enough to secure a detection in polarization.</u>

This does <u>not</u> mean that one should only stack wide surveys.

Corollaries

1. Polarization stacking that detects **1% polarization** of a **1 mJy** source at the 3-sigma level is within reach with POSSUM and VLASS combined with a deep, wide, target catalog of 10^5 to 10^6 galaxies. Derive depolarization factor of spiral galaxy disks between S band and L band.

2. In case of a non-detection in stacked p, it is usually **not** possible to derive a meaningful upper limit to the sample median $p_{0,med}$

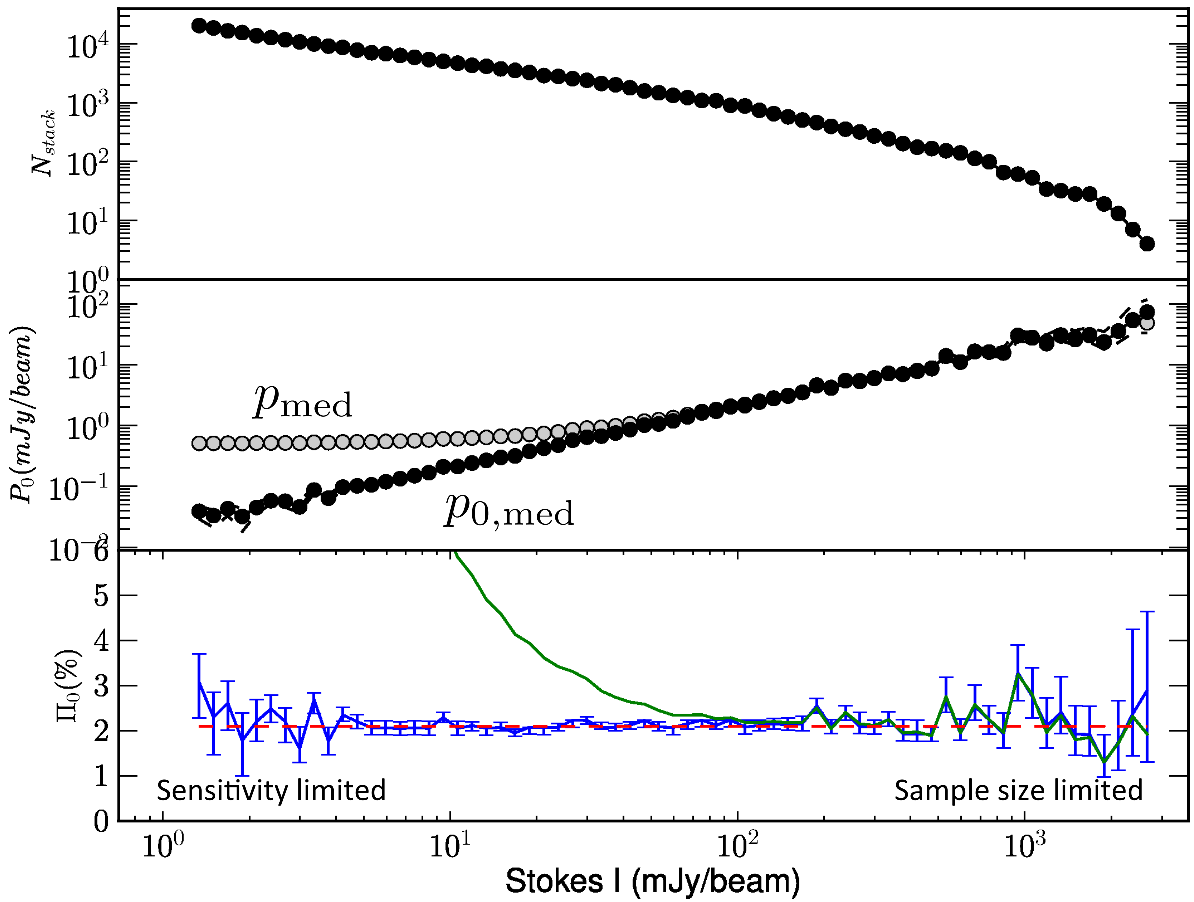


Simulated NVSS-like survey for testing of polarization stacking. George et al. (2011), Stil et al. (2014).

Simulated survey included 600 independent fields.

Polarization Bias Correction

Simulation



Stack of polarized intensity of simulated NVSS-like survey.

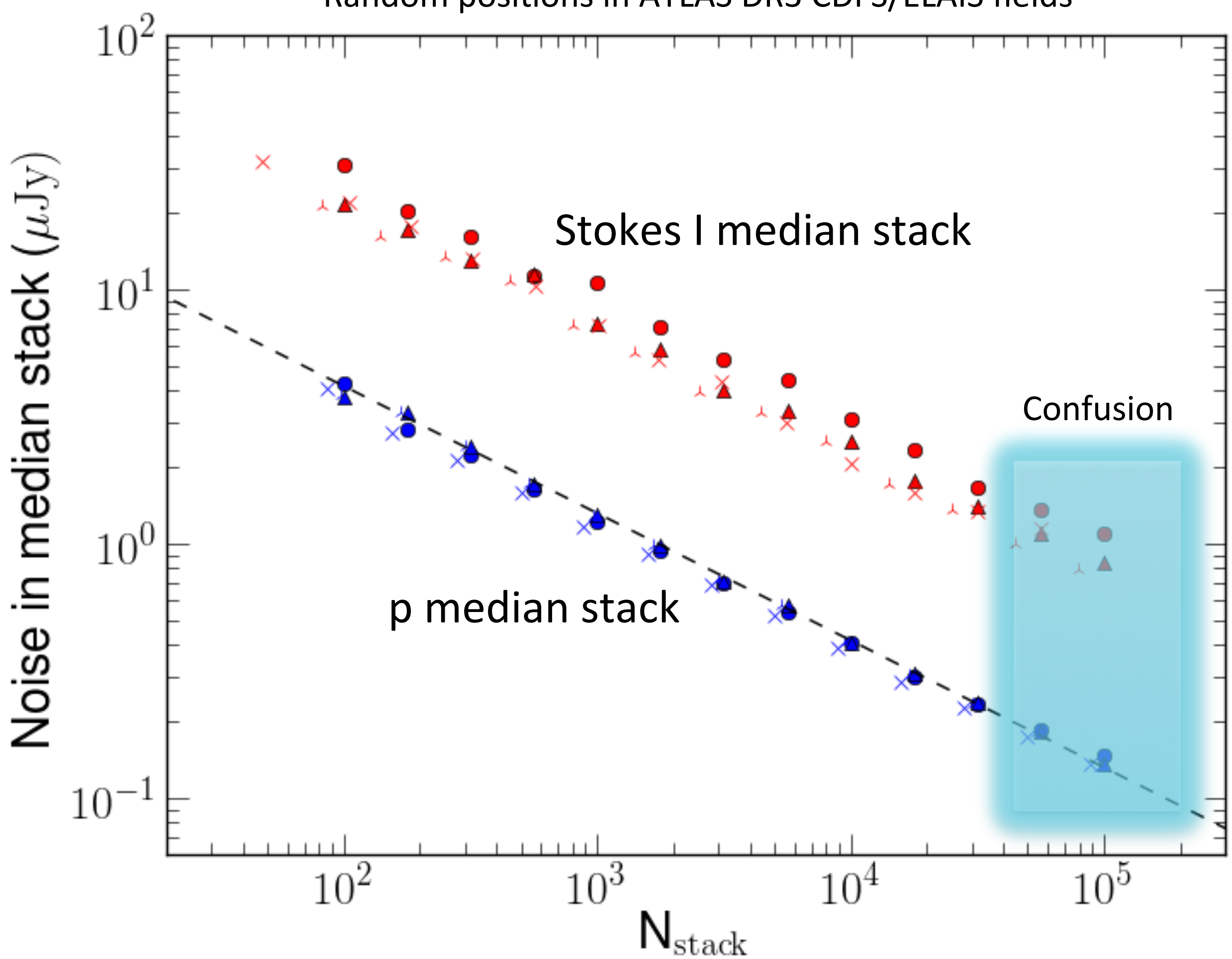
Monte-Carlo simulations recover the true median of the sample $p_{0,med}$ down to the detection limit in total intensity.

Error bars include all knowledge about noise statistics, sample size, etc.

For a description of Monte Carlo simulations that correct for polarization bias, see Stil et al. (2014)

Noise in ATLAS DR3 Stacked I and PI
Does Rejection of Noisy Regions Help?

Random positions in ATLAS DR3 CDFS/ELAIS fields



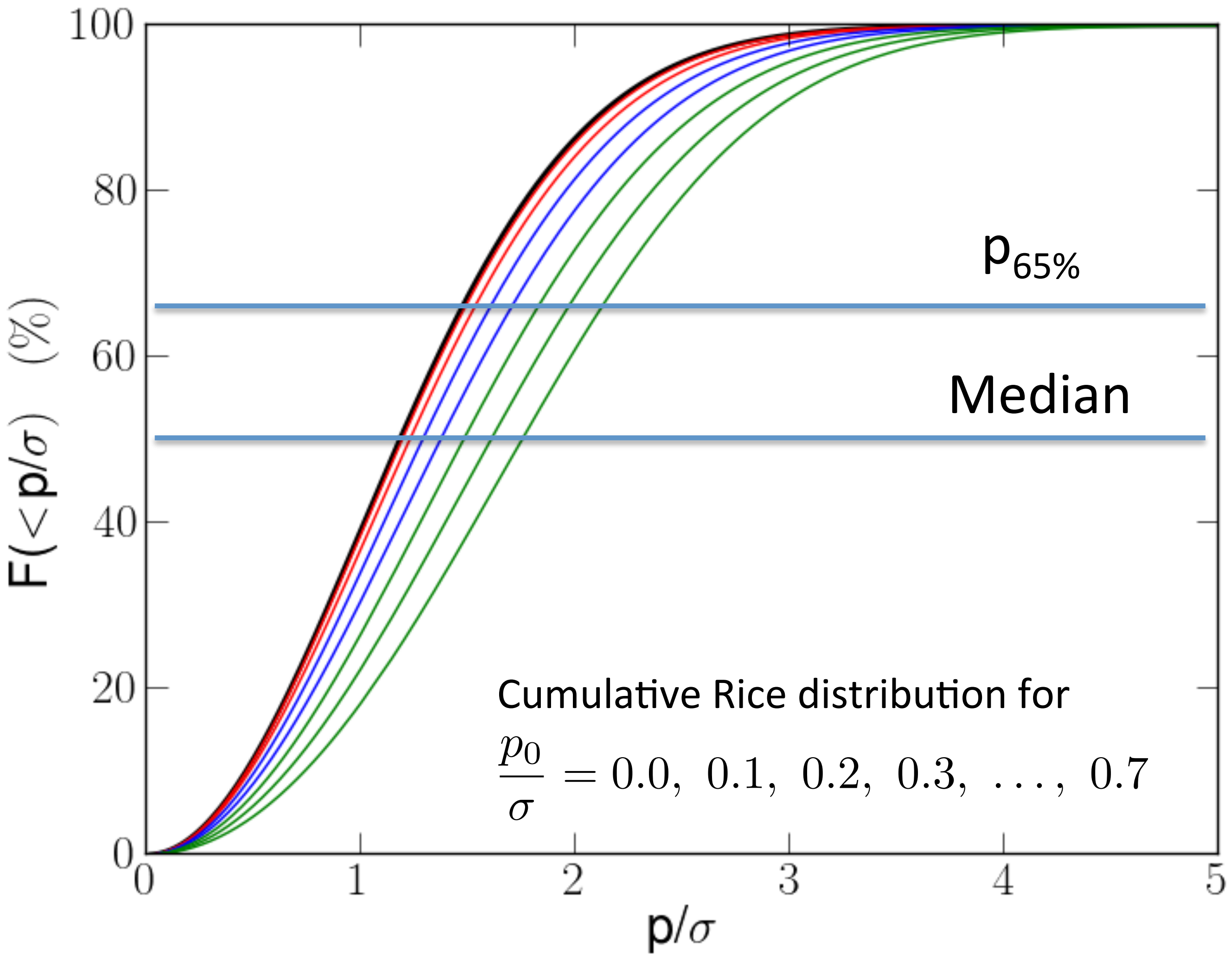
Find consistent $\sqrt{N}$ improvement of noise for **random positions**. Stacking depth eventually limited by confusion.

Reduction of noise by rejecting areas most affected by side lobes is negated by smaller sample size. Little effect in p.

Noise in Stokes Q and U is nearly Gaussian in off-source pixels.

Optimizing Sensitivity in Stacked Polarized Intensity
Percentile Stacking of Polarized Intensity

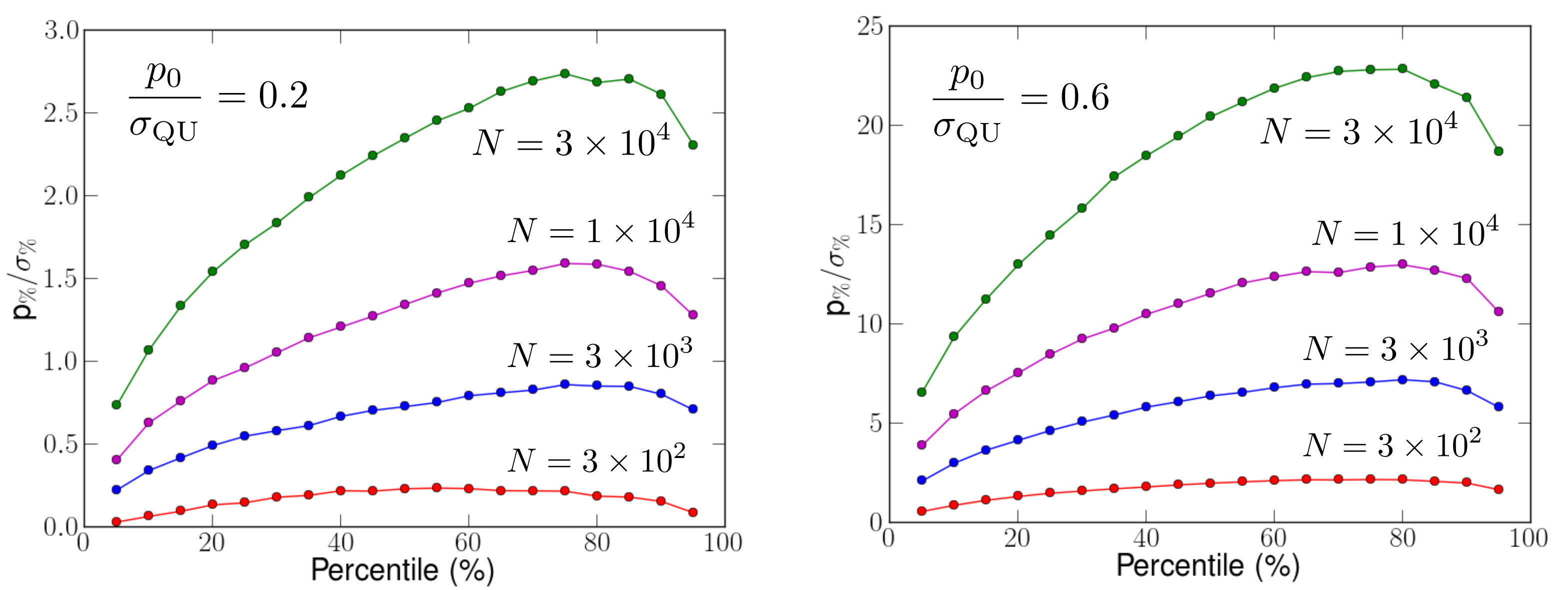
Cumulative Rice Distribution



Median Stacking Does Not Optimize S/N in Polarized Intensity

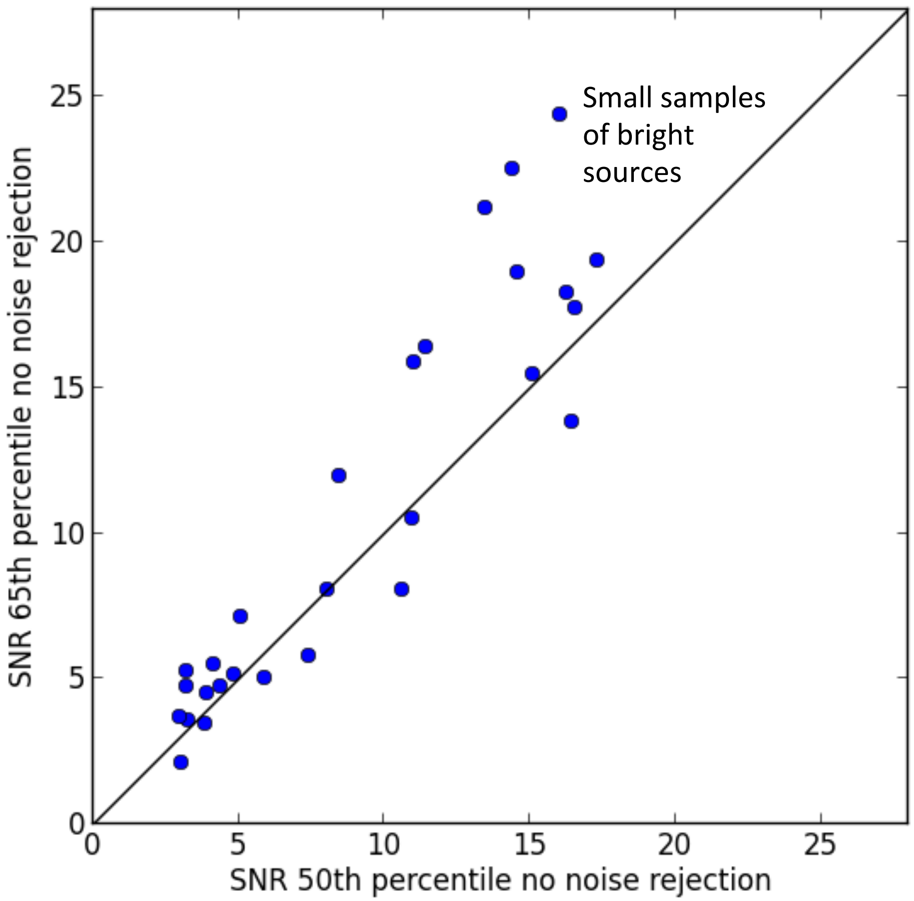
What is the realized signal-to-noise ratio in the stacked image as a function of the percentile?

Trade-off: Using higher percentiles can increase sensitivity to outliers and to the shape of the distribution of fractional polarization of the sample.



Curves derived from Monte-Carlo realizations of stacking percentiles 5% to 95%

Actual S/N Improvement in ATLAS Stacked Polarized Intensity 65th percentile versus median

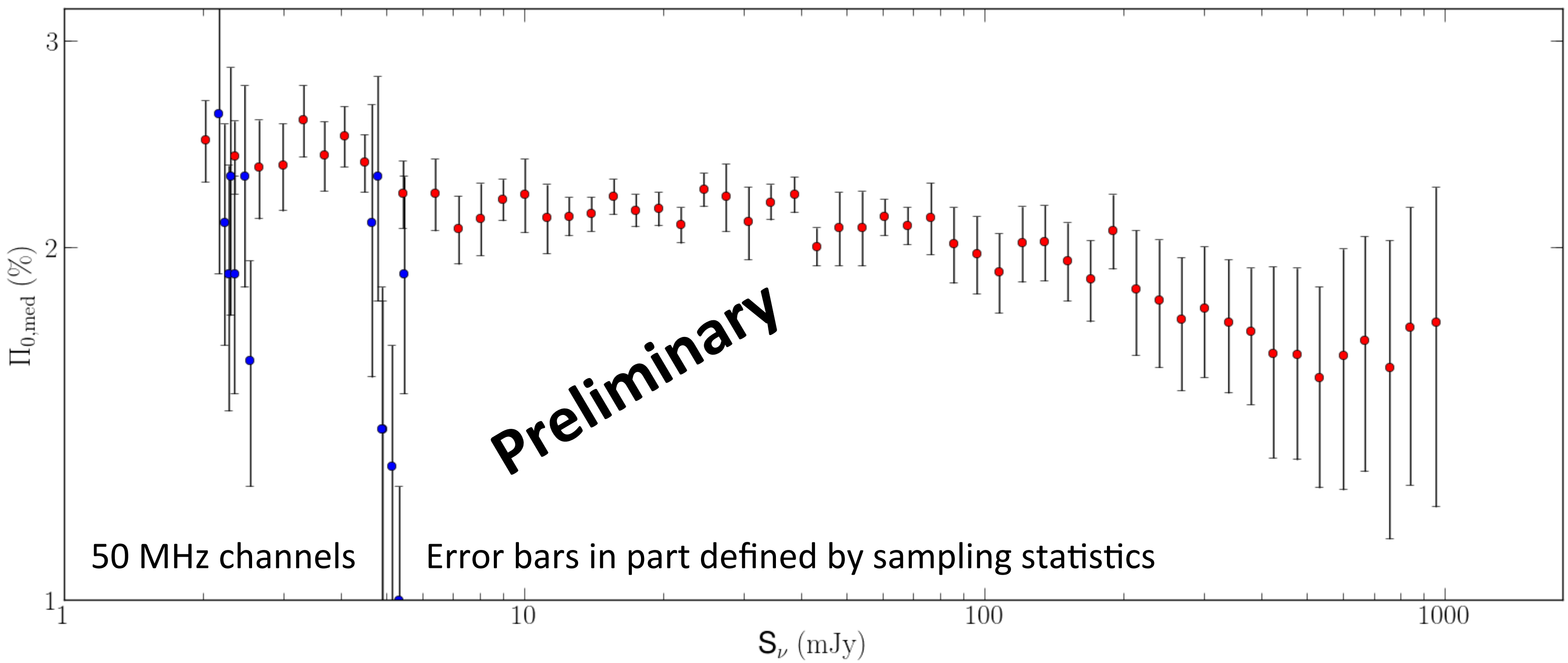


Measured signal-to-noise ratio for polarized intensity in stacking ATLAS DR3 fields, median stacking versus 65th percentile.

Details:

- Sample: Stokes I source list
- Three flux bins 0.4 dex in I, starting at 2 mJy
- Averaged to 50 MHz channels.

Fractional Polarization From Stacking ATLAS Polarized Intensity



Blue: Median fractional polarization from ATLAS ELAIS S1 and CDFS stacking p_{65}
Red: Median fractional polarization from NVSS stacking (Stil et al. 2014).

Monte-Carlo statistics include actual flux distribution within a flux bin

Conclusions

1. The biggest challenge in stacking is understanding and controlling systematics

2. Stacking polarized intensity is is different from stacking intensity:
   - Signal to noise ratio in stacked image increases slower under ideal circumstances.
   - The number of sources required to secure a 3-sigma detection in the median p_{med} is

$$N = 100\left(\frac{p_0}{\sigma}\right)^{-4}$$

3. The median estimator does not optimize signal-to-noise ratio in the stacked image. We used p_{65} here in a trade-off with sensitivity to outliers and the details of the distribution of fractional polarization.
4. In case of a non-detection in stacked p_{med}, an upper limit to the sample median $p_{0,med}$ is seldom useful.