Bayesian Inference for Radio Observations (Some applications)

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Bayesian Analysis (in the uv-plane)

- Upcoming interferometers like the Square Kilometre Array (SKA), will be systematics limited due to their high sensitivity.
- A detailed treatment of the systematics will be necessary.
- Current methods lead to inadequate uncertainty estimates and biased results.
- Using Bayesian inference methods, the uncertainties and the correlations between the various source and instrumental parameters can be estimated with high accuracy (Lochner et al., 2015).

Bayesian inference for radio observations

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Bayesian Analysis (in the uv-plane)

- Examples:
  - Receiver noise, source confusion and other relevant noise terms
  - Source flux, position, shape parameters
  - Time / frequency variability, polarisation
  - Pointing errors, primary beams
- Avoid image reconstruction artefacts (choice of weighting, CLEAN bias etc.,)
- Noise is Gaussian and uncorrelated in the visibility domain.
- Bayesian hypothesis testing can help discriminate between different source structures (among other things).

Bayes' Theorem

- Probability as a measure of the degree of belief in a proposition.
- Bayes' Theorem:

$$Posterior = \frac{Likelihood \times Prior}{Evidence} \qquad P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

- Initial belief (prior) updated by observed data (likelihood) and normalized by the normalization factor (evidence) to obtain new and updated belief (posterior).

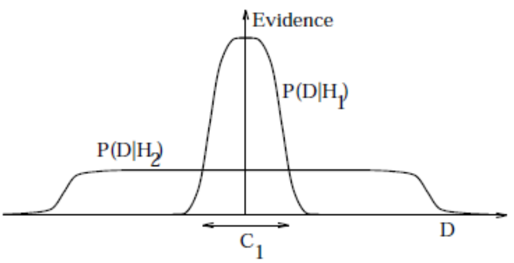
Bayesian Analysis

- Two levels of inference (Mackay, 1991)
  - Parameter Estimation:
    - Assume that the model is known and estimate the parameters of the model.
  - Model Comparison / Hypothesis Testing:
    - Odds in favour of a hypothesis, by computing the ratio between the evidences for models H1 and H2.

Model Selection

- Occam's Razor – Among competing hypotheses, the one that makes the fewest assumptions must be the favoured one.

- The Bayesian evidence automatically incorporates Occam's razor by penalising unnecessarily complex models and favouring ones that are simpler.

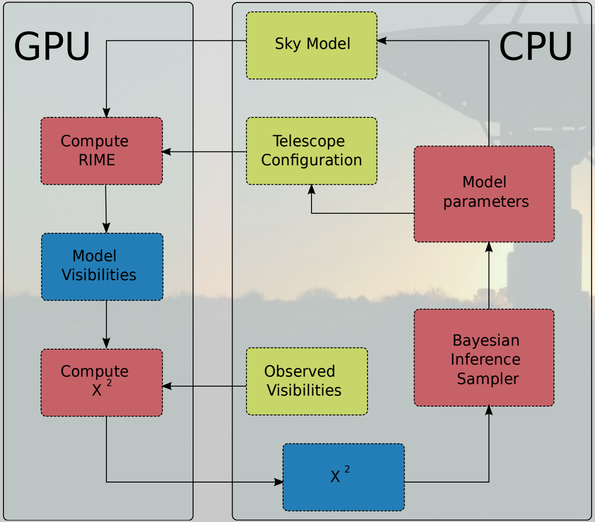


(Mackay, 1991) Bayes' theorem rewards models in proportion to how strongly they predicted the data that occurred. Assuming equal priors, H1 will be favoured.

Software Setup

- We use *MeqTrees* (Noordam & Smirnov, 2010) for interferometric simulations and *MultiNest* (Feroz et al., 2009) for the Bayesian analysis.
- The model parameters with physically meaningful priors and a likelihood function are specified by the user and passed on to MultiNest.
- The likelihood computation happens in a GPU which is facilitated by *montblanc* (Perkins et al., 2015), a GPU implementation of the RIME.
  - Implemented in python and uses PyCUDA with a numpy-like API.
  - About 250x faster than the CPU implementation.

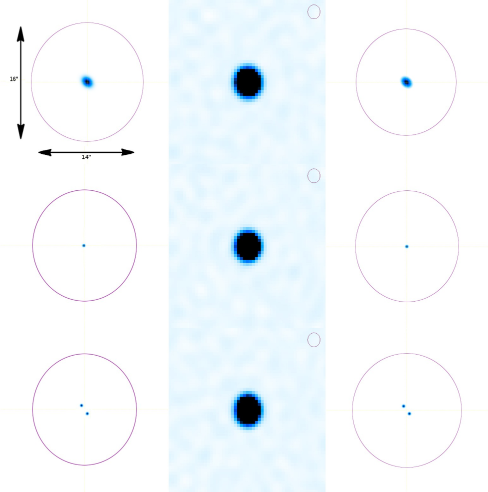
Software Setup



- Workflow diagram for BIRO with montblanc.
- Red boxes indicate computation, green boxes input and blue boxes output.

  (Perkins et al., 2015)
- Code available at https://github.com/ska-sa/montblanc

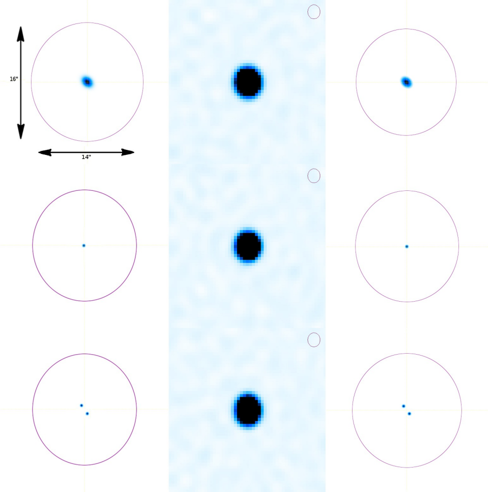
Case I. WSRT Simulations



- Details of the simulations:
  - 12 hour observation time
  - Integration time of 30 seconds
  - Simulated at a frequency of 1.4 GHz
  - Bandwidth of 512 kHz.
  - Simulation noise of 0.1 Jy/visibility

- Parametrisation - flux *S*, position *(l,m)* and shape *(lproj, mproj, min/maj)*.

WSRT Simulations



- *(Left)* Simulated Skies:
  - Extended (Gaussian) source of size 4'' x 3''
  - Point source
  - Two point sources
- *(Middle)* CLEANed images.

  CLEAN beam ~ 16'' x 14''
- *(Right)* Reconstructed skies from estimated parameters for the best-fit model.

Results – Evidence Matrix

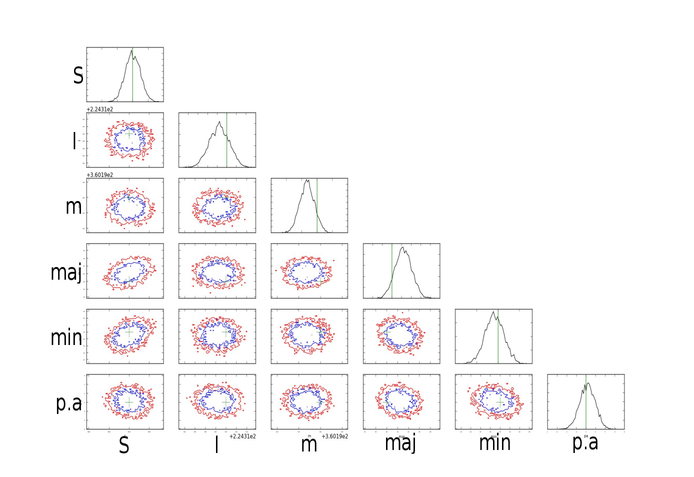
- Each simulation was tested against all three models and the evidences were computed for each case.
- The resulting relative evidence matrix expressed as the odds ratio:

	Hypothesis		
Simulation	Two point sources	Extended source	One point source
Two point sources	1 : 1	10^{593} : 1	10^{7200} : 1
Extended source	10^{993} : 1	1 : 1	10^{5079} : 1
One point source	62 : 1	857 : 1	1 : 1

- In each row, the model with the ratio 1:1 is the one with the highest evidence and the one that was used in the simulation.
- 'Strong' or 'decisive' (> 100) in all cases, according to Jeffrey's criterion (Kass & Raftery, 1985).
- The maximum error in evidence is less than 1.5 in all cases.

Results – Posterior probability distribution

- Posterior probability distribution for the Gaussian case:

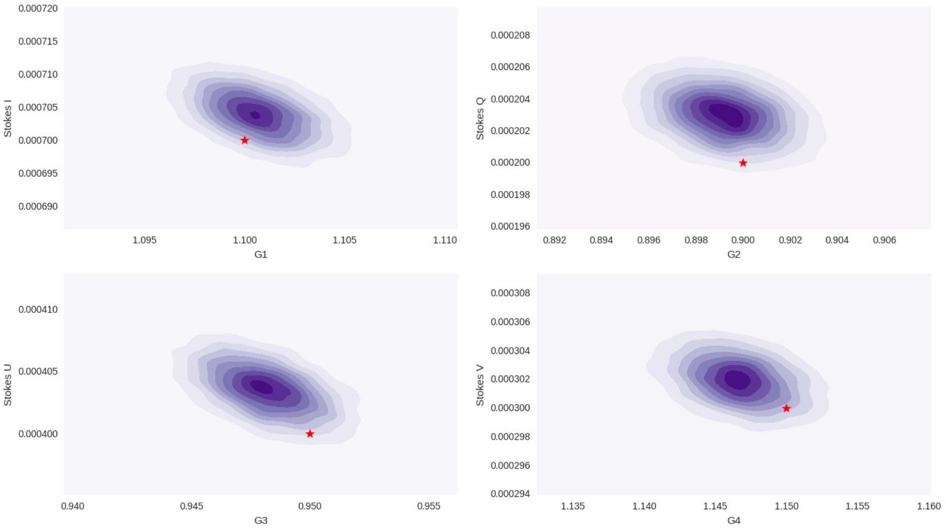


- S : 0.85 ± 2.5e-4 Jy
- l : 224.32 ± 2e-3''
- m : 360.21 ± 2e-3''
- maj : 4 ± 4e-2''
- min : 3 ± 4e-2''
- p.a : 45 ± 2°

Case II. Source + Instrumental Effects

- Simultaneous estimation of source parameters and antenna gains.
- Simulated two extended sources – each smaller than the PSF - with gain corruptions (one amplitude term per antenna).
- The flux density of the central extended source is assumed to be known (the calibrator), thus breaking the degeneracy between the flux densities and the gain terms.
- Estimated the shape of the calibrator, the polarisation & shape of the other extended source and the per antenna G-terms (26 parameters).

Case II. Source + Instrumental Effects

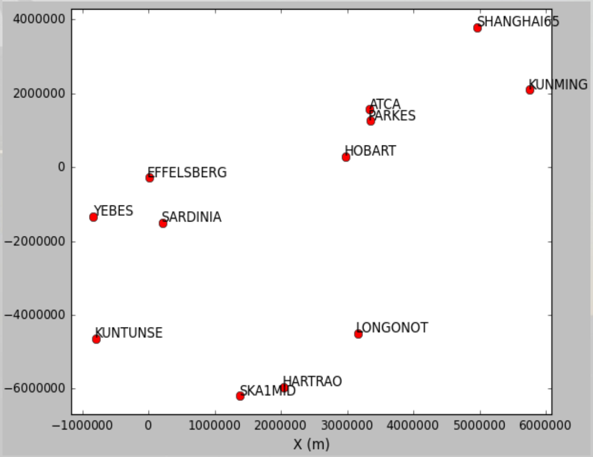


Case III. Evidence cross-over for SKA-VLBI

- What happens when we keep reducing the size of the extended source? Or bring the two sources closer together?
- We expect an evidence cross-over point at which our method starts favouring the wrong model for a given SNR.
- Can be applied to high resolution VLBI observations.

SKA-VLBI Simulations

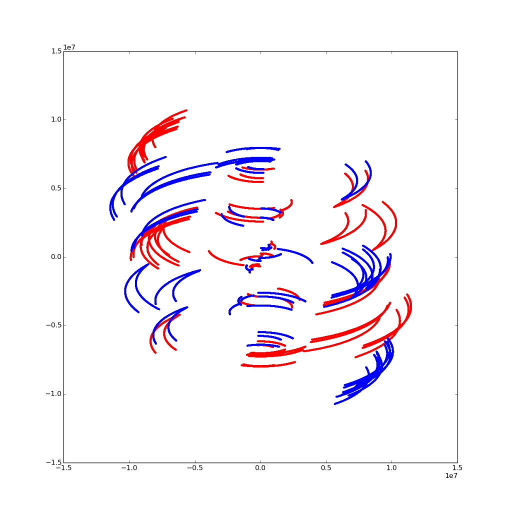
- Very Long Baseline Interferometry (VLBI) with the Square Kilometre Array will provide milli-arcsecond resolution imaging with high signal-to-noise ratio (Zsolt et al., 2014).
- At a frequency of observation of 8.4 GHz, this gives us a PSF of size about 0.6 milli-arcseconds (mas).
- We simulated SKA-VLBI observations of extended sources of varying sizes (all smaller than the PSF) and performed model selection between extended & point source models.

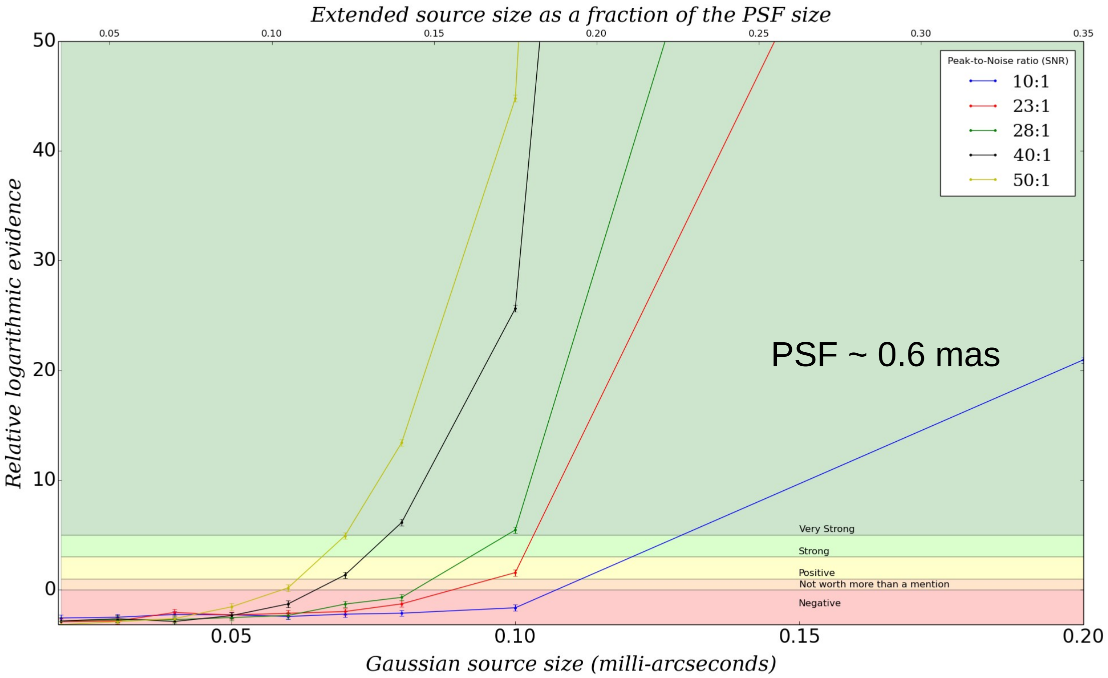


SKA-VLBI Simulations

- Details of the observations:
  - Total observation time – 4 hours
  - Frequency of observation – 8.4 GHz
  - Extended (Gaussian) sources of sizes varying from 0.01 mas to 0.2 mas
  - PSF ~ 0.6 milli-arcseconds (mas)
  - All data below 10º elevation flagged (~20%)
  - Baseline-dependent noise (SEFD varies with station)

- Repeated the simulations for multiple SNR levels.





Extended source size as a fraction of the PSF size
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Peak-to-Noise ratio (SNR)
10:1
23:1
28:1
40:1
50:1
PSF ~ 0.6 mas
Very Strong
Strong
Positive
Not worth more than a mention
Negative
Relative logarithmic evidence
50
40
30
20
10
0
0.05
0.10
0.15
0.20
Gaussian source size (milli-arcseconds)

Ongoing and future work

- montblanc:
  - Full G and E term support is being implemented and tested.
  - Plans to distribute computation among multiple GPUs and compute nodes.
- Applications
  - VLBI observation of quasars at high red-shift (Paragi +).
  - ATCA observations of Circinus X-1: Presence of a jet (Coriat +)


The End?